Computational Complexity of Repair Problems on Simple Temporal Networks with Uncertainty

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Abstract

Simple Temporal Networks with Uncertainty (STNUs) are a well-established formalism for reasoning about temporal plans involving uncontrollable durations, known as contingent links. This model enables checking the controllability of plans under different assumptions about when uncertainties are revealed (Weak, Dynamic, and Strong Controllability). Recent work has also introduced methods to repair uncontrollable STNUs by adjusting contingent bounds, which is relevant e.g., in multi-agent systems, or resource-aware settings where some external flexibility can be negotiated. Nevertheless, there remains a lack of formal understanding regarding the computational complexity of such repair problems. This paper fills that gap by formally defining the problems and assessing the complexity of a number of them. In particular, we show that for a number of settings, repair is no harder than controllability.

1 INTRODUCTION

In many domains, it is crucial to reason about activities that may overlap, span durations, or synchronize with timestamped events. This is especially true in planning and scheduling, where constraint-based models are widely used [Cesta et al., 2002, Verfaillie et al., 2010, Valentini et al., 2020, Bit-Monnot et al., 2020].

Simple Temporal Networks (STN) [Dechter et al., 1991] is a well-known framework for managing temporal constraints and checking temporal consistency of plans through propagation algorithms which prune inconsistent values. They have been extended to STNs with Uncertainty (STNU) [Vidal and Fargier, 1999] to represent uncertain (*contingent*) durations, redefining consistency as *controllability*: an STNU is controllable if a strategy exists for executing the plan, whatever values taken by the contingent durations. Three levels of controllability exist, depending on when the contingent durations are known: just before execution (*Weak*), during execution (*Dynamic*), or never (*Strong*).

A lot of attention has been devoted to checking controllability of STNUs [Sumic and Vidal, 2024, Hunsberger and Posenato, 2022, Vidal and Fargier, 1999], providing efficient constraint propagation algorithms, and less to repairing uncontrollable networks, since contingent durations are out of the control of the planning agent, hence non negotiable, making the repair hardly relevant.

However, the repair problem has recently attracted renewed interest, considering application domains where contingent links can be adjusted, possibly at some cost. The first case arises in single-agent systems where durations may derive from optimized upstream decisions, which might be revised at the cost of a reduced utility [Sumic et al., 2024a].

For instance, the uncertain duration of a satellite data download may depend on the (exogenous) size of the data: reducing the upper bound could mean sending incomplete or less detailed data, thus lowering plan quality. Similarly, in manufacturing cells, increasing production speed may reduce output quality (see a more detailed example in Section 4).

The most compelling case emerged in multi-agent planning where uncertainty may arise from other agent's decisions, which has been studied in the extended Multiple Interdependent STNUs model[Sumic et al., 2024b].

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For instance, the radiology department waits for a patient's surgery, performed by the emergency department, to finish. The duration of the surgery is hence uncertain for the former as it will be set by the latter, but it may be communicated at a time subject to the surgery department's policy. Then, in case of temporally non controllable plans, both services might negotiate changes in the pre-decided durations.

Overall, some works addressed the repair problem of STNUs for one agent [Sumic et al., 2024a, Hunsberger and Posenato, 2025] and for multiple agents [Sumic et al., 2024b]. Yet, no formal results exist on its computational complexity. Moreover, only limited optimality criteria and no distinctive repair capabilities among activities or agents have been considered so far. In this paper, we wish to address those two limitations, by introducing distinctive STNU repair problem variants and conducting an in-depth theoretical complexity analysis of those. One of our main findings is that for many variants, computing a repair is computationally no harder than deciding controllability.

Sections 2 and 3 provide the necessary background and discuss motivation and related work. The repair problems are then formally defined in Section 4; their complexity is analyzed in Sections 5 and 6.

2 BACKGROUND

A *Simple Temporal Network* (STN) is a tuple $\langle \mathcal{V}, E \rangle$: $\mathcal{V} = \{v_0, \ldots, v_n\}$ is a set of real-valued variables called *timepoints*, E is a set of temporal constraints between timepoints called *requirements* [Dechter et al., 1991]. Specifically, each requirement $e_k \in E$ is of the form $v_j - v_i \in [l, u]$, also written $v_i \xrightarrow{[l,u]} v_j$, with $v_i, v_j \in \mathcal{V}$, $l \in \mathbb{R} \cup \{-\infty\}$, and $u \in \mathbb{R} \cup \{+\infty\}$. Bounds l and u represent the minimal and maximal possible temporal distance between v_i and v_j.

An STN is *consistent* if it has a *solution*, called a *schedule*, $\delta : \mathcal{V} \to \mathbb{R}$ which satisfies all the constraints, that is, $\delta(v_j) - \delta(v_i) \in [l, u]$ holds for all requirements $v_i \xrightarrow{[l,u]} v_j \in E$.

Dechter et al. [1991] show that an STN $\mathcal{X}$ is consistent if and only if its *distance graph* $G_{\mathcal{X}}$ contains no negative cycle, which can be detected in polynomial time using shortest-path algorithms. The distance graph of $\langle \mathcal{V}, E \rangle$ is the directed, valued graph with $\mathcal{V}$ as vertices, $v_i \xrightarrow{u} v_j$ and $v_j \xrightarrow{-l} v_i$ for each requirement $v_i \xrightarrow{[l,u]} v_j \in E$ as edges.

An STN with Uncertainty (STNU) is an STN augmented with additional *contingent links* between timepoints: their duration is decided by an external entity (*Nature*) and the controller can only observe them [Vidal and Fargier, 1999]. Formally, an STNU $\mathcal{X} = \langle \mathcal{V}, E, C \rangle$ is a tuple where $\langle \mathcal{V}, E \rangle$ is an STN; $\mathcal{V}$ is partitioned into controllable ($\mathcal{V}_c$) and uncontrollable ($\mathcal{V}_u$) timepoints; and C is a set of contingent links c_k each of the form $v_j - v_i \in [l, u]$, also written $v_i \overset{[l,u]}{\dashrightarrow} v_j$, with $v_i \in \mathcal{V}$, $v_j \in \mathcal{V}_u$, and $0 \leqslant l \leqslant u$. The duration $\omega_k = v_j - v_i$ is set by an external entity before or during the execution: once v_i is executed, the value of v_j is $v_i + \omega_k$. A *schedule* for an STNU is a mapping $\delta \colon \mathcal{V}_c \to \mathbb{R}$.

A setting (by Nature) of a duration for each contingent constraint defines a *situation* ω, and induces an STN since it instantiates all uncertain durations.

Definition 1 (Situation, Projection)**.** *A* situation *for an STNU $\mathcal{X} = \langle \mathcal{V}, E, C \rangle$ is a function $\omega : C \to \mathbb{R}$ satisfying $\omega(v_i \overset{[l,u]}{\dashrightarrow} v_j) \in [l, u]$. The* projection *of $\mathcal{X}$ onto ω is the STN $\mathcal{X}_\omega := \langle \mathcal{V}, E \cup C_\omega \rangle$, with $C_\omega := \{v_i \xrightarrow{[\omega(c), \omega(c)]} v_j \mid c = v_i \overset{[l,u]}{\dashrightarrow} v_j \in C\}$.*

A situation ω and a schedule δ for an STNU $\mathcal{X}$ together uniquely define the *time of occurrence* $t_{\delta,\omega}(v)$ of each timepoint v: $t_{\delta,\omega}(v_0) := 0$; $t_{\delta,\omega}(v) := \delta(v)$ if v is controllable; and $t_{\delta,\omega}(v) := t_{\delta,\omega}(v') + \omega(c)$ if v is uncontrollable with incoming contingent link $c = v' \overset{[l,u]}{\dashrightarrow} v$.

Three *controllability levels* are defined by Vidal and Fargier [1999]. *Strong Controllability* (SC) requires a single schedule to be a solution of $\mathcal{X}_\omega$ for all situations ω, which is relevant when external events durations cannot be observed (conformant planning). *Weak Controllability* (WC) requires a solution of $\mathcal{X}_\omega$ to exist for each situation ω, which is relevant when all contingent durations are known before execution starts (oracle). In-between, *Dynamic Controllability* (DC) requires schedules consisting of sequential decisions that may differ according to previous observations but do not depend on future observations.

Definition 2 (Strong Controllability)**.** *An STNU $\mathcal{X}$ is said to be* strongly controllable *(SC) if there exists a schedule δ, called a* strong schedule*, such that for all situations ω, δ is a solution of $\mathcal{X}_\omega$.*

Definition 3 (Weak Controllability)**.** *An STNU $\mathcal{X}$ is said to be* weakly controllable *(WC) if for all situations ω, there exists a solution δ_ω of $\mathcal{X}_\omega$.*

Definition 4 (Dynamic Controllability)**.** *An STNU $\mathcal{X}$ is said to be* dynamically controllable *(DC) if there exists a solution δ_ω of $\mathcal{X}_\omega$ for all situations ω, and for all ω_1, ω_2, v, t the following holds: if $\delta_{\omega_1}(v) = t$ and for all timepoints v', $t_{\delta_{\omega_1},\omega_1}(v') < t \Leftrightarrow t_{\delta_{\omega_2},\omega_2}(v') < t$ and $t_{\delta_{\omega_1},\omega_1}(v') = t' < t \Rightarrow t_{\delta_{\omega_2},\omega_2}(v') = t'$, then $\delta_{\omega_2}(v) = t$.*

Dynamic and Strong Controllability have been shown to be decidable in polynomial time [Vidal and Fargier, 1999, Hunsberger and Posenato, 2022], whereas deciding Weak Controllability is coNP-complete [Morris and Muscettola, 1999]. Algorithms for DC and WC leverage the search for negative cycles in the *distance graph* of the STNU [Hunsberger and Posenato, 2022, Sumic et al., 2024a].

3 MOTIVATION AND RELATED WORK

The first notion of repair emerged from the idea of relaxation, where requirement bounds are adjusted to recover Dynamic Controllability, or to maintain DC while optimising these bounds [Cui et al., 2015, Wah and Xin, 2004].

Further work by Akmal et al. [2020] introduced quantitative metrics to measure the degree of non-controllability in STNUs. They also proposed an incomplete Linear Programming-based method to repair non-DC STNUs by resolving interdependent negative cycles.

The repair problem was formally defined by Sumic et al. [2024a], using Satisfiability Modulo Theory (SMT) solvers to repair non-WC/SC STNUs by minimizing contingent reductions. These methods exhibit scalability limitations due to the sensitivity of SMT solving to the number of contingencies.

The first mention of repair in multi-agent settings was by Casanova et al. [2016], introducing the Multi-agent STNU (MaSTNU) model, where agents have independent plans but share exogenous contingent constraints. They proposed a Mixed-Integer Linear Programming (MILP) formulation for DC checking and suggested that the same formulation could be adapted for repair by tightening contingent links.

In all of the above approaches, the actual relevance of repairing a contingent link was not fully motivated, since as already mentioned uncertainty is assumed to be exogenous (set by *Nature*), and hence cannot be changed.

The recent Multiple Interdependant STNUs (MISTNU) model changed the game, extending STNUs to multi-agent paradigms [Sumic et al., 2024b], introducing the notion of a *contract* as a duration controlled by one agent but contingent for (compliant) agents that depend on that duration. Non-controllable compliant agents can now repair their plan by negotiating these inter-agent contingencies, resulting in agents sharing their temporal flexibility.

Broadly, two approaches to repair exist in multi-agent systems. The centralized approach merges local repair problems into a global formulation, typically expressed as a first-order formula or a system of linear inequations, as done by Casanova et al. [2016], Sumic et al. [2024b]. In such cases, the problem remains computationally equivalent to repairing a single STNU, since the solution is computed centrally with complete information. The distributed case is more challenging as agents must first compute repair solutions locally and then reach agreement on the bounds of contingent constraints, which may depend on some optimization criteria (e.g., communication cost, temporal flexibility) and/or the nature of agent interactions (cooperative vs. selfish).

Anyway, the relevance of single-agent repair has been overlooked. As mentioned, we claim that such cases exist and need to be considered. All in all, all those contexts raise a number of new and challenging ways of defining the repair problem under optimization criteria. Those claims will be further illustrated and developed in the next sections.

4 REPAIR PROBLEMS

We now formally define the repair problems.

Definition 5 (Tightening)**.** *A* tightening *of an STNU* $\mathcal{X} = \langle \mathcal{V}, E, C \rangle$ *is a function* $\rho : C \to \mathbb{R}^2$ *such that for all* $c = v_i \overset{[l,u]}{\dashrightarrow} v_j$*, if* $\rho(c) = (l', u')$*, then* $l \leqslant l' \leqslant u' \leqslant u$ *holds.*

For consistency, we write $\rho(c) = [l', u']$, and also abuse notation by using $\rho(c)$ for $v_i \overset{[l',u']}{\dashrightarrow} v_j$. Finally, we write $\mathcal{X} \oplus \rho$ for the STNU $\langle \mathcal{V}, E, C_\rho \rangle$, with $C_\rho := \{\rho(c) \mid c \in C\}$.

A special case of tightenings is *single-value tightenings* over $C' \subseteq C$, which satisfy $\rho(c) = [b_c, b_c]$ for all $c \in C'$.

Definition 6 (Repair)**.** *Let* $\tau \in \{\mathsf{S}, \mathsf{D}, \mathsf{W}\}$ *be a consistency level. A* τ-repair *of an STNU* $\mathcal{X}$ *is a tightening of* $\mathcal{X}$ *such that* $\mathcal{X} \oplus \rho$ *is* τ*-controllable.*

Definition 7 (Repair Problem)**.** *Let* $\tau \in \{\mathsf{S}, \mathsf{D}, \mathsf{W}\}$ *be a controllability level.* τ-REPAIR *is the decision problem:*

Input: *An STNU* $\mathcal{X}$
Question: *Is there a* τ*-repair* ρ *of* $\mathcal{X}$*?*

Definition 7 is the basic formulation of a satisfaction problem. However, this may not be sufficient in more realistic settings. Depending on the environment (such as the influence of *Nature* or other agents) or specific optimization criteria and performance metrics relevant to the agent's objectives, more demanding definitions may be required.

In order to define such refined definitions, we first introduce additional notation. Let $\mathcal{X}$ be an STNU and ρ be a tightening of $\mathcal{X}$. We write

- $\mathrm{Supp}(\rho)$ for the set $\{c \in C \mid \rho(c) \neq c\}$, i.e., the set of constraints that are actually (strictly) repaired by ρ;
- $\mathrm{Cost}(\rho)$ for the quantity $\sum_c (l'_c - l_c) + (u_c - u'_c)$ (writing $c = v_i \overset{[l_c,u_c]}{\dashrightarrow} v_j$ and $\rho(c) = [l'_c, u'_c]$), i.e., the sum of interval reductions over all contingent constraints.

We introduce three more refined repair problems (in all definitions, τ is any controllability level).

Definition 8. PARTIAL-τ-REPAIR *is defined by:*

Input: *An STNU* $\mathcal{X} = \langle \mathcal{V}, E, C \rangle$ *and* $R \subseteq C$
Question: *Is there a* τ*-repair* ρ *of* $\mathcal{X}$ *with* $\mathrm{Supp}(\rho) \subseteq R$*?*

The partial repair problem is relevant when only a subset of contingent durations can be reduced, as with multiple agents when it is not possible to negotiate with some of them (due to communication or privacy issues), or even for a single agent when some contingent durations depend on external (*Nature*) conditions, while others can be tailored if needed, by changing the speed of a moving robot for instance.

Definition 9. k-BUDGET-τ-REPAIR *is defined by:*

Input: *An STNU $\mathcal{X}$ and a rational number k*
Question: *Is there a τ-repair ρ of $\mathcal{X}$ with* $\mathrm{Cost}(\rho) \leqslant k$*?*

The *k-budget* repair problem is particularly relevant in scenarios where optimizing a specific parameter is important, such as minimizing cost [Cui et al., 2015, Wah and Xin, 2004], or maximizing flexibility or fairness among agents in multi-agent settings [Sumic et al., 2024b].

Definition 10. k-CONSTRAINT-τ-REPAIR *is defined by:*

Input: *An STNU $\mathcal{X}$ and an integer k*
Question: *Is there a τ-repair ρ of $\mathcal{X}$ with* $|\mathrm{Supp}(\rho)| \leqslant k$*?*

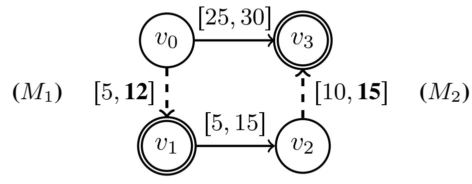
The *k-constraint* repair problem is also relevant for optimization in multi-agent settings, where minimizing communication costs amounts to looking for the smallest set of contingent links to negotiate.

To better illustrate the STNU model and the different repair problems, we provide a realistic single-agent example below, developed from the case raised in the introduction.

Example 1. *Bob operates on a production line involving two machines, M_1 and M_2. Each day, a manager determines the production requirements following predefined regulations. The process begins with M_1 transforming raw materials into components. Bob then performs a quality check before passing the components to M_2, which completes the final product. The durations of tasks executed by M_1 and M_2 are uncertain, ranging from 5–12 minutes and 10–15 minutes, respectively, while Bob's task is controllable and takes between 5–15 minutes. The overall process must be completed within a time window of 25 to 30 minutes. To guarantee feasibility, the manager may adjust the machines' speed before execution, but at a cost.*

Figure 1 illustrates Bob's scheduling problem. The STNU is not weakly controllable, and several W-repairs are shown with new bounds on M_1 and M_2. We show: in the first column, a non-optimized repair; in the second one, a partial repair when only M_1 can be modified; in the third one, a repair when only reducing by no more than 2 units in total is allowed; in the last one, a repair when no more than one contingent duration can be altered (no matter which one).

The next two sections provide complexity results for the different types of repair problem (Definitions 7 to 10) according to the semantics of the three controllability levels. Our results are summarized in Table 1.



c_k	Repair	Partial R. (M_1)	2-Budget R.	1-Cons. R.
M_1	**[10, 10]**	**[7, 7]**	**[5, 11]**	[5, 12]
M_2	**[10, 10]**	[10, 15]	**[10, 14]**	**[10, 10]**

Figure 1: The STNU in Example 1 where nodes represent time-points (doubly circled ones are uncontrollable), edges are requirements and contingent constraints (the later relating to the tasks of machines M_1 and M_2). The STNU is not WC due to the highlighted projection $\omega = \{\omega_1 = 12, \omega_2 = 15\}$. The table highlights a solution to each type of repair problem.

Problem	Strong	Dynamic	Weak
Controllability	P	P	coNP-comp.
Repair	P	P	P
Partial repair	P	in NP	coNP-comp.
k-budget repair	P	in NP	coNP-comp.
k-constraint repair	NP-comp.	NP-comp.	Σ_2^{P}-comp.

Table 1: Complexity of controllability and repair problems (where "comp" is short for "complete").

5 MEMBERSHIP RESULTS

We begin with the membership results in Table 1 by giving several general and useful lemmas.

The first one is the intuitive result that tighter bounds on the contingent links can only make an STNU more controllable; in particular, if we can repair an STNU, then we can *a fortiori* repair it with a single-value tightening. Notice that each single-value tightening corresponds to a situation ω: the repaired STNU is the projection $\mathcal{X}_\omega$, which is an STN.

Lemma 1. *Let $\tau \in \{\mathsf{S}, \mathsf{D}, \mathsf{W}\}$ be a controllability level. If an STNU $\mathcal{X}$ has a τ-repair with support C', then it also has a single-value τ repair over C'.*

Proof. By Definitions 2 to 4, tightenings conserve τ-controllability: if an STNU is controllable for some set of situations Ω, then the STNU remains controllable for every $\Omega' \subseteq \Omega$, in particular for every $\{\omega\} \subset \Omega$. $\square$

Lemma 2. *For every $\tau \in \{\mathsf{S}, \mathsf{D}, \mathsf{W}\}$,* PARTIAL-$\tau$-REPAIR *can be reduced to* k-BUDGET-τ-REPAIR *by a polynomial-time many-one reduction.*

Proof. Let $\langle \mathcal{X}, R \rangle$ be an instance of PARTIAL-τ-REPAIR, with $\mathcal{X} := \langle V, E, C \rangle$. Define the instance $\langle \mathcal{X}', k \rangle$ of k-BUDGET-τ-REPAIR as follows (see an example on Figure 2). First, $\mathcal{X}' := \langle V', E', C' \rangle$ is defined by $V' := V \cup \{v^c \mid c \in R\}$, $E' := E \cup \{v^c \xrightarrow{[0,0]} v' \mid c = v \overset{[l,u]}{\dashrightarrow} v' \in R\}$, and

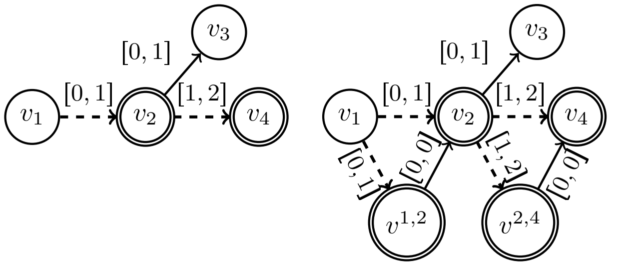


Figure 2: Construction of a new STNU in the reduction from PARTIAL-τ-REPAIR to k-BUDGET-τ-REPAIR.

$C' = C \cup \{v \overset{[l,u]}{\dashrightarrow} v^c \mid c = v \overset{[l,u]}{\dashrightarrow} v' \in R\}$. In other words, each $c = v \overset{[l,u]}{\dashrightarrow} v' \in R$ is duplicated into a path from v to v', consisting of a contingent link with the same bounds followed by a requirement to synchronize the auxiliary time point v^c with v'. Second, the budget k is defined to be $2 \times \sum\{u - l \mid v \overset{[l,u]}{\dashrightarrow} v' \in R\}$.

If $\mathcal{X}$ has a τ-repair ρ with $\mathrm{Supp}(\rho) \subseteq R$, then choose ρ to be single-value over $\mathrm{Supp}(\rho)$ (Lemma 1). Define the repair of $\mathcal{X}'$ to map each $c \in R$ and its "duplicate" $v \overset{[l,u]}{\dashrightarrow} v^c$ to $\rho(c)$. Then clearly this is a k-budget τ-repair of $\mathcal{X}'$.

Conversely, if ρ is a τ-repair of $\mathcal{X}'$ with cost at most k, then ρ must tighten each $c \in R$ and its duplicate to the same singleton interval $[b_c, b_c]$: otherwise, the network would not be controllable due to the required synchronicity between v^c and v'. Hence, ρ must spend the whole budget of k on the contingent links in R and their duplicates; it follows that no other constraint is repaired. Therefore, tightening each $c \in R$ to $[b_c, b_c]$ is a τ-repair of $\mathcal{X}$ with R as support. □

5.1 UNCONSTRAINED REPAIR

We first study the unconstrained repair problem τ-REPAIR, i.e., without any optimization criterion. Intuitively, a single-value repair, if far from being optimal, is enough for repairability.

Proposition 1. *The problems* S-REPAIR*,* W-REPAIR*, and* D-REPAIR *are all solvable in polynomial time.*

Proof. Given an STNU $\mathcal{X} = \langle \mathcal{V}, E, C \rangle$, consider the STN $\mathcal{X}' = \langle \mathcal{V}, E \cup C \rangle$, i.e., contingent links in $\mathcal{X}$ are all regarded as requirements in $\mathcal{X}'$. For $\tau \in \{\mathsf{S}, \mathsf{D}, \mathsf{W}\}$, we show that $\mathcal{X}$ is τ-repairable if and only if $\mathcal{X}'$ is consistent, which can be checked in polynomial time [Dechter et al., 1991].

If $\mathcal{X}$ is τ-repairable, let ρ be a single-value τ-repair (Lemma 1). Then ρ corresponds to a situation ω such that the STN $\mathcal{X}_\omega$ is consistent. A schedule for $\mathcal{X}_\omega$ also satisfies all the constraints in $\mathcal{X}'$, hence $\mathcal{X}'$ is consistent.

Now if δ is a schedule for $\mathcal{X}'$, let ρ be the single-value tightening of $\mathcal{X}$ defined by $\rho(c) = [\delta(v_j) - \delta(v_i), \delta(v_j) - \delta(v_i)]$ for each contingent $c = v_i \overset{[l,u]}{\dashrightarrow} v_j$ of $\mathcal{X}$. Then δ is a schedule for the STN $\mathcal{X} \oplus \rho$, hence $\mathcal{X}$ is τ-repairable. □

Remark 1. *Since a single-value tightening completely removes all the contingencies in an STNU, the problem* τ-REPAIR *is easy to solve regardless of which* τ *we choose. Inspired by this, one may be tempted to introduce a variant of* τ-REPAIR *in which the duration of each contingent link cannot be reduced to be shorter than some* ε *given in the input. However, it can be shown that this* ε*-preserving* τ-REPAIR *is a special case of* PARTIAL-τ-REPAIR.

Indeed, for an STNU, we can replace every contingent link $v_i \overset{[l,u]}{\dashrightarrow} v_j$ *by a repairable link* $v_i \overset{[l,u]}{\dashrightarrow} v'_j$ *followed by a non-reparable link* $v'_j \overset{[0,\varepsilon]}{\dashrightarrow} v_j$*. It is straightforward to verify that for the same controllability level, the new STNU is partially repairable if and only if the original STNU is* ε*-preserving repairable.*

5.2 REPAIR FOR STRONG CONTROLLABILITY

We now show that k-CONSTRAINT-S-REPAIR is in NP while PARTIAL-S-REPAIR and k-BUDGET-S-REPAIR are solvable in polynomial time, based on a technique of checking Strong Controllability by solving a system of linear inequations encoding an STNU [Vidal and Fargier, 1999].

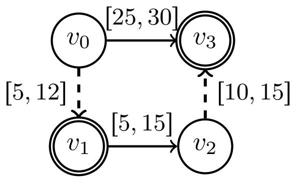
For a requirement $v_i \xrightarrow{[l,u]} v_j$, there are three cases in which an uncontrollable timepoint is involved: (1) $v_j \in \mathcal{V}_u$, (2) $v_i \in \mathcal{V}_u$, and (3) $v_i, v_j \in \mathcal{V}_u$. A requirement in case (1) can be encoded by the linear inequation $l \leqslant (v_k + \omega_j) - v_i \leqslant u$, where ω_j represents the uncontrollable duration of the contingent link with v_j as endpoint ($v_k \overset{[l',u']}{\dashrightarrow} v_j$). Other cases can be encoded similarly, resulting in a system of linear inequations of polynomial size.

By Definition 2, a strong schedule must satisfy this system regardless of the durations $\omega_1, \ldots, \omega_n$. Vidal and Fargier [1999] show that for a schedule to be a strong schedule, it suffices for this schedule to satisfy the system when each ω_i takes one of its worst possible values, which are the lower and upper bounds of the corresponding contingent link. For example, to satisfy $l \leqslant (v_k + \omega_j) - v_i \leqslant u$ for every $\omega_j \in [l', u']$, it suffices to satisfy $l \leqslant (v_k + l') - v_i$ and $(v_k + u') - v_i \leqslant u$. Figure 3 (bottom left and bottom right) illustrates this construction for the STNU in Figure 1.

Proposition 2. PARTIAL-S-REPAIR *and* k-BUDGET-S-REPAIR *are solvable in polynomial time.*

Proof. We modify the construction of Vidal and Fargier [1999] as follows:

- For partial repairability: for each numeric value corresponding to a lower (resp. upper) bound of a *repairable* contingent link $c = v \overset{[l,u]}{\dashrightarrow} v'$ in the linear program



$$\begin{cases} 25 \leqslant (v_2 + \mathbf{L}) - v_0 \\ (v_2 + \mathbf{U}) - v_0 \leqslant 30 \\ 5 \leqslant v_2 - (v_0 + \mathbf{12}) \\ v_2 - (v_0 + \mathbf{5}) \leqslant 15 \\ 10 \leqslant L \leqslant U \leqslant 15 \end{cases}$$

$$\begin{cases} 25 \leqslant (v_2 + \omega_2) - v_0 \\ (v_2 + \omega_2) - v_0 \leqslant 30 \\ 5 \leqslant v_2 - (v_0 + \omega_1) \\ v_2 - (v_0 + \omega_1) \leqslant 15 \end{cases} \qquad \begin{cases} 25 \leqslant (v_2 + \mathbf{10}) - v_0 \\ (v_2 + \mathbf{15}) - v_0 \leqslant 30 \\ 5 \leqslant v_2 - (v_0 + \mathbf{12}) \\ v_2 - (v_0 + \mathbf{5}) \leqslant 15 \end{cases}$$

Figure 3: LP for Strong Controllability of the STNU in Figure 1. The bottom left group of inequations is the abstract view; the bottom right group (with the instantiated worst bounds in boldface) is an LP the solutions of which are strong schedules; the top right group is an LP encoding PARTIAL-S-REPAIR when only constraint M_2 can be repaired.

from the construction of Proposition 2 (e.g., Figure 3, bottom right), we replace it by a new variable L (resp. U) and we add the linear inequation $l \leqslant L \leqslant U \leqslant u$ (e.g., Figure 3, top right).

- For k-budget repairability: we similarly replace the numeric values of *all* contingent constraints by new variables and add new inequations of the form $l \leqslant L \leqslant U \leqslant u$, and additionally this linear inequation:

$$\sum \{(L - l) + (u - U) \mid v \overset{[l,u]}{\dashrightarrow} v' \in C\} \leqslant k. \quad (1)$$

In both cases, the linear program (LP) thus obtained is of polynomial size. It follows from the same argument as Vidal and Fargier [1999] that the solutions to the modified LP are in one-to-one correspondence with the pairs (ρ, δ) such that ρ is a S-repair (with the required property) for $\mathcal{X}$ and δ is a strong schedule for $\mathcal{X} \oplus \rho$. □

Corollary 1. k-CONSTRAINT-S-REPAIR *is in* NP.

Proof. We first guess a set of k constraints to repair, then solve PARTIAL-S-REPAIR with this set of constraints, which is in P by Proposition 2. □

5.3 REPAIR FOR WEAK CONTROLLABILITY

Proposition 3. k-BUDGET-W-REPAIR *is in* coNP.

Proof. We prove this by providing an NP algorithm to solve the complement of k-BUDGET-W-REPAIR with the help of Lemma 3 below. Let $\mathcal{X} = \langle \mathcal{V}, E, C \rangle$ be an STNU. For every $c \in C$ with bounds $[l_c, u_c]$, let $[L_c, U_c] \subseteq [l_c, u_c]$. Then the function ρ defined for all $c \in C$ by $\rho(c) := [L_c, U_c]$ is a tightening of $\mathcal{X}$.

Lemma 3. *Weak Controllability of the repaired STNU $\mathcal{X} \oplus \rho$ is equivalent to the satisfiability of an exponential number (in the size of the input $\mathcal{X}$) of linear inequations involving constants from $\mathcal{X}$ and bound variables L_c, U_c for every c.*

Proof. Let $\Omega' := \times_{c \in C} \{L_c, U_c\}$, which is finite (of size $2^{|C|}$). Vidal and Fargier [1999, Property 4.2] show that $\mathcal{X} \oplus \rho$ is weakly controllable if and only if for every $\omega \in \Omega'$, $(\mathcal{X} \oplus \rho)_\omega$ is a consistent STN, i.e., its distance graph G has no cycle of negative length. Since the length of a cycle is a sum of some constants from $\mathcal{X}$ and bound variables, this means the Weak Controllability of $\mathcal{X} \oplus \rho$ is equivalent to the satisfiability of a set of linear inequations involving such constants and bound variables, one inequation for every pair of $\omega \in \Omega'$ and a minimal cycle (i.e., a cycle containing no strictly smaller cycle) in G. Hence, the number of linear inequations is exponential in the size of the input $\mathcal{X}$. Actually, by the same argument from the proof of Proposition 2 (cf. Figure 3, bottom left and bottom right), one inequation per minimal cycle suffices. □

Now we present an NP algorithm which decides whether a given $\langle \mathcal{X}, k \rangle$ is a *negative* instance of k-BUDGET-W-REPAIR by exploiting Lemma 3.

Notice that k-budget W-repairs for $\mathcal{X}$ are solutions to the LP consisting of: the variables L_c and U_c for every $c \in C$; inequation (1) representing the budget constraint; one inequation for every minimal cycle from Lemma 3.

$\langle \mathcal{X}, k \rangle$ is a negative instance if and only if this LP has no solution, meaning that the half-spaces in $\mathbb{R}^{2|C|}$ defined by these inequations have an empty intersection. By Helly's theorem [Danzer et al., 1963], there is a subcollection of $n = 2|C| + 1$ half-spaces with an empty intersection.

Consider now the following algorithm. First, it guesses $2|C| + 1$ inequations among the budget constraint and the cycle constraints (the number of which is exponential in the size of the input $\mathcal{X}$, hence the guessing is in polynomial time). Then it checks that the sub LP with these $2|C| + 1$ inequations has no solution, which can be performed in polynomial time. Therefore, this is an NP algorithm for solving the complement of k-BUDGET-W-REPAIR. □

Corollary 2. PARTIAL-W-REPAIR *is in* coNP *and* k-CONSTRAINT-W-REPAIR *is in* Σ_2^{P}.

Proof. By Proposition 3 and Lemma 2, PARTIAL-W-REPAIR is in coNP. To solve k-CONSTRAINT-W-REPAIR in Σ_2^{P}, we can first guess a set of k constraints to repair, then solve PARTIAL-W-REPAIR with a coNP algorithm. □

5.4 REPAIR FOR DYNAMIC CONTROLLABILITY

Proposition 4. k-BUDGET-D-REPAIR *is in* NP.

Proof. Cui et al. [2015] show that k-BUDGET-D-REPAIR can be formulated as a mixed-integer linear program, which is solvable in NP [Papadimitriou, 1981].[1] □

Corollary 3. PARTIAL-D-REPAIR *and* k-CONSTRAINT-D-REPAIR *are in* NP.

Proof. By Proposition 4 and Lemma 2,PARTIAL-D-REPAIR is in NP. k-CONSTRAINT-D-REPAIR can be solved in NP by first guessing a set of k constraints to repair then solving PARTIAL-D-REPAIR with this set of constraints. □

General Linear Cost for Repairs We define k-BUDGET-τ-REPAIR with a uniform cost function (tightening costs the same for every contingent link). However, the same upper bounds from Propositions 2 to 4 hold for a more general version of k-BUDGET-τ-REPAIR with respect to an arbitrary but given cost function which is linear in the repaired bounds: it suffices to replace the linear constraint related to the budget in the proofs. Similarly, a more general version of k-CONSTRAINT-τ-REPAIR with an arbitrary additive cost function has the same upper bounds (Corollaries 1 to 3).

6 HARDNESS RESULTS

We now prove the hardness results in Table 1. Note that no NP-hardness result is provided for PARTIAL-D-REPAIR or k-BUDGET-D-REPAIR.

Remark 2. *Wah and Xin [2004] proved the* NP*-hardness of maximally reducing requirement bounds (hence making the STNU less controllable) while maintaining DC. We know no polynomial-time reduction from their setting to ours, which is minimally tightening contingent bounds (hence making the STNU more controllable) to recover DC. Hence, their NP-hardness result does not seem to give a proof of the NP-hardness of* k-BUDGET-D-REPAIR. *On the other hand, we establish the NP-hardness of* k-CONSTRAINT-D-REPAIR, *the equivalent of which has not been studied by them.*

Proposition 5. *Both* k-CONSTRAINT-S-REPAIR *and* k-CONSTRAINT-D-REPAIR *are* NP*-hard.*

Proof. We give a reduction from SUBSET-SUM, which is defined as follows and known to be NP-complete [Garey and Johnson, 1979, Sec. A3.2].

Input: A multiset of strictly positive integers $\{\{n_1, \ldots, n_\ell\}\}$ and an integer N
Question: Is there $I \subseteq \{1, \ldots, \ell\}$ satisfying $N = \sum_{i \in I} n_i$?

[1]Cui et al. [2015] also allow tightenings of requirements and an arbitrary objective function given as input which is linear in the bound variables of the requirement and contingent links.

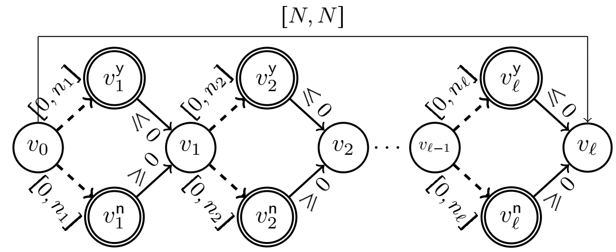


Figure 4: The STNU $\mathcal{X}_S$ for an instance S of SUBSET-SUM.

Given $S = \langle \{\{n_1, \ldots, n_\ell\}\}, N \rangle$, we define an STNU $\mathcal{X}^S$ as follows (see Figure 4):[2]

- the sets of controllable and uncontrollable timepoints are $V_c^S := \{v_0, v_1, \ldots, v_\ell\}$ and $V_u^S := \{v_1^{\mathsf{y}}, \ldots, v_\ell^{\mathsf{y}}\} \cup \{v_1^{\mathsf{n}}, \ldots, v_\ell^{\mathsf{n}}\}$, with v_0 as the reference timepoint;
- the set of requirement constraints is $E^S = \{e^N\} \cup \{e_1^{\mathsf{y}}, \ldots, e_\ell^{\mathsf{y}}\} \cup \{e_1^{\mathsf{n}}, \ldots, e_\ell^{\mathsf{n}}\}$, with $e^N = v_0 \xrightarrow{[N,N]} v_\ell$, $e_i^{\mathsf{y}} = v_i^{\mathsf{y}} \xrightarrow{[-\infty,0]} v_i$, and $e_i^{\mathsf{n}} = v_i^{\mathsf{n}} \xrightarrow{[0,+\infty]} v_i$;
- the set of contingent constraints is given by $C^S = \{c_1^{\mathsf{y}}, \ldots, c_\ell^{\mathsf{y}}\} \cup \{c_1^{\mathsf{n}}, \ldots, c_\ell^{\mathsf{n}}\}$, with $c_i^{\mathsf{y}} = v_{i-1} \xrightarrow{[0,n_i]} v_i^{\mathsf{y}}$ and $c_i^{\mathsf{n}} = v_{i-1} \xrightarrow{[0,n_i]} v_i^{\mathsf{n}}$.

Clearly, $\mathcal{X}^S$ can be constructed in polynomial time given S. We now show that (1) S is a positive instance of SUBSET-SUM if and only if (2) $\mathcal{X}^S$ is ℓ-constraint strongly repairable if and only if (3) $\mathcal{X}^S$ is ℓ-constraint dynamically repairable, which will allow us to conclude the NP-hardness of both k-CONSTRAINT-S-REPAIR and k-CONSTRAINT-D-REPAIR.

To show (1) implies (2), assume that I is a solution to S, i.e., $N = \sum_{i \in I} n_i$. We define the ℓ-constraint tightening ρ^I by $\rho^I(c_i^{\mathsf{y}}) := [n_i, n_i]$ for $i \in I$ and $\rho^I(c_i^{\mathsf{n}}) := [0, 0]$ for $i \notin I$. Clearly, $|\mathrm{Supp}(\rho^I)| = \ell$. It is easy to see that the schedule defined by $\delta(v_i) = \sum_{j \leqslant i, j \in I} n_j$ for $i = 0, \ldots, \ell$ (in particular, $\delta(v_0) = 0$) is a strong schedule for $\mathcal{X}^S \oplus \rho^I$. Hence, ρ^I is a ℓ-constraint S-repair of $\mathcal{X}^S$.

(2) trivially implies (3) since Strong Controllability implies Dynamic Controllability by definition.

To show (3) implies (1), assume that ρ is a ℓ-constraint D-repair of $\mathcal{X}^S$, with δ_ω as a solution for each situation ω. For $1 \leqslant i \leqslant \ell$ and every situation ω in $\mathcal{X}^S \oplus \rho$, δ_ω must satisfy

$$\delta_\omega(v_{i-1}) + \omega(c_i^{\mathsf{n}}) \leqslant \delta_\omega(v_i) \leqslant \delta_\omega(v_{i-1}) + \omega(c_i^{\mathsf{y}}). \quad (2)$$

In particular, $\omega(c_i^{\mathsf{n}}) \leqslant \omega(c_i^{\mathsf{y}})$ must hold for every ω. Hence, ρ must repair at least one of c_i^{y} and c_i^{n}. Given the budget of ℓ constraints, ρ must repair exactly one for each $i = 1, \ldots, \ell$. In addition:

- If ρ repairs c_i^{y}, then there is a situation ω such that

[2]In Figure 4, $\leqslant 0$ and $\geqslant 0$ stand for $]-\infty, 0]$ and $[0, +\infty[$, respectively.

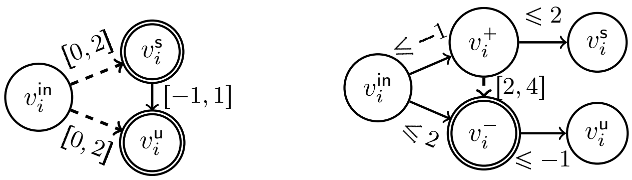


Figure 5: STNU $\mathcal{X}_i$ for each variable x_i (left) and y_i (right)

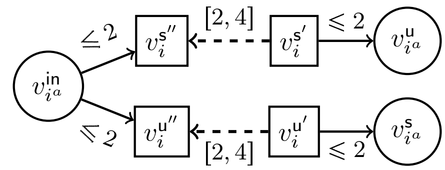


Figure 7: STNU $\mathcal{X}_i^a$ for occurrence a of variable x_i or y_i in ϕ

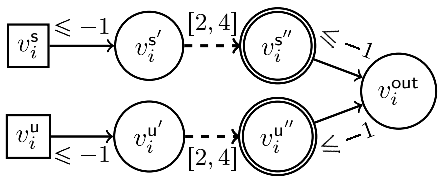


Figure 6: STNU $\mathcal{X}_i^{\mathsf{p}}$ for each variable x_i or y_i

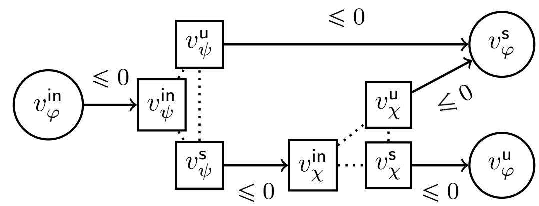


Figure 8: STNU $\mathcal{X}_\varphi$ for subformula $\varphi = \psi \uparrow \chi$

$\omega(c_i^{\mathsf{n}}) = n_i$. Combining with eq. (2) and the constraint $\omega(c_i^{\mathsf{y}}) \leqslant n_i$, we have $\delta_\omega(v_i) = \delta_\omega(v_{i-1}) + n_i$.

- If ρ repairs c_i^{n}, then there is a situation ω such that $\omega(c_i^{\mathsf{y}}) = 0$. Combining with eq. (2) and the constraint $\omega(c_i^{\mathsf{n}}) \geqslant 0$, we have $\delta_\omega(v_i) = \delta_\omega(v_{i-1})$.

Let $I \subseteq \{1, \ldots, \ell\}$ be the set of i such that ρ repairs c_i^{y}. Consider a situation ω such that $\omega(c_i^{\mathsf{n}}) = n_i$ for $i \in I$ and $\omega(c_i^{\mathsf{y}}) = 0$ for $i \notin I$ (notice that ρ repairs c_i^{n}). Then δ_ω must satisfy $\delta_\omega(v_\ell) - \delta_\omega(v_0) = \sum_{i \in I} n_i$. By constraint e^N, $\delta_\omega(v_\ell) - \delta_\omega(v_0) = N$, meaning that $\sum_{i \in I} n_i = N$ holds and S is a positive instance of SUBSET-SUM. ☐

Proposition 6. PARTIAL-W-REPAIR *and* k-BUDGET-W-REPAIR *are* coNP*-hard.*

Proof. Notice that Weak Controllability, which is known to be coNP-complete [Morris and Muscettola, 1999], is a special case of both PARTIAL-W-REPAIR (with $R = \varnothing$) and k-BUDGET-W-REPAIR (with $k = 0$). ☐

Proposition 7. k-CONSTRAINT-W-REPAIR *is* Σ_2^{P}*-hard.*

Proof. Our proof is inspired by the construction of Morris and Muscettola [1999] for showing the coNP-hardness of Weak Controllability. We give a reduction from the problem of deciding the validity of a formula Φ of the form $\exists x_1 \ldots x_\ell \forall y_{\ell+1} \ldots y_m \phi(x_1, \ldots, x_\ell, y_{\ell+1}, \ldots, y_m)$, where we assume without loss of generality that ϕ only uses the NAND connective $\uparrow$ (with $a \uparrow b := \neg(a \wedge b)$).

Given Φ, we build the STNU $\mathcal{X}_\Phi$ in polynomial time.[3]

- For each x_i (resp. y_i), $\mathcal{X}_\Phi$ contains the timepoints and constraints in Figure 5 (left, resp. right) and Figure 6.
- For each occurrence x_{i^a} or y_{i^a} of an atom in ϕ, $\mathcal{X}_\Phi$ contains the timepoints and constraints in Figure 7.

[3] In the figures, $\leqslant a$ stands for $]-\infty, a]$, and a rectangle indicates that the node is shared with a previously defined STNU.

- For each subformula $\varphi := \psi \uparrow \chi$ of ϕ, $\mathcal{X}_\Phi$ contains the timepoints and constraints in Figure 8; note that ψ, χ can be occurrences x_{i^a} or y_{i^a} of atoms.
- Finally, $\mathcal{X}_\Phi$ contains requirements $v_i^{\mathsf{out}} \xrightarrow{[-\infty,0]} v_{i+1}^{\mathsf{in}}$ for $i = 1, \ldots, m-1$, $v_m^{\mathsf{out}} \xrightarrow{[-\infty,0]} v_\phi^{\mathsf{in}}$, and $v_\phi^{\mathsf{u}} \xrightarrow{[-\infty,-\frac{1}{2}]} v_0^{\mathsf{in}}$.

The intuition is that assignments to x_i are mapped to tightenings of the STNUs $\mathcal{X}_i$; assignments to Y satisfying (resp. falsifying) φ are in one-to-one correspondence to paths with length 0 in the distance graph induced by some situation of the repaired STNU.

Assume that ρ is a ℓ-constraint W-repair of $\mathcal{X}_\Phi$. For every x_i ($1 \leqslant i \leqslant \ell$), $\mathcal{X}_i$ is clearly uncontrollable: ρ must therefore repair at least one contingent link in it. It follows that ρ repairs exactly one such link per $\mathcal{X}_i$. In addition, the repaired links must be repaired to $[1,1]$. We write μ_ρ^X for the assignment to $x_1, \ldots, x_\ell$ defined by $\mu_\rho^X(x_i) = \top$ (resp. $\bot$) if ρ repairs the link to v_i^{u} (resp. v_i^{s}). Now, suppose towards contradiction that Φ is not valid. Then there is an assignment μ^Y to $y_{\ell+1}, \ldots, y_m$ such that $\mu_\rho^X \cdot \mu^Y$ falsifies ϕ. Let:

$$\omega(v_i^{\mathsf{in}} \dashrightarrow v_i^{\mathsf{s}}) = 0 \text{ if } \mu_\rho^X(x_i) = \bot, \text{ else } 1;$$
$$\omega(v_i^{\mathsf{in}} \dashrightarrow v_i^{\mathsf{u}}) = 0 \text{ if } \mu_\rho^X(x_i) = \top, \text{ else } 1;$$
$$\omega(v_j^{+} \dashrightarrow v_j^{-}) = 2 \text{ if } \mu_\rho^X(y_j) = \bot, \text{ else } 4;$$
$$\omega(v_i^{\mathsf{s}'} \dashrightarrow v_i^{\mathsf{s}''}) = 2 \text{ if } \mu_\rho^X(x_i)/\mu^Y(y_i) = \top, \text{ else } 4;$$
$$\omega(v_i^{\mathsf{u}'} \dashrightarrow v_i^{\mathsf{u}''}) = 2 \text{ if } \mu_\rho^X(x_i)/\mu^Y(y_i) = \bot, \text{ else } 4.$$

ω thus defined is a situation of $\mathcal{X}_\Phi \oplus \rho$. Then it is easy to show that in the distance graph induced by ω, there is a path of length 0 from v_0^{in} to v_ϕ^{u} traversing v_φ^{s} (resp. v_φ^{u}) for every subformula φ satisfied (resp. falsified) by $\mu_\rho^X \cdot \mu^Y$. Hence, there is a negative cycle of length $-1/2$ in this distance graph, contradicting the assumption that ρ is a W-repair of $\mathcal{X}_\Phi$; Φ is therefore valid.

Conversely, assume that Φ is valid with an assignment μ^X to

$x_1, \ldots, x_\ell$ as witness. Let ρ be the tightening of $\mathcal{X}_\Phi$ which tightens the link to v_i^{u} (resp. v_i^{s}) to $[1, 1]$ if $\mu^X(x_i) = \top$ (resp. $\perp$). Then ρ only tightens ℓ constraints. Now, suppose towards contradiction that ρ is not a W-repair and let ω be a situation of $\mathcal{X}_\Phi \oplus \rho$ which induces a negative cycle. Such a cycle must consist of a path π of length 0 from v_0^{in} to v_ϕ^{u} and the edge of length $-\frac{1}{2}$ from v_ϕ^{u} back to v_0^{in}. Let μ^Y be the assignment to Y defined by $\mu^Y(y_i) = \top$ (resp. $\perp$) if π traverses v_i^{s} (resp. v_i^{u}) in $\mathcal{X}_i$. Then it is easy to show that π traverses v_φ^{s} (resp. v_φ^{u}) for every subformula φ satisfied (resp. falsified) by $\mu_\rho^X \cdot \mu^Y$. Since π traverses v_ϕ^{u}, $\mu^X \cdot \mu^Y$ falsifies ϕ, contradicting the assumption that Φ is valid; ρ is therefore a ℓ-constraint W-repair. $\square$

7 CONCLUSION

We studied the repair problem of STNUs with respect to all three controllability levels and introduced four definitions of this problem to tackle extended heterogeneity or optimization concerns. We discussed their relevance for practical applications, in particular for networks controlled by different agents. For most of these repair problems and controllability levels, we determined the complexity of deciding whether a given STNU is repairable. It turns out repairability is no more difficult than controllability in many cases.

Future work includes the exact complexity of partial and k-budget repairs for dynamic controllability (we only showed their membership in NP); the practical use of linear programming and NP or coNP solvers for repair problems; and the design of fixed-parameter tractable or approximation algorithms. Finally, we also envision studying the complexity of the repair problems for higher classes of Temporal Networks with Uncertainty, for both single-agent and multi-agent settings.

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