

# DDPM Score Matching and Distribution Learning (Extended Abstract)\*

**Sinho Chewi**

*Yale University*

SINHO.CHEWI@YALE.EDU

**Alkis Kalavasis**

*Yale University*

ALKIS.KALAVASIS@YALE.EDU

**Anay Mehrotra**

*Stanford University*

ANAYMEHROTRA1@GMAIL.COM

**Omar Montasser**

*Yale University*

OMAR.MONTASSER@YALE.EDU

**Editors:** Steve Hanneke and Tor Lattimore

## Abstract

Score estimation is the backbone of score-based generative models (SGMs), and particularly denoising diffusion probabilistic models (DDPMs). A fundamental theoretical result in this area is that, given access to accurate score estimates, SGMs can efficiently generate from any realistic data distribution (Chen, Chewi, Li, Li, Salim, and Zhang, ICLR’23; Lee, Lu, and Tan, ALT’23). This can be viewed as a result on distribution learning, where the learned distribution is implicit as the law of the output of a sampler. However, it is unclear how score estimation relates to more classical forms of distribution learning, such as parameter estimation and density estimation.

We present a framework reducing the other two forms of distribution learning to score estimation, which has various implications in statistical and computational learning theory:

- ▶ (Parameter Estimation) Recent work has shown that for parametric estimation, a variant of score matching known as implicit score matching is provably statistically inefficient for multimodal densities, common in practice (Koehler, Heckett, and Risteski, ICLR’23). In contrast, under mild conditions, we show that denoising score matching in DDPMs is asymptotically efficient, i.e., the DDPM estimator is asymptotically normal with a covariance matrix given by the inverse Fisher information.
- ▶ (Density Estimation) Given the reduction from generation to score estimation, there is a large volume of work providing statistical and computational guarantees for learning the score of a distribution. Using our framework, we can lift the estimated scores to a  $(\epsilon, \delta)$ -PAC density estimator, i.e., a function that  $\epsilon$ -approximates the target log-density in all but a  $\delta$ -fraction of the space. To illustrate our framework, we provide two results: (i) minimax rates for density estimation over Hölder classes of densities in the standard  $L^1$  risk and (ii) a quasi-polynomial PAC density estimation algorithm for the classical Gaussian location mixture model. For the latter result, our result builds on and answers an open problem in the recent work of Gatmiry, Kelner, and Lee (COLT’25).
- ▶ (Lower Bounds for Score Estimation) Using the power of PAC density estimation, we can prove computational lower bounds for score estimation for general distribution families. As an application, we prove cryptographic lower bounds for score estimation of general Gaussian mixture models, conceptually recovering the results of Song (NeurIPS’24) and making progress on Song’s key open problem.

**Keywords:** diffusion models, density estimation, score estimation

---

\*. Extended abstract. Full version appears as arXiv:2504.05161.

## References

- Sitan Chen, Sinho Chewi, Jerry Li, Yuanzhi Li, Adil Salim, and Anru R. Zhang. Sampling is as easy as learning the score: theory for diffusion models with minimal data assumptions. In *International Conference on Learning Representations*, 2023.
- Khashayar Gatzmiry, Jonathan Kelner, and Holden Lee. Learning mixtures of Gaussians using diffusion models. In *Proceedings of Thirty Eighth Conference on Learning Theory*, volume 291 of *Proceedings of Machine Learning Research*, pages 2403–2456. PMLR, 2025.
- Frederic Koehler, Alexander Heckett, and Andrej Risteski. Statistical efficiency of score matching: the view from isoperimetry. In *The Eleventh International Conference on Learning Representations*, 2023.
- Holden Lee, Jianfeng Lu, and Yixin Tan. Convergence of score-based generative modeling for general data distributions. In *International Conference on Algorithmic Learning Theory*, volume 201 of *Proceedings of Machine Learning Research*, pages 946–985. PMLR, 2023.
- Min Jae Song. Cryptographic hardness of score estimation. In *The Thirty-Eighth Annual Conference on Neural Information Processing Systems*, 2024.