MTH229 Notes

Professor Kevin O'Bryant

Spring 2025

First Day Checklist

Lab Rules: Attendance is required, each and every lab period for the full lab period. You are not to use a phone in the lab for texting, calling, nor browsing. No goofing off, that is.

Checklist for the bare minimum first day work.

☐ Log into the campus network SLAS.

☐ Log into Brightspace and see if there are any announcements for this lab class.

☐ Make sure the email address that Brightspace has is one that works for you.

☐ Make sure the email address that CUNYFirst has for you is one that you monitor.

☐ Take a look at the Syllabus, also located on Brightspace.

☐ WeBWorK is the service that we will use to assign and grade homework. Log in to WeBWorK

`https://www.math.csi.cuny.edu/webwork2/`

and select your professor's name, or log in by going to the math department website

`https://www.math.csi.cuny.edu/`

and finding the "WeBWorK" link at the lower right, and then selecting our class. You will find your username and initial password described on that page (it is your first initial, last name, and 4 digits from ID). If you can't log in, your professor can help. There is often trouble for students who have added the class recently, and sometimes the username just gets malformed.

☐ Change WeBWorK password. WeBWorK is not secure: you should not re-use a password from a system you care about. It's under "User Settings", as is an email address that will be used by WeBWorK. Change it, too, if you want. Course related emails will go through the CUNYFirst address or the Brightspace address, but exchanges about specific homework problems will go through WeBWorK. If you ever forget your password, your professor can reset it.

☐ Contact someone else in the classroom by sending them a chat message, or a text, or just using your old-fashioned voice. Say hello, exchange email addresses, etc. It's okay to do this with several people.

☐ Open "Problem 1" of the first homework set, "01-calculator".

☐ Use the "Juliabox" button at the top of the problem. Your login name and Juliabox password are both set to your WeBWorK username. It works best if you are using Chrome.

☐ Open the "01-calculator.ipynb" file. This opens our textbook. Read through this material. The first line is "Read about this material here" with a link. That link will take you to a *more* detailed and in-depth explanation — I advise you to skip it at least until you are stuck on a problem. After giving this material a read-through, you will come to a blank input line at the bottom. Look at the WeBWorK problem. Try to do it in the blank input line at the bottom. If you can't, get in touch with your instructor.

☐ When you have the value for (a), copy-and-paste it into the WeBWorK blank. Press the "Submit Answers" button at the bottom of the WeBWorK page. This saves your answer, and also tells you if it was correct or incorrect. You have unlimited attempts for most problems, including this one.

☐ You are expected to do your own work, but you are not expected to do it alone. The best experience for this class is to work in parallel with someone, as it helps to have someone to bounce ideas around and have fresh eyes, occasionally. Do not split the work in half: everyone needs to do and understand every problem. There's plenty of time.

☐ After completing Problem 1, make notes about the problem on the page of this notebook in Chapter 1. Actually, we have given you a head-start by pre-making notes, so you get an idea of the level of formality (low) needed, and the kinds of comments you might want to make. If you got an error message (and you definitely did) while getting this problem, put the error message and how you resolved it in the "What I was supposed to learn" section. You should put something into each box for each problem in the entire course.

☐ You should do several problems from this project on the first day. Some people are able to finish it, even. It should definitely be completed by (at the latest) the end of the second lab period. Note that the due dates shown in WeBWorK are deadlines: they will not be postponed under (almost) any circumstances. You are expected to finish the projects a week or more *ahead* of the deadlines.

☐ At the end of lab period, navigate to the first problem that you have not completed (or the last problem, if you've completed them all), and press the "email instructor" button. In the message, include the names of the people sitting next to you. This email will count as your attendance at the first lab period.

Installing Julia

It is in principle easy to install `Julia` on any computer, including laptops. This is sometimes broken, as there are too many types of computers with too many possible combinations of software on them.

The best advice I can give for installing `Julia` is to check out

`www.julialang.org`

and download the version that seems most appropriate for your situation.

After downloading and installing `Julia`, you will want to download our specific packages and files for this course. The instructions for doing that are kept up-to-date at

`https://mth229.github.io/`

If you follow these directions and have trouble, or in hindsight think the directions could be worded differently, please email your recommendations to your professor.

Chapter 1

`01-calculator`

1. Problem 1: Title: "Using `Julia` like a calculator"

   (a) Summary of the problem:

   > evaluating simple expressions

   (b) What I was supposed to learn:

   > adding, subtracting, multiplying in `Julia`
   >
   > When I want to multiply, I should use $*$
   >
   > I should put parentheses around numerators and denominators if they have more than one term.

   (c) Date started, finished, and time spent

   > started 25-2-27, finished 25-2-27, 10 minutes

   (d) Difficulty:

   > 1 out of 10

2. Problem 2: Title: "Precedence (PEMDAS)"

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

7. Problem 7: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

8. Problem 8: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

Chapter 2

02-functions

Define the function

$$f(x) = \frac{(x-1)^2}{x^2+3x+2} + \tan^{-1}(x) + \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

and use your function to find $f(0), f(1), f(-3/2)$.

Solution: We input the following:

```
f(x) = (x-1)^2 / (x^2+3x+2) + atan(x) + 1/sqrt(2*pi)*exp(-x^2/2)
f(0),f(1),f(-3/2)
```

and the output is the following:

```
(0.8989422804014326, 1.0273688879165916, -25.853276127581438)
```

Thus,

$$\begin{aligned} f(0) &\approx 0.8989422804014326 \\ f(1) &\approx 1.0273688879165916 \\ f(-3/2) &\approx -25.853276127581438 \end{aligned}$$

1. Problem 1: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

2. Problem 2: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

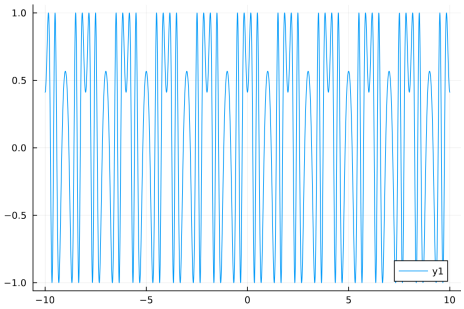
Chapter 3

03-graphics

Make a plot of the periodic function $f(x) = \sin(9\cos(2\sin(\frac{\pi x}{2})))$ over one period.

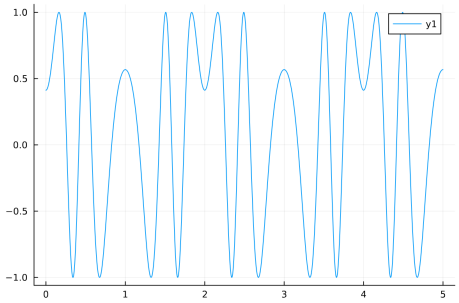
Solution: We input the code on the left and are rewarded with the picture on the right.

```
using MTH229, Plots
f(x) = sin(9*cos(2*sin(pi*x/2)))
plot(f,-10,10)
```



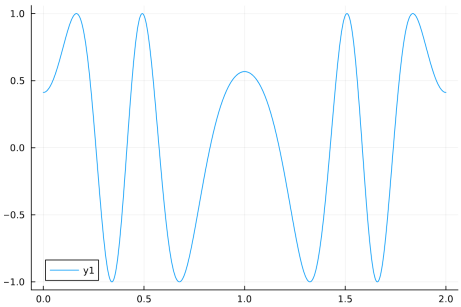
From that image, we count 9 or 10 periods. We decide to look at the graph over the domain $[0, 5]$

```
plot(f,0,5)

```



We graphically conclude that the period is 2, and make the required graph.

```
plot(f,0,2)

```



1. Problem 1: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

2. Problem 2: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

7. Problem 7: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

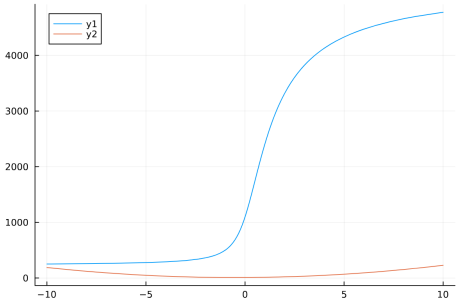
Chapter 4

04-zeros

Solve the equation $\exp(7 + \tan^{-1} x) = 2x^2 + 2x + 9$.

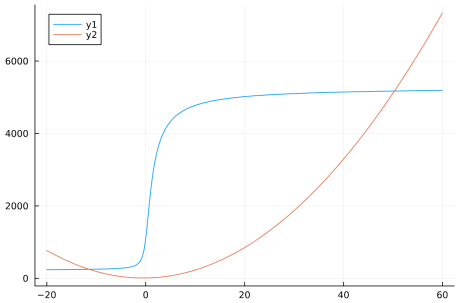
Solution: First, we define the functions and plot them together.

```
using MTH229, Plots
left(x) = exp(7+atan(x))
right(x) = 2x^2 + 2x + 9
plot(left, -10, 10)
plot!(right)
```



We think: As $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$, it must be that $e^{7-\frac{\pi}{2}} \leq \exp(7 + \arctan x) \leq e^{7+\frac{\pi}{2}}$. Also, we know that $y = 2x^2 + 2x + 9$ is a parabola. We visualize it arcing up to where it crosses above $y = e^9$. With that in mind, we make a picture over $[-100, 100]$, and from that picture decide to instead use $[-50, 50]$. From that picture, we decide to use $[-20, 60]$.

```
plot(left, -20, 60)
plot!(right)
```



Now we can see 2 solutions, and we see that $[-20, 0]$ is a bracketing interval for one, and $[40, 60]$ is a bracketing interval for the other. To use `bisection`, we need a function that has the solutions as roots. I name this function "`aux`" for "auxiliary".

```
aux(x) = left(x) - right(x)
bisection(aux, -20, 0) , bisection(aux, 40, 60)
```

The output is the pair of solutions, the first in $[-20, 0]$ and the second in $[40, 60]$.

```
(-11.459032702506452, 50.30855414302505)
```

Note that the solutions are not points, but just x-values. We invented left, right, y, aux, to help us solve the problem, but the original problem is just about x-values.

1. Problem 1: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

2. Problem 2: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

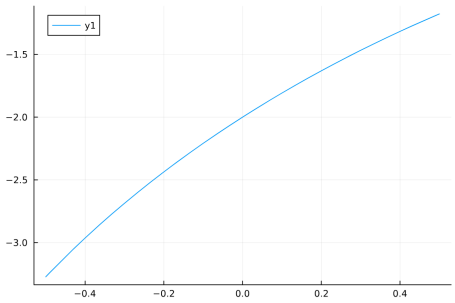
Chapter 5

05-limits

Compute $\lim_{x\to 0} \dfrac{\tan(x)(\sin(x)-\cos(x)-1)}{\exp(x)-1}$ three different ways.

Solution: First, we define a function, and then we plot it on an interval around 0 (because it is a limit as $x \to 0$):

```
using MTH229, Plots
top(x) = tan(x) * (sin(x)-cos(x)-1)
bot(x) = exp(x) -1
f(x) = top(x) / bot(x)
plot(f, -0.5, 0.5 )
```



From the image, we conclude that the limit is about -2.

Alternatively, we could make a table showing inputs getting close to 0 and see what the outputs are getting close to.

```
    lim(f, 0 )

```

c	L?
0.100000	-1.8080220187207678
0.010000	-1.980083002344296
0.001000	-1.9980008330001504
0.000100	-1.999800008332135
0.000010	-1.9999800001027366
0.000001	-1.9999980000767992
⋮	⋮
-0.000001	-2.000002000032556
-0.000010	-2.000020000821873
-0.000100	-2.00020000833364
-0.001000	-2.0020008336669637
-0.010000	-2.0200836690448423
-0.100000	-2.208692087969323

We see in the second column that L wants to be -2.

Alternatively, we could use `SymPy`, which is loaded with the `MTH229` package.

```
@syms x
limit(f(x), x => 0 )
```

which outputs a simple "-2".

1. Problem 1: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

2. Problem 2: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

Chapter 6

06-derivatives

1. Problem 1: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

2. Problem 2: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

7. Problem 7: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

8. Problem 8: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

9. Problem 9: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

Chapter 7

`07-newton`

1. Problem 1: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

2. Problem 2: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

Chapter 8

08-first second derivatives

1. Problem 1: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

2. Problem 2: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

7. Problem 7: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

Chapter 9

`09-extrema`

1. Problem 1: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

2. Problem 2: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

7. Problem 7: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

8. Problem 8: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

Chapter 10

10-integration

1. Problem 1: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

2. Problem 2: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

3. Problem 3: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

4. Problem 4: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

5. Problem 5: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

6. Problem 6: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

7. Problem 7: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

8. Problem 8: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

9. Problem 9: Title: _____

   (a) Summary of the problem

   (b) What I was supposed to learn

   (c) Date started, finished, and time spent

   (d) Difficulty:

10. Problem 10: Title: _____

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

Appendix A

Various Functions

A.1 Trigonometric Functions

Julia Function	Description
`sin(x)`	Sine of x
`cos(x)`	Cosine of x
`tan(x)`	Tangent of x
`cot(x)`	Cotangent of x
`sec(x)`	Secant of x
`csc(x)`	Cosecant of x

A.2 Inverse Trigonometric Functions

Julia Function	Description
`asin(x)`	Inverse sine (arcsine) of x
`acos(x)`	Inverse cosine (arccosine) of x
`atan(x)`	Inverse tangent (arctangent) of x
`atan(y, x)`	Inverse tangent (arctangent) of $\frac{y}{x}$
`acot(x)`	Inverse cotangent (arccotangent) of x
`asec(x)`	Inverse arcsecant (arcsecant) of x
`acsc(x)`	Inverse arccosecant (arccosecant) of x

A.3 Hyperbolic Functions

Julia Function	Description
`sinh(x)`	Hyperbolic sine of x
`cosh(x)`	Hyperbolic cosine of x
`tanh(x)`	Hyperbolic tangent of x
`coth(x)`	Hyperbolic cotangent of x
`sech(x)`	Hyperbolic secant of x
`csch(x)`	Hyperbolic cosecant of x

A.4 Inverse Hyperbolic Functions

Julia Function	**Description**
`asinh(x)`	Inverse hyperbolic sine of x
`acosh(x)`	Inverse hyperbolic cosine of x
`atanh(x)`	Inverse hyperbolic tangent of x
`acoth(x)`	Inverse hyperbolic cotangent of x
`asech(x)`	Inverse hyperbolic secant of x
`acsch(x)`	Inverse hyperbolic cosecant of x

A.5 Exponential and Logarithmic Functions

Julia Function	**Description**
`exp(x)`	e^x
`exp2(x)`	2^x
`exp10(x)`	10^x
`expm1(x)`	$e^x - 1$
`log(x)`	Natural logarithm of x
`log2(x)`	Base-2 logarithm of x
`log10(x)`	Base-10 logarithm of x
`log1p(x)`	$\log(1 + x)$

A.6 Root Functions

Julia Function	**Description**
`sqrt(x)`	Square root of x
`cbrt(x)`	Cube root of x
`hypot(x, y)`	$\sqrt{x^2 + y^2}$

A.7 Power Functions

Julia Function	**Description**
`x^y`	x raised to the power of y
`x^2`	Square of x
`x^3`	Cube of x
`x^(-1)`	reciprocal of x

A.8 Special Functions

Julia Function	**Description**
`abs(x)`	Absolute value of x
`abs2(x)`	Squared absolute value
`sign(x)`	Sign of x
`factorial(n)`	Factorial of n
`binomial(n, k)`	Binomial coefficient
`gamma(x)`	Gamma function at x
`lgamma(x)`	Log-gamma function at x

A.9 Rounding Functions

Julia Function	**Description**
`round(x)`	Round to nearest integer
`ceil(x)`	Round up to integer
`floor(x)`	Round down to integer
`trunc(x)`	Truncate to integer

Appendix B

SymPy

B.1 Setup and Basic Usage

```
# Installing SymPy
using Pkg
Pkg.add("SymPy")

# Loading the package
using SymPy

# Creating symbolic variables
@vars x y z
@syms a b c real=true  # Declaring real variables
@syms n::Integer       # Integer variable
```

B.2 Algebraic Manipulation

```
# Basic operations
expr = x^2 + 2*x + 1
expanded = expand((x + 1)^2)
factored = factor(x^2 + 2*x + 1)

# Substitution
expr = x^2 + y
subs(expr, x => 2)         # Replace x with 2
subs(expr, Dict(x => 2, y => 3))  # Multiple substitutions

# Simplification
simplified = simplify((x^2 + 2*x + 1) / (x + 1))
trigsimp(sin(x)^2 + cos(x)^2)     # Trigonometric simplification
collect(x^2*y + x*y^2 + x^2 + x, x)  # Collect terms with x

# Polynomial operations
p1 = Poly(x^2 + 2*x + 1, x)
p2 = Poly(x + 1, x)
p1 + p2    # Addition
p1 * p2    # Multiplication
div(p1, p2)  # Division
```

B.3 Calculus

```
# Differentiation
diff(x^2 + 2*x + 1, x)     # First derivative
diff(x^2 + 2*x + 1, x, 2)  # Second derivative
diff(sin(x)*exp(x), x)     # Product rule automatically applied

# Partial differentiation
f = x^2 + 2*y^2 + 3*x*y
diff(f, x)                 # Partial with respect to x
diff(f, y)                 # Partial with respect to y
diff(f, x, y)              # Mixed partial derivative

# Integration
integrate(x^2 + 2*x + 1, x)     # Indefinite integration
integrate(x^2 + 2*x + 1, (x, 0, 1))  # Definite integration

# Limits
limit(sin(x)/x, x => 0)
limit((1 + 1/x)^x, x => oo)  # Limit as x approaches infinity

# Series expansion
series(sin(x), x, 0, 5)    # Taylor series around x=0 up to x^5
```

B.4 Solving Equations

```
# Solving a single equation
solve(x^2 - 4, x)           # Solve x^2 - 4 = 0
solve(sin(x) - cos(x), x)   # Trigonometric equation

# Systems of equations
eqs = [x + y - 2, x - y - 0]
solve(eqs, [x, y])

# Solving inequalities
solve_univariate_inequality(x^2 - 4 < 0, x)

# Differential equations
@vars y(x)
diffeq = Eq(diff(y(x), x, 2) + y(x), sin(x))
dsolve(diffeq, y(x))
```

B.5 Linear Algebra

```
# Creating matrices
A = [x 1; 1 1]  # 2x2 symbolic matrix
B = Matrix{Sym}([1 2; 3 4])  # Matrix from numeric values

# Matrix operations
A + B         # Addition
A * B         # Matrix multiplication
A^2           # Matrix power
transpose(A)  # Transpose
inv(A)        # Inverse

# Determinant and eigenvalues
det(A)        # Determinant
eigenvals(A)  # Eigenvalues
eigenvects(A) # Eigenvectors

# Solving linear systems
sols = solve_linear_system(A, [1, 2], [x, y])
```

B.6 Numerical Evaluation

```
# Converting symbolic to numeric
N(pi)              # Default precision
N(pi, 50)          # 50 digits of precision

# Evaluating expressions
expr = sin(pi/3)
float(expr)        # Convert to floating point
complex(expr)      # Convert to complex

# Using with standard Julia
f(x) = 2*x^2 + 3*x + 1
sym_f = lambdify(f(x))    # Convert to Julia function
sym_f(2)                  # Evaluate at x = 2
```

B.7 Plotting with SymPy and Plots.jl

```julia
using Plots

# Converting symbolic expressions for plotting
f(x) = sin(x) * exp(-0.1*x)
sym_f = lambdify(f(x))

# Create plot
x_range = 0:0.1:10
plot(x_range, sym_f.(x_range),
title="Symbolic Function Plot",
label="sin(x)*exp(-0.1x)",
xlabel="x", ylabel="f(x)")
```

B.8 Common Functions and Constants

```julia
# Constants
PI       # pi = 3.14159...
E        # Euler's number e = 2.78182828...
oo       # Infinity
I        # Imaginary unit

# Functions
sin(x), cos(x), tan(x)    # Trigonometric
asin(x), acos(x), atan(x) # Inverse trigonometric
sinh(x), cosh(x), tanh(x) # Hyperbolic
exp(x)                    # Exponential
log(x), log(x, b)         # Natural log, log base b
sqrt(x)                   # Square root
factorial(n)              # Factorial
binomial(n, k)            # Binomial coefficient
```

B.9 Tips and Tricks

```
# Assumption handling
@vars x real=true positive=true
simplify(sqrt(x^2))  # Returns x, not |x|

# Comparing expressions
a = (x + 1)^2
b = x^2 + 2*x + 1
a == b               # Structural equality
simplify(a - b) == 0 # Mathematical equality

# Converting to Julia expressions
expr = x^2 + sin(y)
convert(Expr, expr)  # Convert to Julia Expr

# Function for numerical calculations with uncertainties
@vars x y
f = x^2 + y^2
subs(f, Dict(x => 3 +/- 0.1, y => 2 +/- 0.2))
```

Appendix C

Extra Credit

> These instructions step you through using Julia[1] to create a good looking scientific document. You will enter the questions, solve them, and print your solutions. You will email the completed file, which should have extension "`.ipynb`" and filename your name without spaces. The project is due before your final exam. Your grade will be based, in part, on the beautiful appearance of your document.
>
> Your professor should provide a value of m specific to you with $m \in [3, 10) \setminus \{\pi\}$.

Note that this is a take-home outside-the-lab experience. You are allowed to use your notes, past homework and exams, and to google questions. You may *not* use any live-answer site like Chegg, StackExchange, Slader, et cetera. In fact, anyone who would use such a site, or would ask you for help, is totally lost and should not be allowed to skew the curve that applies to students who have actually done the work, like you. Please report any such activity you are aware of; you will be rewarded (if there's evidence) and you will be doing the right thing.

1. Start `Julia`.
2. Open a new notebook using `Julia`.
3. Type "MTH229 Final Exam" into the cell. Then hit the escape button, then 1, which will make the cell into the title. Then use escape-m, and press shift-enter. If these directions do not work with how you are using Julia, contact your instructor immediately for discussion and advice.
4. Insert a cell below that, and type your name, followed by a comma, followed by your student ID number, followed by a comma, followed by the value of m you will receive when you email your professor. Then use escape-2 to make this cell a subtitle, escape-m to make it markdown, and shift-enter to evaluate it. Don't put off getting your value of m until the last minute!
5. In a new cell, type "`using MTH229, Plots`", and use shift-enter to evaluate the cell.

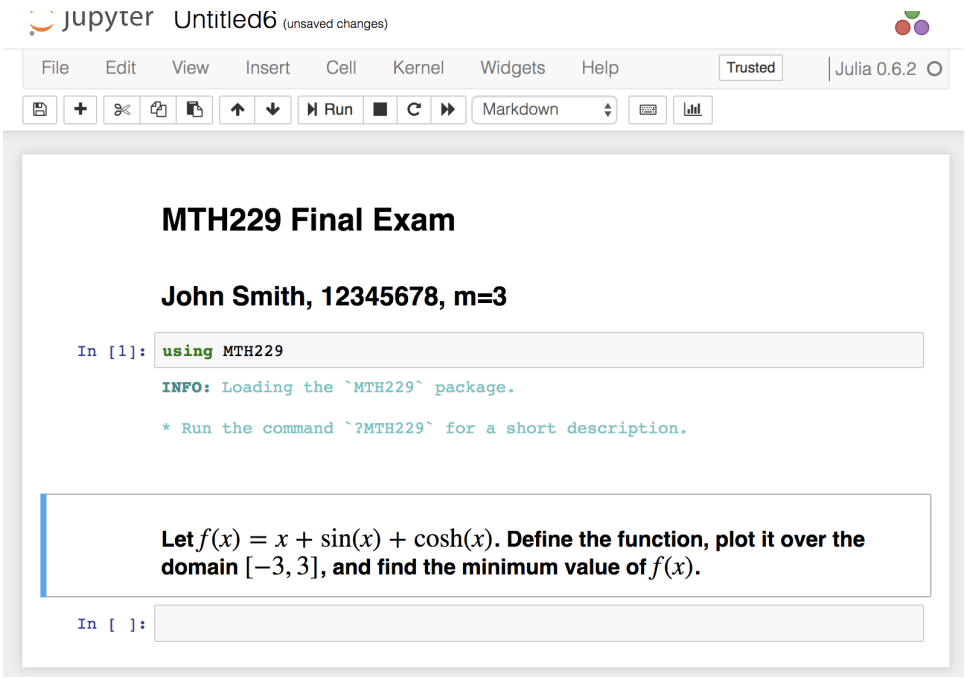
[1] Actually, Jupyter notebooks.

6. **FIRST PROBLEM:** In a new cell, type

```
Let $f(x) = x+4\sin(x)+\cosh(x)$. Define the function, plot it over
the domain $[-3,3]$, and find the maximum value of $f(x)$.

```

   Include all of the dollar signs and backslashes *exactly* as above. Use escape-m to convert the cell to markdown (which means it's text, not code). Then escape-3 to make it bold, and then shift-enter the cell to render it.

7. Insert a new cell below this. Your screen should look similar to mine:



8. This is a good moment to figure out how to save your work (use your name as the file name), and how to locate the `.ipynb` file. Especially if you are using Binder, make sure you have work saved where it won't disappear.

9. Solve the problem. Don't be shy about creating your own markdown cells (but don't use escape-3 to make them bold) to explain what you are doing, or what you are concluding. Real mathematicians use complete sentences.

10. In a cell below your solution, enter

    ***

11. Save your work.

12. **SECOND PROBLEM:** In a new cell, type (replacing m with the value assigned to you).

```
Let $$g(x) = \sin(x)\cos(m x)\sin(\pi x).$$ Graph $g$ on the interval $[0,2]$,
 and also the tangent line at $x=\sqrt{2}$. Compute the roots of $g$ in the
interval $[0,2]$. Find the minimum and maximum values of $g$ on $[0,2]$.

```

Convert the cell to markdown (escape-m), and bold (escape-3), and typeset (shift-enter).

13. Solve the problem. Don't be shy about creating your own markdown cells to explain what you are doing, and what you have concluded. Below your solution, create a markdown cell with "$***$", which gives a horizontal line.

14. **THIRD PROBLEM:** In a new cell, type (replacing m with the value assigned to you).

```
Let $g(x) = \sin(x)\cos(m x)\sin(\pi x)$. Compute $\int_0^{1/\pi} g(x)\,dx$.

```

Convert the cell to markdown (escape-m), and bold (escape-3), and typeset (shift-enter).

15. Solve the problem. Don't be shy about creating your own markdown cells to explain what you are doing, and what you have concluded. Below your solution, create a horizontal line. Save your work.

16. **FOURTH PROBLEM:** In a new cell, type (replacing m with the value assigned to you).

```
Find all solutions to the equation $\sqrt[3]{x} = \frac{x^2}{m} - 229$.

```

Warning: that's a cube root, not a square root. Convert the cell to markdown (escape-m), and bold (escape-3), and typeset (shift-enter).

17. Solve the problem.

18. **BONUS PROBLEM:** In a new cell, type (replacing m with the value assigned to you).

```
Let $$r(x) = \frac{m/\pi}{\log(1+ \sqrt m)} \frac{\tan^{-1} x}{x (m+x^2)}.$$
Compute the derivative $r'(x)$ symbolically. Compute $\int_{-1}^1 r(x) \, dx$
numerically. Find estimates for the inflection points of $r(x)$.

```

Convert the cell to markdown (escape-m), and bold (escape-3), and typeset (shift-enter).

19. Solve the problem. Don't be shy about creating your own markdown cells to explain what you are doing, and what you have concluded. Below your solution, create a horizontal line. Save your work.

C.1 The problems

FIRST: Let $f(x) = x + 4\sin(x) + \cosh(x)$. Define the function, plot it over the domain $[-3, 3]$, and find the maximum value of $f(x)$.

SECOND: Let $g(x) = \sin(x)\cos(mx)\sin(\pi x)$. Graph g on the interval $[0, 2]$, and also the tangent line at $x = \sqrt{2}$. Compute the roots of g in the interval $[0, 2]$. Find the minimum and maximum values of g on $[0, 2]$.

THIRD: Let $g(x) = \sin(x)\cos(mx)\sin(\pi x)$. Compute $\int_0^{1/\pi} g(x)\,dx$.

FOURTH: Find all solutions to the equation $\sqrt[3]{x} = \frac{x^2}{m} - 229$.

BONUS: Let $r(x) = \dfrac{m/\pi}{\log(1+\sqrt{m})} \dfrac{\tan^{-1} x}{x(m+x^2)}$. Compute the derivative $r'(x)$ symbolically. Compute $\int_{-1}^{1} r(x)\,dx$ numerically. Find estimates for the inflection points of $r(x)$.