

1 ANGIOGRAPHNET: REAL-TIME PREDICTION OF VESSEL EXPANSION IN 2 CORONARY ANGIOPLASTY USING GEOMETRIC DEEP LEARNING

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4 **Abstract.** Coronary atherosclerosis can impede blood flow in the heart, necessitating revascularization proce-
5 dures such as angioplasty. Coronary angioplasty success relies on the complex interplay between stent deployment
6 and patient-specific micro-morphology. To facilitate intervention planning, we propose a graph convolutional neu-
7 ral network (GCNN) that leverages geometric deep learning for real-time prediction of vessel expansion based on
8 the 3D arterial structure. This approach significantly reduces computational time, enabling the use of a surrogate
9 model to optimize coronary interventions, enhancing decision-making processes, and improving patient outcomes in
10 time-sensitive clinical settings.

11 **Key words.** Computational Cardiology, Medical Imaging, Machine Learning, Numerical Simulation, Virtual
12 Angioplasty, Graph Convolutional Networks

13 **1. Introduction.** Coronary artery disease (CAD) remains the foremost global cause of mor-
14 tality, primarily attributed to the buildup of atherosclerotic plaque within the coronary wall, which
15 subsequently leads to reduced blood flow to the heart muscle[8]. These plaques exhibit a wide vari-
16 ety of micro-anatomical phenotypes, with their complex micro-morphology and micro-topology
17 significantly impacting the efficacy of clinical interventions [10]. To restore blood flow and alleviate
18 the detrimental effects of CAD, coronary angioplasty is performed. This revascularization proce-
19 dure involves the insertion and expansion of a stent within the artery, with its success hinging on
20 the intricate interaction between the device and the patient’s unique coronary micro-morphology,
21 such as the presence of calcium deposits [2].

22 Cardiologists currently base their interventional decisions on invasive and non-invasive imag-
23 ing technologies, such as optical coherence tomography (OCT), to analyze plaque composition and
24 determine stent size, placement, inflation pressure, and vessel preparation techniques [7]. While
25 OCT provides high-resolution, 3D image stacks of atherosclerotic lesions, enabling the differentia-
26 tion of various plaque components based on their optical appearance, relying solely on these broad
27 morphological indicators may not provide adequate information to predict device success [3, 13].
28 Consequently, understanding and utilizing 3D micro-anatomical information to guide percutaneous
29 coronary intervention (PCI) strategies is of paramount importance to optimize clinical outcomes.

30 Computational cardiology endeavors to address this limitation by converting intravascular 3D
31 OCT images of coronary arteries into patient-specific, multi-material digital twins. These digital
32 representations serve as the foundation for virtual angioplasty simulations, which predict vessel
33 expansion in response to stent deployment parameters[3, 13]. These simulations employ partial
34 differential equations to model the complex interplay of soft tissue biomechanics, contact mechanics,
35 and metal alloy plasticity. However, while highly accurate, such numerical simulations are time-
36 consuming and computationally intensive, preventing their use in catheter labs, where critical
37 interventional parameter decisions are made within minutes of obtaining intravascular images.

38 To bridge this gap, recent advancements in scientific machine learning have fostered the de-
39 velopment of surrogate machine learning models, trained on numerical simulation data, to predict
40 the dynamic state of organic physical systems more rapidly than traditional numerical simula-
41 tions [9]. This paves the way for real-time applications in clinical settings. Previous surrogate
42 models for PDEs primarily relied on convolutional neural networks (CNNs), which were limited
43 by their inability to handle non-Euclidean data, non-homogeneous prediction resolutions, and low
44 resolutions[11]. In contrast, graph-based approaches predict system dynamics using a graph repre-
45 sentation of computational meshes, which serve as the substrate for numerical simulations. These
46 approaches accommodate anisotropic resolutions and non-Euclidean data handling, offering a more
47 versatile solution for complex, patient-specific geometries[11].

48 In this course project, we propose leveraging graph convolutional neural networks (GCNN)
49 to predict vessel expansion in coronary angioplasty based on the morphological representation of
50 3D arterial structures. This approach would significantly reduces the time taken to predict vessel
51 response, enabling the use of a real-time surrogate model for optimizing coronary interventions. By
52 providing a more comprehensive understanding of the intricate interactions between cardiovascular
53 morphology and device-based interventions, our proposed GCNN has the potential to accelerate

54 decision-making processes and patient outcomes in coronary angioplasty. With this novel method-
 55 ology, cardiologists will be better equipped to tailor interventions to individual patients, resulting in
 56 more successful revascularization procedures and improved long-term prognosis. Our contributions
 57 are therefore as follows:

- 58 • We develop a parametric model of calcified coronary arteries to produce meshes with
 59 different morphological parameters
- 60 • We develop and leverage a virtual angioplasty platform that numerically simulates stent
 61 expansion to calculate vessel displacement
- 62 • We introduce a novel method of condensing the three-dimensional atherosclerotic mor-
 63 phology onto a 2D mesh manifold embedded in three dimensions, which preserves the
 64 information necessary to predict vessel expansion in response to virtual angioplasty
- 65 • We leverage a graph convolutional architecture to take the manifold representation of a
 66 coronary artery to predict nodal displacements on the inner surface.
- 67 • We trained and validated our graph convolutional network on numerical simulations of
 68 virtual angioplasties applied to a wide range of coronary morphologies.

69 **2. Methodology.** In the methodology section of our study, we delineate the systematic pro-
 70 cess undertaken to evaluate the practicality of employing Graph Neural Networks (GNNs) in
 71 predicting the outcomes of virtual stent angioplasty procedures. The methodology comprises of
 72 several phases: 1) constructing the computational models for virtual stenting (section 2.1), 2) the
 73 implementation of virtual stent angioplasty simulations (section 2.3), 3) the generation of training
 74 data for the GNN (section 2.4), 4) the development of the GNN architecture (section 2.5), and 5)
 75 the conduction of validation experiments section (section 2.6).

76 2.1. Computational Models.

77 **2.1.1. Baseline Geometries.** The virtual angioplasty platform consists of three compo-
 78 nents, a computational model of a calcified coronary artery, a stent, and a balloon. The stent has
 79 a nominal (starting) diameter of 3mm, is 20mm long and has a radial thickness of $70\mu\text{m}$. The
 80 multi-folded angioplasty balloon measures 23mm in length and has an outer diameter of 0.9mm. It
 81 is designed with a nominal diameter of 3mm. The baseline arterial model is partitioned into three
 82 distinct segments, two of which represent healthy sections located at either extremity, and one
 83 diseased segment positioned centrally. The healthy sections are configured to have a cylindrical ge-
 84 ometry with a diameter of 2.9mm, a thickness of 0.66mm, and a length of 40mm. In these healthy
 85 portions, the media and adventitia layers are respectively 0.32mm and 0.34mm thick, adhering
 86 to the empirical data obtained from the analysis of 13 fresh human cadaveric hearts by Holzapfel
 87 et al [6]. Interposed between these healthy portions is the diseased segment, characterized by a
 88 stenosis degree of 60%. This value was derived from an in-depth analysis of an Optical Coherence
 89 Tomography (OCT) pullback from a patient diagnosed with Coronary Artery Calcification (CAC).

90 **2.1.2. Parametric variations of the artery model.** The parametric variations of the
 91 artery can be seen in Figure 1. Lumen stenosis diameter (L) ranges from 0 to 2, the calcium
 92 thickness (T) ranges from 0.1mm to 2mm, and calcium arclength (A) span from 10 to 360 degrees.

93 **2.1.3. Meshing.** The artery was meshed with first order tetrahedra. The number of elements
 94 varies between the different cases but is usually on the order of 100,000. The balloon has meshed
 95 with 9600 4-node quadrilateral membranes. The stent has been meshed with 266,588 first order
 96 linear bricks

97 2.2. Material Properties.

98 **2.2.1. Artery properties.** The artery was defined as hyperelastic/plastic and the properties
 99 were obtained from Poletti et. al [12]. The hyperelastic behaviour is modeled as a polynomial strain
 100 energy function:

$$101 \quad (2.1) \quad \psi = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3 + C_{40}(I_1 - 3)^4 + C_{50}(I_1 - 3)^5 + C_{60}(I_1 - 3)^6$$

102 Where, C_{xx} are the material coefficients, and I_1 is the first invariant of the cauchy deformation
 103 tensor. The hyperelastic coefficients for the plastic and elastic behaviour can be found in Tables 1
 104 and 2.

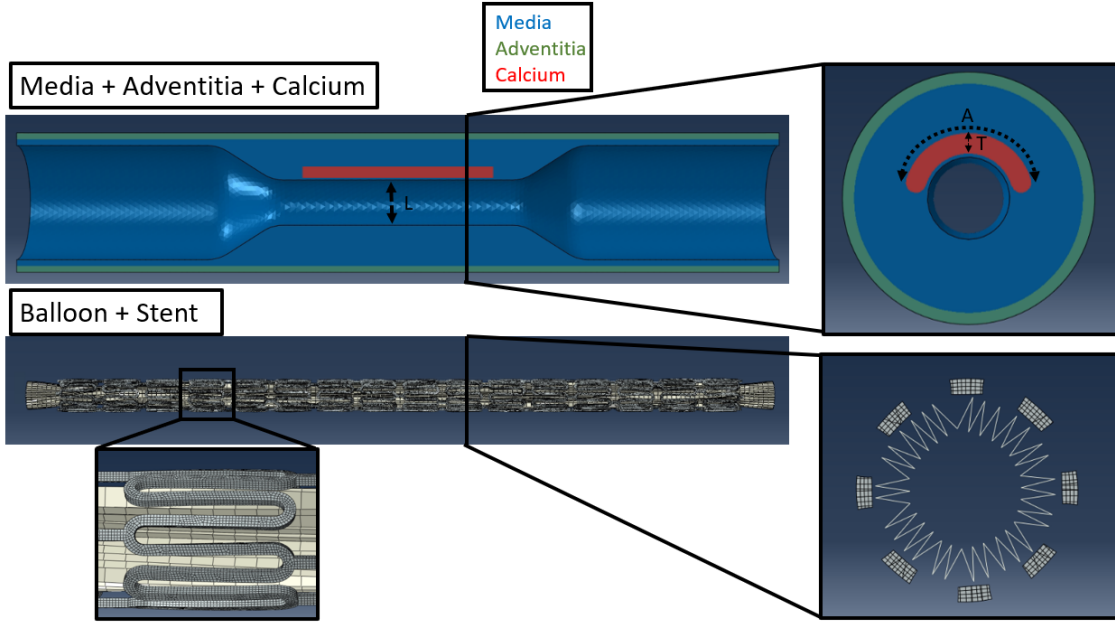


Fig. 1: A visual representation of the computational models for virtual angioplasty. Top row: Longitudinal inset of parametrized geometry of calcified coronary artery, which consists of media (blue), adventitia (green) and embedded calcified plaque (red). Right inset shows cross section of artery with associated calcium deposit. Bottom row: balloon+stent angioplasty system after crimping operation, left inset shows zoom in on crimped stent mesh, right inset shows cross section of balloon folding pattern.

| | C_{10} | C_{20} | C_{30} | C_{40} | C_{50} | C_{60} |
|-------------------|-----------------------|--------------------|---------------------|--------------------|---------------------|--------------------|
| Adventitia | 2.60×10^{-1} | 4.76×10^1 | -4.09×10^3 | 5.29×10^5 | -2.69×10^7 | 5.65×10^8 |
| Media | 7.29×10^{-2} | 3.71×10^0 | -1.56×10^2 | 9.18×10^3 | -2.61×10^5 | 2.91×10^6 |

Table 1: Hyperelastic Behavior of Artery Tissue

105 **2.2.2. Balloon and Stent Properties.** The balloon was assumed to have a first order
 106 Ogden hyperelastic behavior (Table 3:

107 (2.2)
$$\Psi = \frac{\mu_1}{\alpha_1} (\lambda_1^{\alpha_1} + \lambda_2^{\alpha_1} + \lambda_3^{\alpha_1} - 3)$$

108 Where λ_X are the principle stretches along each direction. The stent was assumed to have an
 109 elastic-plastic material behavior as can be seen in tables 4 and 5.

110 **2.3. Virtual Angioplasty.**

111 **2.3.1. Crimping procedure.** The stent was initially 3D-drawn at its nominal diameter of
 112 3 mm to match the standard received by cardiologists. However, coronary arteries can vary in
 113 diameter from 3.7 mm to 1.9 mm. Therefore, in order to deliver the stent through these arteries,
 114 it needs to be crimped or compressed.

115 To achieve crimping, 16 rigid planes were placed in an orthoradial pattern, maintaining a
 116 distance equal to the stent radius. This arrangement ensures immediate contact from the beginning
 117 of the simulation. To effectively crimp the stent, a radial displacement to a diameter of 1.1 mm
 118 was imposed on the crimping planes. This process reduces the stent diameter, enabling it to pass
 119 through the coronary arteries. The resulting configuration represents the final crimped stent.

120 **2.3.2. Inflation and Deflation.** In the simulation setup, both the proximal and distal ends
 121 of the artery and balloon are pinned, meaning they can rotate locally but not translate. The

| Artery Tissue | Plastic Strain (-) | Plastic Stress (MPa) |
|---------------|--------------------|----------------------|
| Adventitia | 0 | 1.60 |
| | 0.07 | 2.30 |
| | 0.40 | 4.00 |
| Media | 0 | 0.70 |
| | 0.07 | 1.10 |
| | 0.40 | 2.00 |

Table 2: Plastic Behavior of Artery Tissue

| μ_1 (MPa) | α_1 (-) |
|---------------|----------------|
| 80 | -15 |

Table 3: Hyperelastic behavior of the Balloon

122 balloon is inflated with a pressure of 1.4 MPa. The length of the stent was intentionally chosen to
 123 overlap a portion of each healthy end of the artery, ensuring that the stent spans the entire length
 124 of the lesion. Once the balloon is inflated, it is then correspondingly deflated, leaving only the
 125 stent to keep it open.

126 In terms of contact behavior, tangential contact is assumed to be frictionless, while normal
 127 contact is assumed to be hard, meaning no penetration is allowed. These contact properties were
 128 applied between every part of the model. Considering the highly non-linear dynamics of this
 129 simulation, which includes contact and significant deformation, Abaqus/Explicit (Abaqus Inc.,
 130 Providence RI, USA) was utilized for the analysis. Abaqus/Explicit is well-suited for such simula-
 131 tions due to its ability to handle explicit dynamics and large deformation scenarios. We run each
 132 simulation on a node with 28/40 cpus and 128 GB of ram, we utilize domain parallelization, which
 133 divides the computational mesh into separate domains for each cpu, and loop parallelization, which
 134 utilizes multiple cpus to quickly solve certain parallelizable loop procedures. Figure 2 demonstrates
 135 the results of a single stent angioplasty procedure

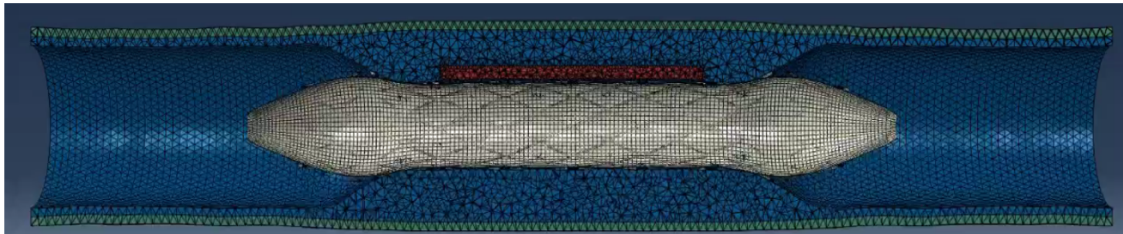


Fig. 2: A visual representation of a virtual stent angioplasty at the end of the inflation step.

136 **2.4. Production of Training Data.** The virtual angioplasty produces a a vector prediction
 137 of the displacement at every node in the mesh. While this mesh and the nodewise displacement
 138 vector can be fed into the graph neural network, they prove quite large to process on GPU's. We
 139 therefore instead choose to project the 3D calcium onto the luminal surface of the artery mesh.
 140 Producing a 2D triangular mesh embedded in 3D space. We project the calcium by shooting 1 ray
 141 per node from the surface of the lumen in an outwardly radial fashion. We measure the thickness
 142 of the calcium by finding the intersection points between the nodal rays and calcium elements. We
 143 then assign each node a thickness value. Similarly, we found the xyz displacement vector associated
 144 with each node on the surface. This can be visualized in figure 3

145 **2.5. Geometric Deep Learning.**

146 **2.5.1. Problem definition.** We define vessel expansion as a node-level displacement predic-
 147 tion problem. We are given as input a 2D surface mesh that is a homogeneous graph $G = (\mathcal{V}, \mathcal{E})$

| Young's modulus (GPa) | Poisson's ratio (-) |
|-----------------------|---------------------|
| 203 | 0.3 |

Table 4: Elastic Behavior of the stent

| Plastic stress (MPa) | Plastic strain (-) |
|----------------------|--------------------|
| 203 | 0.3 |
| 480 | 0 |
| 1208 | 0.35 |
| 1300 | 0.62 |
| 2300 | 1.09 |

Table 5: Plastic behavior of the stent

148 with nodes in the vertex set $v_i \in \mathcal{V}$, edges $e_{i,j} = (v_i, v_j)$ in the edge set \mathcal{E} , where v_i is called the
 149 head/source node and v_j is called the tail/target node. Each node also has an initial embedding,
 150 which we denote as $\mathbf{h}_i^{(0)}$, that contains various expertly curated features. For each node, we would
 151 like to predict a two-dimensional vector $\mathbf{h}_i^{(n)} = (x, y)$ that represents the displacement of mesh at
 152 that node in the respective coordinate directions.

153 **2.5.2. Message passing in graph convolutional layers.** Given a graph of an artery, we
 154 aim to learn a numerical vector $\mathbf{h}_i^{(n)} = (x, y)$ for each node such that it captures vessel expansion
 155 at that point in a manner that is physiological sound and consistent with other parts of the
 156 artery. This is achieved by transforming initial node embeddings through several layers of local
 157 graph-based non-linear function transformations to generate predictions [5]. These functions are
 158 optimized iteratively, given a loss function to gradually minimize the error of making poor vessel
 159 expansion predictions. Upon convergence, optimized functions generate an optimal set of node
 160 displacements.

161 **Step 1: Initialization.** We denote the input node embedding \mathbf{X}_i for each node i , which is
 162 initialized using curated features such as the thickness of calcium at the node, and the position of
 163 the node. For every layer l of message-passing, there are the following three stages:

164 **Step 2: Propagating relation-specific neural messages.** We calculate a transformation
 165 of the embedding at each node from the previous layer $\mathbf{h}^{(l-1)}$, where the first layer $\mathbf{h}^{(0)} = \mathbf{X}$. This
 166 is achieved via applying a weight matrix $\mathbf{W}_M^{(l)}$ on the previous layer's embedding:

$$167 \quad \mathbf{m}_i^{(l)} = \mathbf{W}_M^{(l)} \mathbf{h}_i^{(l-1)}$$

168 **Step 3: Aggregating local network neighborhoods.** For each node v_i , we aggregate on
 169 the incoming messages $\{\mathbf{m}_j^{(l)} | j \in \mathcal{N}_i\}$ from neighboring nodes denoted as \mathcal{N}_i by taking the average
 170 of these messages:

$$171 \quad \widetilde{\mathbf{m}}^{(l)}_i = \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \mathbf{m}_j^{(l)}$$

172 **Step 4: Updating network embeddings.** We then combine the node embedding trans-
 173 formed from the previous layer and the aggregated messages to obtain the new node embedding:

$$174 \quad \mathbf{h}_i^{(l)} = \mathbf{h}_i^{(l-1)} + \widetilde{\mathbf{m}}^{(l)}_i$$

175 After L layers of propagation, we arrive at our encoded node embeddings $\mathbf{h}_i^{(L)}$ for each node i .
 176 The final node embeddings $\mathbf{h}_i^{(N)}$ represent the displacement prediction at the given node.

177 **2.5.3. Graph U-Net Architecture.** We employ the Graph U-Net architecture [4], inspired
 178 by the conventional U-Net architecture [14] for image segmentation. This Graph U-Net archite-
 179 cture effectively captures and decodes hierarchical topological and spatial information in graph-

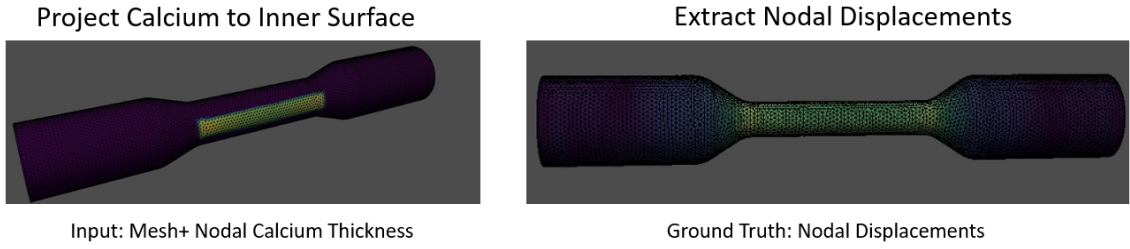


Fig. 3: A visual representation of the input and output representations to be fed into a graph neural network. Left: calcium is projected onto the inner surface of the lumen. Right: The nodal displacements of the inner surface are extracted and used as ground truth to supervise network training.

180 structured data, enabling the efficient learning of complex patterns across large, densely-connected
 181 graphs [4].

182 The encoder effectively captures both local and global features of the input graph at varying
 183 mesh resolutions by employing alternating graph convolutional layers and graph pooling layers.
 184 The pooling layer samples a subset of important nodes to facilitate the enlargement of nodal
 185 receptive fields [4]. Subsequently, the decoder reconstructs the graph to its original size using
 186 a series of graph convolutional layers and unpooling operations. The unpooling layer restores
 187 the original graph structure by using the position information of nodes in the original graph and
 188 performing the inverse operation of the corresponding pooling layer [4]. To facilitate information
 189 transfer across layers, skip connections from the encoding phase are integrated into the decoder.
 190 A representation of this architecture, applied to arterial meshes, is presented in Figure 4.

191 The Graph U-Net architecture is particularly effective at propagating information across the
 192 entire mesh structure through its pooling and unpooling operations, unlike traditional graph neural
 193 networks that tend to be constrained to a few-hop neighborhood around a node [15]. This ability
 194 is particularly important for our vessel expansion problem, as the model must recognize long-range
 195 dependencies along the artery to generate physiologically plausible displacement predictions for a
 196 given node.

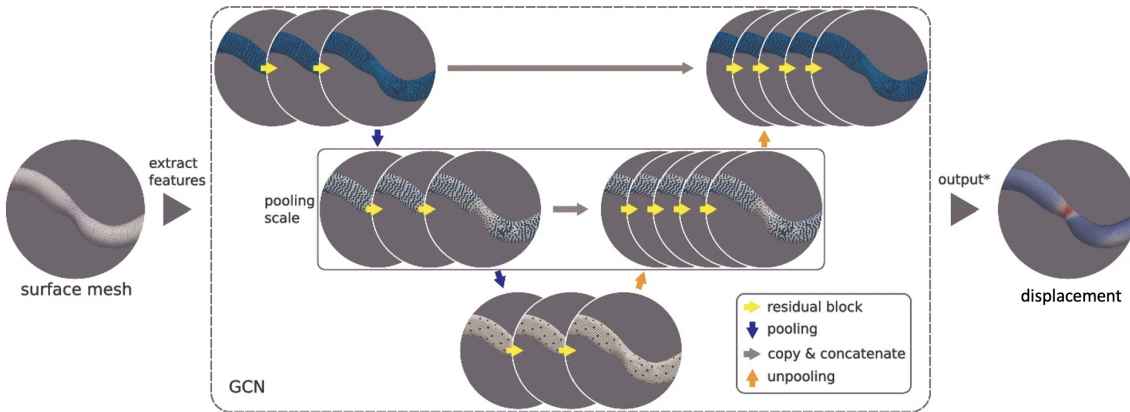


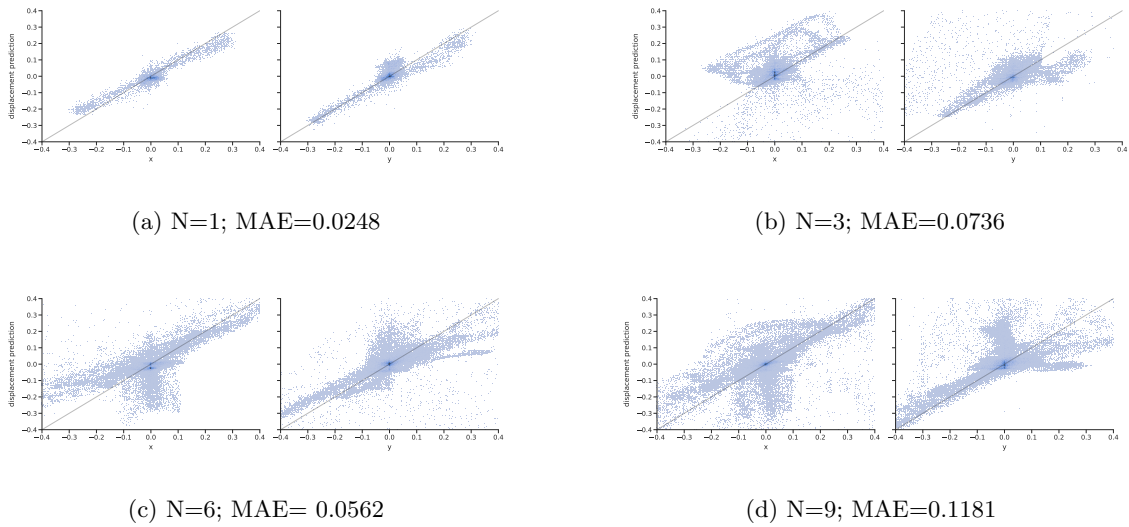
Fig. 4: Graph U-Net architecture used to estimate displacements during coronary angiography. Adapted from [15].

197 **2.6. Experiments.** We implement the Graph-UNet architecture using FeaSt convolutions
 198 [16], as employed in the existing literature for modeling arterial meshes [15]. A FeaSt convolution
 199 [16] represents a graph-convolutional message-passing layer that leverages an attention mechanism
 200 to dynamically determine local filters based on the features present in the preceding network layer.
 201 The network consists of approximately 800,000 trainable parameters.

202 Our dataset, comprising 13 arteries, is randomly partitioned into training and testing sets,
 203 forgoing a validation set due to data constraints. The graph neural networks are trained to predict
 204 a two-dimensional radial displacement vector at each vertex on the surface mesh. All models are
 205 trained using a mean absolute error loss and the Adam optimizer. The implementation of this
 206 architecture is adapted from the code in [1, 15]. Inference on a new artery featuring unseen mesh
 207 geometry requires only a few seconds, as opposed to the equivalent finite element simulation, which
 208 took 48 hours.

209 **3. Results.** The results are shown below in the following figures. we present the distributions
 210 of predicted displacements vs ground truth displacements, and mean absolute error values for the
 211 training distribution (Figure 5) and test distribution (Figure 6). The training error increases with
 212 the number of training arteries, while the test error trends downwards with the number of training
 213 arteries. Indicating the model was overfitting on small datasets but is slowly learning to generalize.

Fig. 5: Consistent distribution of predicted displacements and increasing mean absolute error (MAE) during training as training set (N) grows



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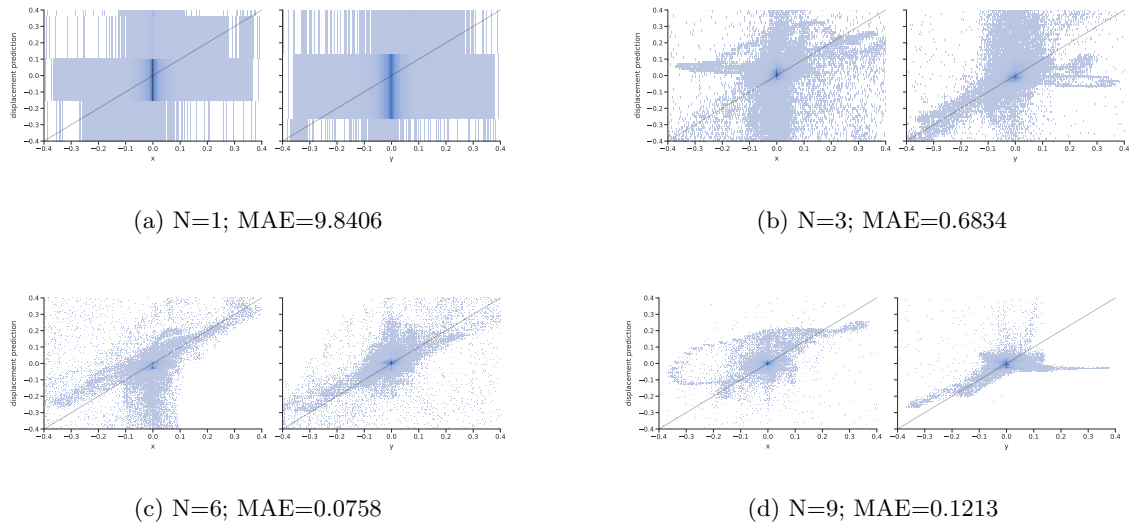
215 **4. Discussion.** In this study, we present a proof of concept for using geometric deep learning
 216 to predict vessel expansion during coronary angioplasty, showcasing promising results in the
 217 preliminary stage. Our neural network employs geometric deep learning in conjunction with a
 218 U-Net architecture, effectively analyzing the 3D arterial mesh to predict radial displacement. Our
 219 approach significantly reduces computational time and could potentially enable the real-time prediction
 220 of vessel expansion to optimize coronary interventions.

221 Despite the encouraging results, our training dataset was limited to a mere 13 arteries, which
 222 impeded a comprehensive assessment of the graph neural network’s performance and its capacity
 223 to generalize. To mitigate this limitation, we aim to simulate additional meshes to enhance training
 224 and facilitate a more robust evaluation.

225 Moreover, our current model is not equivariant to transformations in the input data, implying
 226 that predictions may be influenced by translations, rotations, or the introduction of noise. To
 227 enhance the model’s performance, we plan to generate a larger dataset with such augmentations
 228 and train the model using this expanded dataset.

229 Finally, our evaluation in this study relies on a random training and testing split of the dataset.
 230 However, in clinical practice, we would prefer to train models on a specific cohort of patients and
 231 generalize the model to unseen patients with potentially different morphologies. As a result,
 232 future evaluations should take into account data splits based on morphology to gain a deeper
 233 understanding of the graph neural network’s performance in real-world settings.

Fig. 6: Improving distribution of predicted displacements and decreasing mean absolute error (MAE) during testing as training set (N) grows



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