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A Complexity Measure for Active Learning in Multi-group Mean Estimation

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Abstract

We study a *max-risk* objective for active learning in d-armed bandits: a learner adaptively allocates a budget of T samples across d groups to minimize the worst-case per-group uncertainty index $\max_{k\in[d]} \sigma_k^2/n_k$. We develop a local minimax framework and prove the first general lower bound for this objective, valid for any finite-variance hypothesis class $\mathcal{H}$. The bound separates difficulty into three orthogonal factors: a *budget* term, a *heteroscedasticity* index measuring how unevenly the uncertainty is spread across arms, and a model-dependent curvature functional, the *Variance Local Curvature* (VLC), which captures how much information a local change of variance creates inside $\mathcal{H}$. For smooth classes, the VLC is a reparametrization of a variance–Fisher information, with closed-form values for common families. Benchmarking against the strongest available upper bound (Aznag et al., 2025) shows near-optimality up to logarithmic factors in broad regimes, and pinpoints a systematic gap in highly heterogeneous instances. Our proof introduces two key ingredients: a loss-induced ℓ_1 geometry on the decision space, and a representation-based instance generator that reduces hard-instance construction to an explicit random matrix calculation.

Keywords: Active learning, multi-armed bandits, information-theoretic lower bounds, non-additive regrets.

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