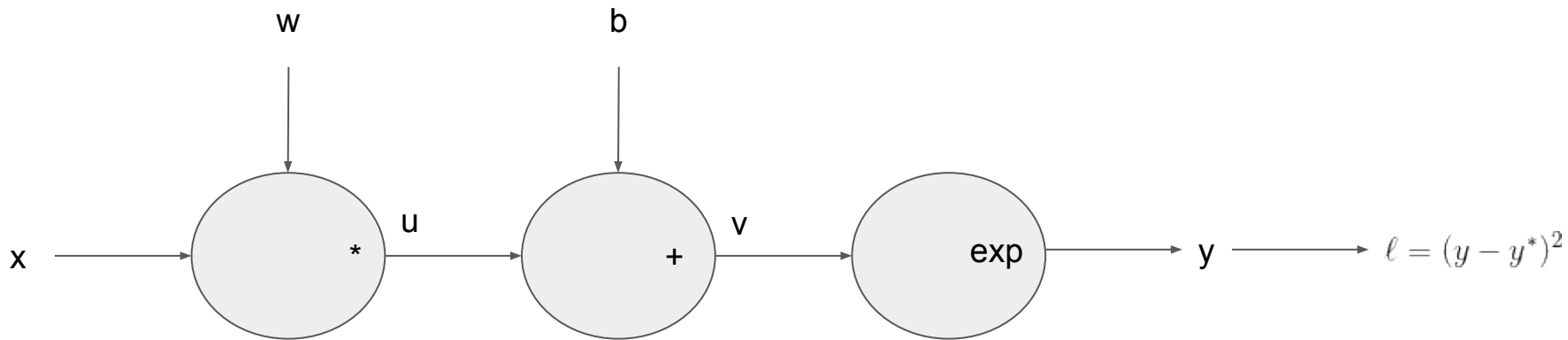


Lesson 2: how automatic differentiation works

on an example

FORWARD PASS

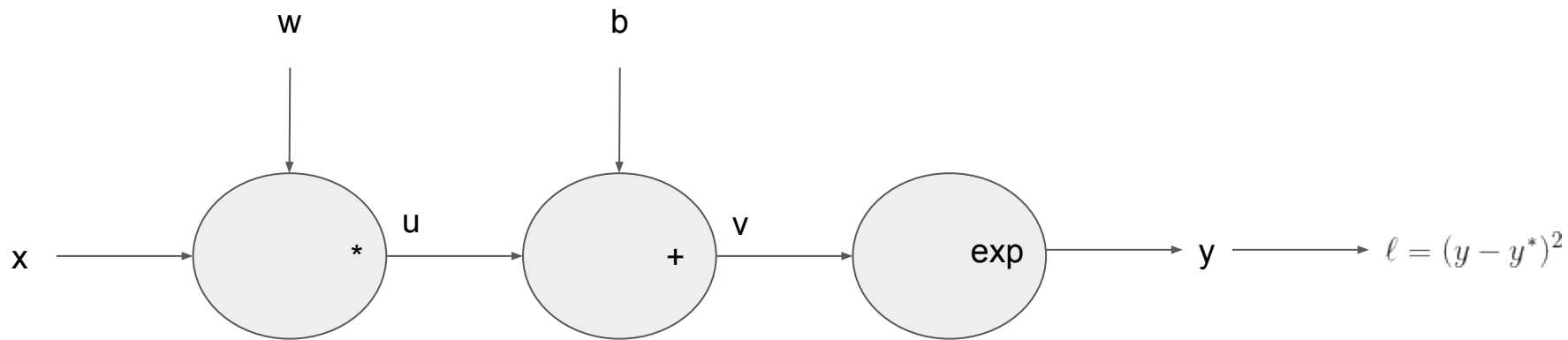


$$u = wx$$

$$v = u + b$$

$$y = e^v$$

$$\ell = (y - y^*)^2$$



$$u = wx$$

$$v = u + b$$

$$y = e^v$$

$$\ell = (y - y^*)^2$$

$$\frac{\partial u}{\partial w} = x$$

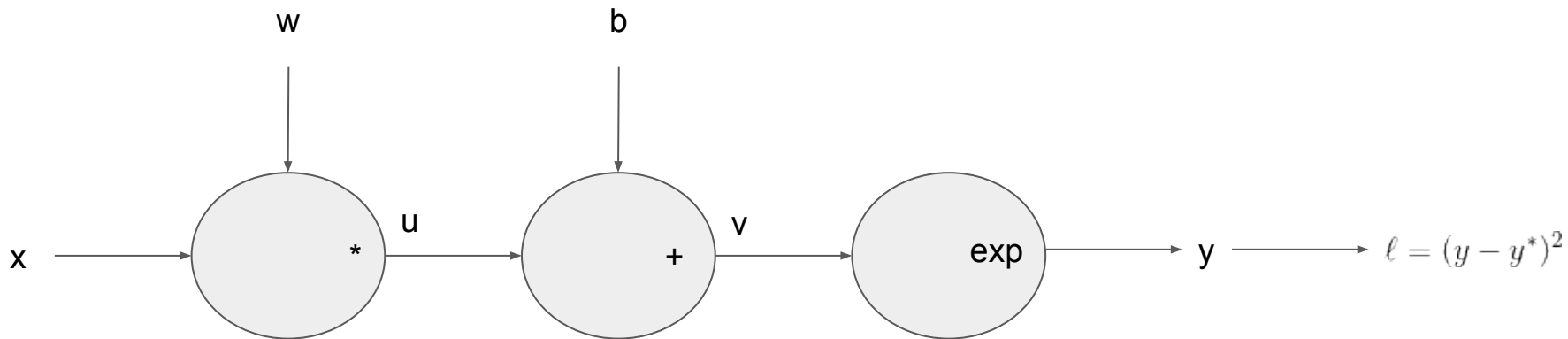
$$\frac{\partial v}{\partial b} = 1, \quad \frac{\partial v}{\partial u} = 1$$

$$\frac{\partial y}{\partial v} = e^v = y.$$

$$\frac{\partial \ell}{\partial w} = \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial w}$$

$$\frac{\partial \ell}{\partial b} = \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial b}$$

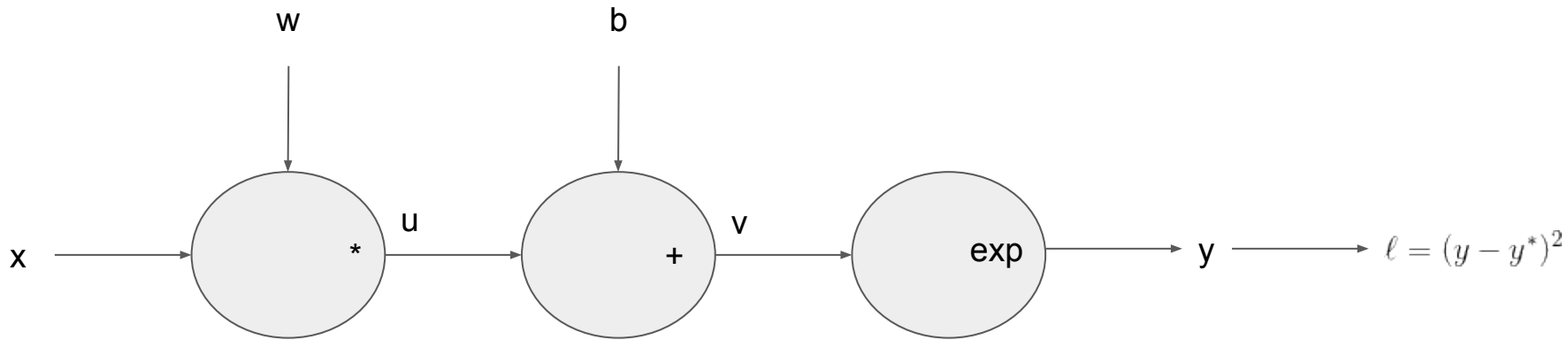
CHAIN RULE



$$\begin{aligned}u &= wx \\v &= u + b \\y &= e^v \\ \ell &= (y - y^*)^2\end{aligned}$$

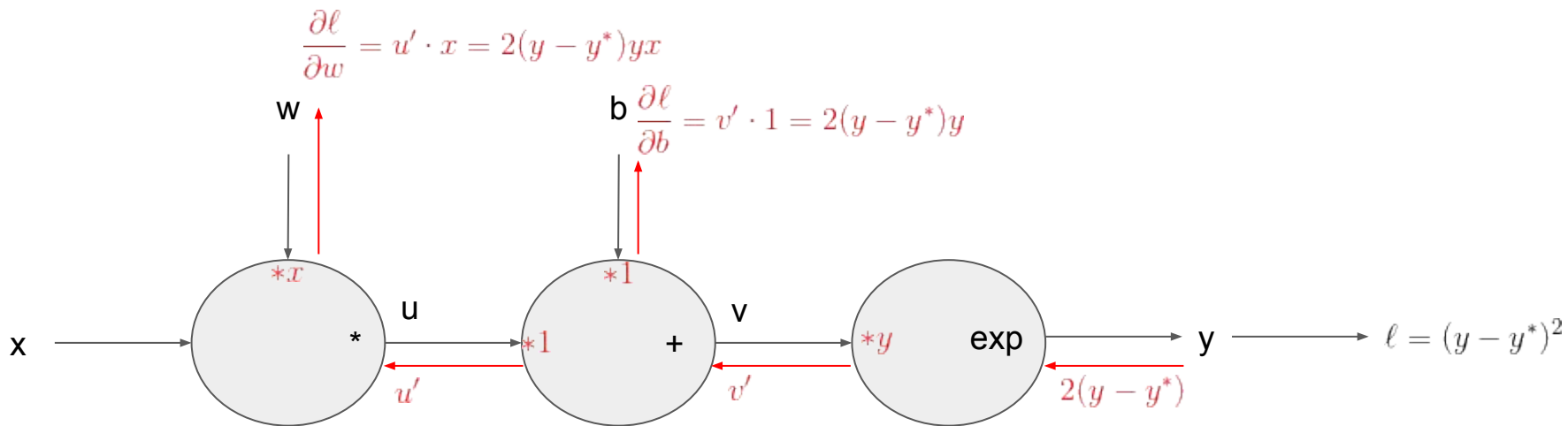
$$\begin{aligned}\frac{\partial u}{\partial w} &= x \\ \frac{\partial v}{\partial b} &= 1, \quad \frac{\partial v}{\partial u} = 1 \\ \frac{\partial y}{\partial v} &= e^v = y.\end{aligned}$$

$$\begin{aligned}\frac{\partial y}{\partial w} &= \frac{\partial y}{\partial v} \frac{\partial v}{\partial u} \frac{\partial u}{\partial w} & \frac{\partial \ell}{\partial w} &= \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial w} \\ \frac{\partial y}{\partial b} &= \frac{\partial y}{\partial v} \frac{\partial v}{\partial b} & \frac{\partial \ell}{\partial b} &= \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial b}\end{aligned}$$



$u = wx$	$\frac{\partial u}{\partial w} = x$	$\frac{\partial \ell}{\partial w} = 2(y - y^*) \cdot y \cdot 1 \cdot x$	$\frac{\partial y}{\partial w} = \frac{\partial y}{\partial v} \frac{\partial v}{\partial u} \frac{\partial u}{\partial w}$	$\frac{\partial \ell}{\partial w} = \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial w}$
$v = u + b$	$\frac{\partial v}{\partial b} = 1, \quad \frac{\partial v}{\partial u} = 1$	$\frac{\partial \ell}{\partial b} = 2(y - y^*) \cdot y \cdot 1$	$\frac{\partial y}{\partial b} = \frac{\partial y}{\partial v} \frac{\partial v}{\partial b}$	$\frac{\partial \ell}{\partial b} = \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial b}$
$y = e^v$				
$\ell = (y - y^*)^2$				

BACKWARD PASS



$$\begin{aligned}
 u &= wx \\
 v &= u + b \\
 y &= e^v \\
 \ell &= (y - y^*)^2
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial u}{\partial w} &= x \\
 \frac{\partial v}{\partial b} &= 1, \quad \frac{\partial v}{\partial u} = 1 \\
 \frac{\partial y}{\partial v} &= e^v = y.
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \ell}{\partial w} &= 2(y - y^*) \cdot y \cdot 1 \cdot x \\
 \frac{\partial \ell}{\partial b} &= 2(y - y^*) \cdot y \cdot 1
 \end{aligned}$$

$$\begin{aligned}
 v' &= 2(y - y^*) \cdot y \\
 u' &= v' \cdot 1
 \end{aligned}$$