

Stable algorithms Lower Bounds for Estimation from MMSE Discontinuities: Extended Abstract

Xifan Yu

XIFAN.YU@YALE.EDU

Ilias Zadik

ILIAS.ZADIK@YALE.EDU

Yale University

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Abstract

Recent works in average-case complexity have identified stable (noise-stable) algorithms as a central class. Specifically, in average-case optimization, the class is conjectured to capture the power of polynomial-time computation for many problems. This perspective has been supported by establishing variants of the Overlap Gap Property (OGP) phase transitions just below conjectured polynomial-time thresholds. Yet, it was recently challenged by Schramm and Li (2025), who showed that Shortest Path in random graphs exhibits the OGP—and hence all stable algorithms fail—despite being solvable in polynomial time. This counterexample has also been particularly curious as it appeared rather distinct from other classical “noiseless” counterexamples, such as solving random linear systems.

By contrast, the power of stable methods in statistical estimation has remained unclear. A central difficulty is the absence of an OGP-type phenomenon that can uniformly exclude all stable methods. Instead, existing lower bounds largely focus on the related class of low-degree polynomials and are confined to restricted models, such as Gaussian additive models, reflecting the high technical difficulty of controlling the minimum mean-squared error (MMSE) of low-degree estimators.

In this work, we show that for all statistical estimation problems, a natural MMSE instability (discontinuity) condition implies the failure of stable algorithms, serving as a version of OGP for estimation tasks. Using this criterion, we establish separations between stable and polynomial-time algorithms for the following MMSE-unstable tasks (i) Planted Shortest Path, where Dijkstra’s algorithm succeeds, (ii) random Parity Codes, where Gaussian elimination succeeds, and (iii) Gaussian Subset Sum, where lattice-based methods succeed. For all three, we further show that all low-degree polynomials are stable, yielding separations against low-degree methods and a new method to bound the low-degree MMSE. In particular, our technique highlights that MMSE instability is a common feature for Shortest Path and the noiseless Parity Codes and Gaussian subset sum.

Last, we highlight that our work places rigorous algorithmic footing on the long-standing physics belief that first-order phase transitions—which in this setting translates to MMSE instability—impose fundamental limits on classes of efficient algorithms.¹

Keywords: High Dimensional Statistical Estimation, Stable Algorithms, Low-degree Polynomials, Computational-Statistical Trade-offs

1. Extended abstract. Full version appears as <https://arxiv.org/abs/2603.22192v1>.

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