

# HADAMARD DOMAIN TRAINING WITH INTEGERS FOR CLASS INCREMENTAL QUANTIZED LEARNING

Martin Schiemer<sup>1\*</sup>, Clemens JS Schaefer<sup>2\*</sup>, Mark J Horeni<sup>2</sup>, Yu Emma Wang<sup>3</sup>, Juan Ye<sup>1</sup> & Siddharth Joshi<sup>2</sup>

<sup>1</sup> Department of Computer Science, University of St Andrews, St Andrews, Scotland, United Kingdom  
{ms400, jy31}@st-andrews.ac.uk

<sup>2</sup> Department of Computer Science and Engineering, University of Notre Dame, Notre Dame, IN, USA  
{cschae6, mhoreni, sjoshi2}@nd.edu

<sup>3</sup> Google LLC Mountain View, CA, USA  
yuemmawang@google.com

## ABSTRACT

Continual learning (CL) implemented directly on-device is crucial for practical deployment of applications to battery powered devices, where privacy needs must be balanced with personalization, and agility in adapting to new data. Existing CL techniques can be cost-prohibitive on such devices requiring quantized operations to enable practical deployment. However, as we show commonly used fully quantized training (FQT) solutions do not converge when applied to CL with low-precision hardware. We propose Hadamard Domain Quantized Training (HDQT), that uses the Hadamard transform to facilitate FQT with 4-bit integer operands. HDQT enables low-precision, on-device training where other FQT solutions fail. An examination of gradient alignment reveals that for early feature detection layers, HDQT gradients are better aligned to the unquantized baselines than those generated by other FQT methods. This improved alignment translates to consistently better performance over the course of learning, reflected in the training trajectories through the model loss-landscape. Numerical experiments conducted on Human Activity Recognition (HAR) datasets reveal a  $\ll 1\%$  average accuracy reduction across various competitive CL methods, even under aggressive 4-bit quantization with 8-bit accumulators. Additionally, on CIFAR100 there is no loss of accuracy observed when the accumulator precision is relaxed to 12 bits for competitive non-dynamic architecture CL methods.

## 1 INTRODUCTION

Humans have a remarkable ability to continuously acquire new information throughout their lives. Enabling such capability in neural networks (NNs) would vastly increase their effectiveness and applicability, since tasks can change over time, and data distributions can shift. Even when a task remains constant, the integration of new data is crucial for improving predictive outcomes. To accomplish this, NNs must strike a delicate balance, being adaptable enough to accommodate new tasks and data, while also remaining stable enough to preserve previously acquired knowledge. This inherent tension between adaptability and stability is referred to as the “Stability-Plasticity Dilemma” (Mermilod et al., 2013), and is the subject of Continual Learning (CL). A neural networks’ inability to acquire new tasks without compromising their performance on previously learned tasks is commonly referred to as *Catastrophic Forgetting* (CF). Addressing CF stands as a significant obstacle in the pursuit of more capable autonomous agents, as highlighted by McCloskey & Cohen (1989) and Zhou et al. (2023). Techniques to mitigate CF include regularization to prevent information loss (Li & Hoiem, 2016), memory replay (Rebuffi et al., 2017), or dynamic architectures (Zhou et al., 2022). Harun et al. (2023a) found that many CL algorithms are resource-expensive, with some of them being more costly than training the same model from scratch. The cost of using CL techniques is further exacerbated when additional constraints such as low-latency, or low-energy are imposed. Such constraints are typically encountered on edge devices in applications of HAR in mobile/AR/VR scenarios, drones, and miniaturized autonomous robots.

Operating with reduced precision tensors for an NN through quantization simultaneously increases energy-efficiency of computation and reduces memory footprint and bandwidth requirements. These improvements can be leveraged by custom compute primitives typically present in machine learning (ML) accelerators (Choquette, 2023), custom field programmable gate arrays (Jain et al., 2020), and custom integrated circuits (Garofalo et al., 2022). Additionally, since

\* Shared first authorship.

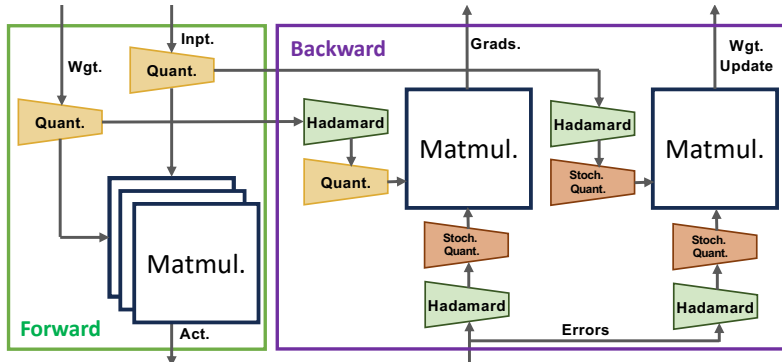


Figure 1: Overview of our proposed methodology, illustrating Hadamard transform and stochastic rounding in the backward pass and tiled matrix multiplication in the forward pass to ease the burden of the forward-pass accumulator.

quantization can be applied together with any model architectural improvements, gains from quantization are typically multiplicative with architectural improvements. Previous work on fully quantized training (FQT), *e.g.*, accelerating matrix multiplication through quantization in forward- and backward-pass of the backpropagation algorithm, have shown promising results in non-CL scenarios (NoCL) through the use of custom floating point formats, stochastic rounding, or adaptive scales (Chmiel et al., 2023; Lee et al., 2022; Xi et al., 2023; Sun et al., 2020).

There has been little work in using FQT for CL to enable its application in resource-constrained environments. This naturally leads to the following question: *Can FQT be applied for CL and to what extent are they compatible?* To answer this question, we show that existing FQT techniques fail when tensors are quantized to 4 bits. Next, we overcome the limitations of existing FQT approaches through a new quantization algorithm which we call Hadamard Domain Quantized Training (HDQT). Our main contribution are:

- We propose a quantized training technique where the backward-pass matrix multiplications are implemented in the Hadamard domain to make efficient use of quantization ranges (overview in Figure 1). We additionally determine which tensors in the backward-pass are sensitive to lower precision and employ stochastic rounding to mitigate this effect. We propose tile-based matrix multiplication for the forward pass to enable low bit-width accumulation; that is, enabling training with standard 4-bit integers as inputs and 8-bit accumulators.
- We study the interaction between FQT and CL, specifically class incremental learning (CIL). We observe a general accuracy reduction owing to HDQT, with dynamic models incurring greater accuracy loss. No specific interaction was observed in the learning dynamics across the various CL techniques. This indicates HDQT’s suitability to applying CL in compute-constrained environments.
- We evaluate our approach with five well-known CL techniques on the example of CIFAR100 a classic CL benchmark as well as three popular human activity recognition datasets. For a 4-bit training with 8-bit accumulators we find, +0.45%, +0.25%, and -0.36% in the final accuracy changes for the HAR datasets (DSADS, PAMAP2, and HAPT) over the investigated algorithms compared to a 16-bit floating point baseline. For CIFAR100 if we increase the accumulator bit-width to 12 while keeping input bit-width constant we even get a 1.00% performance boost given the BIC CL technique over the floating point implementation.

## 2 BACKGROUND

### 2.1 CLASS INCREMENTAL LEARNING (CIL)

Addressing catastrophic forgetting (CF) is the central challenge in continual learning. Three classes of techniques tackle this: *regularization*, *memory replay*, and *dynamic architectures* (Parisi et al., 2018). A notable example of a technique that uses regularization, *Learning without forgetting* (LwF) (Li & Hoiem, 2016) leverages Knowledge Distillation (KD) (Hinton et al., 2015) to penalize large parameter updates during continual learning, mitigating CF. Memory replay approaches periodically update the network with a combination of the current training set with holdout data samples subsampled from previous tasks’ training data (Hetherington & Seidenberg, 1989; Robins, 1993; 1995). Examples include *incremental classifier and representation learning* (iCaRL) (Rebuffi et al., 2017), *maximal interfered retrieval* (MIR) (Aljundi et al., 2019), *learning a unified classifier incrementally via rebalancing* (LUCIR) (Hou et al., 2019), and *look-ahead MAML* (La-MAML) (Gupta et al., 2020). When storage is limited, generative techniques like *replay through feedback* (RtF) (van de Ven et al., 2020) can augment or replace stored samples. Dynamic architectures modify the model architecture as needed to prevent CF. These techniques can grow the network as with *progressive networks*

(Rusu et al., 2016), *dynamically expandable representation* (DER) (Yan et al., 2021) and *memory-efficient expandable model* (MEMO) (Zhou et al., 2022) or isolate and freeze parameters like *packnet* (Mallya & Lazebnik, 2018). Finally, there has been some recent research indicating that that CF is largely caused by a bias in the weights towards new classes, with successful application in *bias correction* (BiC) (Wu et al., 2019), *weight alignment* (WA) (Zhao et al., 2019), or *vector normalization and rescaling* (WVN-RS) (Kim & Kim, 2020).

Given the complexity of these techniques, there have been several studies focusing on efficient continual learning. Hayes & Kanan (2022) define criteria for CL algorithms on the edge and evaluate popular models’ performance using CNNs designed for the edge. Wang et al. (2022) create SparCL a framework for continual learning on the edge. They leverage weight sparsity, effective data selection and gradient sparsity to reduce the computational demand achieving 23× less training FLOPs while improving SOTA results.

Human activity recognition (HAR) community (Ye et al., 2019) has attracted increasing interest as an application area for on-device CL. Jha et al. (2021) have conducted an empirical assessment on a wide array of state-of-the-art continual learning techniques to HAR datasets. The results have demonstrated promising performance to support incremental learning on new activities. Leite & Xiao (2022) have proposed an efficient strategy for HAR that leverages expandable networks, which dynamically grow with the incoming new classes. They utilize the replay of compressed samples selected for their maximal variability. Zhang et al. (2022) have explored multi-modal learning in a continuous fashion for HAR. Their approach combines EWC (Kirkpatrick et al., 2017) with canonical correlation analysis to harness correlations among various activity sensors, with the aim of mitigating catastrophic forgetting. Schiemer et al. (2023) present an online continual learning scenario for HAR where their technique *OCL-HAR* has to perform real-time inference on streaming sensor data while simultaneously discovering, labeling and learning new classes.

Given the need to efficiently deploy CL for models on low-power, energy-efficient devices, existing work has studied quantization, primarily in the context of weight quantization and feature compression for replay memory. Shi et al. (2021) introduce weight quantization to CL through bit-level information preserving, where they quantize to 20-bits and use quantization behavior as a signal to identify important task parameters. Pietróń et al. (2023) use weight quantization to create efficient task sub-networks. Ravaglia et al. (2021) employ quantized latent replay through 7-bit quantization of the frozen old model, prototyping their idea on system-on-a-chip (VEGA). They note a 65× speedup while being 37× more energy efficient compared to STM32 L4 baseline. The most prominent application is not parameter quantization but feature compression (vector quantization) using auto-encoders to optimize sample memory storage; e.g. Hayes et al. (2019); Caccia et al. (2020); Srivastava et al. (2021); Wang et al. (2021); Harun et al. (2023b). While vector quantization certainly improves learning efficiency, it is not a holistic resource optimization approach.

## 2.2 FULLY QUANTIZED TRAINING (FQT)

There has been a renewed interest in quantized learning due to the increasing costs associated with training larger and more capable models. Several techniques have seen widespread use including specific floating-point formats, adaptive gradient-scaling, modified rounding schemes for quantization, amongst others. We provide a short overview of related work here, and a more extensive overview in the Appendix A.1.

Different floating-point representation formats such as the radix-4 format introduced in Sun et al. (2020) have been widely studied; for example, recent work on FP8 training with commodity hardware van Baalen et al. (2023). The authors in Sun et al. (2020) also include an adaptive scaling scheme with layerwise updates of the gradient scales to minimize value clipping. They also employ a multi-phase rounding scheme for their weights and error gradients to effectively increase resolution leading to a less than 1% drop in accuracy on ImageNet and several NLP tasks. Chmiel et al. (2020) achieve iso-accuracy for 6-bit quantized gradients on ImageNet, by leveraging the distribution differences between forward pass tensors and gradients. By modeling the distribution of gradients as log-normal, the authors derive specific values for the mantissa and exponent in their floating-point format. They then build on these results in Chmiel et al. (2023) to develop an exponent-only format for the gradients using stochastic rounding to unbiased gradient estimation. This delivers accuracy within 1.1% of a floating-point ResNet50 for 4-bit quantization.

Lee et al. (2022) developed a principled approach to determine an optimized floating point format by examining the misalignment in weight gradient errors compared to a high-precision backward pass and introducing quantization hysteresis. This results in a 4-bit weight, 8-bit activation and 8-bit gradient ResNet18 training to within 0.2% of their floating-point baseline accuracy. FQT has also been examined for language models with Xi et al. (2023) proposing to use a Hadamard transform during the *forward pass* to better capture outlier values in the activations with bit-splitting in the *backward-pass* to speedup training performance on GPUs by 35.1%. Note, that this approach does not use the Hadamard transform in the backward pass, instead adopting a bit-splitting scheme coupled with standard quantization.

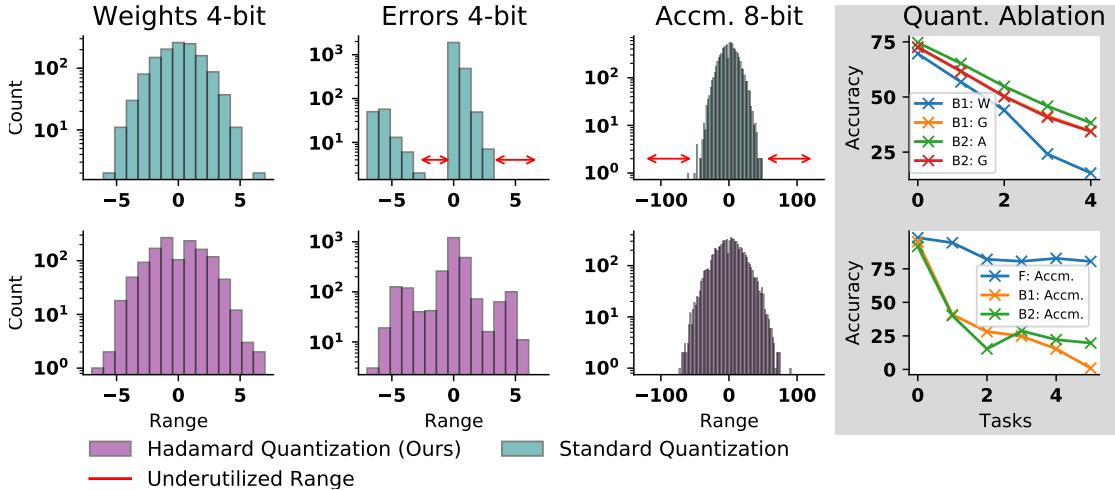


Figure 2: Comparing Hadamard domain quantized matrix-multiply with quantized matrix-multiply for 4-bit inputs (weight and errors) and 8-bit accumulation during CIFAR100 CIL training. The Hadamard domain operations make better use of the quantization range, with fewer empty bins. We observe this to be consistent over all tensors, with the difference accentuated for error tensors and the output of the matrix-multiply (Accm.). On the right grey background, we show results from an ablation study for quantizing tensors in the backward pass (top) and accumulator quantization (bottom). The model accuracy is most sensitive to quantizing the gradients and activation-tensor. We also observe that not quantizing the accumulator in the forward pass has an outsized impact on accuracy, compared to not quantizing the accumulators in the backward pass. Without Hadamard transformation training is not successful (see A.4).

### 3 METHODOLOGY

CL must simultaneously accommodate changing distributions in data, weights and gradients while preventing CF, which can be sensitive to small perturbations (Zhao et al., 2023). We propose a lightweight FQT approach that directly addresses these challenges in CL. Our proposed FQT approach enables matrix multiplication of the form  $\mathbf{Z} = \mathbf{X}\mathbf{W}$ , where  $\mathbf{Z} \in \mathbb{R}^{N \times C}$ ,  $\mathbf{X} \in \mathbb{R}^{N \times D}$  and  $\mathbf{W} \in \mathbb{R}^{D \times C}$ . These operations are required for both forward- and backward-pass. We emulate hardware quantization in software:

$$Q(x) = \text{round}(\text{clip}(\alpha \cdot x) \cdot 2^{b-1}) \cdot 2^{-(b-1)} \cdot \alpha^{-1}$$

where  $Q$  is the quantization function which takes in a high precision value  $x$  and converts it to a  $b$ -bit integer value.  $\alpha$  scales the values to lie between  $\pm 1$  by clipping saturated values outside this range and rounding returns the nearest integer. We calibrate  $\alpha$  using the maximum value in that tensor.

During training, most tensors especially the gradients vary over a wide dynamic range, requiring an adaptive quantization scheme to be used for FQT. Typically, the range of tensors is addressed through custom floating-point formats that trade-off precision for an improved representable range (Lee et al., 2022). This aligns with observations that gradient distributions exhibit power-law heavy tails (Xie et al., 2022). However, for a given number of bits, floating-point operations are more expensive than the equivalent integer operation. Instead, we avoid the precision-dynamic range trade-off and operate only with integers by implementing computations in a transform domain. We choose Hadamard transform as it is an energy-preserving, orthogonal, linear transform that can be implemented using only  $n \log(n)$  additions. This principle was also recently independently applied to post-training quantization methods for large language models, where the Hadamard transform produces incoherent tensors that are more amenable to lower-precision operation in the forward-pass (Xi et al., 2023; Tseng et al., 2024). For a more in-depth introduction to the Hadamard transform, we refer the reader to Aгаian et al. (2011). The information-spreading property of Hadamard transform allows us to make better use of all the available quantization levels, capturing information from the tails of the distribution. The Sylvester form (Sylvester, 1867) of the Hadamard transforms is defined as:

$$\mathbf{H}_0 = [1], \quad \mathbf{H}_k = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{H}_{k-1} & \mathbf{H}_{k-1} \\ \mathbf{H}_{k-1} & -\mathbf{H}_{k-1} \end{bmatrix},$$

where  $k$  denotes the matrix size  $2^k \times 2^k$ . Hadamard matrices are orthogonal and symmetric  $\mathbf{H}_k = \mathbf{H}_k^T = \mathbf{H}_k^{-1}$ , so  $\mathbf{H}_k \mathbf{H}_k = k\mathbf{I}$ . As a result, given an input  $X$  and a weight matrix  $W$ , we have:

$$\mathbf{X}\mathbf{H}\mathbf{H}^{-1}\mathbf{W} = k\mathbf{X}\mathbf{W}$$

Table 1: Results for standard learning (NoCL) and CIL with 20 classes (CIFAR100) or 2 classes (DSADS, PAMPA2, or HAPT) per task quantized (4-bit inputs and 8-bit accumulators) and unquantized (FP). Numbers are final accuracy percentages averaged over 20 runs of the HAR datasets and 5 runs for CIFAR100 (small numbers in brackets are standard deviation). Note that for quantized CIL on HAR datasets, iCaRL is the best CIL method and for CIFAR100 BiC (see underlined numbers).

	CIFAR100		DSADS		PAMAP2		HAPT		$\mathbb{E}[\Delta\text{Acc.}]$
	FP	4-bit	FP	4-bit	FP	4-bit	FP	4-bit	
NoCL	68.61 ( $\pm 0.24$ )	59.43 ( $\pm 1.88$ )	84.44 ( $\pm 4.36$ )	82.71 ( $\pm 4.45$ )	85.81 ( $\pm 4.91$ )	84.35 ( $\pm 7.66$ )	93.15 ( $\pm 2.11$ )	92.08 ( $\pm 2.10$ )	3.36
LwF	33.92 ( $\pm 0.91$ )	29.32 ( $\pm 1.28$ )	12.06 ( $\pm 6.01$ )	12.33 ( $\pm 7.68$ )	36.34 ( $\pm 8.03$ )	32.84 ( $\pm 10.25$ )	32.78 ( $\pm 8.74$ )	28.65 ( $\pm 10.22$ )	2.99
iCaRL	43.20 ( $\pm 1.21$ )	37.25 ( $\pm 2.42$ )	72.83 ( $\pm 4.84$ )	72.74 ( $\pm 4.59$ )	77.52 ( $\pm 5.50$ )	<b>77.87</b> ( $\pm 5.02$ )	82.98 ( $\pm 5.12$ )	82.56 ( $\pm 4.51$ )	1.53
BiC	48.54 ( $\pm 3.95$ )	46.07 ( $\pm 2.64$ )	67.55 ( $\pm 6.05$ )	63.79 ( $\pm 5.86$ )	77.04 ( $\pm 7.00$ )	75.46 ( $\pm 5.79$ )	81.42 ( $\pm 4.32$ )	78.26 ( $\pm 5.14$ )	2.74
DER	62.00 ( $\pm 0.88$ )	<b>51.39</b> ( $\pm 1.43$ )	74.02 ( $\pm 4.61$ )	<b>74.47</b> ( $\pm 5.40$ )	74.36 ( $\pm 6.73$ )	74.96 ( $\pm 6.90$ )	86.31 ( $\pm 1.63$ )	<b>85.95</b> ( $\pm 2.73$ )	2.48
MEMO	55.75 ( $\pm 0.82$ )	47.94 ( $\pm 1.14$ )	46.18 ( $\pm 8.46$ )	44.95 ( $\pm 11.18$ )	58.89 ( $\pm 5.16$ )	31.05 ( $\pm 12.66$ )	69.90 ( $\pm 7.05$ )	58.01 ( $\pm 24.41$ )	12.19
$\mathbb{E}[\Delta\text{Acc.}]$	6.77		1.02		5.57		3.51		

Due to this property, the products of Hadamard-transformed tensors need not be transformed back from the Hadamard domain. Since Hadamard matrices are only defined for powers of two, like [Xi et al. \(2023\)](#), we use block-diagonal transformation matrices  $\mathbf{H} \in \mathbb{R}^{D \times D} : \text{BlockDiag}(\mathbf{H}_k, \dots, \mathbf{H}_k)$ . Here, the highest  $k$  is chosen to deliver a valid block-diagonal matrix for the given tensor dimension (*e.g.*, dimensions of  $\mathbf{X}$  or  $\mathbf{W}$ ). [Figure 2](#) compares the distribution of quantized tensors, with and without the Hadamard transform. As seen by the underutilized bins (red arrows) when directly quantizing the errors to 4-bits, the Hadamard transform is more efficient in using available quantization levels. We also observe consistently that this spreading allows better use of the full dynamic range available for accumulation (8-bits shown in [Figure 2](#)).

To identify which tensor quantization impacts model accuracy the most, we conduct an ablation study to quantize all but one tensor in the backward pass (inputs, weights, activations, and gradients). [Figure 2](#) (right) shows that the best performance is achieved when the two gradient tensors and the activation tensor in the backward pass are unquantized. To mitigate the impact of bias when quantizing these sensitive tensors, we use stochastic rounding ([Chmiel et al., 2023](#); [Liu et al., 2022](#)). [Appendix A.9](#) discusses the impact of stochastic rounding on model quality in greater detail.

We also observed that model accuracy is sensitive to the precision with which partial sums are calculated during the forward pass. As shown in [Figure 2](#) (right, bottom), performance is better when we do not quantize the forward pass accumulator of the matrix multiply units. To mitigate, we employ tiled quantization, which reduces the dynamic range of partial sums, limiting computations to tiles of size  $s$  (similar to ([Wang et al., 2018b](#))). We did not observe a similar effect during the backward pass, and currently hypothesize that gradient sparsity might have had a mitigating effect.

## 4 EXPERIMENTS

We focus on supervised class incremental learning where each incoming task has disjoint classes and task identities are unknown during testing. Let  $T = [t_1, t_2, \dots, t_n]$  be a sequence of tasks where each task  $t_i$  holds a finite set of classes  $C_i$  and their  $n_i$  training samples  $\{(x^j, y^j) | y^j \in C_i\}_{j=1}^{n_i}$  are disjoint from previous tasks  $t_k < t_i$ , with  $k \in 1, \dots, i-1$ . The goal is to integrate new information into an existing model without regressing performance on prior tasks. Our method is applicable to different CL scenarios *e.g.*, online Continual Learning.

To evaluate our performance we choose average class accuracies  $a_c$  for each task and the forgetting measure. The forgetting score by [Chaudhry et al. \(2018\)](#) measures the difference between the best and current accuracies, indicating how much knowledge of previous tasks has been lost when learning new tasks. Forgetting at task  $t_i$  is defined as:  $F_i = \frac{1}{|C|} \sum_{c=1}^C f_c$  where  $f_c = \max_{l \in 1, \dots, i-1} (a_c^l) - a_c^i$ . We evaluate HDQT on CIFAR100 ([Krizhevsky, 2009](#)) and three

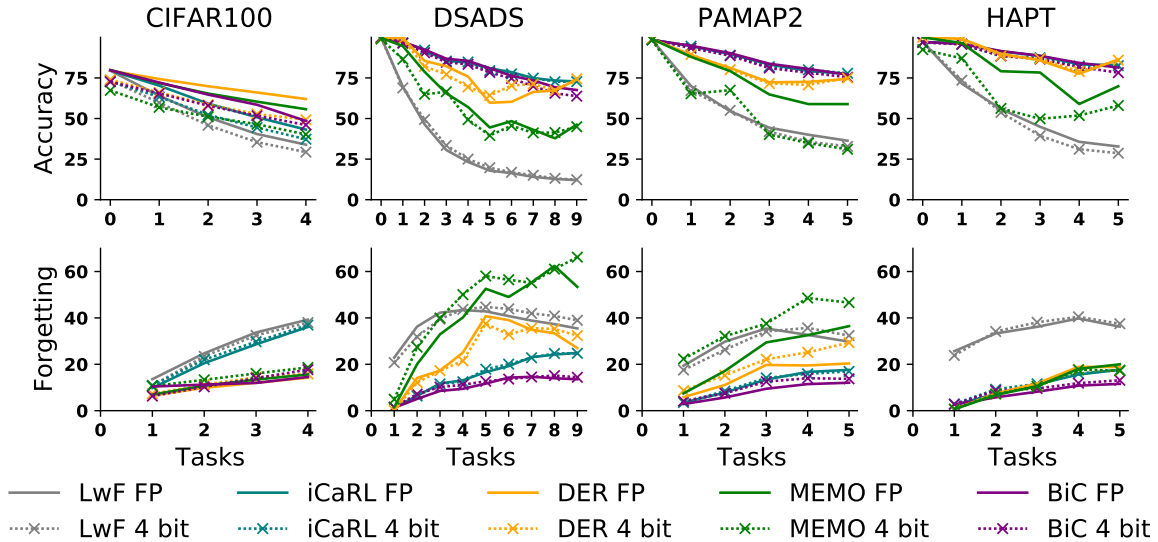


Figure 3: Step accuracy (first row) and forgetting (second row) of quantized (matrix multiplications in forward and backward-pass with 4-bit inputs and 8-bit accumulators) and FP models for CIFAR100, DSADS, PAMAP2, and HAPT datasets averaged over 5 (CIFAR100) and 20 (HAR datasets) runs with different class orders. Note the intersection of the forgetting lines of quantized and FP models for the BiC on CIFAR100, LwF on DSADS and PAMAP2, indicate model learning capacity saturation mid-training in contrast to other models.

popular HAR datasets: PAMAP2, DSADS, and HAPT [Jha et al. \(2021\)](#); [Schiemer et al. \(2023\)](#). These datasets have been commonly used as a baseline to evaluate CIL algorithms [Rebuffi et al. \(2017\)](#); [Wu et al. \(2019\)](#) and a brief introduction of HAR datasets can be found in Appendix A.3. We study the interaction between FQT and continual learning through 5 representative CIL techniques: LwF ([Li & Hoiem, 2016](#)), iCaRL ([Rebuffi et al., 2017](#)), BiC ([Wu et al., 2019](#)), DER ([Yan et al., 2021](#)) and MEMO ([Zhou et al., 2022](#)) with the implementations from [Zhou et al. \(2023\)](#). Appendix A.2 and A.8 presents the description of each technique and additional results.

For CIFAR100, we use ResNet-32 with hyperparameters consistent with [Zhou et al. \(2023\)](#). For HAR datasets, three-layered fully connected networks (FCN) are chosen, with each layer containing the same number of neurons as the input layer. The output layer, adjusted as new tasks arrive, aligns its neuron count with the currently seen classes. HDQT introduces two quantization-specific hyperparameters: a block-size for accumulators (32) and a scaling factor (0.975) to remove outliers before integer quantization in the forward pass for weights and activations. Additional hyperparameter settings are outlined in Appendix A.5.

#### 4.1 RESULTS

Table 1 summarizes the final accuracy of 4-bit quantized models and FP models. For non-dynamic CL techniques we observed negligible degradation in model accuracy on HAR datasets with quantized iCaRL, the largest drop being 0.42% on HAPT. BiC achieves the best performance on CIFAR100 of the non-dynamic models with and without quantization, consistent with previous results ([Wu et al., 2019](#)). Quantization effects are more pronounced for CIFAR100 (-10.61% for BiC) vs. HAR datasets (DSADS +0.45% & HAPT -0.36% for DER and PAMAP2 +0.35% for iCaRL). Generally, we observe a lower loss in accuracy from quantization when using CIL compared to models trained with NoCL (-3.36% NoCL vs. -2.99% LwF). In the case of dynamic models, we observe that quantization exacerbated performance loss. This is likely due to the task-wise “feature stacking” layer after the feature extractor layers. This layer seems less resilient to quantization-induced information loss which leads to classes being less distinguishable. This is especially pronounced for MEMO where a task-wise common feature extractor produces worse features over-all. The standard deviation in model accuracy for CIL through HDQT is similar to what is achieved on FP-trained models, indicating minimal impact in variance for HDQT with the exception of MEMO.

Figure 3 shows average class accuracy and forgetting per step using 4-bit weights and 8-bit accumulation for HDQT and baseline FP models. For CIFAR100, HDQT impacts the initial model accuracy with, *e.g.*, a 7.08% drop on BiC at Task 0. However, the per-task learning trajectory remains similar for various CL techniques in quantized and unquantized models across datasets. We calculated the correlation coefficient between FP and HDQT on averaged trajectories, observing 1.00 LwF, 0.99-1.00 iCaRL, 0.99-1.00 BiC, 0.95-1.00 DER and 0.88-1.00 MEMO. This suggests that combining CIL and HDQT does not further degrade model accuracy.

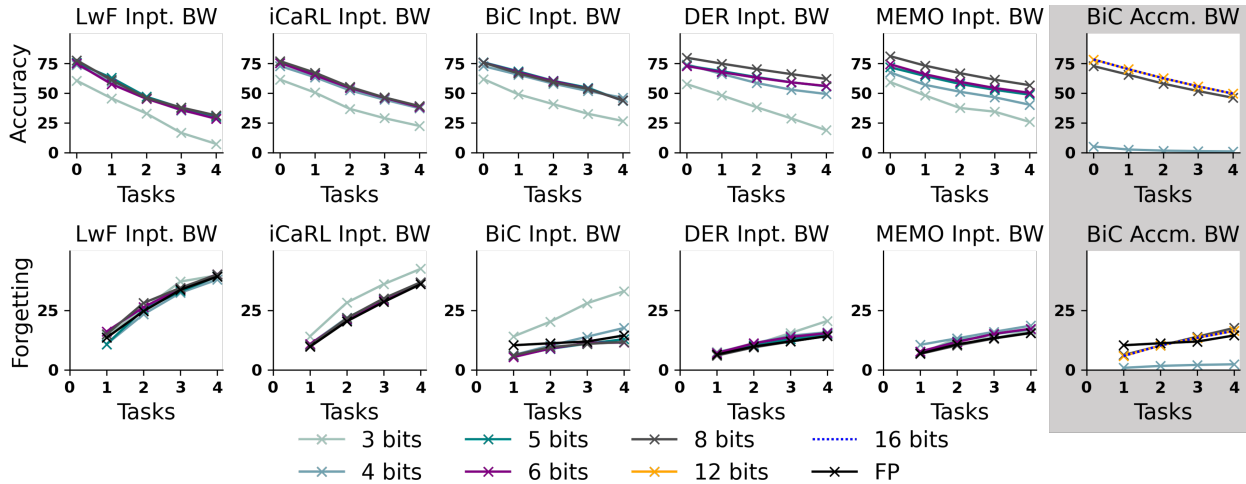


Figure 4: Step accuracy averaged over five runs on CIFAR100 for LwF, iCaRL, BiC, DER and MEMO with different bit-widths ranging from 3b–6b, and 8b for inputs and 4b, 8b, 12b and 16b for accumulators (grey background).

Figure 4 shows the effect of varying input bit-widths for all five CIL techniques on CIFAR100 while keeping the accumulator bit-widths as double of the input bit-width to account for ‘bit-growth’<sup>1</sup> during accumulation. For all techniques, we observe that quantizing inputs to 3-bits causes a severe drop in accuracy (*e.g.*, -19.7% on CIFAR100 with BiC compared to 4-bits). Input precision higher than 4-bits shows no additional benefit except for the dynamic models which display a continuous performance increase the higher the input bit-width. The accumulator bit-width sweep (Fig. 4 grey hue) exhibits the effects of bit-growth during dot product operations. We fix the input bit-width at 4 and observed accuracy over multiple settings of accumulator-bit-width. When constrained to 4-bits, HDQT fails, with accuracy no better than random. Increasing accumulator bit-width over our default 8-bits (12- and 16-bits) leads to only  $\sim 3\%$  improvement in the final accuracy, suggesting a favorable trade-off in precision and efficiency at 8-bits. We observe that different degrees of quantization still result in similar learning over different tasks (similar change in accuracy per step). Again this confirms that HDQT reduces initial training accuracy, but minimally impacts CIL.

Despite initial accuracy variations, the training curves of FP and HDQT models display strong similarity, indicating comparable learning performance. However, distinct forgetting trajectories reveal changes in representational quality at each incremental step. By examining both models’ learning and forgetting patterns, qualitative differences in features learned through FQT become apparent. The consistent accuracy change between tasks (1.00 LwF, 0.99-1.00 iCaRL, 0.99-1.00 BiC, 0.95-1.00 DER and 0.88-1.00 MEMO) for HDQT and FP implies that despite initial accuracy differences, both the FP and HDQT models learn at similar rates.

To better analyze their difference, we categorize forgetting behaviors into distinct scenarios. Scenario 1: the quantized model forgets more than the unquantized model, indicating that the quantized model has reached its representational limit. Scenario 2: the quantized model forgets less, indicating it has not yet reached its representational limit. We analyze forgetting curves in Fig. 3 to further understand how FQT on CIL algorithms impact the representational limit. Consider LwF on CIFAR100, here the FP model always forgets more than the HDQT model hinting that the quantized model has not yet reached its representational limit. On the other hand, in the same figure, consider iCaRL, DER or MEMO for CIFAR100, the HDQT model always forgets more than the FP model, indicating it is already at its representational limit. When the forgetting curves intersect, we can infer that the HDQT model has reached its representational limit (*e.g.*, see BiC on Task 3 for CIFAR100, suggesting that the learning technique has reached its representational limit for the quantized model during that task). Beyond this point, acquiring new knowledge may necessitate overwriting prior knowledge at higher rates for HDQT when compared to the baseline (FP) CIL.

In Figure 4, we see the capacity changes over different bit-widths where the crossover points between the FP model and the quantized models shift rightward given more bits. This is especially apparent for BiC, which we infer has a higher representational limit for our model.

Compared to CIFAR100, the HAR datasets show a smaller forgetting gap between the FP and HDQT models, for all static techniques. As shown in Table 1, HDQT on HAR datasets has negligible performance impact when using iCaRL, indicating that iCaRL’s 4-bit network poses sufficient learning capacity to produce on-par results to FP models. The results in CIFAR100 for iCaRL and BiC are also reflected in the HAR datasets with BiC and LwF. In contrast,

<sup>1</sup>Bit-growth occurs when mathematical operations increase a result’s precision beyond a system’s representational limit.

Table 2: Final accuracy and forgetting values for five CIL techniques using LUQ (Chmiel et al., 2023), FP134 (Lee et al., 2022), and our method given 8- and 12-bit accumulators on CIFAR100 averaged over five runs. ‡ indicates that we only took the average over four runs because one run resulted in random accuracy due to numerical issues. \*Denotes when we use a modification to the LUQ code (see Section A.7 in the Appendix for detailed discussion). § denotes at least one failed run omitted from result.

	Inpt. BW	Accm. BW	NoCL Acc.↑	LwF Acc.↑ / Forg. ↓	iCaRL Acc.↑ / Forg. ↓	BiC Acc. ↑ / Forg. ↓	DER Acc. ↑ / Forg. ↓	MEMO Acc. ↑ / Forg. ↓
Base (NoQ)	16	16	68.61	33.92 / 39.25	43.20 / 36.10	48.54 / 14.53	62.00 / 14.26	55.75 / 15.62
LUQ	4	16	65.30	33.23 / 36.56	42.46 / 35.65	48.58 / 13.58	57.04 / 18.77	52.26 / 20.24
LUQ*	4	16	65.14	33.66 / 35.60	41.72 / 36.24	47.91 / 14.20	56.94 / 18.92	51.99 / 20.39
LUQ*	4	12	Fail	33.68 / 34.74	41.42 / 36.41	49.37 / 15.88	56.16 / 19.57	51.79 / 20.37
LUQ*	4	8	Fail	Fail	Fail	Fail	Fail	Fail
FP134	8	16	65.91§	35.55 / 25.60	38.80 / 35.35	48.43‡ / 13.35‡	57.11 / 12.26	49.93 / 16.42
FP134	4/8	16	65.81§	35.15 / 24.70	38.99 / 35.65	47.49‡ / 13.86‡	57.88 / 16.49	50.30 / 12.15
FP134	4	16	Fail	Fail	Fail	Fail	Fail	Fail
HDQT	4	16	61.96	30.21 / 39.56	39.07 / 37.70	49.00 / 17.01	51.39 / 15.95	47.94 / 17.46
HDQT	4	12	62.31	29.90 / 40.00	39.26 / 37.80	49.54 / 16.47	51.14 / 16.08	47.81 / 17.95
HDQT	4	8	59.43	29.32 / 38.00	37.25 / 36.93	46.07 / 17.69	49.25 / 15.88	40.25 / 18.64

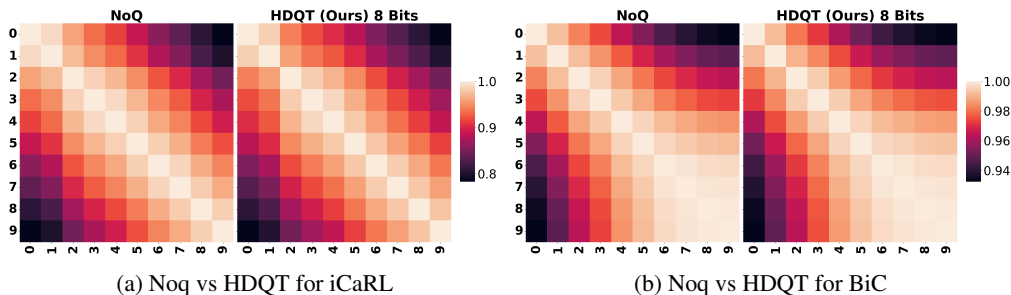


Figure 5: Correlation matrices between model weights of each task for DSADS training, plotted for iCaRL (a) and BiC (b). High similarity confirms that HDQT preserves underlying learning characteristics.

the dynamic models show a more erratic learning pattern and higher performance degradation through quantization. Especially in case of MEMO, the inconsistent HAR results indicate that the common generalized fully connected backbone as a feature extractor is not able to produce features that are generalisable between classes. Thus, MEMO’s use-case for this type of data is limited. We observe that HDQT saturates earlier (*e.g.*, see the forgetting curve of LwF on DSADS at task 4), indicating that new knowledge may have replaced old knowledge, leading to a higher average forgetting than the FP equivalent. LwF’s low performance is consistent with previous research that regularization alone is not enough to mitigate forgetting (Knoblauch et al., 2020; Jha et al., 2021).

## 4.2 EFFECTS OF FORGETTING FROM QUANTIZATION AND CIL

We study the impact of quantization on the learning characteristics of CL algorithms. Figure 5 shows the correlation of model weights after each training step, finding that quantization mostly preserves the correlation. This aligns with prior work that indicated weight importance correlation across tasks was critical to continual learning performance Zenke et al. (2017). Similar in spirit to Hooker et al. (2019), we also study how HDQT affects forgetting in CIL. Here, we look at class-specific accuracy changes in quantization and continual learning on an exemplary single DSADS dataset run. On quantizing a NoCL model directly using HDQT, we observe a minimal change in accuracy of 1.73%. Notably, CIL causes a greater total accuracy reduction of 11.61%. We do not observe any consistent change in accuracy due to HDQT applied with CIL. Although there are examples where quantization and CIL combined amplify forgetting, *e.g.*, A (Sitting) or D (Lying - Right Side), there are others where the opposite effect is observed, *e.g.*, C (Lying - Back). On comparing the results for CIL with those for CIL combined with HDQT, there is no distinct trend in forgetting observed for earlier classes in the training sequence (I, S, J ...), indicating that HDQT does not amplify forgetting of the earlier classes. Additional information for DSADS and other datasets are depicted in Figure 10, 11 and 12 in Appendix A.13.



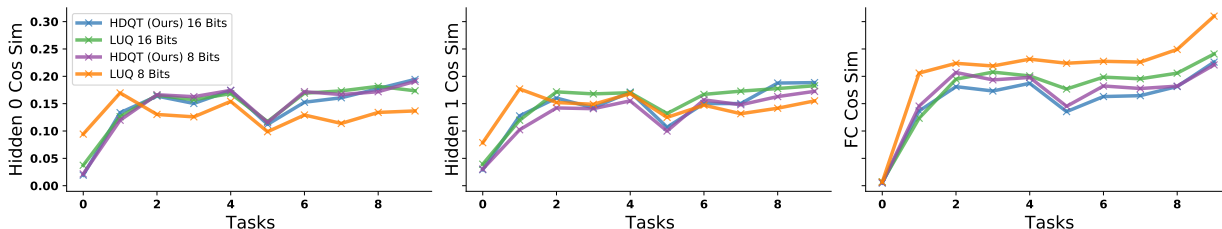


Figure 6: Mean cosine similarity between NoQ and quantization methods on DSADS. LUQ deviates in upper layers.

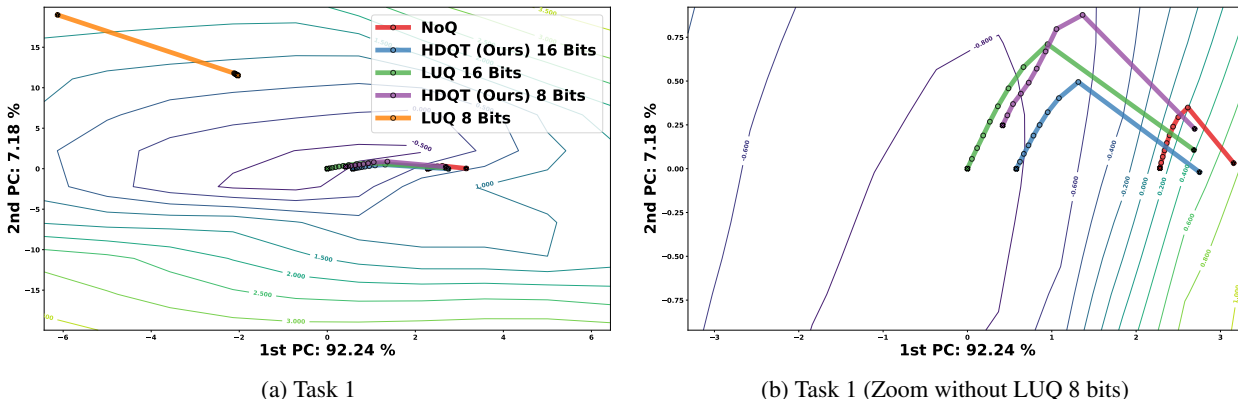


Figure 7: Loss landscapes (LL) (Li et al., 2018) visualizations of CL training for DSADS during Task 1, after an initial training step from common initialization. (a) shows LL for all models, while (b) expands that view, removing the 8bit LUQ. HDQT experiences minimal deviation from NoQ for both 8 and 16 accumulator bits, while LUQ with 8 bit performs worse. LLs were generated using LUQ 16 bits model as base.

### 4.3 COMPARISON TO OTHER QUANTIZATION WORK

To the authors’ knowledge, there is no other work applying FQT to CIL. Thus, we ported two competitive FQT techniques, logarithmic unbiased quantization (LUQ) (Chmiel et al., 2023) and FP134 (Lee et al., 2022), into our CIL framework to enable a comparative study. Table 2 shows the resulting final accuracies of these two algorithms compared to HDQT over different accumulator bit-widths. LUQ uses 4-bit integer inputs to matrix multiplications in the forward pass and a specific 4-bit radix-2 format for the gradients with no accumulator quantization. Our method uses a standard 4-bit integer as input with 8-bit integer accumulators in forward and backward-pass. For fair comparison, we also include an 8/12-bit accumulator in the LUQ forward pass, which results in numerical instabilities for NoCL, preventing learning. The authors of Lee et al. (2022) primarily use 8-bit floating point formats, and in addition, they include results from FQT with 4-bit floating point weights. We include both techniques in Table 2 with all inputs reduced to 4 bits and varying accumulators bits. However, this resulted in numerical instability at low bit-widths, leading to the model performance being no different from random classification. Given an increased bit-width for the accumulator from 8- to 12 as well as 16-bits our method outperforms others (including our floating point baseline) when using BiC training. In Figure 6 we investigate our better performance at lower bit-widths. HDQT delivers better gradient approximates for early layers as the cosine similarity to the unquantized gradients is higher than LUQ. Despite observing the opposite in the FC layer, this performance loss is unrecovered as information has already been lost. Figure 7 visualizes the loss landscape (Li et al., 2018) of the different training configurations. The learning progress of the worse performing LUQ (8-bit accumulators) is distinctive from different NoQ and HDQT configurations. Figure 8 showcases a Pareto frontier for DSADS training, comparing HDQT and LUQ on final accuracy and bit-width over different runs. We find that HDQT is the dominant strategy when bit-width is minimized and accuracy maximized.

### 4.4 ENERGY CONSIDERATIONS

We used an architectural performance estimation tool (Wu et al., 2019; Parashar et al., 2019), calibrated to a commercially available FinFET process node to better capture the impact of using HDQT. We use the architectural cost model for Nvidia’s SIMBA (Shao et al., 2019) accelerator configured to 32 KB weight buffer, 8 kB input buffer, 64 kB global buffer with 64 MAC units (8 lanes of 8 wide vector macs). Conclusions drawn from performance estimates across

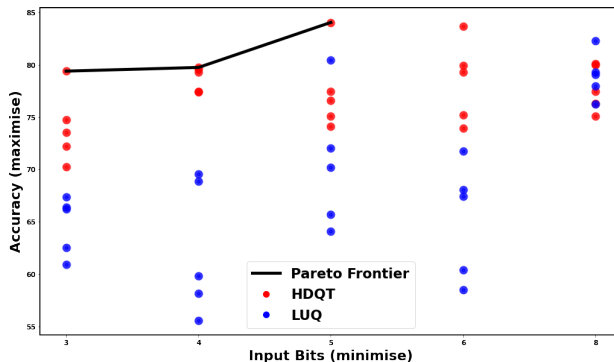


Figure 8: Pareto frontiers of the top-5 accuracy runs for HDQT and LUQ across input bit-widths of 3-8 bits. Accumulator bits are doubled relative to input bits to accommodate for bit growth. Input bits are plotted against accuracy, where minimal input bits and maximal accuracy are better. HDQT emerges as the dominant strategy. We compare HDQT and LUQ in terms of operations in Table 7 in Appendix A.14.

multiple hardware platforms are very similar due to the quantization overhead being little and differences are dominated by the implementation of matrix multiplications. We analyze results over different numerical precisions for the hidden and output layers for HAPT training. When constraining the architecture to the same cycle-count, our estimations indicate a  $4.5\times$  improvement in energy efficiency in implementing the hidden layer and  $3.8\times$  for the output layer. These are driven by a  $18.7\times$  reduction in forward pass energy for the hidden layer and  $6.4\times$  in the output layer. The backward pass has a  $3.3\times$  and  $3.1\times$  lower energy consumption for the hidden and output layer respectively. Applying this to LwF results in energy-efficiency improvements in the forward and backward pass as well as in the inference step of the previous model used for KD. We show up to  $4.5\times$  energy savings when reducing accumulators precision from 16 bit, indicating a favorable trade-off. A detailed breakdown of the performance estimates and other datasets can be found in Table 5 in A.10. The energy overhead of using the Hadamard Transform is tabulated in Table 6 in A.12. Because Timeloop does not currently support recursive and butterfly operations, our estimates are likely under-counting the gains obtained through HDQT.

## 5 CONCLUSIONS

We introduced HDQT, an FQT quantization method for CL which leverages the Hadamard domain to make efficient use of quantization ranges in the backward pass. HDQT delivered promising results on vision (CIFAR100) and HAR (DSADS, PAMAP2, HAPT) datasets, with a 6.77% degradation on vision and 3.37% for HAR with 4-bit inputs. Per competitive CL technique we measure  $\ll 3\%$  averaged over datasets. We demonstrate low-precision operation in both accumulators and multipliers, with competitive results at precisions for which other techniques fail. Further, while there is need to further study FQT for dynamic models, to better accommodate their specifics, HDQT shows competitive performance across multiple models and datasets. Additionally, we demonstrate that HDQT with additional accumulation bits consistently outperforms even the floating point model with BiC. This indicates that we can exceed existing models by exploring the trade-offs between energy efficiency and accuracy, paving the road for learning continually on energy-efficient platforms. While we have looked at a single element of CL, for future work we will study how FQT might be applied to a variety of CL tasks, including the impact of task sequences and imbalanced data batches resulting from continual learning. Prior work has noted that severe class imbalance can adversely affect quantization results in the non-continual setting. Extending FQT to the general CL setting might potentially exacerbate information loss, impeding the network’s ability to discern representative features, particularly within minority classes (Hooker et al., 2019). This investigation could shed further light on the interplay between continual learning dynamics and the quantization processes, offering valuable insights for optimizing model performance in real-world scenarios.

## ACKNOWLEDGMENTS

This work is partly funded by Leverhulme Research Project Grant RPG-2021-355 and the NSF Grant SHF-2340799.

## REFERENCES

Sos S. Aghaian, Hakob G. Sarukhanyan, Karen O. Egiazarian, and Jaakko Astola. *Hadamard Transforms*. SPIE, aug 2011. ISBN 9780819486479. doi: 10.1117/3.890094. URL <https://spiedigitallibrary.org/>

[ebooks/PM/Hadamard-Transforms/eISBN-9780819486486/10.1117/3.890094](https://ebooks/PM/Hadamard-Transforms/eISBN-9780819486486/10.1117/3.890094).

- Rahaf Aljundi, Eugene Belilovsky, Tinne Tuytelaars, Laurent Charlin, Massimo Caccia, Min Lin, and Lucas Page-Caccia. Online Continual Learning with Maximal Interfered Retrieval. In H Wallach, H Larochelle, A Beygelzimer, F Alché-Buc, E Fox, and R Garnett (eds.), *Advances in Neural Information Processing Systems 32*, pp. 11849–11860. Curran Associates, Inc., 2019.
- Kerem Altun, Billur Barshan, and Orkun Tunçel. Comparative study on classifying human activities with miniature inertial and magnetic sensors. *Pattern Recognition*, 43(10):3605–3620, 2010. ISSN 00313203. doi: 10.1016/j.patcog.2010.04.019.
- Lucas Caccia, Eugene Belilovsky, Massimo Caccia, and Joelle Pineau. Online learned continual compression with adaptive quantization modules. In *37th International Conference on Machine Learning, ICML 2020*, volume Part F16814, pp. 1217–1227, 2020. ISBN 9781713821120.
- Arslan Chaudhry, Puneet K. Dokania, Thalaiyasingam Ajanthan, and Philip H.S. Torr. Riemannian Walk for Incremental Learning: Understanding Forgetting and Intransigence. In *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, 2018. ISBN 9783030012519. doi: 10.1007/978-3-030-01252-6\_33.
- Jianfei Chen, Yu Gai, Zhewei Yao, Michael W Mahoney, and Joseph E Gonzalez. A statistical framework for low-bitwidth training of deep neural networks. *Advances in neural information processing systems*, 33:883–894, 2020.
- Brian Chmiel, Liad Ben-Uri, Moran Shkolnik, Elad Hoffer, Ron Banner, and Daniel Soudry. Neural gradients are near-lognormal: improved quantized and sparse training. *arXiv preprint arXiv:2006.08173*, 2020.
- Brian Chmiel, Ron Banner, Elad Hoffer, Hilla Ben-Yaacov, and Daniel Soudry. Accurate neural training with 4-bit matrix multiplications at standard formats. In *The Eleventh International Conference on Learning Representations*, 2023.
- Jack Choquette. Nvidia hopper h100 gpu: Scaling performance. *IEEE Micro*, 2023.
- Marios Fournarakis and Markus Nagel. In-hindsight quantization range estimation for quantized training. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 3063–3070, 2021.
- Angelo Garofalo, Yvan Tortorella, Matteo Perotti, Luca Valente, Alessandro Nadalini, Luca Benini, Davide Rossi, and Francesco Conti. Darkside: A heterogeneous risc-v compute cluster for extreme-edge on-chip dnn inference and training. *IEEE Open Journal of the Solid-State Circuits Society*, 2:231–243, 2022.
- Gunshi Gupta, Karmesh Yadav, and Liam Paull. La-MAML: Look-ahead meta learning for continual learning. In *Advances in Neural Information Processing Systems*, volume 2020-Decem, 2020.
- Md Yousuf Harun, Jhair Gallardo, Tyler L. Hayes, and Christopher Kanan. How Efficient Are Today’s Continual Learning Algorithms? 2023a. ISSN 21607516. doi: 10.1109/CVPRW59228.2023.00241. URL <http://arxiv.org/abs/2303.18171>.
- Md Yousuf Harun, Jhair Gallardo, Tyler L. Hayes, Ronald Kemker, and Christopher Kanan. SIESTA: Efficient Online Continual Learning with Sleep. 2023b. URL <http://arxiv.org/abs/2303.10725>.
- Tyler L. Hayes and Christopher Kanan. Online Continual Learning for Embedded Devices. 2022. URL <http://arxiv.org/abs/2203.10681>.
- Tyler L. Hayes, Kushal Kafle, Robik Shrestha, Manoj Acharya, and Christopher Kanan. REMIND Your Neural Network to Prevent Catastrophic Forgetting. oct 2019. URL <http://arxiv.org/abs/1910.02509>.
- P.A. Hetherington and M.S. Seidenberg. Is there “catastrophic interference” in connectionist networks. In *Proceedings of the 11th annual conference of the cognitive science society*, volume 26, pp. 33, 1989. URL <http://scholar.google.com/scholar?q=related:6OvJaVLsTzwJ:scholar.google.com/{&hl=en{&as{&#}3>.
- Geoffrey Hinton, Oriol Vinyals, and Jeff Dean. Distilling the Knowledge in a Neural Network. In *NIPS 2014 Workshop*, mar 2015. URL <http://arxiv.org/abs/1503.02531>.

- Sara Hooker, Aaron Courville, Gregory Clark, Yann Dauphin, and Andrea Frome. What do compressed deep neural networks forget? *arXiv preprint arXiv:1911.05248*, 2019.
- Saihui Hou, Xinyu Pan, Chen Change Loy, Zilei Wang, and Dahua Lin. Learning a unified classifier incrementally via rebalancing. In *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 2019. ISBN 9781728132938. doi: 10.1109/CVPR.2019.00092.
- Tian Huang, Tao Luo, Ming Yan, Joey Tianyi Zhou, and Rick Goh. Rct: Resource constrained training for edge ai. *IEEE Transactions on Neural Networks and Learning Systems*, 2022.
- Sambhav Jain, Albert Gural, Michael Wu, and Chris Dick. Trained quantization thresholds for accurate and efficient fixed-point inference of deep neural networks. *Proceedings of Machine Learning and Systems*, 2:112–128, 2020.
- Saurav Jha, Martin Schiemer, Franco Zambonelli, and Juan Ye. Continual learning in sensor-based human activity recognition: An empirical benchmark analysis. *Information Sciences*, 575:1–21, oct 2021. ISSN 00200255. doi: 10.1016/j.ins.2021.04.062. URL <https://linkinghub.elsevier.com/retrieve/pii/S0020025521003911>.
- Byungju Kim and Junmo Kim. Adjusting Decision Boundary for Class Imbalanced Learning. *IEEE Access*, 8: 81674–81685, 2020. ISSN 2169-3536. doi: 10.1109/ACCESS.2020.2991231. URL <https://ieeexplore.ieee.org/document/9081988/>.
- James Kirkpatrick, Razvan Pascanu, Neil Rabinowitz, Joel Veness, Guillaume Desjardins, Andrei A. Rusu, Kieran Milan, John Quan, Tiago Ramalho, Agnieszka Grabska-Barwinska, Demis Hassabis, Claudia Clopath, Dharshan Kumaran, and Raia Hadsell. Overcoming catastrophic forgetting in neural networks. *Proceedings of the National Academy of Sciences*, 114(13):3521–3526, mar 2017. ISSN 0027-8424. doi: 10.1073/pnas.1611835114. URL <https://pnas.org/doi/full/10.1073/pnas.1611835114>.
- Jeremias Knoblauch, Hisham Husain, and Tom Diethe. Optimal Continual Learning has Perfect Memory and is NP-HARD. In *37th International Conference on Machine Learning, ICML 2020*, volume PartF16814, pp. 5283–5293, 2020. ISBN 9781713821120.
- Alex Krizhevsky. Learning Multiple Layers of Features from Tiny Images. ... *Science Department, University of Toronto, Tech. ...*, pp. 1–60, 2009. ISSN 1098-6596. doi: 10.1.1.222.9220. URL <http://scholar.google.com/scholar?hl=en{%&btnG=Search{%&q=intitle:Learning+Multiple+Layers+of+Features+from+Tiny+Images{#}0>.
- Sunwoo Lee, Jeongwoo Park, and Dongsuk Jeon. Toward efficient low-precision training: Data format optimization and hysteresis quantization. In *International Conference on Learning Representations*, 2022.
- Clayton Frederick Souza Leite and Yu Xiao. Resource-Efficient Continual Learning for Sensor-Based Human Activity Recognition. *ACM Transactions on Embedded Computing Systems*, 21(6):1–25, nov 2022. ISSN 1539-9087. doi: 10.1145/3530910. URL <https://dl.acm.org/doi/10.1145/3530910>.
- Bingrui Li, Ziteng Wang, Jianfei Chen, and Jun Zhu. Training neural networks with low-precision model memory. 2022.
- Hao Li, Zheng Xu, Gavin Taylor, Christoph Studer, and Tom Goldstein. Visualizing the loss landscape of neural nets. In *Advances in Neural Information Processing Systems*, volume 2018-Decem, pp. 6389–6399, 2018.
- Zhizhong Li and Derek Hoiem. Learning Without Forgetting. In *European Conference on Computer Vision 2016*, pp. 614–629, 2016. doi: 10.1007/978-3-319-46493-0\_37. URL [http://link.springer.com/10.1007/978-3-319-46493-0\\_{\\_}37](http://link.springer.com/10.1007/978-3-319-46493-0_{_}37).
- Chang Liu, Xishan Zhang, Rui Zhang, Ling Li, Shiyi Zhou, Di Huang, Zhen Li, Zidong Du, Shaoli Liu, and Tianshi Chen. Rethinking the importance of quantization bias, toward full low-bit training. *IEEE Transactions on Image Processing*, 31:7006–7019, 2022.
- Arun Mallya and Svetlana Lazebnik. PackNet: Adding Multiple Tasks to a Single Network by Iterative Pruning. In *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, pp. 7765–7773, 2018. ISBN 9781538664209. doi: 10.1109/CVPR.2018.00810.

- Michael McCloskey and Neal J. Cohen. Catastrophic Interference in Connectionist Networks: The Sequential Learning Problem. *Psychology of Learning and Motivation - Advances in Research and Theory*, 1989. ISSN 00797421. doi: 10.1016/S0079-7421(08)60536-8.
- Martial Mermillod, Aurélie Bugaiska, and Patrick Bonin. The stability-plasticity dilemma: investigating the continuum from catastrophic forgetting to age-limited learning effects. *Frontiers in Psychology*, 4, 2013. ISSN 1664-1078. doi: 10.3389/fpsyg.2013.00504. URL <http://journal.frontiersin.org/article/10.3389/fpsyg.2013.00504/abstract>.
- Angshuman Parashar, Priyanka Raina, Yakun Sophia Shao, Yu-Hsin Chen, Victor A Ying, Anurag Mukkara, Rangharajan Venkatesan, Brucek Khailany, Stephen W Keckler, and Joel Emer. Timeloop: A systematic approach to dnn accelerator evaluation. In *2019 IEEE international symposium on performance analysis of systems and software (ISPASS)*, pp. 304–315. IEEE, 2019.
- German I. Parisi, Ronald Kemker, Jose L. Part, Christopher Kanan, and Stefan Wermter. Continual Lifelong Learning with Neural Networks: A Review. feb 2018. doi: 10.1016/j.neunet.2019.01.012. URL <http://arxiv.org/abs/1802.07569><http://dx.doi.org/10.1016/j.neunet.2019.01.012>.
- Marcin Pietroń, Dominik Żurek, Kamil Faber, and Roberto Corizzo. Ada-QPacknet – adaptive pruning with bit width reduction as an efficient continual learning method without forgetting. 2023. URL <http://arxiv.org/abs/2308.07939>.
- Leonardo Ravaglia, Manuele Rusci, Davide Nadalini, Alessandro Capotondi, Francesco Conti, and Luca Benini. A TinyML Platform for On-Device Continual Learning With Quantized Latent Replays. *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, 11(4):789–802, dec 2021. ISSN 2156-3357. doi: 10.1109/JETCAS.2021.3121554. URL <https://ieeexplore.ieee.org/document/9580920/>.
- Sylvestre Alvisé Rebuffi, Alexander Kolesnikov, Georg Sperl, and Christoph H. Lampert. iCaRL: Incremental classifier and representation learning. In *Proceedings - 30th IEEE Conference on Computer Vision and Pattern Recognition, CVPR 2017*, 2017. ISBN 9781538604571. doi: 10.1109/CVPR.2017.587.
- Attila Reiss and Didier Stricker. Introducing a New Benchmarked Dataset for Activity Monitoring. In *2012 16th International Symposium on Wearable Computers*, pp. 108–109. IEEE, jun 2012. ISBN 978-0-7695-4697-1. doi: 10.1109/ISWC.2012.13. URL <http://ieeexplore.ieee.org/document/6246152/>.
- Jorge-L. Reyes-Ortiz, Luca Oneto, Albert Samà, Xavier Parra, and Davide Anguita. Transition-Aware Human Activity Recognition Using Smartphones. *Neurocomputing*, 171:754–767, jan 2016. ISSN 09252312. doi: 10.1016/j.neucom.2015.07.085. URL <https://linkinghub.elsevier.com/retrieve/pii/S0925231215010930>.
- A. Robins. Catastrophic forgetting in neural networks: the role of rehearsal mechanisms. In *Proceedings 1993 The First New Zealand International Two-Stream Conference on Artificial Neural Networks and Expert Systems*, pp. 65–68. IEEE Comput. Soc. Press, 1993. ISBN 0-8186-4260-2. doi: 10.1109/ANNES.1993.323080. URL <http://ieeexplore.ieee.org/document/323080/>.
- Anthony Robins. Catastrophic Forgetting, Rehearsal and Pseudorehearsal. *Connection Science*, 1995. ISSN 13600494. doi: 10.1080/09540099550039318.
- Andrei A. Rusu, Neil C. Rabinowitz, Guillaume Desjardins, Hubert Soyer, James Kirkpatrick, Koray Kavukcuoglu, Razvan Pascanu, and Raia Hadsell. Progressive Neural Networks. jun 2016. URL <http://arxiv.org/abs/1606.04671>.
- Martin Schiemer, Lei Fang, Simon Dobson, and Juan Ye. Online continual learning for human activity recognition. *Pervasive and Mobile Computing*, pp. 101817, 2023. ISSN 15741192. doi: 10.1016/j.pmcj.2023.101817.
- Yakun Sophia Shao, Jason Clemons, Rangharajan Venkatesan, Brian Zimmer, Matthew Fojtik, Nan Jiang, Ben Keller, Alicia Klinefelter, Nathaniel Pinckney, Priyanka Raina, et al. Simba: Scaling deep-learning inference with multi-chip-module-based architecture. In *Proceedings of the 52nd Annual IEEE/ACM International Symposium on Microarchitecture*, pp. 14–27, 2019.
- Yujun Shi, Li Yuan, Yunpeng Chen, and Jiashi Feng. Continual learning via bit-level information preserving. In *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, pp. 16669–16678, 2021. ISBN 9781665445092. doi: 10.1109/CVPR46437.2021.01640.

- Shikhar Srivastava, Mohammad Yaqub, Karthik Nandakumar, Zongyuan Ge, and Dwarikanath Mahapatra. Continual Domain Incremental Learning for Chest X-Ray Classification in Low-Resource Clinical Settings. In *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, volume 12968 LNCS, pp. 226–238, 2021. ISBN 9783030877217. doi: 10.1007/978-3-030-87722-4\_21.
- Xiao Sun, Naigang Wang, Chia-Yu Chen, Jiamin Ni, Ankur Agrawal, Xiaodong Cui, Swagath Venkataramani, Kaoutar El Maghraoui, Vijayalakshmi Viji Srinivasan, and Kailash Gopalakrishnan. Ultra-low precision 4-bit training of deep neural networks. *Advances in Neural Information Processing Systems*, 33:1796–1807, 2020.
- James Joseph Sylvester. Lx. thoughts on inverse orthogonal matrices, simultaneous signsuccessions, and tessellated pavements in two or more colours, with applications to newton’s rule, ornamental tile-work, and the theory of numbers. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 34(232):461–475, 1867.
- Albert Tseng, Jerry Chee, Qingyao Sun, Volodymyr Kuleshov, and Christopher De Sa. Quip#: Even better llm quantization with hadamard incoherence and lattice codebooks. *arXiv preprint arXiv:2402.04396*, 2024.
- Mart van Baalen, Andrey Kuzmin, Suparna S Nair, Yuwei Ren, Eric Mahurin, Chirag Patel, Sundar Subramanian, Sanghyuk Lee, Markus Nagel, Joseph Soriaga, et al. Fp8 versus int8 for efficient deep learning inference. *arXiv preprint arXiv:2303.17951*, 2023.
- Gido M. van de Ven, Hava T. Siegelmann, and Andreas S. Tolia. Brain-inspired replay for continual learning with artificial neural networks. *Nature Communications*, 11(1):4069, aug 2020. ISSN 2041-1723. doi: 10.1038/s41467-020-17866-2. URL <https://www.nature.com/articles/s41467-020-17866-2>.
- Jindong Wang, Yiqiang Chen, Lisha Hu, Xiaohui Peng, and Philip S. Yu. Stratified Transfer Learning for Cross-domain Activity Recognition. In *2018 IEEE International Conference on Pervasive Computing and Communications (PerCom)*, pp. 1–10. IEEE, mar 2018a. ISBN 978-1-5386-3224-6. doi: 10.1109/PERCOM.2018.8444572. URL <https://ieeexplore.ieee.org/document/8444572/>.
- Kai Wang, Joost van de Weijer, and Luis Herranz. ACAE-REMIND for online continual learning with compressed feature replay. *Pattern Recognition Letters*, 150:122–129, 2021. ISSN 01678655. doi: 10.1016/j.patrec.2021.06.025.
- Naigang Wang, Jungwook Choi, Daniel Brand, Chia-Yu Chen, and Kailash Gopalakrishnan. Training deep neural networks with 8-bit floating point numbers. *Advances in neural information processing systems*, 31, 2018b.
- Zifeng Wang, Zheng Zhan, Yifan Gong, Geng Yuan, Wei Niu, Tong Jian, Bin Ren, Stratis Ioannidis, Yanzhi Wang, and Jennifer Dy. SparCL: Sparse Continual Learning on the Edge. 2022. ISSN 10495258. URL <http://arxiv.org/abs/2209.09476>.
- Max Welling. Herding dynamical weights to learn. In *Proceedings of the 26th Annual International Conference on Machine Learning - ICML ’09*, pp. 1–8, New York, New York, USA, 2009. ACM Press. ISBN 9781605585161. doi: 10.1145/1553374.1553517. URL <http://portal.acm.org/citation.cfm?doid=1553374.1553517>.
- Yannan N. Wu, Joel S. Emer, and Vivienne Sze. Accelerly: An Architecture-Level Energy Estimation Methodology for Accelerator Designs. In *IEEE/ACM International Conference On Computer Aided Design (ICCAD)*, 2019.
- Yue Wu, Yinpeng Chen, Lijuan Wang, Yuancheng Ye, Zicheng Liu, Yandong Guo, and Yun Fu. Large Scale Incremental Learning. In *2019 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*, pp. 374–382. IEEE, jun 2019. ISBN 978-1-7281-3293-8. doi: 10.1109/CVPR.2019.00046. URL <https://ieeexplore.ieee.org/document/8954008/>.
- Haocheng Xi, Changhao Li, Jianfei Chen, and Jun Zhu. Training transformers with 4-bit integers. *arXiv preprint arXiv:2306.11987*, 2023.
- Zeke Xie, Qian-Yuan Tang, Zheng He, Mingming Sun, and Ping Li. Rethinking the structure of stochastic gradients: Empirical and statistical evidence. *arXiv preprint arXiv:2212.02083*, 2022.
- Kaixin Xu, Jie Lin, Zhe Wang, Peng Hu, and Ziyuan Zhao. Improved fully quantized training via rectifying batch normalization. 2022.
- Shipeng Yan, Jiangwei Xie, and Xuming He. DER: Dynamically Expandable Representation for Class Incremental Learning. In *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, pp. 3013–3022, 2021. ISBN 9781665445092. doi: 10.1109/CVPR46437.2021.00303.

- Guo Yang, Daniel Lo, Robert Mullins, and Yiren Zhao. Dynamic stashing quantization for efficient transformer training. *arXiv preprint arXiv:2303.05295*, 2023.
- Juan Ye, Simon Dobson, and Franco Zambonelli. Lifelong Learning in Sensor-Based Human Activity Recognition. *IEEE Pervasive Computing*, 18(3):49–58, jul 2019. ISSN 1536-1268. doi: 10.1109/MPRV.2019.2913933. URL <https://ieeexplore.ieee.org/document/8903481/>.
- Friedemann Zenke, Ben Poole, and Surya Ganguli. Continual learning through synaptic intelligence. In *International conference on machine learning*, pp. 3987–3995. PMLR, 2017.
- Xiao Zhang, Hongzheng Yu, Yang Yang, Jingjing Gu, Yujun Li, Fuzhen Zhuang, Dongxiao Yu, and Zhaochun Ren. HarMI: Human Activity Recognition Via Multi-Modality Incremental Learning. *IEEE Journal of Biomedical and Health Informatics*, 26(3):939–951, mar 2022. ISSN 2168-2194. doi: 10.1109/JBHI.2021.3085602. URL <https://ieeexplore.ieee.org/document/9444883/>.
- Bowen Zhao, Xi Xiao, Guojun Gan, Bin Zhang, and Shutao Xia. Maintaining Discrimination and Fairness in Class Incremental Learning. nov 2019. URL <http://arxiv.org/abs/1911.07053>.
- Haiyan Zhao, Tianyi Zhou, Guodong Long, Jing Jiang, and Chengqi Zhang. Does continual learning equally forget all parameters? *arXiv preprint arXiv:2304.04158*, 2023.
- Da-Wei Zhou, Qi-Wei Wang, Han-Jia Ye, and De-Chuan Zhan. A Model or 603 Exemplars: Towards Memory-Efficient Class-Incremental Learning. 2022. URL <http://arxiv.org/abs/2205.13218>.
- Da-Wei Zhou, Qi-Wei Wang, Zhi-Hong Qi, Han-Jia Ye, De-Chuan Zhan, and Ziwei Liu. Deep Class-Incremental Learning: A Survey. 2023. URL <http://arxiv.org/abs/2302.03648>.

## A APPENDIX

### A.1 DETAILED FQT BACKGROUND

The authors in [Chen et al. \(2020\)](#) propose the idea that updates using FQT should align with updates from quantization-aware training (QAT), where weights and activations are quantized but the backward passes use floating point. They formulate FQT as a statistical estimator of QAT, capturing the performance loss due to the variance of the estimator. Building on this, they devise a variance minimization scheme using a 5-bit block Householder quantizer, resulting in a 0.5% accuracy loss on ResNet50.

[Huang et al. \(2022\)](#) highlight that especially quantization-aware training methods are costly because they require two copies of the weights and incur a lot of data movement. They propose resource constraint training which only keeps quantized weights which heavily reduces the burden for training and enables training on the edge. Their method adjusts the per-layer bit-width dynamically to train efficiently with just the right amount of bits necessary to facilitate learning. They show energy saving for general matrix multiplications of 86% and data movement while incurring less than a 1% accuracy penalty on various models such as BERT (SQuAD and GLUE) or ResNet18.

Most FQT techniques use dynamic quantization where ranges are computed on the fly given current data. [Fournarakis & Nagel \(2021\)](#) argue that this is very expensive and suggest that it is better to statically quantize data given range parameters estimated from previous training steps. They demonstrate minimal accuracy loss of MobileNets and ResNet18 while estimating a 50-400% memory cost difference between static quantization methods and their hindsight quantization.

Given the challenging nature of low-precision optimizers, e.g. keeping model parameters and gradient accumulators at low precision, [Li et al. \(2022\)](#) do a thorough analysis and construct an optimizer called low-precision model memory (LPMM). They claim that the biggest challenge is underflow and use stochastic rounding and microbatching to alleviate it. They use their optimizer to train up a ResNet18 and ResNet50 to SOTA with 70% reduced model memory.

Alternatively, [Xu et al. \(2022\)](#) analysis FQT for models with batch normalization. They find that batch normalization can amplify gradient noise. They add a rectification loss which reduces noise the bachtnorm noise amplification and show competitive results on 8, 4, and 2-bits.

In contrast, [Liu et al. \(2022\)](#) argue that the issue with quantization gradient noise is not variance but bias. Additionally, they claim that commonly used stochastic rounding can not prevent the “training crash” sometimes observed with fully quantized training. To enable FQT they propose a new adaptive quantization method called piecewise fixed point (PWF), which divides gradients into gradients close to zero and ones that with bigger magnitudes and both are quantized with different step sizes. They demonstrate that their method can enable 8-bit quantization for weights, activations, gradients and errors on ImageNet (image classification), COCO (vision detection), BLEU (machine translation), optical character recognition, and test classification while achieving  $1.9 - 3.5\times$  speed up over full precision training with accuracy loss of less than 0.5%.

Another example for LLM training quantization is [Yang et al. \(2023\)](#) introduce dynamic stashing quantization, which quantizes operations and intermediate results to reduce DRAM traffic at the beginning with low precision and toward the end of the training the bit-width increases. They are able to reduce the number of arithmetic operations by  $20.95\times$  and the number of DRAM operations by  $2.55\times$  on IWSLT17.

### A.2 CONTINUAL LEARNING TECHNIQUE DETAILS

The techniques used in our investigation are representative of a variety of research directions and are often used as reliable baselines for continual learning. **Learning without Forgetting** [Li & Hoiem \(2016\)](#)’s objective is to maintain the similarity between the output of the old tasks in the new network to the output from the original model. This is achieved by using the KD loss as a regularization term. KD is a way to transfer knowledge from a complex *teacher* model to a simpler *student* model by minimizing the loss on the output class probabilities from the teacher model ([Hinton et al., 2015](#)). KD has found widespread application in various continual learning techniques to distill knowledge learned from previous tasks to the model for new tasks. It is defined as follows:

$$\mathcal{L}_{KD}(y_o, \hat{y}_o) = - \sum_{i=1}^l y_o^{(i)} \log \hat{y}_o^{(i)}, \quad (1)$$



where  $l$  is the count of class labels, and  $y_o^{(i)}$  and  $\hat{y}_o^{(i)}$  are temperature-scaled *recorded* and *predicted* probabilities for the current sample for an old class label  $i$ . Temperature is used to address over-confident probabilities that the teacher model produces on their prediction. That is:

$$y_o^{(i)} = \frac{(y_o^{(i)})^{1/T}}{\sum_j (y_o^{(j)})^{1/T}} \text{ and } \hat{y}_o^{(i)} = \frac{(\hat{y}_o^{(i)})^{1/T}}{\sum_j (\hat{y}_o^{(j)})^{1/T}}.$$

The loss  $\mathcal{L}_{KD}$  is combined with the cross-entropy loss on a new task’s samples  $\mathcal{L}_{CE}^n$ ; that is,

$$\mathcal{L}(y_n, \hat{y}_n, y_o, \hat{y}_o) = \lambda_o \mathcal{L}_{KD}(y_o, \hat{y}_o) + \mathcal{L}_{CE}(y_n, \hat{y}_n), \quad (2)$$

$$\mathcal{L}_{CE}^n(y_n, \hat{y}_n) = -y_n \log \hat{y}_n, \quad (3)$$

where  $y_n$  represents the one-hot encoded vector of the ground-truth label,  $\hat{y}_n$  is the predicted logits (i.e., the softmax output of the network), and  $\lambda_o$  is a loss balance weight. A higher  $\lambda_o$  will prioritize the old task performance over the new task. During training, a new batch gets passed through the old network to record its outputs, which then are used in  $\mathcal{L}_{KD}$  to promote that the network updates do not deviate from the old network.

**Incremental Classifier and Representation Learning (iCaRL)** (Rebuffi et al., 2017) is an early attempt at rehearsal approaches for class-incremental learning. It leverages memory replay and KD regularization. After each task’s training, it utilizes the herding sampling technique (Welling, 2009) to select a small set of exemplars (i.e., representative samples) from the current task’s training data and stores them in memory. Herding works by choosing samples that are closest to the centroid of each old class. When the next task becomes available, the in-memory exemplars will be combined with the new task’s training data to update the network. The loss function of iCaRL is the same as Equation 1 in LwF, which is a combination of cross-entropy (CE) loss on new classes and the KD loss on old classes to allow knowledge transfer. Additionally, Rebuffi et al. (2017) added a nearest-mean-of-exemplars (NME) classifier which performs a forward pass through the model for the holdout data and classifies new samples by finding the nearest class mean.

**Bias Correction (BiC)** (Wu et al., 2019) introduces a BiC correction layer after the last FC layer to adjust the weights in order to tackle the imbalance problem. There are two stages of training. In the first stage, the network will be trained with new task’s data and in-memory samples of old tasks using the CE and KD loss similar to iCaRL. In the second stage, the layers of the network are frozen and a linear BiC layer is added to the end of the network and trained with a small validation set consisting of samples from both old and new tasks. The linear model of the BiC layer corrects the bias on the output logits for the new classes:

$$\tilde{y}_k = \begin{cases} \hat{y}_k & 1 \leq i \leq n \\ \alpha \hat{y}_k + \beta, & n + 1 \leq i \leq n + m \end{cases}$$

where  $\hat{y}_k$  is the output logits on the  $k$ th class, the old classes are  $[1, \dots, n]$  and the new classes are  $[n + 1, \dots, n + m]$ .  $\alpha$  and  $\beta$  are the bias parameters of the linear model, which are optimized in the following loss function:

$$L_{BiC} = - \sum_{k=1}^{n+m} \delta_{y=k} \log \text{softmax}(\tilde{y}_k),$$

where  $\delta_{y=k}$  is the indicator function to check if the ground-truth label  $y$  is the same as a class label  $k$ . The intuition is that a balanced small validation set for all seen classes can counter the accumulated bias during the training of the new task. As the non-linearities in the data are already captured by the model’s parameters, the linear function proves to be quite effective in estimating the bias in the output logits of the model arising.

**Dynamically Expandable Representation (DER)** (Yan et al., 2021) proposes to address CF through the creation of a new backbone for a new task is encountered. The features of all backbones are then aggregated through a fully connected layer. During model update phases old backbones are frozen. Naturally, this approach results in relatively large resource costs.

**Memory-efficient Expandable Model (MEMO)** (Zhou et al., 2022) tries to rectify memory problems of expandable networks. They identify and leverage differences between shallow and deep NN layers. Deep layers are class specific, while shallow layers are more generalizable. MEMO opts to decouple backbones into a generalized block for all tasks and only expands specialized deep layers. Hence, MEMO’s memory requirements are generally lower than other expandable approaches like DER.

### A.3 HAR DATASETS

*Physical Activity Monitoring* (PAMAP2) (Reiss & Stricker, 2012) contains 18 physical activities, including a wide range of everyday, household and sport activities, from 9 users. The sensor data is collected on accelerometers worn

on the chest, dominant arm and side ankle, totaling 10 hours of data with a sampling frequency of 100Hz. We use features generated by Wang et al. (2018a) which contain 12 classes and 6 users. Their feature generation protocol was that 27 features are extracted per sensor on each body part which results in 81 per body part, including mean, standard deviation, and spectrum peak position. *Daily and Sports Activities Dataset* (DSADS) (Altun et al., 2010) contains 19 activity classes such as running, rowing, and sitting. Each user performs each of these activities for 5 minutes at a 25Hz sampling frequency. The data samples are collected on 8 users with 5 accelerometer units on each user’s torso and extremities. The subjects were not instructed on how the activities should be performed. Here we use the same set of feature generation protocol as PAMAP2 (Wang et al., 2018a). *Human Activity Recognition Dataset* (HAPT) (Reyes-Ortiz et al., 2016) contains 12 activities such as standing or walking and postural transitions such as stand-to-sit. It is collected from 30 subjects wearing a smartphone (Samsung Galaxy S II) on their waist with a 50Hz sampling frequency. The data is split with a fixed-width sliding window of 2.56 seconds and 50% overlapping. After filters and transformations, features like mean, absolute difference, skewness and more are generated. In the end, Reyes-Ortiz et al. (2016) extracted 561 features from accelerometer and gyroscope sensor readings. To ensure uniform evaluation across all users and classes, we apply a filter to each dataset. This involves removing users who do not have data for all classes and eliminating classes that are significantly underrepresented. As a result 1 class and 3 users respective 2 users are removed from PAMAP2 and HAPT.

#### A.4 WITHOUT HADAMARD TRANSFORMATION

If we do not apply the Hadamard transformation in the backward pass, our performance drops to an averaged accuracy of 48.11 with a standard deviation of 21.4 over 5 runs for iCaRL on HAPT. One can see that without the Hadamard transform the performance is significantly compromised in terms of accuracy and stability.

#### A.5 HYPERPARAMETERS

For the CIFAR datasets we used the hyperparameters reported in Zhou et al. (2023), and did not employ hyperparameter search techniques for any reported results. We use 0.05 as learning rate for CIFAR100 and 0.01 for the HAR datasets. We apply a decay of 0.1 at the 60th, 100th and 140th epoch for CIFAR100 and at the 50th for HAR with a weight decay of 0.0002 for both. The batch size is 128. For techniques that employ memory replay, CIFAR100 memory size is set to 2000 and HAR’s too 200 as the datasets are smaller. The number of samples per class are reduced at every step to make space for new classes. CIFAR100 models are trained for 170 epochs per task and 100 for HAR. Both use SGD with a momentum of 0.9. The temperature value for KD is set to two as hinted by Hinton et al. (2015) and the lambda multiplier to three. BiC’s split ratio is 0.1. CIFAR100 models are introduced to 20 classes per task and HAR to 2.

#### A.6 TILE SIZE VARIATIONS

To determine a good tile-size, we performed a hyperparameter search with iCaRL on HAPT. Table 3 shows that there is a stable performance until tile-size 32. Larger ones start to compromise accuracy performance and increase variance.

Table 3: Results for HDQT using different tile sizes. Numbers are final accuracy percentages for iCaRL averaged over 5 runs on the HAPT dataset.

	Tile-size							
	1	2	4	8	16	32	64	128
iCaRL	82.04	83.18	82.26	81.64	81.64	82.39	80.17	41.16
	(±2.62)	(±2.88)	(±1.71)	(±1.07)	(±3.41)	(±3.68)	(±4.24)	(±37.19)

#### A.7 LUQ CODE MODIFICATIONS

We modified the LUQ code (found in the supplementary materials) to account for: i) difference between `max()` and `max(abs())` which resulted in incorrect scaling from intended dynamic range (see `Luq supplementary/LUQ_code/vision/models/modules/LUQ.py` line 103), ii) LUQ having 17 distinct levels for 4-bit due to overcounting (see `Luq supplementary/LUQ_code/vision/models/modules/LUQ.py` lines 24, 25, 27, and 48), and iii) the omission of resetting the positive values upper bound when of negative activations are detected, resulting in 25 distinct representation levels for four-bit quantization (see

Luq supplementary/LUQ\_code/vision/models/modules/LUQ.py line 48). For ii we attempt to retain their symmetry at the cost of one quantization level. Results obtained with the modified version of their code are marked with a \* in Table 2.

### A.8 NME

For our experiments, we have removed the NME classifiers from iCaRL because we are interested in the network learning interaction between quantization and CIL solely. Table 4 shows a comparison for iCaRL with NME with no additional cross influence with quantization.

Table 4: Result for standard learning (NoCL) and CIL with 20 classes (CIFAR100) or 2 classes (DSADS, PAMPA2, or HAPT) per task quantized (4-bit inputs and 8-bit accumulators) and unquantized (FP). Numbers are final accuracy percentages averaged over 20 runs of the HAR datasets and 5 runs for CIFAR100. Small numbers in brackets indicate standard deviation. Note that NME gives iCaRL a straight forward performance boost that is not influenced by quantization.

	CIFAR100		DSADS		PAMAP2		HAPT		$\mathbb{E}[\Delta \text{Acc.}]$
	FP	4-bit	FP	4-bit	FP	4-bit	FP	4-bit	
NoCL	68.61 ( $\pm 0.24$ )	59.43 ( $\pm 1.88$ )	84.44 ( $\pm 4.36$ )	82.71 ( $\pm 4.45$ )	85.81 ( $\pm 4.91$ )	84.35 ( $\pm 7.66$ )	93.15 ( $\pm 2.11$ )	92.08 ( $\pm 2.10$ )	3.36
iCaRL NME	52.84 ( $\pm 0.74$ )	47.15 ( $\pm 1.75$ )	76.38 ( $\pm 4.06$ )	77.19 ( $\pm 4.22$ )	80.03 ( $\pm 5.57$ )	79.96 ( $\pm 4.53$ )	85.80 ( $\pm 2.93$ )	84.94 ( $\pm 3.24$ )	1.45
iCaRL	43.20 ( $\pm 1.21$ )	37.25 ( $\pm 2.42$ )	72.83 ( $\pm 4.84$ )	72.74 ( $\pm 4.59$ )	77.52 ( $\pm 5.50$ )	77.87 ( $\pm 5.02$ )	82.98 ( $\pm 5.12$ )	82.56 ( $\pm 4.51$ )	1.53

### A.9 GRADIENT BIAS AND STANDARD DEVIATION W/WO STOCHASTIC ROUNDING

Figure 9 shows that stochastic rounding reduces the gradient bias as well as the standard deviation of HDQT training and brings it closer to when no quantization is applied.

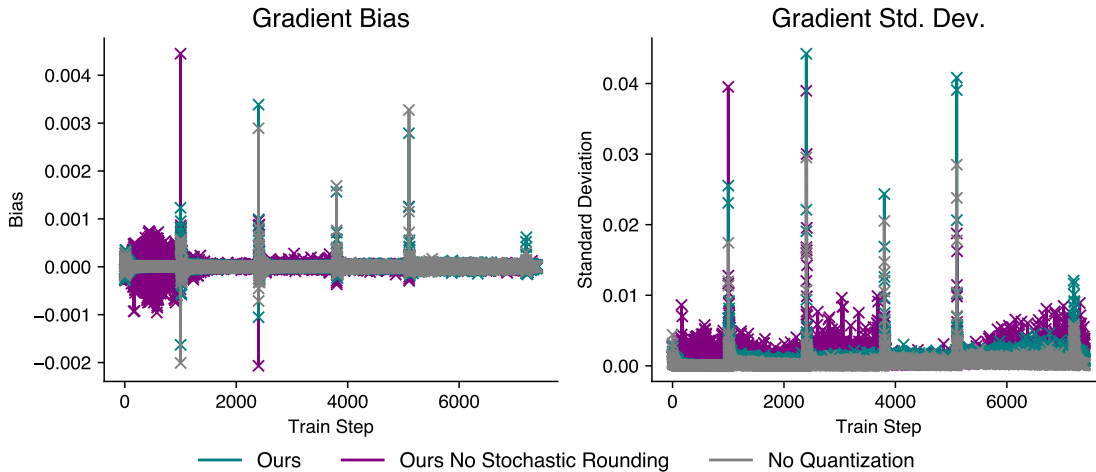


Figure 9: Showing the development of the gradient bias and standard deviation over CL training. Note how especially at the beginning the bias when not using stochastic rounding is different from an unquantized baseline which is remedied through stochastic rounding. Additionally, stochastic rounding keeps the standard deviation smaller.

## A.10 COMPREHENSIVE ENERGY ESTIMATION

Table 5 displays the energy estimations using HDQT with different accumulator bit widths compared to a FP baseline for different HAR datasets. We see substantial energy savings between  $3.2\times$  and  $13.5\times$ .

Table 5: Accelerator cost-model derived estimates on the SIMBA platform for the energy-cost incurred for implementing two representative layers of the FCN for DSADS, PAMAP2, and HAPT. These include all operations required, such as max or FHT. **H** stands for hidden layer and **O** for output layer.

Layer	Inpt. BW	Accm. BW	Inpt. Dim.	Wgt. Dim.	FWD Energy $\mu$ J	BWD Energy $\mu$ J	Total Energy $\mu$ J	Improvement (vs FP16)
DSADS <b>H</b>	FP	FP	$128 \times 405$	$405 \times 405$	905.3	1840.0	2745.3	1.0
	4	8	$128 \times 405$	$405 \times 405$	46.7	803.6	850.3	3.2
DSADS <b>O</b>	FP	FP	$128 \times 405$	$405 \times 19$	49.5	99.6	149.1	1.0
	4	8	$128 \times 405$	$405 \times 19$	5.6	23.2	28.8	5.1
PAMAP2 <b>H</b>	FP	FP	$128 \times 243$	$243 \times 243$	585.4	904.0	1489.4	1.0
	4	8	$128 \times 243$	$243 \times 243$	17.0	93.4	110.4	13.5
PAMAP2 <b>O</b>	FP	FP	$128 \times 243$	$243 \times 11$	20.1	37.7	57.8	1.0
	4	8	$128 \times 243$	$243 \times 11$	2.9	12.9	15.8	3.7
HAPT <b>H</b>	FP16	FP16	$128 \times 561$	$561 \times 561$	1630.6	3319.8	4950.4	1.0
	4	16	$128 \times 561$	$561 \times 561$	99.6	1412.5	1512.1	3.3
	4	12	$128 \times 561$	$561 \times 561$	93.6	1358.8	1452.4	3.4
	4	8	$128 \times 561$	$561 \times 561$	87.0	991.3	1078.3	4.5
HAPT <b>O</b>	FP16	FP16	$128 \times 561$	$11 \times 561$	45.0	88.9	133.8	1.0
	4	16	$128 \times 561$	$11 \times 561$	6.8	33.4	40.2	3.3
	4	12	$128 \times 561$	$11 \times 561$	6.6	30.6	37.2	3.6
	4	8	$128 \times 561$	$11 \times 561$	6.4	28.7	35.1	3.8

## A.11 NO HADAMARD REQUANTIZATION

If the Hadamard re-quantization is left out we achieve the average final accuracy of 81.18 with a standard deviation of 3.67 over 5 runs for iCaRL on HAPT. Note that compared to the re-quantized performance in table 2 this difference is not significant.

## A.12 NO HADAMARD ENERGY

Table 6 displays energy numbers if no Hadamard transform is used.

Table 6: Accelerator cost-model derived estimates for the energy incurred to compute two representative layers of the FCN for HAPT. We include all operations, *e.g.*, max, FHT, etc. Results indicate an improvement of up to  $4.0\times$  over the FP16 baseline architecture. \* indicates results for integer quantization without Hadamard transformation. **H** stands for hidden layer and **O** for output layer.

Layer	Inpt. BW	Accm. BW	Inpt. Dim.	Wgt. Dim.	FWD Energy $\mu$ J	BWD Energy $\mu$ J	Total Energy $\mu$ J	Improvement (vs FP16)
HAPT <b>H</b>	FP16	FP16	$128 \times 561$	$561 \times 561$	1630.6	3319.8	4950.4	1.0
	4*	16*	$128 \times 561$	$561 \times 561$	86.2	496.1	582.3	8.5
	4*	12*	$128 \times 561$	$561 \times 561$	80.3	482.7	563.0	8.8
	4*	8*	$128 \times 561$	$561 \times 561$	73.7	149.1	222.8	22.2
HAPT <b>O</b>	FP16	FP16	$128 \times 561$	$11 \times 561$	45.0	88.9	133.8	1.0
	4*	16*	$128 \times 561$	$11 \times 561$	4.0	8.1	12.1	11.1
	4*	12*	$128 \times 561$	$11 \times 561$	3.9	7.8	11.7	11.4
	4*	8*	$128 \times 561$	$11 \times 561$	3.7	7.6	11.3	11.8

A.13 ADDITIONAL FIGURES: EFFECTS OF FORGETTING FROM QUANTIZATION AND CIL

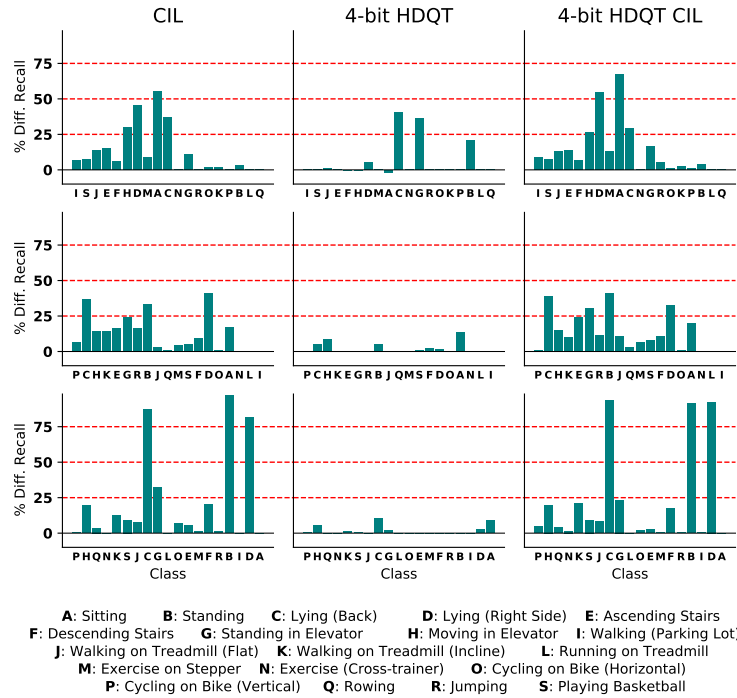


Figure 10: Class accuracy difference comparison of an unquantized CIL model with models trained with either only HDQT or with CIL and HDQT. Training was performed on all 19 classes of the DSADS dataset (more examples in A.13). Classes are sorted in incoming order. Note that quantization does not add or amplify CF *e.g.*, class accuracy for H (Moving in Elevator) increases while A (Sitting) goes down when combined. Additionally, an overview of per-class forgetting due to CIL and quantization for three different seeds on DSADS.

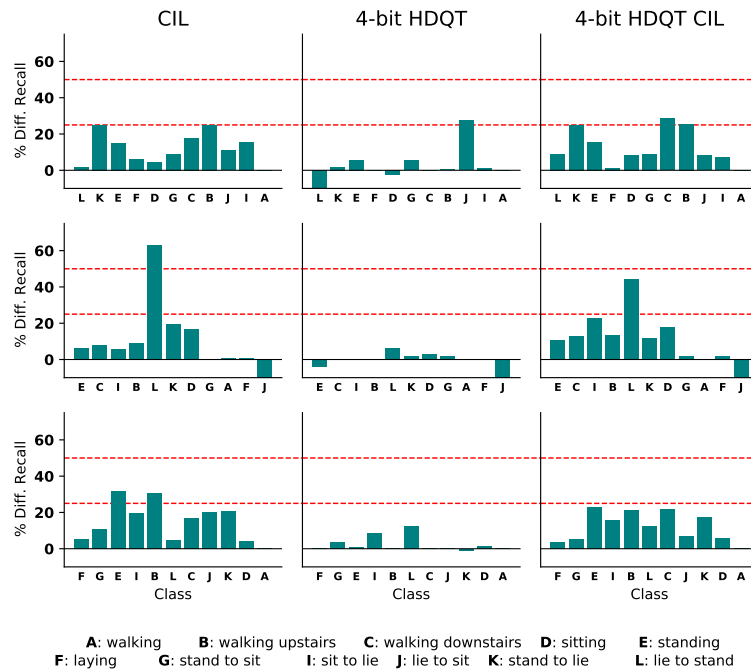


Figure 11: Additional overview of per-class forgetting due to CIL and quantization for three different seeds on HAPT.

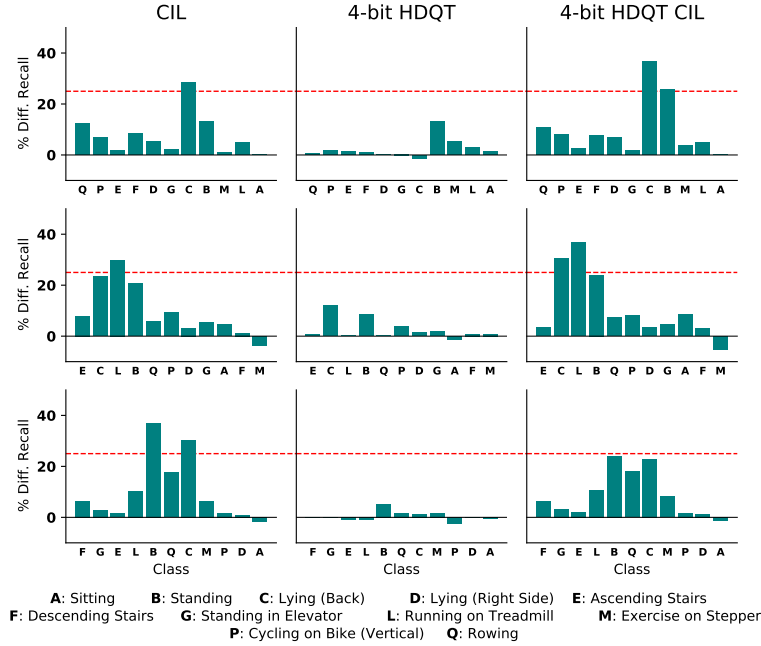


Figure 12: Additional overview of per class forgetting due to CIL and quantization for three different seeds on PAMAP2.

Table 7: Quantization operation comparison between HDQT and LUQ. \*Denotes that more efficient implementations could reach  $N^2 \log N$  complexity.

Step	Target	Operation	Precision	Operation Count	HDQT	LUQ
FWD	Weights	Max	bf16	$N$	1×	-
		Input squared	accm bw	$N$	-	1×
		Mean	accm bw	$N$	-	2×
	Activations	Max	bf16	$N$	1×	-
		Input squared	accm bw	$N$	-	1×
		Mean	accm bw	$N$	-	2×
BWD	Weights	Hadamard*	in/w bw	$N^3$	1×	-
		Max	in/w bw/ bf16	$N$	1×	1×
		Where	accm bw	$N$	-	2×
		Subtract	bf16	$N$	-	1×
	Activations	Hadamard*	in/w bw	$N^3$	1×	-
		Max	in/w bw/ bf16	$N$	1×	1×
		Where	accm bw	$N$	-	2×
		Subtract	bf16	$N$	-	1×
	G1	Hadamard*	in/w bw	$N^3$	1×	-
		Max	in/w bw/ bf16	$N$	1×	1×
		Where	accm bw	$N$	-	2×
		Subtract	bf16	$N$	-	1×
G2	Hadamard*	in/w bw	$N^3$	1×	-	
	Max	in/w bw/ bf16	$N$	1×	1×	
	Where	accm bw	$N$	-	2×	
	Subtract	bf16	$N$	-	1×	

## A.14 OPERATION COMPARISON

Table 7 compares the operations used in HDQT and LUQ based on Table 8 which shows the energy breakdown per individual operation.

Table 8: Energy consumption breakdown for a single layer (4/8 bitwidth). \*Note: Access counts for 128x405x405 layer at 4/8 bitwidth. Changing bitwidth affects memory usage. † number from Table 7

Component	Input Accesses	Weight Accesses	Output Accesses	Computes	Energy (pj/bit)
DRAM	$128 \times 405 \times 3$	$405 \times 405$	$128 \times 405$	-	8.0000
Global Buffer	$128 \times 405 \times 3$	-	$128 \times 405 \times 26$	-	0.2071
PE Input Buffer	$128 \times 405 \times 135$	-	-	-	0.1548
PE Weight Buffer	-	$405 \times 405$	-	-	0.0928
PE Accm Buffer	$128 \times 405 \times 9$	-	-	-	0.0625
Register	$128 \times 405 \times 3$	-	-	-	0.0025
MAC	-	-	-	$128 \times 405 \times 405$	0.0375
Adder	-	-	-	†	0.0038