On a Geometry of Interbrain Networks

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Abstract

Effective analysis in neuroscience benefits significantly from robust conceptual frameworks. Traditional metrics of interbrain synchrony in social neuroscience typically depend on fixed, correlation-based approaches, restricting their explanatory capacity to descriptive observations. Inspired by the successful integration of geometric insights in network science, we propose leveraging discrete geometry to examine the dynamic reconfigurations in neural interactions during social exchanges. Unlike conventional synchrony approaches, our method interprets inter-brain connectivity changes through the evolving geometric structures of neural networks. This geometric framework is realized through a pipeline that identifies critical transitions in network connectivity using entropy metrics derived from curvature distributions. By doing so, we significantly enhance the capacity of hyperscanning methodologies to uncover underlying neural mechanisms in interactive social behavior.

Keywords: Discrete Geometry, Graph Curvature, Inter-brain Networks, Hyperscanning, Social Neuroscience, Network Dynamics

1. Introduction

Interbrain synchrony (IBS) metrics, such as the Phase Locking Value (PLV), have dominated social neuroscience research, providing practical but fundamentally descriptive measures of neural interactions (Hakim et al., 2023). These methods typically neglect dynamic transitions between brain network states that could provide insight into social interaction mechanisms. Recent advancements in geometric machine learning highlight discrete geometric methods as powerful tools for characterizing complex network structures and dynamics (Weber, 2025). The present opinion piece is motivated by the idea that transient connectivity patterns govern flexible cognitive processes (Sporns, 2010). Such processes have been previously analyzed with geometric tools; however, these works explored primarily intra-brain structural and functional networks (Chatterjee et al., 2021; Weber et al., 2019). In this article, we propose the application of geometric methods to time-varying interbrain networks during social interaction. Specifically, our proposed approach leverages discrete graph curvatures to address the unique challenges of dynamic interbrain networks in hyperscanning research (Hinrichs et al., 2025); it aims to overcome the limitations of correlation-based metrics by providing richer, more mechanistic insights into how brain networks dynamically reorganize during social interactions.

2. A Graph Geometry Toolkit

Central to our proposal are discrete curvatures, one example of which is the Forman-Ricci curvature (FRC). Developed initially to characterize geometric properties of discrete spaces parametrized as cell complexes (Forman, 2003), a specialization of FRC to graphs quantifies the expansion and contraction of information across the network by examining the network's connectivity patterns. Specifically, the FRC of an edge e connecting nodes i and j in a weighted network is defined as

$$F(e) = w_e \left(\frac{z_i}{w_e} + \frac{z_j}{w_e} - \sum_{e_i \sim i, e_i \neq e} \frac{z_i}{\sqrt{w_e w_{e_i}}} - \sum_{e_j \sim j, e_j \neq e} \frac{z_j}{\sqrt{w_e w_{e_j}}} \right), \tag{1}$$

where z_i, z_j represent node weights and w_e denotes edge weights corresponding to neural connectivity strength. Positive curvature values typically identify edges in densely connected regions, whereas negative curvature highlights edges that bridge highly connected network modules.

Ollivier-Ricci curvature (ORC) represents an alternative notion of discrete Ricci curvature (Ollivier, 2009), which provides a comparable characterization of network geometry (Samal et al., 2018); we defer a formal definition to Appendix A. Its definition via Markov chains lends itself to another interpretation in the context of inter-brain connectivity: The curvature of an edge provides a proxy for its tendency to *attract* information flow, in the sense that negative curvature indicates more attraction (Wang et al., 2022). Regions with a high density of edges with low (negative) curvature promote shortest-path traversal, while regions with higher (positive) curvature promote diffusion.

In the next section, we investigate how a toolkit based on discrete Ricci curvature can be fruitfully applied to social neuroscience.

3. The Case of Hyperscanning

Hyperscanning, defined as the simultaneous recording of neural signals from interacting individuals (Montague et al., 2002), has reshaped social neuropsychology (Schilbach and Redcay, 2025) and clinical neuroscience alike (Adel et al., 2025). Despite these advances, the analytical methods applied in hyperscanning remain heavily reliant on purely correlational approaches (Hamilton, 2021), inherently restricting their explanatory power. *We contend that the curvature-based analysis of interbrain coupling networks can move hyperscanning studies closer toward mechanistic explanations.*

3.1. Interbrain Networks and their Synchrony

Interbrain networks represent the joint neural connectivity of two or more individuals as interconnected nodes within weighted graphs, constructed via hyperscanning, with each node typically corresponding to a neural region and the edge weights derived by computing IBS metrics (e.g., PLV) from the neural activity in these regions (Hakim et al., 2023). These studies have been limited in the mechanistic inferences they afford researchers; at best, correlations between brain regions of interacting subjects can be interpreted in terms of the computational-cognitive roles ascribed to these regions, with detailed mechanisms and their dynamic evolution – as social behavior unfolds over time – remaining speculative. We propose extending studies of time-varying interbrain networks with graph curvatures to detect meaningful phase transitions in interpersonal neural dynamics and provide insight into the information routing strategies interbrain networks use to accomplish joint behavioral tasks. We explore these applications in detail in the following sections.

Operationally, a curvature-based hyperscanning analysis proceeds in four steps. First, after standard preprocessing and artifact rejection, interbrain connectivity is estimated in a time-resolved fashion, for example by computing IBS measures in sliding windows and in task-relevant frequency bands. Second, each window is represented as a weighted bipartite or multilayer graph whose nodes correspond to regions in each participant and whose edges reflect interbrain coupling strength. Third, discrete curvatures are computed for edges or nodes in each graph, yielding a time series of curvature distributions that summarizes how interbrain geometry evolves. Finally, summary statistics of these distributions (e.g., entropy, moments, or the proportion of strongly negative edges) are aligned with behavioral events and task structure, and evaluated against appropriate null models (such as trial shuffling or surrogate coupling matrices) to determine whether observed transitions exceed those expected from stationary dynamics or measurement noise.

3.2. Capturing Phase Transitions

Suppose the timing of task-related behavioral transitions or events, such as cooperative engagements, misunderstandings, or conflict resolutions, is synchronized with the timing of phase transitions in interbrain networks as identified by graph curvatures. In that case, investigators can more confidently make inferences about the neural mechanisms of behavior (Steyn-Ross and Steyn-Ross, 2010). To capture significant dynamic shifts in network configurations, we examine divergences over time in the differential entropy of graph curvature distributions of IBS, H_{RC}, defined as

$$H_{RC}(G_t) = -\int_{\mathbb{R}} f^t_{RC}(x) \log\left[f^t_{RC}(x)\right]\, dx, \tag{2}$$

where $f^t_{RC}(x)$ describes the probability density of discrete curvature values across the network configuration G_t at time t (Znaidi et al., 2023). In Figure 1, we apply this method to detect phase transitions in a toy model of time-varying brain networks with small-world topology. We show that as the rewiring probability used to generate the networks evolves from zero to unity, the differential entropy of the FRC distribution undergoes a divergence between $p = 10^{-3}$ and $p = 10^{-1}$ as the network transitions between a regular lattice and a random network. Panels E–F show a sharp rise in entropy once $p \gtrsim 10^{-2}$ and a widening curvature distribution (95th-percentile jump), due to increased neighborhood overlap and shortcut formation, marking a transition from a segregated, lattice-like topology to a more integrated small-world/random regime; see Table 1 in Appendix B for modality- and condition-specific expectations that map these geometric signatures to EEG, fNIRS, and fMRI hyperscanning in resting and experimental task conditions.

In empirical hyperscanning data, the rewiring probability p is replaced by latent changes in interpersonal coordination driven by the task. The curvature-entropy trace $H_{RC}(G_t)$ can be inspected for sharp excursions, but in practice these should be quantified by change-point detection, clustering of network states, or model comparison between stationary and non-stationary curvature processes. Critically, the interpretation of a detected "phase transition" depends on its alignment with independently measured behavioral markers (onset of joint action, breakdowns in coordination, feedback delivery) and on comparison with subject- and task-specific null ensembles. Without these controls, large curvature fluctuations could simply reflect transient changes in signal-to-noise ratio, motion confounds, or common-input effects rather than genuine reconfigurations of interbrain communication structure.

3.3. Capturing Information Routing Strategies

Theoretical work on information routing in brain networks has used Markov chains to model a spectrum of information routing strategies between shortest-path traversal to a target node at one extreme, and random diffusion at the other (Avena-Koenigsberger et al., 2019). Thus, when applied to interbrain networks, the ORC distribution of the network can be interpreted as identifying the information routing strategy adopted by its subnetworks.

Recent work in deep learning has shown that FRC can identify information bottlenecks that distort information flow during message-passing in graph neural networks (Topping et al., 2022; Fesser and Weber, 2023). These results suggest that FRC could be a valuable tool for assessing information flow in brain networks, a key component of the mechanistic models proposed by predictive theories of the brain (Friston et al., 2017).

Practically, ORC-based routing analyses can be implemented by tracking, over time, the fraction of edges whose curvature falls below a chosen negative threshold, or by clustering edges and nodes into modules with similar curvature profiles. Intervals dominated by strongly negative curvature subnetworks would then correspond to regimes in which information is funneled along a small number of backbone paths, whereas intervals with predominantly positive curvature would favor more diffuse, exploratory exchange. In a cooperative joint-action task, for instance, one might expect transient emergence of negatively curved "bridges" between premotor and parietal regions across brains at moments of tight sensorimotor alignment, followed by relaxation toward more positively curved, segregated configurations during periods of rest or independent planning.

4. Towards an Interbrain Geometry

Adopting a geometric framework within neuroscience offers methodological and conceptual advancements over traditional IBS-based analyses. *Geometric hyperscanning* could address the inability of correlation-based metrics alone to capture dynamic network reconfigurations and characterize real-time information routing strategies within and between socially interacting brains.

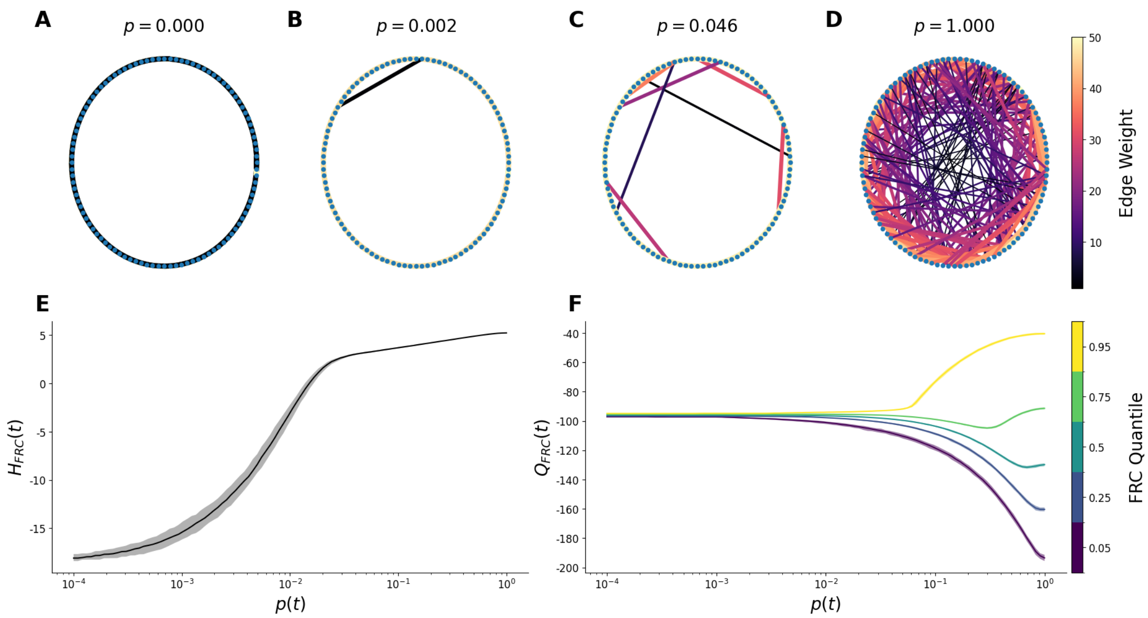

Figure 1: Simulations of time-varying brain networks modeled as weighted small-world networks with varying rewiring probability. **A–D:** Four examples with $N = 100$, mean degree $K = 5$, and different p, generated using (Muldoon et al., 2016). **E:** Entropy of the FRC distribution as p evolves from 0 to 1 for $N = 1000$, $K = 50$ (note phase transition around $p = 10^{-2}$). **F:** Corresponding quantiles of the FRC distribution. Solid curves show the median over 200 replications; shaded areas mark 0.05 and 0.95 quantiles.

Discrete curvature distributions could summarize constraints on network dynamics, with divergences in the entropy of the distribution indicating network reorganization events. While this does not intrinsically resolve the confounding factors that arise in IBS-based approaches, it provides a complementary network-level description of interbrain interactions, enabling further inferences required to construct mechanistic explanations. This direction accords with Kulkarni and Bassett (2024)'s call for minimal, principled models of brain-network complexity and with Sporns (2010)'s emphasis on meso-scale features (hubs, clusters, bridges), reinforcing discrete curvatures as indicators of structural transitions during social interaction.

Several limitations and open questions remain. Curvature estimates are sensitive to how interbrain graphs are constructed, including choices of frequency band, time-window length, edge-weight transformation, and thresholding; systematic benchmarking across these design decisions is still lacking. Different notions of discrete curvature may capture partially non-overlapping aspects of network organization, raising the question of whether a single curvature measure suffices or whether a multi-geometry description is required.

Curvature-based analyses could allow researchers to explore the information routing implications of IBS and how they reorganize dynamically throughout real-time interactions, as captured in hyperscanning data, paving the way for a deeper mechanistic understanding of the social brain.

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Appendix A. Ollivier's Ricci Curvature

We provide a formal definition of Ollivier's Ricci curvature, which was discussed in the main text.

Consider the 1-hop neighborhoods of two adjacent nodes u and v in a network and equip each with uniform measures defined as follows: Let $m_u(i) := \frac{z_i}{\sum_{j \in \mathcal{N}_u} z_j}$, where i is a neighbor of u, z_i its weight, and $\mathcal{N}_u$ denotes u's 1-hop neighborhood. An analogous measure can be defined on the neighborhood of v. The cost of transporting mass between these two node neighborhoods along the edge $e = (u, v)$ is quantified by the Wasserstein-1 distance between the measures, namely

$$W_1(m_u, m_v) = \inf_{m \in \Gamma(m_u, m_v)} \int_{(z,z') \in V \times V} d(z, z')\, m(z, z')\, dz\, dz' \,, \tag{3}$$

where $\Gamma(m_u, m_v)$ is the set of all measures over $V \times V$ whose marginals are m_u and m_v. The *Ollivier-Ricci curvature* (Ollivier, 2009) is then defined as

$$\kappa(u, v) := 1 - \frac{W_1(m_u, m_v)}{d_G(u, v)} \,, \tag{4}$$

with $d_G(u, v)$ denoting the shortest-path distance between u and v in G.

Appendix B. Hyperscanning Modalities Across Conditions

We provide a comparative overview pairing illustrative values to empirical hyperscanning modalities across common conditions, as drawn from our simulations.

Mod./Cond.	**Edge-Weight Range**	**Empiric Implication**
EEG – Task	PLV $\approx$ 0.2–0.6	fast, captures rapid behavior
EEG – Resting	PLV $\approx$ 0.1–0.4	fast, spontaneous activity
fNIRS – Task	Corr. $\approx$ 0.1–0.3	0.1–1 s, suited to slow tasks
fNIRS – Resting	Corr. < 0.2	slow, long-term fluctuations
fMRI – Task	Cohe. $\approx$ 0.2–0.5	1–2 s, block tasks only (too slow for fast actions)
fMRI – Resting	Cohe. < 0.2	very slow, long-term networks

Table 1: Modalities across conditions, their canonical edge-weight ranges, and characteristic spatiotemporal scales with empirical implications.

The modality-dependent spatiotemporal sampling rates and signal strengths frame the core challenge addressed by our pipeline: detecting network reconfigurations only when the neural signal is sampled at a sufficient rate to resolve the target behavior.