Modelling Shared Decision Making Interactions using Influence Diagrams

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Abstract

Shared Decision Making (SDM) has become a predominant element of patient-centered healthcare delivery in recent years. In SDM, multiple agents, including a patient and a clinician interact to make a joint decision that is aligned with the patient's preferences. Despite its popularity, previous SDM studies lack structured decision modeling approaches applied to this problem. This paper presents Influence Diagram (ID) models for SDM agents, and proposes graphical operations for IDs to model the interaction between the agents. Using a case study, we demonstrate that widely used conceptual models for SDM such as the Three Talk Model are aligned with the proposed ID models and operations. The case study also shows that SDM is a cooperative decision making setting that is also present in non-clinical domains. The proposed influence diagrams and interaction operations enable SDM to be studied based on structured and quantitative decision models.

Keywords: Shared Decision Making; Influence Diagrams; Multi-agent Models; Cooperative Decision Making; Clinical Decision Making

1. Introduction

Shared Decision Making (SDM) stands as a cornerstone of patient-centered care, emphasizing collaborative decision-making between patients and clinicians (Charles et al., 1997). It involves at least two agents –a patient and a clinician– who aim to make a joint decision based on the preferences of the patient. The clinician has a richer and more accurate model of the problem domain, but they may have an incomplete understanding of the patient's preferences. The agents interact about the problem domain and preferences to make better decisions. SDM describes a special case of cooperative decision making in which both agents focus on the same decision, one agent's preferences is essential and there is information asymmetry regarding the preferences and random variables.

Despite its recognized importance, the definition of SDM and its associated concepts lacks structured quantitative models as previous SDM studies mainly focussed on qualitative conceptual models (Bomhof-Roordink et al., 2019; Makoul and Clayman, 2006; Stiggelbout et al., 2015). This paper aims to model SDM agents and interactions using Influence Diagrams (IDs). The proposed IDs provides a structured modelling approach that is aligned with previous conceptual models of SDM. We represent SDM agents with separate IDs and present graphical operations to modify these IDs based on the interactions between these agents in SDM. We apply the proposed ID models and operations to a case study to demonstrate their use and alignment with a widely used guideline for SDM practice

(Elwyn et al., 2012, 2017). While several extensions of IDs are available multi-agent settings (Detwarasiti and Shachter, 2005; Suryadi and Gmytrasiewicz, 1999; Koller and Milch, 2003; Gal and Pfeffer, 2008; Zeng and Poh, 2009), these approaches do not capture the properties and interactions that take place throughout SDM. Our approach is the first attempt to model SDM using ID representations, offering a novel perspective on this widely studied topic from both clinical decision making and probabilistic graphical model domains.

In the remainder of this paper, Section 2 provides an overview of SDM, Section 3 reviews existing ID frameworks designed for multi-agent settings. Section 4 presents the ID models and operations for SDM. Sections 5 and 6 applies the proposed ID and operations to a case study, presents our conclusions respectively.

2. Shared Decision Making

The Shared Decision-Making concept was coined in the seminal paper of Charles et al. (1997) but the clinical acceptance of SDM happened more recently (Stiggelbout et al., 2015). Charles et al. (1997) described the four main characteristics of SDM as follows. Firstly, SDM should involve at least two parties: the clinician and the patient. In addition, family members, friends, or other domain experts can also be involved depending on the type of clinical decision. Secondly, there should be information sharing between the clinician and the patient. The clinician provides the relevant medical information such as the outcomes and side-effects of treatment options. The patient can share their preferences related to the treatment options. The clinician assist the patient in comparing the risks and benefits. Thirdly, both parties should be involved in the decision making. The degree of involvement can vary depending on the patient's preferences and the nature of the clinical decision problem. Fourthly, both parties need to discuss and build a consensus on the treatment decision. There have been variations in the SDM definitions (Stiggelbout et al., 2015) and systematic reviews were conducted to unify their key components (Bomhof-Roordink et al., 2019; Makoul and Clayman, 2006). These key components include information exchange, patient participation, learning about the patient's preferences, discussion of options, advocating patient's views, partnership, and patient education.

Computational models have been mainly used for aiding SDM by computing risks of outcome variables, training clinicians in SDM interactions and guiding SDM policy. Quaglini et al. (2013) used decision trees with embedded Markov models to produce expected values of outcome variables to aid SDM. Veloso (2013) used agent-based simulation models to inform Multiple Sclerosis (MS) patients about the possible progress of the disease during SDM. Petukhova et al. (2019) used cognitive agents based on the ACT-R architecture to model the negotiation stage of SDM between patients and doctors. The agents were designed to train doctors and to enhance their social and cognitive negotiation abilities in SDM. Tuncalp et al. (2023) used a simple utility-based model to compare the performances of three decision-making processes, namely SDM, Evidence-Based Medicine, and Case-by-Case, under different paradigms, such as perfect rationality or bounded rationality. Their main finding is that if both parties are perfectly rational, SDM outperforms other decision-making processes.

Several models have been developed to describe the key components of SDM in clinical practice. Hargraves et al. (2020) proposed a five-step process called the SHARE approach in which doctors explore and compare the treatment options in terms of benefits, and harms through interactions about what matters most to the patient. The steps of SHARE are 1) Seek your patient's participation, 2) Help your patient explore and compare treatment options, 3) Assess your patient's values and preferences, 4) Reach a decision with your patient, 5) Evaluate your patient's decision. The SHARE Approach aims to guide clinicians to involve patients into the treatment decision-making process, by focusing on general communication skills and interpersonal behaviors. Elwyn et al. (2012) proposed the Three-Talk Model (TTM) as a guideline for achieving SDM in clinical practice. They later evaluated and revised this model with clinical experts (Elwyn et al., 2017). TTM consists of three steps. The first step is the choice talk in which the clinician gives an overview of the decision making process, briefly describes the treatment options, and asks about the patient's preferences. The second step is the option talk in which the patient and the clinician discuss treatment alternatives and their outcomes. The third step is the decision talk in which the patient and the clinician make a joint decision based on the informed preferences. All these three steps together ensure that treatment decisions are aligned with the patient's values and preferences. TTM aims to guide clinicians based on these three stages, thereby promoting patient-centered care and enhancing treatment decision outcomes. Although these models provide valuable insights into its principles and practice, the application of structured decision modelling approaches to SDM remains largely unexplored. In the following section, we review the influence diagrams and their extensions into multi-agent settings, and assess their advantages and limitations in capturing the SDM process.

3. Influence Diagrams

A Bayesian Network (BN) is a graphical probabilistic model that is composed of a Directed Acyclic Graph (DAG) and local probability distributions (Pearl, 1988). DAG is composed of nodes $\mathcal{X} = \{X_1, \ldots, X_n\}$ representing random variables and edges representing direct dependencies between the nodes. DAG encodes conditional independence assertions between the random variables. A node X_i is conditionally independent of their non-descendants given their parents $\mathrm{par}(X_i)$. The domain of a node describes the possible values it can take $\mathrm{dom}(X) = \{x_1, \ldots, x_k\}$. Every node X_i has a local conditional probability distribution conditioned on its parents $P(X_i \mid \mathrm{par}(X_i))$. These probability distributions are often represented as Conditional Probability Tables (CPTs) when X_i and $\mathrm{par}(X_i)$ are discrete, A BN represents a joint probability distribution that factorizes as follows:

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid \mathrm{par}(X_i))$$

Influence Diagrams (IDs) are an extension of Bayesian Networks (BN) for decision problems (Howard and Matheson, 2005). An ID is a DAG that is composed of decision nodes $\mathcal{D}$, chance nodes $\mathcal{X}$ and utility nodes $\mathcal{U}$ that are often drawn as rectangles, ellipses and rhombuses respectively in graphical illustrations. Chance nodes in IDs are equivalent to the nodes in a BN. Let Y be any chance, decision or utility node in an ID. Parents of Y can be decision or chance nodes $\mathrm{par}(Y) \subseteq \mathcal{D} \cup \mathcal{X}$. Utility nodes cannot be the parent of other nodes. Incoming arcs to a decision node, called information arcs, represent that the state of the parent is known at the time of decision making. Information arcs between decision nodes model the sequential order of decisions. Information arcs from chance nodes to decision nodes denote the chance nodes that are observed before decision making. The domain of a decision node represents the decision alternatives.

IDs model a decision making problem from the perspective of a single decision maker. Several extensions to IDs have been proposed to model multi-agent decision problems. Detwarasiti and Shachter (2005) modeled team decision making with IDs assuming that a team has a common knowledge model and utility function. A team also makes decentralized decisions and has imperfect information sharing. They model these properties as a single ID for the team assuming that the chance, utility, and decision nodes are shared between the team members but information about previous decisions is not shared. The resulting ID represents a Markov Decision Problem with imperfect recall where some of the information links between sequential decisions are missing to reflect decentralized decisions and incomplete information sharing. Although the clinician and the patient in SDM can be seen as a team, Detwarasiti and Shachter (2005)'s model is not suitable for SDM as the clinician and the patient do not have a shared knowledge model and utilities about the decision problem.

Suryadi and Gmytrasiewicz (1999) use IDs to model agents that learn from each other in the multi-agent decision making setting. Each agent has their own ID decision model. The decisions of other agents can be included in those IDs as chance nodes. If an agent has detailed information about the other agent's decision making process, it can be modeled as a nested ID within the agent's decision model. This nested ID represents the agent's mental model of the other agent's decision making process. Suryadi and Gmytrasiewicz also show examples of learning the beliefs, decisions and utilities in this nested ID based on observations of an agent's decisions. For example, when Agent 1 makes a decision that is not available in Agent 2's mental model, a corresponding state can be added to the decision node in this model. When Agent 1 makes a decision that is irrational according to Agent 2's mental model, the chance and utility nodes can be updated in this model. Suryadi and Gmytrasiewicz focus on a competitive decision making setting, and they do not present a general systematic approach that can be fully used for SDM. Yet, their approach for learning the decision and utility nodes provide useful insights about how the treatment and preferences could be learned in different stages of SDM.

Zeng and Poh (2009) model a cooperative multi-agent decision making setting with Multiply Sectioned Influence Diagrams (MSID) and Hyper Relevance Graphs (HRG). In MSID, each agent has a separate ID which is composed of their own belief, decision, and utility nodes. Agents communicate through chance nodes that are shared between those IDs representing public information. HRG models the organizational relationships regarding information sharing between the agents. If the information provided for a chance by one of the agents influences the decision of another agent, this indicates a control type of relationship in HRG. If the information shared is not a prerequisite for a decision, this indicates a communication relationship in HRG. Information sharing is a main element of SDM but modeling it with shared chance nodes does not cover the extent of communication between the SDM parties. The clinician has deeper knowledge about the clinical problem and communicates relevant parts of his knowledge with the patient who updates his own knowledge model accordingly. If we model both the clinician's and patient's domain knowledge as separate DAGs, the clinician will select the most informative parts of their DAG and communicate them to the patient. The patient may modify or expand their DAG based on this information. This kind of communication may require structural DAG operations rather than just using shared chance nodes.

Koller and Milch (2003) present Multi-Agent Influence Diagrams as a general extension of IDs for non-cooperative multi-agent games. Syntactically, a MAID is a DAG that is composed of chance, decision and utility variables as in a conventional ID. Each decision and utility node in a MAID is associated with a certain agent. Miller and Koller presents and algorithm for computing Nash equilibrium in MAIDs by using local independence assertions of the model. MAID assumes that the agents have a shared model of the real world.

Gal and Pfeffer (2008) present Networks of Influence Diagrams (NID) that represent the decision-making processes and beliefs of other agents from an agent's perspective. They represent each agent's mental model and model of the real world from the modeler's perspective as separate MAIDs. These MAIDs are represented as nodes that are connected in a NID. The root of the NID, called top-level, is the real-world model. Mental models of other agents are included as chance nodes in the top-level model. An agent can be uncertain about other agents' decision models and operate based on an incorrect model of the world. In SDM, the clinician has a mental model of the patient's preferences and decision model. The clinician learns the patient's preferences to update his mental model and guides the patient in terms of decision alternatives and outcomes according to this mental model. NIDs provide a suitable representation for modeling the clinician and patient as agents who have an incomplete model of the world and other agents respectively. In order to capture the whole SDM process, mental models in NID may need to be modified and expanded based on what agents learn from each other throughout SDM.

4. Influence Diagrams for Shared Decision Making

We focus on SDM involving two agents: a patient and a clinician. The agents aim to make shared decisions, but their understanding of the problem domain and preferences may differ. Hence, we have separate IDs that represent the decision making problem from each agents' perspective. Decision alternatives, chance nodes, preferences that describe the utility node, and relationships between the nodes may differ between those IDs reflecting the differences in how agents understand the decision problem. Typically, clinicians have more accurate and detailed domain knowledge regarding the decision problem. Patient preferences are central to SDM yet clinicians may lack complete understanding of the variables influencing patient's preferences and utility function. Considering these properties, the main properties of the IDs of SDM agents are : 1) Decision nodes in both IDs represent the same decision but their domain can be different. For example, the patient may not be aware of some possible treatment alternatives that the clinician knows, 2) The parents of the decision nodes in each ID are the same as the agents have access to the same observed variables by the time of decision, 3) Utility node in the patient's ID represents the patient's utility function. Utility node in clinician's ID represents the clinician's understanding of the patient's utility function. If there are multiple utility nodes in an ID they are added.

In SDM, agents interact to have a shared understanding of the decision problem and make shared decisions. We model these agent interactions as graphical operations that revise ID structure and parameters based on the other agent's ID. Figure 1 gives an overview of these operations. The clinician's interaction about the treatment alternatives corresponds

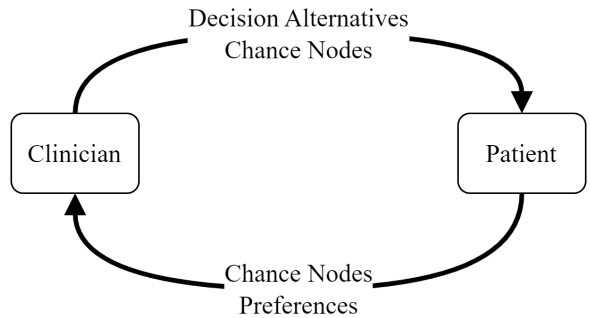


Figure 1: Overview of agent interactions in SDM

to revision of decision alternatives and chance nodes associated with the outcomes and side effects of those alternatives in the patient's ID. The patient's interaction about the preferences corresponds to updating of the utility node and the ancestors of those chance nodes in the clinician's ID. In the following sections, we present the properties of SDM IDs and structured operations for modelling these interactions. In Section 5 we demonstrate that these operations can simulate TTM: a widely used guideline for achieving SDM in practice (Elwyn et al., 2012, 2017).

4.1. IDs for SDM Agents

In SDM, a clinician and a patient have IDs, I^C and I^P, respectively. Decision nodes in I^C and I^P are $\mathcal{D}^C = \{D_1^C, D_2^C \dots\}$ and $\mathcal{D}^P = \{D_1^P, D_2^P, \dots\}$ respectively. D^C and D^P represent the same decisions from the clinician's and patient's point of view. However, their domains can be different as the patient may not be aware of some of the decision alternatives that the clinician knows. The parents of decision nodes are the same in both models as both parties have shared understanding of information available before decisions. $\mathcal{U}^C = \{U_1^C, U_2^C, \dots\}$ and $\mathcal{U}^P = \{U_1^P, U_2^P, \dots\}$ represent the utility nodes in I^C and I^P respectively. In SDM, the patient's preferences are essential. Hence, $\mathcal{U}^P$ represent patient's true preferences, and $\mathcal{U}^C$ represents clinicians understanding of the patient's preferences. The parents of a utility node define the decision and chance node that characterizes the preferences. The parents $\text{par}(U^C)$ and $\text{par}(U^P)$ can be different as the clinician may not be aware of some of the main factors in the patient's preferences. In other words, $\text{par}(U^P)$ may have additional nodes that do not exist in $\text{par}(U^C)$. Chance nodes in I^C and I^P are denoted by $\mathcal{X}^C = \{X_1^C, X_2^C, \dots\}$ and $\mathcal{X}^P = \{X_1^P, X_2^P, \dots\}$. Both I^C and I^P may have chance nodes that do not have an equivalent node in the other model. In SDM, the clinician has a more detailed understanding regarding the disease, hence I^C can have chance nodes that are unknown to the patient. The clinician may not be aware of some of the criteria influencing patient's preferences. Hence, some chance nodes that are parents or ancestors of U^P may not be available in I^C.

Due to these differences in chance, utility and decision nodes, the maximum expected utility decisions in I^P and I^C may not be the same even when both agents reason about the same decisions and they both focus on the patient's preferences. In order to achieve SDM, the patient and clinician interact regarding the decision alternatives, patient preferences and the random variables in the problem domain. In IDs these interactions correspond to revision of the model structure and parameters based on the other agent's ID. In the remainder of this section, we present three operations to revise decision, chance and utility nodes in SDM interactions.

4.2. Decision Alternative Transfer

An agent's interaction about a decision node corresponds to revision of the domain of the decision node in the other agent's ID. If a decision alternative known by a clinician, $d_i \in \mathrm{dom}(D^C)$, is not known by the patient $d_i \notin \mathrm{dom}(D^P)$, we add the decision alternative to the patient's decision node $\mathrm{dom}(D^P) := d_i \cup \mathrm{dom}(D^P)$. When the decision alternative is added, the CPTs of the children of D^P will expand and new conditional probabilities need to be defined. Let X_d^P be a child of D^P. We need to enter the conditional probability distributions for $P(X_d^P \mid d_i \cup (\mathrm{par}(X_d^P) \setminus D^P))$. These conditional probability distributions can be initialised by using uniform distributions, sample a categorical distribution from Dirichlet priors or from a donor pool of probability distributions. Note that, the revision of the CPTs of the chance nodes based on the information from the other agent's ID is described in the following section.

4.3. Chance Node Transfer

An agent's interaction about chance node corresponds to copying or revision of that chance node in the other agent's ID. Suppose the clinician describes the likelihood of a side effect X_i^C which is represented as a chance node I^C. We first check if X_i^P is available in I^P. If not, we add X_i^P and revise its domain $\mathrm{dom}(X_i^P) := \mathrm{dom}(X_i^C)$. For each parent of $X_j^C \in \mathrm{par}(X_i^C)$, we check if the parent node X_j^P is available in I^P. If not, we add the node $\mathcal{X}^P := X_j^P \cup \mathcal{X}^P$, add the arc $X_j^P \to X_i^P$, and revise its domain as $\mathrm{dom}(X_j^P) := \mathrm{dom}(X_j^C)$. When we add a new parent node X_j^P we initialize its probability distributions by using uniform distributions, sample a categorical distribution from Dirichlet priors or from a donor pool of probability distributions. If X_i^P has some parents that are not shared by X_i^C, i.e. $X_k^P \in \mathrm{par}(X_i^P)$ and $X_k^C \notin \mathrm{par}(X_i^C)$, then we remove the arc $X_k^P \to X_i^P$ in I^P. Finally, we copy the CPT of X_i^C to X_i^P as the parents and domains of the nodes are same at this point. When copying the CPT, we can add some random noise to reflect imperfect communication between the agents. A possible option to add randomness is to obtain random samples from a Dirichlet distribution that has the same expected value as the CPT of X_i^C.

4.4. Preference Transfer

An agent's interaction about preferences corresponds to revision of the utility node and its parents in the other agent's ID. Decision criteria that define the preferences of a decision maker are represented as parents of utility nodes in IDs. When the clinician does not know some of the important criteria regarding patients, $\mathrm{par}(U^C)$ may not be the same as $\mathrm{par}(U^P)$. In this case, the patient's interaction about the criteria will be reflected as an updating of $\mathrm{par}(U^C)$ in I^C.

Let $X_u^P \in \text{par}(U^P)$. We first check if parent node X_u^C is available in I^C, if not we add the parent node $\mathcal{X}^C := X_u^C \cup \mathcal{X}^C$, and define its domain $\text{dom}(X_u^C) := \text{dom}(X_u^P)$. We add the arc $X_u^C \rightarrow U^C$. If U^P has some parents that are not shared by U^C, i.e. $X_k^C \in \text{par}(U^C)$ and $X_k^P \notin \text{par}(U^P)$, then we remove the arc $X_k^C \rightarrow U^C$ in I^C as the patient preferences are essential. Finally, when the utility nodes U^C and U^P has the same number of parents representing the same random variables, $\text{par}(U^C) = \{X_1^C, \ldots, X_m^C\}$ and $\text{par}(U^P) = \{X_1^P, \ldots, X_m^P\}$, the utility function can be transferred from patient to clinician $U^C := U^P$. Some noise could be added to model imperfect communication between the clinician and patient such as $U^C := U^P + \mathcal{N}(0, \sigma)$, where $\mathcal{N}(0, \sigma)$ is a Gaussian distribution with mean 0 and standard deviation σ.

In some cases, the patient may not share information about all criteria affecting their preferences. In other words, the patient may share only some of the parents of U^P with the clinician. In this case, U^P will have parents that do not exist in U^C i.e. $\text{par}(U^C) = \{X_1^C, \ldots, X_u^C\}$ and $\text{par}(U^P) = \{X_1^P, \ldots, X_u^P, X_{u+1}^P \ldots, X_m^P\}$. Let $Pd^P = \{X_u^C, X_{u+1}^C \ldots, X_m^C\}$ be the parents of U^P that do not have an equivalent node in I^C. In order to transfer the utility function I^C we need to marginalize the utility function by Pd^P as follows

$$U^C(x_1, \ldots, x_u) := \sum_{(x_{u+1}, \ldots, x_m) \in \text{dom}(Pd^C)} P(x_{u+1}, \ldots, x_m) U^P(x_1, ..., x_m)$$

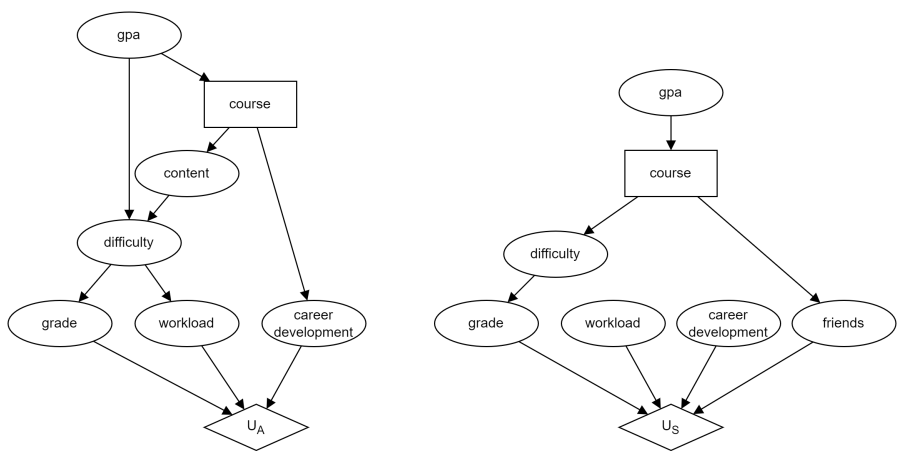
When the utility function is parameterized as a weighted linear function $U^P(x_1, ..., x_m) = w_1^P x_1 + \cdots + w_m^P x_m$, the associated weights are transferred in preference transfer. For example, when the patient shares information about $X_i^P \in \text{par}(U^P)$, this corresponds to transferring of the associated weight $w_i^C := w_i^P$ to U^C. Similarly, some noise could be added to model imperfect communication $w_i^C := w_i^P + \mathcal{N}(0, \sigma)$.

5. Case Study: Course Selection as SDM

This section applies the ID models and interaction operations to a SDM case study. Although SDM is mainly studied in the clinical domain, it is relevant to other domains where at least two agents make joint decisions primarily based on one of the decision makers' preferences. To illustrate this, we present a non-clinical SDM example between a student and an academic advisor regarding university course selection. Our example focuses on a senior undergraduate student in his final semester, seeking guidance from her academic advisor to select an elective course before graduation. The advisor provides expertise on academic content of the decision alternatives and university regulations but she may not have a complete understanding of the student's preferences. Similar to SDM in clinical setting, the student and the advisor collaborate to make decisions that are aligned with the student's preferences and also consistent with the curriculum requirements. Our example is aligned with the SDM properties (Charles et al., 1997) outlined in Section 2 as 1) it involves two agents engaged in collaborative decision-making, 2) there is information exchange between the agents as the student gives information about her preferences and the advisor provides information about course content, academic outcomes and regulations, 3) both parties actively participate in the decision-making process, 4) the final decision is made with mutual agreement. The problem is also aligned with the fundamental concepts of SDM identified in the systematic literature reviews of (Bomhof-Roordink et al., 2019;

Makoul and Clayman, 2006), including learning about the student's preferences, discussion of options and advocating the student's views.

Figures 2(a) and 2(b) show the ID models for the advisor I^A and student I^S that correspond to the clinician and patient IDs models described in Section 4.1 respectively. All chance nodes represent categorical variables in this example. Some chance nodes differ between the models reflecting that may different understanding of the decision problem. In this example, the advisor has more detailed knowledge about the course contents and their potentials for career development, while the student has a more detailed and accurate model in terms of her preferences. The advisor and student interact about decision alternatives, preferences and other factors to reach an SDM regarding course selection. We describe how each ID operation described in Sections 4.2, 4.3 and 4.4 are applied to this case study.



(a) I^A (b) I^S

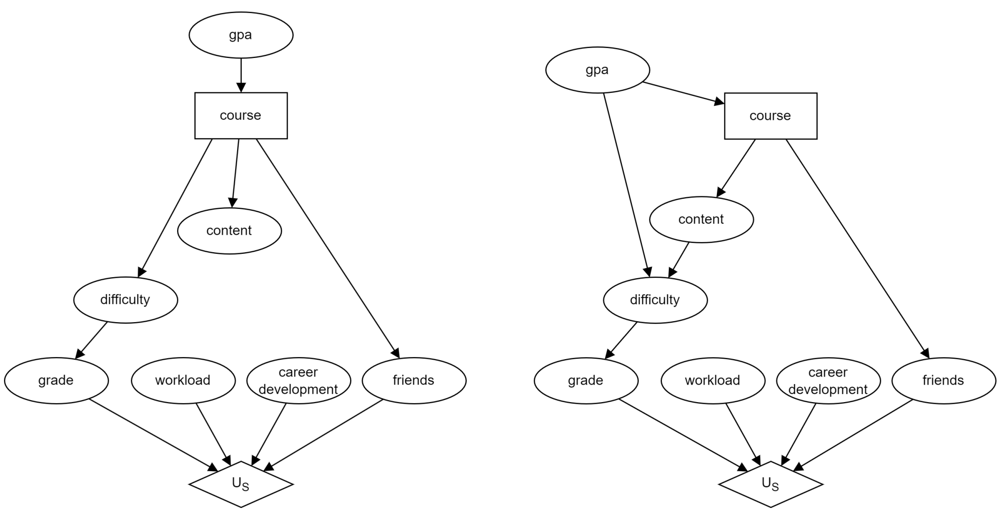
Figure 2: Initial IDs of Advisor and Student

5.1. Decision Alternative Transfer

The decision alternative transfer represents informing an agent about the decision alternatives when she has an incomplete knowledge about the domain of the decision node. For example, when the student does not know some of the courses available to her, the advisor can inform her the possible courses she can take. In our example, let $\text{dom}(course^A) = \{416, 424, 481\}$ and $\text{dom}(course^S) = \{416, 424\}$. Applying decision alternative transfer make $\text{dom}(course^S) := \{416, 424, 481\}$. In other words, the advisor informs the student about the course with code 481 that she can take. The CPTs of the children of $course^S$ need to be revised as we changed the domain of their parent. We need to initialize the conditional probability distribution of $P(friends \mid course = 481)$ and $P(difficulty \mid course = 481)$ in their CPTs. We can use a uniform distribution, predefined distribution or sample a categorical distribution from a Dirichlet prior to initialise these conditional probability distributions.

5.2. Chance Node Transfer

The chance node transfer corresponds to informing an agent about one of the random variables based on the knowledge of the other agent. Suppose the advisor talks about the *content* variable that represents whether the content of a course is suitable for the student's considering their academic background. The possible values that $content^A$ can take are $\text{dom}(content^A) = \{suitable, unsuitable\}$. Since content is not available in I^S, we firstly add this node and make its domain $\text{dom}(content^S) := \{suitable, unsuitable\}$. The parent of $content^A$ is $course^A$ in I^A. The $course^S$ node is also available in I^S and its domain is the same as $course^A$ due to the decision transfer step. Next, we add the arc $course^S \rightarrow content^S$. At this stage, the domains of content and its parents are the same, hence we can copy the CPT of $content^A$ to $content^S$. We can copy the CPT by adding random noise to reflect imperfect communication between the agents. For example, we added random noise from N(0,0.1), clipped them between [0,1] and normalised the results to ensure that they are probability distributions. Figure 3(*a*) shows the revised I^S after transferring $content^A$ from I^A. Note that, chance node transfer does not make any modification to the children of the transferred node. To link, $difficulty^S$ to $content^S$ and update its CPT, we need to apply chance node transfer to $difficulty^S$ next, as shown in Figure 3(*b*).



(a) I^S after transferring *content* (b) I^S after transferring *difficulty*

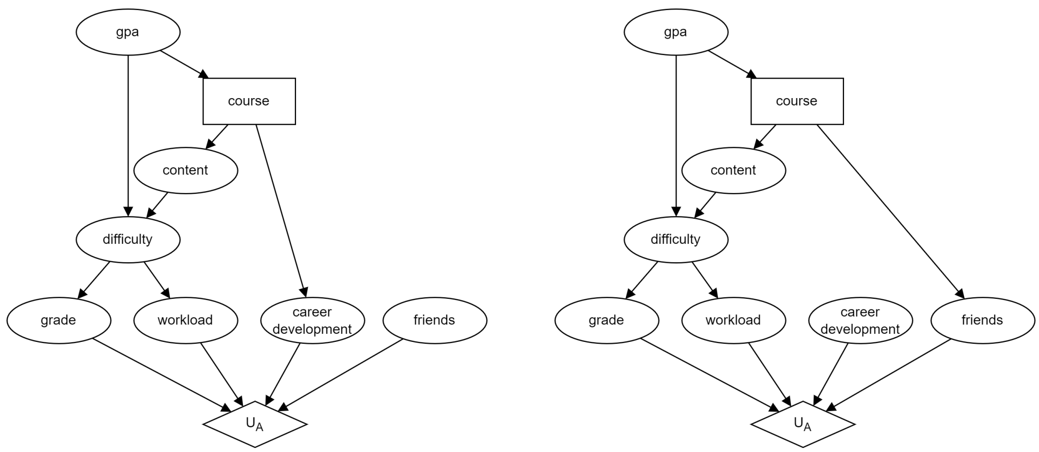
Figure 3: Chance node transfers from I^A to I^S

5.3. Preference Transfer

The preference transfer corresponds to informing an agent about the preferences of the other agent. In our example, this corresponds to communication for understanding the student's preferences. We firstly check if the parents of the utility nodes of both agents are aligned. In our example, $\text{par}(U^S) = \{grade, workload, career\_development, friends\}$, $\text{par}(U^A) = \{grade, workload, career\_development\}$. We copy *friends* to I^A with its domain. Figure 4(*a*) shows the revised I^A after preference transfer. After this operation, U^A and U^S share the same parents and we can copy the utility values from U^S to U^A. We can some random noise to reflect that the student may not clearly express their preferences.

Note that, the CPT of $friends^A$ is initialised randomly when preference transfer is applied. In order to revise the CPTs of the parents of U^A, further chance node transfer operations can be applied. Figure 4(*b*) shows the revised I^A after applying chance node transfer operations to $friends$ and $career\_development$.

When the student does not inform the advisor about all criteria associated with her preferences, only some of the parents of U^S are transferred to U^A. In this case we need to marginalise the utility distribution U^S before copying it to U^A as described in Section 4.4



(a) I^A after preference transfer

(b) I^A after transferring $friends$ and $career\_development$ from I^S

Figure 4: Preference and chance node transfers to I^A

5.4. Simulating Three Talk Model using Influence Diagrams

The operations shown in Sections 4.2, 4.3 and 4.4 can be used to model all stages of TTM which describes the core elements of SDM (see Section 2). Note that, the student and advisor in our case study corresponds to the patient and clinician in TTM respectively.

The first stage in the TTM is the choice talk that involves the advisor presenting the available options to the student. In our model, this corresponds to Decision Alternative Transfer operation, consolidating decision node states into a joint decision node across both. The second stage of TTM is the option talk. During this stage, the student and advisor discuss the decision alternatives and their potential outcomes. The main role of the advisor is getting to know which factors affect the student's course preferences, and updating any misinformation about those factors. In IDs, the option talk corresponds to applying preference transfer operation to match the advisor's utility node and its parents with I^S. Then, we apply chance node transfers recursively starting from the parents of utility nodes I^A for all chance nodes. Since we assume that the student's preferences are essential, and the advisor has a richer domain knowledge in SDM, we apply the preference transfer to I^S and chance node transfers to I^A. After the preference and chance node transfers, the advisor's utility node will be consistent with the student's preferences, and student's chance nodes will be aligned with the advisor leading to the same maximum expected utility decisions – shared decisions – from both IDs. Note that, communication is imperfect and time is limited to talk about the entire knowledge of the advisor. Hence, noise could be added, and limited nodes could be transferred respectively to reflect these conditions. The last step of TTM is the decision talk, in which the advisor and student makes a joint decision. This corresponds to computing the maximum expected utility decisions from both IDs and comparing them.

6. Conclusion

This paper proposed a new ID modelling approach for SDM. We modelled SDM agents with separate ID models, and proposed graphical operations to reflect interactions between those agents enabling them to make a shared decision. We illustrated the use of proposed operations in a case study and showed that they can model all stages of TTM which is a widely used guideline describing the key components of SDM practice. Our case study also showed that SDM is a cooperative decision making setting that is not specific to the clinical domain. Although IDs have been extended to competitive multi-agent settings, they have not been extended for SDM or similar cooperative decision-making problems (see Section 3). One of the key contributions of this paper is to capture the iterative nature of SDM interactions in ID models. SDM involves multiple stages including information exchange, preference elicitation and joint decision making. The proposed operations models these stages as iterative revisions between the IDs.

A limitation of our approach is that the agents do not prioritize which nodes to communicate through the proposed operations. In future work, we plan to incorporate decision theoretic metrics such as the value of information to examine the effect of prioritising communication in SDM. Direct transfer of utility function in our approach is another limitation as humans often find it easier to express their preferences in indirect ways such as ranking or sorting alternatives. The preference transfer operation can be expanded with indirect preference elicitation approaches to overcome this limitation.

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