
Average Controlled and Average Natural Micro Direct Effects in Summary Causal Graphs

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Abstract

In this paper, we investigate the identifiability of average controlled direct effects and average natural direct effects in causal systems represented by summary causal graphs, which are abstractions of full causal graphs, often used in dynamic systems where cycles and omitted temporal information complicate causal inference. Unlike in the traditional linear setting, where direct effects are typically easier to identify and estimate, non-parametric direct effects, which are crucial for handling real-world complexities, particularly in epidemiological contexts where relationships between variables (*e.g.*, genetic, environmental, and behavioral factors) are often non-linear, are much harder to define and identify. In particular, we give sufficient conditions for identifying average controlled micro direct effect and average natural micro direct effect from summary causal graphs in the presence of hidden confounding. Furthermore, we show that the conditions given for the average controlled micro direct effect become also necessary in the setting where there is no hidden confounding and where we are only interested in identifiability by adjustment.

1 INTRODUCTION

The identification and estimation of direct effects are critical in many applications. For instance, epidemiologists aim to measure how smoking affects lung cancer risk independently of genetic susceptibility [Zhou et al., 2021]. In addition, it has been shown that estimating direct effects can help diagnose system failures by comparing the direct impacts of different components before and after failure [Assaad et al., 2023].

In the framework of Structural Causal Models (SCMs)

[Pearl, 2009], there has been significant progress in identifying direct effects using fully specified causal graphs. However, constructing a fully specified causal graph is a challenging task, as it requires precise knowledge of the causal relationships between all pairs of observed variables. In complex, high-dimensional settings such as in medico-administrative databases, this level of detail is often unavailable, limiting the practical application of causal inference. As a result, there has been growing interest in the use of partially specified causal graphs in recent years [Perkovic, 2020, Anand et al., 2023, Ferreira and Assaad, 2024, Assaad et al., 2024, Wahl et al., 2024]. One notable example is the cluster-ADMG (Acyclic Directed Mixed Graph) introduced by Anand et al. [2023], which offers a coarser representation of causal relationships. In this graph, vertices represent clusters of variables and therefore it simplifies the representation of complex systems. However, one main limitation of this type of graph is its acyclicity assumption which does not necessarily hold even if the underlying fully specified graph is acyclic. A closely related graph type is the summary causal graph (SCG) [Peters et al., 2013], which is mainly used in systems involving time and which relaxes the assumption of acyclicity and represents causal relationships where each cluster corresponds to a time series or repeated measurements of the same variable in longitudinal studies. This flexibility makes SCGs particularly useful in scenarios involving temporal data and cycles.

In SCGs, two types of causal effects may be of interest. The first is the macro causal effect, which captures the impact of one set of time series on another set of time series. The second is the micro causal effect, which concerns the effect of one specific time point on another. This paper focuses on micro causal effects, with particular emphasis on micro direct effects. To date, the problem of identifying micro direct effects from SCGs has primarily been addressed under the assumptions of no hidden confounding and linear SCMs [Ferreira and Assaad, 2024]. However, many real-world applications involve hidden confounding and relationships between causes and effects that are not necessarily

linear. In a non-parametric setting, defining direct effects becomes more complex. The literature of causal inference primarily offers two definitions: the controlled direct effect and the natural direct effect [Robins and Greenland, 1992, Pearl, 2001]. The former isolates the impact of the treatment variable by controlling on mediators or on the parents of the response variable, while the natural direct effect captures how the treatment variable influences the response variable while fixing mediators to their natural values, making it more realistic but harder to identify and estimate.

This paper focuses on the identifiability of direct effects from SCGs in a non-parametric setting. Our contributions are threefold: first, we present sufficient conditions that characterizes cases where the average controlled micro direct effect is graphically identifiable from an SCG in the presence of hidden confounding. Next we show that those conditions become necessary in the absence of hidden confounding and when considering only identifiability by adjustment. Finally, we establish sufficient conditions for the identifiability of the average natural micro direct effect from an SCG with hidden confounding.

The remainder of the paper is organized as follows: Section 2 reviews related work, while Section 3 introduces preliminaries and tools necessary for the subsequent sections. Section 4 provides identifiability conditions for average controlled micro direct effects, and Section 5 addresses identifiability conditions for average natural micro direct effects. Finally, Section 8 concludes the paper.

2 RELATED WORKS

There is a substantial body of work in the literature focused on identifying causal effects using partially specified graphs. For instance, the studies by Maathuis and Colombo [2013], Perkovic et al. [2016], Perkovic [2020], Wang et al. [2023] provided conditions for identifying total effects through partially directed graphs that represent all graphs within the Markov equivalence class of the true causal graph, including CPDAGs, MPDAGs, and PAGs. Furthermore, Flanagan [2020] provided conditions for identifying average controlled direct effects and average natural direct effects using CPDAGs and MPDAGs assuming no hidden confounding.

In the context of Cluster-ADMGs (acyclic directed mixed graphs where each vertex represents a group of variables), Anand et al. [2023] extended Pearl’s do-calculus [Pearl, 1995] to establish necessary and sufficient conditions for identifying total effects between clusters. Building on this, Ferreira and Assaad [2025] extended these results to SCGs, where each cluster represents a time series and which unlike Cluster-ADMGs, can contain cycles. However, both studies focus on macro causal effects (*i.e.*, effects between clusters), whereas the present paper is concerned with micro-level causal effects within clusters.

Tikka et al. [2023] investigated ways of clustering variables in a way that preserves identifiability and provided sufficient conditions of non-identifiability in a cluster DAG. Tabell et al. [2026] extended this work to the case of data fusion.

For micro causal effects, Eichler and Didelez [2007] provided sufficient conditions for identifying micro total effects in time series, applicable to SCGs under the assumption of no instantaneous relations. When instantaneous relations are allowed, Assaad et al. [2023] showed that both micro total and micro direct effects are identifiable from SCGs in a linear setting, assuming no hidden confounders and that cycles in the SCG cannot be of size greater than one. Subsequently, Ferreira and Assaad [2024] and Assaad et al. [2024] extended these results, establishing conditions for identifying micro total effects in a non-parametric setting and micro direct effects in a linear setting, even when cycles larger than one exist in the SCG. More recently, Assaad provided sufficient conditions for identifying total effects in the presence of hidden confounding. However, identifying micro direct effects in a non-parametric setting or with hidden confounding from SCGs (or any graph with cycles) remains an open challenge. This is not surprising, as direct effects are inherently more complex than total effects in non-parametric setting.

It is worth highlighting that in order to reach our goal of identifying a direct effect from a summary causal graph, one could discover the true full-time graph Assaad et al. [2022] to use existing identification results. This can rely on untestable assumptions and thus our objective in this works is not to discover the full-time graph but to identify direct effects from the information contained in the summary causal graph.

3 PRELIMINARIES

In this section, we introduce the tools, terminology, and key theoretical results that will be essential for the remainder of the paper.

In the remainder, uppercase letters are used to denote variables, lowercase letters represent specific values of those variables, and blackboard bold letters indicate sets of variables. For any given graph, $Pa(\cdot, \cdot)$, $An(\cdot, \cdot)$, and $De(\cdot, \cdot)$ will be used to represent the parents, ancestors, and descendants of a vertex, respectively. The first argument refers to the specific vertex under consideration, while the second argument corresponds to the graph in which these relationships are being examined. For instance, $Pa(X, \mathcal{G})$ refers to the parents of vertex X in graph $\mathcal{G}$. By convention, we will treat any vertex as an ancestor and a descendant of itself. The mutilated graph $\mathcal{G}_{\overline{WZ}}$ is obtained from $\mathcal{G}$ by removing all edges with an arrowhead pointing to W (*e.g.*, $X \rightarrow W$, $X \longleftrightarrow W$), as well as all edges with a tail originating from Z (*e.g.*, $X \leftarrow Z$). In a graph, a path is said to be acti-

vated by $\mathbb{W}$ if every collider W (i.e., a vertex with structure $\rightarrow W \leftarrow$) on the path, or any of its descendants, is in $\mathbb{W}$, and no non-collider on the path is in $\mathbb{W}$. A path that is not activated is said to be blocked. If all paths between X and Y are blocked by a set $\mathbb{W}$, then X is said to be m-separated from Y by $\mathbb{W}$ in the graph $\mathcal{G}$, denoted as $X \perp\!\!\!\perp_{\mathcal{G}} Y \mid \mathbb{W}$. The strongly connected component of a vertex X in a graph $\mathcal{G}$, denoted by $Scc(X, \mathcal{G})$, is the maximal subset of vertices that includes X , such that every vertex in this subset has a directed path to every other vertex in the subset. By convention, $Scc(X, \mathcal{G}) = \emptyset$ if there is no cycle in the graph that includes X and $Scc(X, \mathcal{G}) = \{X\}$ if the only cycle that includes X is a self loop on X . We refer to $\Pr$ as the probability distribution, and $\Pr(\cdot \mid \text{do}(X = x))$ as the interventional distribution, where the $\text{do}(\cdot)$ operator represents the intervention.

In this study, we consider an unknown discrete-time dynamic structural causal model, a specific variant of the structural causal model [Pearl, 2009], designed to account for temporal dynamics.

Definition 1 (Discrete-time dynamic structural causal model (DTDSCM)). *A discrete-time dynamic structural causal model is a tuple $\mathcal{M} = (\mathbb{L}, \mathbb{V}, \mathbb{F}, \Pr(\mathbb{I}))$, where $\mathbb{L} = \bigcup \{\mathbb{L}^{v_i} \mid i \in [1, d], t \in [t_0, t_{max}]\}$ is a set of exogenous variables, which cannot be observed but affect the rest of the model. $\mathbb{V} = \bigcup \{\mathbb{V}^i \mid i \in [1, d]\}$ such that $\forall i \in [1, d], \mathbb{V}^i = \{V_t^i \mid t \in [t_0, t_{max}]\}$, is a set of endogenous variables, which are observed and every $V_t^i \in \mathbb{V}$ is functionally dependent on some subset of $\mathbb{L}^{v_i} \cup \mathbb{V}_{\leq t} \setminus \{V_t^i\}$ where $\mathbb{V}_{\leq t} = \{V_{t'}^j \mid j \in [1, d], t' \leq t\}$. $\mathbb{F}$ is a set of functions such that for all $V_t^i \in \mathbb{V}$, f^{v_i} is a mapping from $\mathbb{L}^{v_i}$ and a subset of $\mathbb{V}_{\leq t} \setminus \{V_t^i\}$ to V_t^i . $\Pr(\mathbb{I})$ is a joint probability distribution over $\mathbb{L}$.*

Some of the findings in this paper consider an unknown, general DTDSCM, while others focus on a specific sub-family of DTDSCM that are constrained by the following assumption known as causal sufficiency.

Assumption 1. $\forall L, L' \in \mathbb{L}, \Pr(L, L') = \Pr(L)\Pr(L')$ and $\forall V_t^i \neq V_{t'}^j, \mathbb{L}^{v_i} \cap \mathbb{L}^{v_j} = \emptyset$, i.e., there is no hidden confounding.

Furthermore, we suppose that in every unknown DTDSCM, there exists a known maximal lag between causes and effects, denoted as γ_{max} ($\exists \gamma_{max}, \forall U_{t-\gamma_U} \rightarrow V_t, \gamma_U \leq \gamma_{max}$). This assumption is fairly natural and is often associated with the causal stationarity of the DTDSCM. Additionally, we assume that the probability distribution $\Pr$ over the observed variables is positive. Finally, we suppose that each DTDSCM induces a full-time acyclic directed mixed graph (FT-ADMG), a specific type of ADMG [Richardson, 2003], where bidirected dashed arrows represent hidden confounding. Under Assumption 1, the FT-ADMG reduces to a full-time directed acyclic graph (FT-DAG).

Definition 2 (Full-Time Acyclic Directed Mixed Graph). *Consider a DTDSCM $\mathcal{M}$. The full-time acyclic directed mixed graph (FT-ADMG) $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ induced by $\mathcal{M}$ is defined in the following way:*

$$\begin{aligned} \mathbb{E}^1 &:= \{X_{t-\gamma} \rightarrow Y_t \mid \forall Y_t \in \mathbb{V}, X_{t-\gamma} \in \mathbb{X} \subseteq \mathbb{V}_{\leq t} \setminus \{Y_t\} \\ &\quad \text{such that } Y_t := f^{y_t}(\mathbb{X}, \mathbb{L}^{y_t}) \text{ in } \mathcal{M}\}, \\ \mathbb{E}^2 &:= \{X_{t-\gamma} \longleftrightarrow Y_t \mid \forall Y_t, \forall X_{t-\gamma} \in \mathbb{V} \setminus \{Y_t\} \\ &\quad \text{such that } \Pr(\mathbb{L}^{x_{t-\gamma}} \cap \mathbb{L}^{y_t}) \neq \Pr(\mathbb{L}^{x_{t-\gamma}})\Pr(\mathbb{L}^{y_t})\}, \end{aligned}$$

where $\mathbb{E} = \mathbb{E}^1 \cup \mathbb{E}^2$.

The findings of this study, when viewed in isolation, do not rely on any additional assumptions. However, following the identification of the direct effect, the estimation phase involves using actual data. If the data consist of time series (e.g., various time series describing a specific patient), the causal stationarity assumption becomes necessary to satisfy the positivity assumption (i.e., $\forall v, \Pr(\mathbb{V} = v) > 0$). Conversely, the causal stationarity assumption is not required when working with spatio-temporal or cohort data.

In the context of DTDSCM and FT-ADMG, both macro and micro causal effect questions can be posed [Ferreira and Asaad, 2025]. A macro causal effect refers to the influence of an entire time series (or multiple time series) on another series, whereas a micro causal effect focuses on the influence of specific time points (or multiple time points) on other time points. This work specifically focuses on micro causal effects, with a particular emphasis on micro direct effects. In a linear setting (where all functions in $\mathbb{F}$ are linear), the micro direct effect of $X_{t-\gamma}$ on Y_t is represented by the path coefficient [Wright, 1920, 1921] of $X_{t-\gamma}$ in f^{y_t} within the DTDSCM. However, in a non-parametric setting, defining the direct effect becomes more complex. The literature distinguishes two types of non-parametric direct effects, the first being the controlled direct effect, which is defined as follows.

Definition 3 (Average Controlled Micro Direct Effect [Pearl, 2001]). *Given a FT-ADMG $\mathcal{G} = (\mathbb{V}, \mathbb{E})$, $Y_t, X_{t-\gamma} \in \mathbb{V}$ and $\mathbb{Z} = Pa(Y_t, \mathcal{G}) \setminus \{X_{t-\gamma}\}$. The controlled micro direct effect of $X_{t-\gamma}$ on Y_t contrasting x and x' while keeping $\mathbb{Z}$ at value z is defined as*

$$\begin{aligned} CDE(X_{t-\gamma}^{x, x'}, Y_t, z) &= E(Y_t \mid \text{do}(X_{t-\gamma} = x'), \text{do}(\mathbb{Z} = z)) \\ &\quad - E(Y_t \mid \text{do}(X_{t-\gamma} = x), \text{do}(\mathbb{Z} = z)) \end{aligned}$$

In general, average controlled micro direct effects cannot be used for effect decomposition and therefore are not useful in mediation analysis. Specifically, the difference between the total effect and the average controlled micro direct effect cannot typically be interpreted as an indirect effect [Robins and Greenland, 1992, Pearl, 2001, Kaufman et al., 2004, Vanderweele, 2011]. This issue can be solved when we set all values of parents of Y_t to whatever value they would have

obtained under $do(X_{t-\gamma} = x)$. This is known as the natural direct effect [Pearl, 2001] or the pure direct effect [Robins and Greenland, 1992]).

Definition 4 (Average Natural Micro Direct Effect [Pearl, 2001]). *Given a FT-ADMG $\mathcal{G} = (\mathbb{V}, \mathbb{E})$, $Y_t, X_{t-\gamma} \in \mathbb{V}$ and $\mathbb{Z} = Pa(Y_t, \mathcal{G}) \setminus \{X_{t-\gamma}\}$. The natural micro direct effect of $X_{t-\gamma}$ on Y_t contrasting x and x' is defined as*

$$NDE(X_{t-\gamma}^{x,x'}, Y_t) = E(Y_t \mid do(X_{t-\gamma} = x'), do(\mathbb{Z} = z_x)) - E(Y_t \mid do(X_{t-\gamma} = x))$$

where z_x is the counterfactual value of $\mathbb{Z}$ under the setting $X_{t-\gamma} = x$.

Note that in Pearl [2014], $\mathbb{Z}$ is not the parents of Y_t but the set of intermediate variables (*i.e.*, mediators) between $X_{t-\gamma}$ and Y_t . In this work we will focus on the set of parents of the effect, as in Pearl [2001], since it allows a better parallel with the definition of the average controlled micro direct effect that we adopted. Both definitions are equivalent as any parent of Y_t which is not a mediator between $X_{t-\gamma}$ and Y_t is not a descendant of $X_{t-\gamma}$ and thus its counterfactual value under intervention on $X_{t-\gamma}$ is its factual value.

There has been substantial work on identifying both average controlled and average natural direct effects from observational data using ADMGs, and by extension, FT-ADMGs. These direct effects are considered identifiable if they can be expressed solely in terms of conditional probabilities and expectations over observed variables. Notably, the task of identifying these effects can, in some cases, be fully reduced to identifying interventional distributions alone. For instance, in the case of the average controlled direct effect, identification relies entirely on identifying the interventional distribution $\Pr(y_t \mid do(X_{t-\gamma} = x), do(\mathbb{Z} = z))$. Meanwhile, for the average natural direct effect, it partially depends on identifying the interventional distribution $\Pr(y_t \mid do(X_{t-\gamma} = x))$ and additionally requires identifying a counterfactual term. However, as demonstrated by Pearl [2001] and as we will show in Section 5, even this counterfactual term can sometimes be identified by relating it back to specific interventional distributions. The do-calculus, introduced by Pearl [1995] provides a symbolic machinery that can be used to identify interventional distributions. It comprises three rules, each establishing specific graphical criteria that dictate when substitutions can be applied to an interventional distribution. The most important aspect of the do-calculus is that it is sound and complete: any interventional distribution is identifiable if and only if there exists a sequence of the rules of the do-calculus that transforms the interventional distribution into an expression containing only conditional probabilities and expectations over observed variables [Pearl, 1995, Shpitser and Pearl, 2006, Huang and Valtorta, 2006]. The three rules of the do-calculus are as

follows:

$$\begin{aligned} \mathbf{R1}: & \Pr(Y = y \mid do(\mathbb{Z} = z), \mathbb{X} = x, \mathbb{W} = w) \\ &= \Pr(Y = y \mid do(\mathbb{Z} = z), \mathbb{W} = w) \text{ if } \mathbb{Y} \perp_{\mathcal{G}_{\mathbb{Z}}} \mathbb{X} \mid \mathbb{Z}, \mathbb{W} \\ \mathbf{R2}: & \Pr(Y = y \mid do(\mathbb{Z} = z), do(\mathbb{X} = x), \mathbb{W} = w) \\ &= \Pr(Y = y \mid do(\mathbb{Z} = z), \mathbb{X} = x, \mathbb{W} = w) \text{ if } \mathbb{Y} \perp_{\mathcal{G}_{\mathbb{Z}}} \mathbb{X} \mid \mathbb{Z}, \mathbb{W} \\ \mathbf{R3}: & \Pr(Y = y \mid do(\mathbb{Z} = z), do(\mathbb{X} = x), \mathbb{W} = w) \\ &= \Pr(Y = y \mid do(\mathbb{Z} = z), \mathbb{W} = w) \text{ if } \mathbb{Y} \perp_{\mathcal{G}_{\mathbb{Z} \times (\mathbb{W})}} \mathbb{X} \mid \mathbb{Z}, \mathbb{W} \end{aligned}$$

where $\mathbb{X}(\mathbb{W})$ is the set of vertices in $\mathbb{X}$ that are non-ancestors of any vertex in $\mathbb{W}$ in the mutilated graph $\mathcal{G}_{\mathbb{Z}}$.

The interventional distribution is said to be identifiable by adjustment when it can be expressed as a sum of probabilities of the response conditioned on the treatment and a subset of observed variables $\mathbb{W}$, weighted by the probability of observing those values of $\mathbb{W}$, *i.e.*, $\sum_w \Pr(Y_t = y \mid X_{t-\gamma} = x, \mathbb{W} = w) \Pr(\mathbb{W} = w)$.

However, it is challenging for practitioners in fields like epidemiology to provide or validate a FT-ADMG, as this requires detailed knowledge of temporal dynamics at every time point, which can be both complex and impractical. Moreover, causal discovery algorithms [Spirtes et al., 2000] (*i.e.*, algorithms which infer a causal graph from observational data under strong untestable assumptions) are still largely ineffective on real-world data [Ait-Bachir et al., 2023]. Therefore, it is often more feasible to use summary causal graphs, which simplify the causal structure into a more manageable form. This allows practitioners to focus on the key pathways and relationships. For instance, when studying the long-term effects of smoking on lung disease, epidemiologists can outline the main causal pathways—like smoking leading to lung damage—without needing to detail the exact timing of each intermediate step.

Definition 5 (Summary Causal Graph with possible latent confounding). *Consider an FT-ADMG $\mathcal{G} = (\mathbb{V}, \mathbb{E})$. The summary causal graph (SCG) $\mathcal{G}^s = (\mathbb{S}, \mathbb{E}^s)$ compatible with $\mathcal{G}$ is defined in the following way:*

$$\begin{aligned} \mathbb{S} &:= \{V^i = (V_{t_0}^i, \dots, V_{t_{max}}^i) \mid \forall i \in \llbracket 1, d \rrbracket\}, \\ \mathbb{E}^{s1} &:= \{X \rightarrow Y \mid \forall X, Y \in \mathbb{S}, \exists t' \leq t \in \llbracket t_0, t_{max} \rrbracket \\ &\quad \text{such that } X_{t'} \rightarrow Y_t \in \mathbb{E}\}, \\ \mathbb{E}^{s2} &:= \{X \longleftrightarrow Y \mid \forall X, Y \in \mathbb{S}, \exists t', t \in \llbracket t_0, t_{max} \rrbracket \\ &\quad \text{such that } X_{t'} \longleftrightarrow Y_t \in \mathbb{E}\}. \end{aligned}$$

where $\mathbb{E}^s = \mathbb{E}^{s1} \cup \mathbb{E}^{s2}$.

A SCG is an abstraction of the FT-ADMG obtained by clustering all temporal instants of time series into a single vertex. Figures 1-4 present many examples of SCGs with compatible FT-ADMGs. The abstraction of SCGs implies that, while a given FT-ADMG corresponds to exactly one

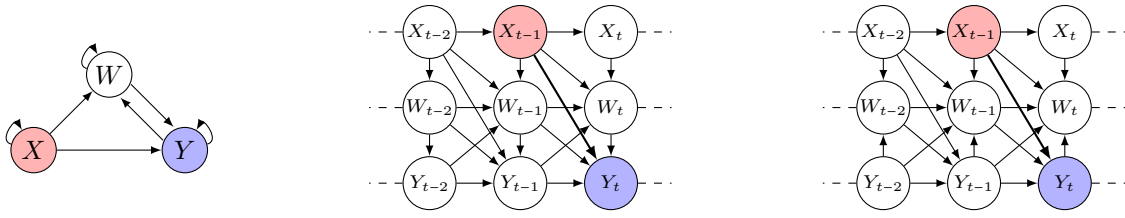


Figure 1: An SCG with two compatible FT-ADMGs. Each pair of red and blue vertices represents the micro direct effect we are interested in. Both the controlled direct effect and the natural direct effect are not identifiable using the conditions given in this paper.



Figure 2: An SCG with a compatible FT-ADMG. Each pair of red and blue vertices represents the micro direct effect we are interested in. Both the controlled direct effect and the natural direct effect are not identifiable using the conditions given in this paper.

SCG, multiple FT-ADMGs can be compatible with the same SCG. As a result, the do-calculus as introduced by Pearl [1995], is not directly applicable to SCGs to identify micro causal effects [Ferreira and Assaad, 2025]. In the following, the term "identifiability" will refer to the capability of recovering the causal effect of interest knowing the SCG and the joint observed probability distribution. The objective of this paper is to establish specific graphical conditions within an SCG that guarantee the existence of a sequence of do-calculus rules that can be applied to any FT-ADMG compatible with the SCG, yielding an identical expression involving only conditional probabilities and expectations over observed variables. A key insight about SCGs that aids in obtaining these conditions is that, although there may not be a strict one-to-one correspondence between the set of parents of a vertex Y in an SCG and the set of parents of Y_t in any particular FT-ADMG, a precise mapping is possible when considering the set of all possible parents across all FT-ADMGs compatible with the SCG. Specifically, this set includes vertices that act as parents in at least one FT-ADMG that aligns with the SCG. Formally, this set of possible parents is defined as follows.

Definition 6 (Possible Parents). *Given an SCG $\mathcal{G}^s = (\mathbb{S}, \mathbb{E}^s)$, a maximal lag γ_{max} and a vertex $Y \in \mathbb{S}$. The set of possible parents of the temporal vertex Y_t is the set of temporal vertices (i.e. vertices in compatible FT-ADMGs) which are parents of Y_t in at least one compatible FT-ADMG. It is written $PP(Y_t, \mathcal{G}^s)$ and is defined as*

$$PP(Y_t, \mathcal{G}^s) = \{P_{t-\gamma} \mid P \in Pa(Y, \mathcal{G}^s), \gamma \in [0, \gamma_{max}]\} \setminus \{Y_t\}$$

We close this section with the following property of possible

parents, which shows that it is possible to replace the set of parents in the definitions of the average controlled micro direct effect and the average natural micro direct effect by the set of possible parents.

Property 1. *Given a FT-ADMG $\mathcal{G} = (\mathbb{V}, \mathbb{E})$, $Y_t \in \mathbb{V}$ and $\mathbb{Z} \supseteq Pa(Y_t, \mathcal{G})$ such that $Y_t \notin \mathbb{Z}$, the following equality holds for all z :*

$$\begin{aligned} & \Pr(Y_t = y \mid do(Pa(Y_t, \mathcal{G}) = \mathbb{p})) \\ &= \Pr(Y_t = y \mid do(\mathbb{Z} = z)). \end{aligned}$$

where $\mathbb{p}$ corresponds to the subset of values in $\mathbb{z}$ associated with $Pa(Y_t, \mathcal{G})$. This equality holds specifically for $\mathbb{Z} = PP(Y_t, \mathcal{G}^s)$ and note that there exists a compatible FT-ADMG $\mathcal{G}'$ in which $PP(Y_t, \mathcal{G}^s) = Pa(Y_t, \mathcal{G}')$.

This property results directly from R3 of the do-calculus. All formal proofs can be found in the supplementary materials.

4 IDENTIFYING CONTROLLED MICRO DIRECT EFFECTS IN SCGS

As stated in the previous section, the average controlled micro direct effect of interest is the one where we intervene on all parents of Y_t . In order to identify this effect, in particular using R2 of the do-calculus, one can find a set of variables of Y_t which m-separates Y_t from its parents in $\mathcal{G}_{Pa(Y_t, \mathcal{G})}$ for all compatible FT-ADMG $\mathcal{G}$. There are two significant challenges in this process. The first challenge arises when there exists a cycle involving Y and another vertex in the

SCG. For example, consider the SCG in Figure 1(a), where Assumption 1 holds. In this case, the cycle between Y and W implies the presence of a compatible FT-ADMG $\mathcal{G}$, as depicted in Figure 1(b), in which $W_t \in Pa(Y_t, \mathcal{G})$. In this FT-ADMG we would like to ultimately remove all the $\text{do}(\cdot)$ operators in the interventional distribution $\Pr(Y_t = y \mid \text{do}(W_t = w), \text{do}(Y_{t-1} = y'), \text{do}(X_{t-1} = x), \text{do}(W_{t-1} = w'))$ but for the sake of illustration let us only focus on removing the $\text{do}(\cdot)$ on W_t . Notice that in this FT-ADMG, we have $Y_t \perp\!\!\!\perp_{\mathcal{G}_{Y_{t-1}X_{t-1}W_{t-1}W_t}} W_t \mid Y_{t-1}, X_{t-1}, W_{t-1}$, which means that by using R2 of the do-calculus, it is possible to write the considered interventional distribution as $\Pr(Y_t = y \mid W_t = w, \text{do}(Y_{t-1} = y'), \text{do}(X_{t-1} = x), \text{do}(W_{t-1} = w'))$. However, there exists another compatible FT-ADMG $\mathcal{G}'$, shown in Figure 1(c) where $W_t \leftarrow Y_t$ and thus it is impossible to use R2 to remove the $\text{do}(\cdot)$ operator on W_t in the same interventional distribution since $\forall \mathbb{W}, Y_t \not\perp\!\!\!\perp_{\mathcal{G}'_{Y_{t-1}X_{t-1}W_{t-1}W_t}} W_t \mid \mathbb{W}$. This indicates that the relationship between W_t and Y_t is ambiguous when only the SCG is considered, preventing the removal of the $\text{do}(\cdot)$ operator on W_t in the considered interventional distribution. The second challenge occurs when hidden confounding exists between Y and another vertex. For instance, consider the SCG in Figure 2(a), in this scenario, by examining the compatible FT-ADMG $\mathcal{G}$ in Figure 2(b), it becomes clear that as in the previous example, it is necessary to remove the $\text{do}(\cdot)$ on W_t in the interventional distribution $\Pr(Y_t = y \mid \text{do}(W_t = w), \text{do}(Y_{t-1} = y'), \text{do}(X_{t-1} = x), \text{do}(W_{t-1} = w'))$ as $W_t \in Pa(Y_t, \mathcal{G})$. However, hidden confounding exists between W_t and Y_t , making R2 and R3 of the do-calculus inapplicable since $\forall \mathbb{W}, Y_t \not\perp\!\!\!\perp_{\mathcal{G}_{Y_{t-1}X_{t-1}W_{t-1}W_t}} W_t \mid \mathbb{W}$ and $Y_t \not\perp\!\!\!\perp_{\mathcal{G}_{Y_{t-1}X_{t-1}W_{t-1}W_t}} W_t \mid \mathbb{W}^1$. Nevertheless, as shown in the following theorem, it is possible to identify the controlled micro direct effect in any SCG where no cycle involving Y and other vertices exists and where hidden confounding between Y and its ancestors is absent.

Theorem 1. *Given an SCG $\mathcal{G}^s = (\mathbb{S}, \mathbb{E}^s)$, $Y \in \mathbb{S}$ and $X_{t-\gamma} \in PP(Y_t, \mathcal{G}^s)$. The controlled direct effect of $X_{t-\gamma}$ on Y_t contrasting x and x' is identifiable if $\text{Scc}(Y, \mathcal{G}^s) \subseteq \{Y\}$ and there does not exist a bidirected dashed arrow between Y and one of its ancestors (i.e. $\nexists Z \in An(Y, \mathcal{G}^s), Z \longleftrightarrow Y$) and we have:*

$$\begin{aligned} CDE(X_{t-\gamma}^{x,x'}, Y_t, z) = & E(Y_t \mid X_{t-\gamma} = x', Z = z) \\ & - E(Y_t \mid X_{t-\gamma} = x, Z = z) \end{aligned}$$

where $Z = PP(Y_t, \mathcal{G}^s) \setminus \{X_{t-\gamma}\}$.

For illustration, we give in Figure 5 three SCGs where the average controlled micro directed effect is identifiable by Theorem 1.

¹The subgraph formed by Y_t and W_t in Figure 2(b) is known as an bow arc graph and when such a structure exists, the interventional distribution $\Pr(Y_t = y \mid \text{do}(W_t = w))$ is known to be non-identifiable [Shpitser and Pearl, 2006]

Theorem 1 offers sufficient conditions for identifying the average controlled micro direct effect supposing a general unknown FT-ADMG. It's important to note, however, that these conditions are not necessary in general. Interestingly, under Assumption 1 (i.e., supposing an unknown FT-DAG) and by only considering identifiability by adjustment, these conditions do become necessary, as demonstrated in the following proposition.

Proposition 1. *Given an SCG $\mathcal{G}^s = (\mathbb{S}, \mathbb{E}^s)$, $Y \in \mathbb{S}$ and $X_{t-\gamma} \in PP(Y_t, \mathcal{G}^s)$. Under Assumption 1, if $\text{Scc}(Y, \mathcal{G}^s) \not\subseteq \{Y\}$ then the controlled direct effect of $X_{t-\gamma}$ on Y_t is not identifiable by adjustment.*

5 IDENTIFYING NATURAL MICRO DIRECT EFFECTS IN SCGS

Pearl [2014] provided sufficient conditions to reframe the problem of identifying the average natural direct effect from one involving counterfactuals to one based on interventions, specifically using the $\text{do}(\cdot)$ operator. The formal result addressed cases where $\mathbb{Z}$ represents all mediators. It was informally observed that the same conditions apply when $\mathbb{Z}$ includes all parent variables. Since this paper focuses on parent variables, we restate and formalize this result in the context of parents.

Lemma 1. *Given an SCG $\mathcal{G}^s = (\mathbb{S}, \mathbb{E}^s)$, let $Y \in \mathbb{S}$, $X_{t-\gamma} \in PP(Y_t, \mathcal{G}^s)$. If $\text{Scc}(Y, \mathcal{G}^s) \subseteq \{Y\}$ and there is no bidirected dashed arrow between Y and its ancestors then the natural direct effect of $X_{t-\gamma}$ on Y_t , $NDE(X_{t-\gamma}^{x,x'}, Y_t)$, is identifiable from the SCG if*

- $\Pr(Y_t = y \mid \text{do}(X_{t-\gamma} = x), \text{do}(Z = z))$ is identifiable from the SCG, and
- $\Pr(Z = z \mid \text{do}(X_{t-\gamma} = x))$ is identifiable from the SCG.

where $Z = PP(Y_t, \mathcal{G}^s) \setminus \{X_{t-\gamma}\}$.

Notice that Lemma 1 shows that the the identifiability of the average natural micro direct effect can be reduced to the identifiability of the average controlled micro direct effect and the identifiability of the interventional distribution $\Pr(Z = z \mid \text{do}(X_{t-\gamma} = x))$. The first identifiability can be directly given by the result of the previous section which means that we need to impose the constraints given for the average controlled micro direct effect to the average natural micro direct effect. In addition, we need to add some conditions to ensure the second identifiability. The SCG in Figure 3(a) represents a case where this second identifiability cannot be achieved. To see this, consider $\Pr(W_{t-1} = w, W_t = w', Y_{t-1} = y \mid \text{do}(X_{t-1} = x))$ and notice that, similarly to Figure 1, because of the cycle $X \rightleftarrows W$ in the SCG, the $\text{do}(\cdot)$ operator cannot be removed using

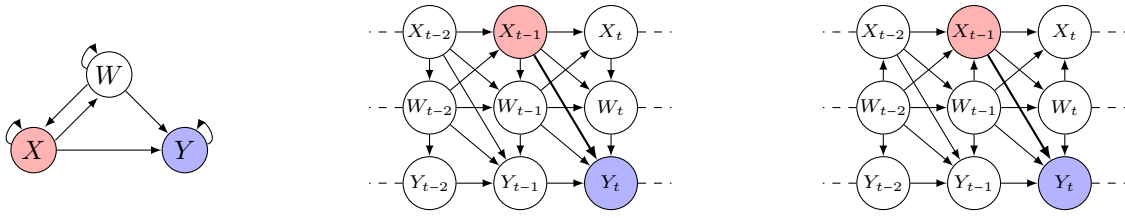


Figure 3: An SCG in (a) with two compatible FT-ADMGs in (b) and (c). Each pair of red and blue vertices represents the micro direct effect we are interested in. The controlled direct effect is identifiable according to our condition but the natural direct effect is not.



Figure 4: An SCG in (a) with a compatible FT-ADMG in (b). Each pair of red and blue vertices represents the micro direct effect we are interested in. The controlled direct effect is identifiable according to our condition but the natural direct effect is not.

the same sequence of do-calculus rules in the first compatible FT-ADMG in Figure 3(b) in which $X_{t-1} \rightarrow W_{t-1}$ and in the second compatible FT-ADMG in Figure 3(c) in which $X_{t-1} \leftarrow W_{t-1}$. The SCG in Figure 4(a) represents another case where this second identifiability cannot be achieved due to hidden confounding. To see this, consider $\Pr(W_{t-1} = w, W_t = w', Y_{t-1} = y \mid \text{do}(X_{t-1} = x))$ and notice that, similarly to Figure 2, because of hidden confounding $X \longleftrightarrow W$ in the SCG, the $\text{do}(\cdot)$ operator cannot be removed in this interventional distribution in the compatible FT-ADMG in Figure 4(b) in which $X_{t-1} \longleftrightarrow W_{t-1}$.

Theorem 2. *Given an SCG $\mathcal{G}^s$, the natural direct effect of $X_{t-\gamma}$ on Y_t , $NDE(X_{t-\gamma}^{x,x'}, Y_t)$, is identifiable from the SCG if*

1. $\text{Scc}(Y, \mathcal{G}^s) \subseteq \{Y\}$,
2. $\text{Scc}(X, \mathcal{G}^s) \subseteq \{X\}$,
3. $PP(X_{t-\gamma}, \mathcal{G}^s) \cap PP(Y_t, \mathcal{G}^s) = \emptyset$,
4. $\nexists Z \in \text{An}(Y, \mathcal{G}^s)$, $Z \longleftrightarrow Y$, and
5. $\nexists Z \in \text{An}(Y, \mathcal{G}^s)$, $Z \longleftrightarrow X$.

When these conditions are satisfied, we have:

$$NDE(X_{t-\gamma}^{x,x'}, Y_t) = \sum_{\mathbb{Z}} \left(CDE(X_{t-\gamma}^{x,x'}, Y_t, \mathbb{Z}) \times \sum_{\mathbb{A}} \Pr(\mathbb{Z} = \mathbb{Z} \mid X_{t-\gamma} = x, \mathbb{A} = \mathbb{A}) \Pr(\mathbb{A} = \mathbb{A}) \right)$$

where $\mathbb{Z} = PP(Y_t, \mathcal{G}^s) \setminus \{X_{t-\gamma}\}$, $\mathbb{A} = PP(X_{t-\gamma}, \mathcal{G}^s)$ and a formula for $CDE(X_{t-\gamma}^{x,x'}, Y_t, \mathbb{Z})$ is given in Theorem 1.

Note that Condition 3 implies either having no self-loop on X in the SCG or choosing $\gamma = \gamma_{max}$. As an illustration, Figure 5(b) presents an SCG where $NDE(X_{t-\gamma_{max}}^{x,x'}, Y_t)$ is identifiable, though this identifiability does not hold when γ_{max} is replaced by $\gamma < \gamma_{max}$. In contrast, Figure 5(c) shows an SCG where $NDE(X_{t-\gamma}^{x,x'}, Y_t)$ is identifiable for all γ .

When there is no hidden confounding, *i.e.*, Assumption 1 is satisfied, it is trivial to check if the average natural indirect effect is identifiable from SCGs by checking if the total effect is identifiable (using Assaad et al., 2024, Theorem 2) and if the average controlled micro direct effect is identifiable using Theorem 1. It is still not completely clear how to identify the natural indirect effect in case there is hidden confounding because the problem of identifying the total effect from SCGs in case there is hidden confounding is yet to be tackled.

6 SIMULATION STUDY

The problems tackled in this article of identifying direct causal effects from summary causal graph is admittedly a complicated one. If the true FT-ADMG is known, then the do-calculus Pearl [1995], Shpitser and Pearl [2008], Huang and Valtorta [2006] is provably complete tool to identify controlled direct effects and the counterfactual calculus Correa and Bareinboim [2025] is a complete tool to identify natural direct effect. However, when the true FT-ADMG is unknown and only the SCG is available much information has been lost and thus it harder to identify the direct effect

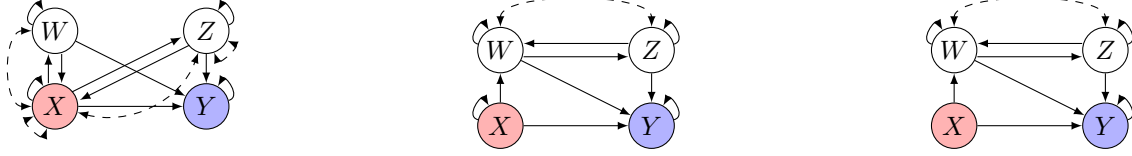


Figure 5: Three SCGs. Each pair of red and blue vertices represents the micro direct effect we are interested in. The controlled direct effect is identifiable in all these SCGs using the conditions given in this paper. The natural direct effect is not identifiable in (a), identifiable if $\gamma = \gamma_{max}$ in (b) and identifiable for all γ in (c).

of interest.

The two main results of this paper (Theorems 1 and 2) are sufficient but not necessary. This means that the conditions are somewhat too restrictive. Therefore, it is necessary to empirically investigate which proportion of direct effects are identifiable according to our Theorems.

For this simulation study we picked at random FT-ADMGs, computed their compatible SCG and chose randomly an outcome Y and a treatment $X_{t-\gamma}$. Then, we checked the conditions of Theorem 1 and Theorem 2 and reported the proportion of direct effects which were identifiable according to our results. We simulate FT-ADMGs with 2 to 15 times series and a maximum lag $\gamma_{max} = 2$ and the edges have a probability of existing between 1% and 15%. For each pair $n \in \llbracket 2, 15 \rrbracket$ and $p \in \llbracket 1\%, 15\% \rrbracket$ we generate 1000 different non-empty FT-ADMGs and report the results in Figure 6. We remove empty graphs as there is no direct effect to identify and we only consider pairs of treatment/outcome such that the treatment is a possible parent of the outcome in order to avoid artificially favoring our results with trivial identifications².

We would like to highlight that while the probability of edge existence seem low ($\leq 15\%$), it is used to generate the FT-ADMG and not the SCG. The density (ratio of edges on nodes) of a SCG is $\gamma_{max} + 1$ times lower in a FT-ADMG than in the compatible SCG. Moreover, under assumption 1 the conditions of Theorem 1 are necessary which is why we did not investigate the causal sufficiency case in this simulation study.

As expected, identifying natural direct effects is harder than controlled direct effects and for both type of direct effects the proportion of identifiable effects drops as the number of nodes grow or as the density of the SCG grow. However, even in the middle to higher range of parameters ($n = 10$, $p = 10$) some effects are identifiable. This simulation study should help the reader know in which range of parameters our theorems will be most useful. While promising, these results show the limit of our work and should motivate the research of complete characterization of identifiability from summary causal graphs. However, in real applications

²The code is available at <https://github.com/CIPHOD/pyCIPHOD/tree/main/reproducibility/uai2026>

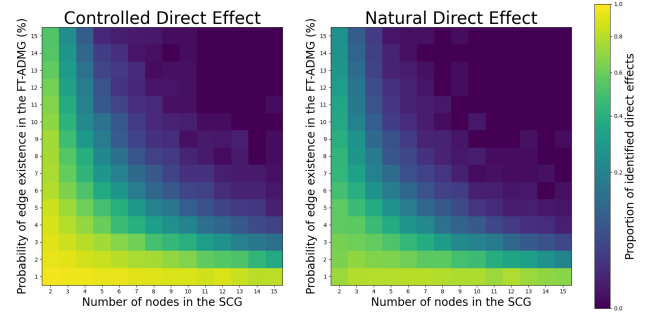


Figure 6: Proportion of effects identified from SCGs of different size and density.

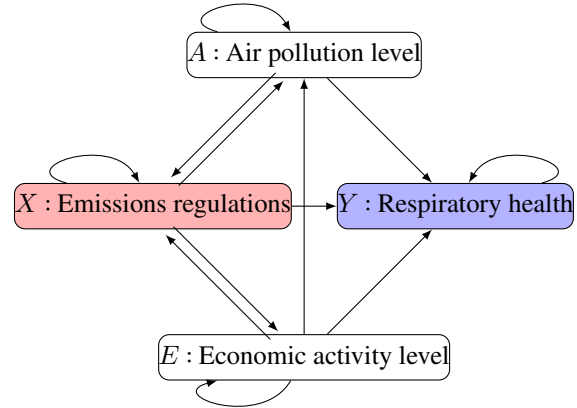


Figure 7: Example in epidemiology

graphs are not picked uniformly at random and thus the usefulness of our results in real world application remains to be investigated. This is the goal of the next section.

7 REAL WORLD EXAMPLE

In the following, we give one example in epidemiology (Figure 7) which illustrates a scenario where our theorem can be applicable.

Suppose we want to assess the effect of introducing stricter emissions regulations (X) in a city at time $t - 1$ on the respiratory health of residents (Y) at time t . There are at least two confounders between the treatment and outcome:

- Economic activity level (E): it affects both pollution levels (which, in turn, impact respiratory health) and the likelihood of implementing emissions regulations. The economic activity level can have a lagged causal relationship with the treatment, or potentially an instantaneous effect, depending on the sampling frequency.
- Air pollution level (A): it affects respiratory health outcomes and the likelihood of emissions regulations. Similarly to the first confounder, the relation between air pollution and X or Y can be instantaneous, depending on the sampling frequency.

At the same time, these two confounders can act as mediators between X and Y as regulations can effect the economy and the air pollution level. At the SCG level, if we disregard the time dimension, we obtain a cyclic relationship between each of the two confounders and the treatment. Therefore, these confounders complicate the estimation of the causal effect of emissions regulations on respiratory health.

Assuming that there are no other confounders, the micro CDE is identifiable using Theorem 1, however the micro NDE is not identifiable using Theorem 2. We emphasize that this does not imply that the NDE is not identifiable in general.

8 CONCLUSION

In this paper, we addressed both average controlled and average natural micro direct effects, providing theoretical results that extend existing work in causal inference using summary causal graphs. By accounting for cycles, hidden confounders and allowing for a non-parametric setting, our results contribute to a broader applicability of causal inference techniques in real-world settings where full causal specification is impractical. Moreover, it is important to note that contradictory to most results regarding identification in summary causal graphs, this work does not require any stationarity assumption. Our findings are particularly relevant in fields such as epidemiology, where accurate measurement of direct effects is crucial for informing public health interventions and policy decisions.

In future work, it will be important to validate whether our conditions hold when focusing on other forms of average controlled and average natural direct effects, such as those that emphasize mediators rather than parents. Additionally, it would be valuable to establish necessary and sufficient conditions for average controlled direct effects that extend beyond simple adjustment. Another future direction would be to derive necessary and sufficient conditions for identifying average controlled direct effects in the presence of hidden confounding, as well as for average natural direct effects. However, we believe that this is more challenging to achieve compared to average controlled direct effects without hidden confounding. Now that total and direct ef-

fects are well understood in the context of summary causal graphs, it would also be interesting to explore the identification of indirect effects and, more broadly, path-specific effects within summary causal graphs. Finally, we plan to apply the findings of this work to real-world applications, particularly in the field of epidemiology.

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