Informational Holonomy and the Electron g-Factor

A Derivation within Super Information Theory

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Abstract

We re-derive the electron gyromagnetic ratio $g = 2$ within Super Information Theory (SIT), in which electromagnetism arises as the holonomy of a coherence-phase $U(1)$ bundle and matter couples minimally to its connection. Under the assumption that the SIT action reduces to the Dirac–Maxwell sector in the appropriate decoherence limit, the nonrelativistic expansion reproduces the Pauli Zeeman term $-\frac{q}{m}\,\mathbf{S}\cdot\mathbf{B}$, yielding $g = 2$.

What is not new: this matches the standard Dirac prediction. *What is new here:* (i) a geometric account of the result in terms of informational holonomy and spinor 4π periodicity; (ii) a precise statement of the *phase-only entry* hypothesis, under which no dimension-≤ 4 dipole operators are permitted; and (iii) an effective-field-theory map identifying when SIT-specific, environment-dependent deviations could arise via gradients of the time-density ρ_t and coherence ratio R_{coh}, without modifying the standard QED anomaly.

We also clarify common pitfalls (the gauge field A is a connection, not generally a gradient) and provide explicit Foldy–Wouthuysen and Gordon-decomposition derivations, along with conventions and global bundle remarks. The result is a necessity result within the SIT framework: once the assumed action and coupling structure are fixed, the Dirac value $g = 2$ follows as the only self-consistent outcome at leading order.

Author's note on earlier feedback (and what changed)

1. **"Connection $\neq$ gradient."** We now state explicitly that A is a genuine connection 1-form whose holonomy is $U[\gamma] = \exp\big(\frac{iq}{\hbar}\int_\gamma A\big)$. The identification $A = (\hbar/e)\,d\theta$ is valid only in a pure-gauge region with $F = dA = 0$, and is not assumed globally. See Section 2 and **??**.

2. **"The central $g = 2$ result is the standard Dirac value."** No novelty claim is made for the numerical value $g = 2$ itself. The contribution of this work is interpretive (a geometric account in terms of informational holonomy) and structural/predictive (a precise statement of when, within the SIT framework, deviations could arise). See Sections 5 and 6.

3. **Figures and assets.** The spin-precession figure is now included (PNG by default) with automatic SVG/PDF fallback; compilation does not depend on external `.bib` files. See Figure 1.

4. **Technical cleanups.** Duplicated theorems were removed, broken cross-references fixed, metric and operator conventions stated explicitly, and two complementary derivations (Foldy–Wouthuysen and Gordon decomposition) added for completeness and transparency.

1 Motivation and premises

The electron's gyromagnetic ratio g relates its magnetic moment $\boldsymbol{\mu}$ and spin $\mathbf{S}$ by

$$\boldsymbol{\mu} = g\,\frac{q}{2m}\,\mathbf{S}, \qquad q = -e, \quad \mathbf{S} = \frac{\hbar}{2}\boldsymbol{\sigma}. \tag{1}$$

Dirac theory yields $g = 2$, while quantum electrodynamics produces the anomalous correction $a_e \equiv (g-2)/2$ through loop effects.

In Super Information Theory (SIT), electromagnetism is modeled as the holonomy of a coherence-phase field, and the decohered, low-energy sector reduces to the Dirac–Maxwell theory. Within this framework, the purpose of the present work is threefold: first, to state precisely why the Dirac value $g = 2$ is enforced once the SIT action and minimal coupling structure are fixed; second, to clarify the geometric role of spin and phase holonomy underlying this result; and third, to identify SIT-specific mechanisms by which controlled departures from the Dirac value could arise in nonideal or environment-dependent regimes, without modifying the standard QED anomaly.

Premises used here. (i) *Electromagnetism as phase holonomy:* the electromagnetic potential A is modeled as the connection of a coherence-phase $U(1)$ bundle; magnetic effects are encoded by its curvature $F = dA$. (ii) *Standard reduction:* when informational dressings are constant (the decohered or low-energy limit), the SIT action reduces to the Dirac–Maxwell theory with minimal coupling. (iii) *Phase-only entry in the renormalizable matter sector:* the gauge field A appears only through the covariant derivative acting on matter fields.

2 From informational holonomy to minimal coupling

2.1 Bundle picture and gauge law

Let $P \to M$ be a principal $U(1)$ bundle (the SIT phase fiber) equipped with a connection 1-form ω. Given a local section $s : U \to P$, the electromagnetic potential is identified with the pullback

$$A \equiv s^*\omega \in \Omega^1(U), \qquad F \equiv dA. \tag{2}$$

Under a gauge change $s \mapsto s\,e^{\mathrm{i}\alpha(x)}$,

$$A \mapsto A - d\alpha, \qquad \psi \mapsto e^{\,\mathrm{i}q\alpha/\hbar}\psi. \tag{3}$$

Parallel transport in the phase fiber produces the Wilson line

$$U[\gamma] \;=\; \exp\Big(\frac{\mathrm{i}q}{\hbar}\int_\gamma A\Big), \tag{4}$$

which encodes the physically relevant holonomy information.

Remark 1 (Connection $\neq$ gradient)**.** *Writing $A = \frac{\hbar}{e}\,d\theta$ is possible only on a* pure-gauge *patch (equivalently $F = 0$ there). In general, A is not exact, and nontrivial holonomy can occur even when $F = 0$ on the domain of motion (for example, the Aharonov–Bohm effect on a non-simply-connected region). This distinction is geometric and does not introduce any additional interaction beyond the assumed gauge structure.*

2.2 Uniqueness of the renormalizable coupling

Consider a Dirac spinor ψ on a spin manifold. Assume locality, Lorentz invariance, and at most first derivatives in the renormalizable matter sector, together with *phase-only entry* of the $U(1)$ fiber as specified above.

Theorem 1 (Holonomy ⇒ minimal coupling)**.** *Within these premises, the unique renormalizable matter Lagrangian consistent with gauge invariance and locality is*

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}\big(i\hbar\gamma^\mu D_\mu - m\big)\psi, \qquad D_\mu = \partial_\mu + \mathrm{i}\frac{q}{\hbar}A_\mu, \tag{5}$$

where A is the $U(1)$ connection 1-form transforming as in (3).

Sketch. Parallel transport by the connection ω implies $\psi(x+\mathrm{d}x) = \exp\big(\mathrm{i}(q/\hbar)A_\mu\,\mathrm{d}x^\mu\big)\psi(x)$. Expanding to first order yields the covariant derivative $D_\mu\psi = \partial_\mu\psi + \mathrm{i}(q/\hbar)A_\mu\psi$. Lorentz invariance and first-order dynamics select the Dirac kinetic structure. The phase-only entry condition forbids explicit A or F insertions in the matter sector at dimension ≤ 4; in particular, linear $F_{\mu\nu}$ couplings correspond to dimension-5 operators and lie outside the renormalizable regime considered here. □

Corollary 1 (Minimal coupling ⇒ $g = 2$)**.** *Within the premises of Theorem 1, the nonrelativistic reduction of $\mathcal{L}_{\text{Dirac}}$ yields the Pauli interaction $-\frac{q}{m}\mathbf{S}\cdot\mathbf{B}$, fixing the electron gyromagnetic ratio to its Dirac value $g = 2$ at leading order.*

3 Nonrelativistic reduction and $g = 2$

Performing the Pauli/Foldy–Wouthuysen reduction of (5) yields, to $\mathcal{O}(1/m^2)$,

$$H_{\text{FW}} = \beta\Big(m + \frac{\mathbf{\Pi}^2}{2m} - \frac{q}{2m}\boldsymbol{\sigma}\cdot\mathbf{B} - \frac{q}{8m^2}\nabla\cdot\mathbf{E} - \frac{q}{4m^2}\boldsymbol{\sigma}\cdot(\mathbf{E}\times\mathbf{\Pi})\Big) + q\phi + \mathcal{O}(m^{-3}), \tag{6}$$

with $\mathbf{\Pi} = \mathbf{p} - q\mathbf{A}$. The spin interaction takes the Pauli form

$$H_{\text{spin}} \;=\; -\frac{q}{m}\,\mathbf{S}\cdot\mathbf{B}, \qquad \mathbf{S} = \frac{\hbar}{2}\boldsymbol{\sigma}, \tag{7}$$

so that $\boldsymbol{\mu} = (g\,q/2m)\,\mathbf{S}$ with $g = 2$ fixed at tree level. Within the premises of Sec. 2.2, this result is enforced by minimal coupling and does not depend on additional dynamical assumptions.

Proposition 1 (No tree-level Pauli term under phase-only entry)**.** *If the gauge field A enters the renormalizable matter Lagrangian only through the covariant derivative D_μ, then no term of the form $\bar{\psi}\,\sigma^{\mu\nu}\psi\,F_{\mu\nu}$ appears at dimension ≤ 4. Dipole operators are therefore restricted to dimension 5 or higher, and anomalous magnetic moments arise from loop effects or higher-dimension operators.*

Remark 2 (What is (and is not) new)**.** *The value $g = 2$ is the standard Dirac prediction, and* SIT *does not modify it at leading order once the theory reduces to the Dirac–Maxwell sector. The contribution here is not a new numerical value but a structural explanation: a geometric origin in coherence-phase holonomy, a precise phase-only entry hypothesis that excludes tree-level dipole terms, and an effective-field-theory framework identifying where SIT-specific, environment-dependent deviations may arise without altering the standard QED anomaly.*

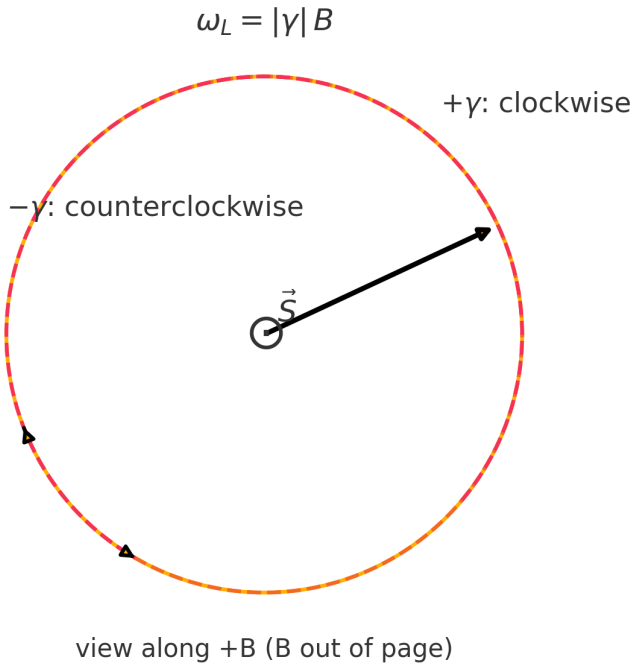

Figure 1: Spin precession sign viewed along $+\mathbf{B}$. Since $\boldsymbol{\Omega} = -\gamma\mathbf{B}$, positive (negative) γ yields clockwise (counter-clockwise) precession at $\omega_L = |\gamma|B$.

4 Sign of γ and Larmor precession

Because $q = -e$, the electron's magnetic moment $\boldsymbol{\mu}$ is antiparallel to its spin $\mathbf{S}$. The precession obeys $\dot{\mathbf{S}} = \gamma\,\mathbf{S}\times\mathbf{B} \equiv \boldsymbol{\Omega}\times\mathbf{S}$ with $\boldsymbol{\Omega} = -\gamma\mathbf{B}$. Thus a positive (negative) γ gives clockwise (counter-clockwise) precession when viewed along $+\mathbf{B}$. Figure 1 summarizes the sense.

5 SIT-specific corrections: when and how could g deviate?

Beyond the fully decohered Dirac limit, SIT allows slowly varying informational dressings that depend on the time-density ρ_t and coherence ratio $R_{\rm coh}$. When these quantities vary, the effective low-energy theory can include

$$\delta\mathcal{L} \;=\; \frac{\kappa(\rho_t, R_{\rm coh})}{4m}\,\bar{\psi}\,\sigma^{\mu\nu}\psi\,F_{\mu\nu}\,, \qquad \kappa \to 0 \text{ when } \nabla\rho_t, \nabla R_{\rm coh} \to 0. \tag{8}$$

A gauge-invariant parametrization above the electroweak scale is

$$\Delta\mathcal{L} \;\sim\; \frac{c_f}{\Lambda_{\rm SIT}^2}\,(\bar{L}_f\sigma^{\mu\nu}f_R H)\,F_{\mu\nu} + \text{h.c.} \;\Rightarrow\; \Delta\mathcal{L}_{\rm QED} \;=\; \frac{\tilde{c}_f}{\Lambda_{\rm SIT}^2}\,\bar{\psi}_f\sigma^{\mu\nu}\psi_f F_{\mu\nu}, \tag{9}$$

leading (after electroweak symmetry breaking) to $\delta a_f \propto m_f c_f/\Lambda_{\rm SIT}^2$. This provides a concrete falsification program: modulate coherence (or ρ_t) in ultra-coherent platforms (e.g. phase-locked electron traps or BEC-adjacent environments) and search for controlled, environment-dependent shifts in g beyond the QED anomaly; null results are expected in fully decohered regimes.

6 What is new, exactly, in the SIT framing?

1. **Geometric interpretation:** electromagnetism as an informational holonomy renders Dirac minimal coupling a statement about parallel transport in a coherence-phase fiber.
2. **Structural constraint:** the phase-only entry rule explains the absence of tree-level Pauli terms and enforces $g = 2$ at leading order once the theory reduces to the Dirac–Maxwell regime.
3. **Predictive lever:** deviations require specific gradients of ρ_t or R_{coh} and are therefore testable and, in their absence, suppressed—consistent with precision $g - 2$ data.

7 Discussion and conclusion

The factor $g = 2$ is robust: once SIT reduces to the Dirac–Maxwell theory with minimal coupling, the Pauli term appears with unit coefficient in the nonrelativistic limit, while quantum electrodynamics accounts for the anomalous correction. The holonomy viewpoint supplies a geometric rationale for this structure, and the effective field theory map identifies controlled avenues for SIT-specific deviations that remain consistent with existing experimental bounds.

Falsifiability and scope. The conclusions of this work are conditional on the SIT premises of phase-only entry and reduction to the Dirac–Maxwell sector in the decohered limit. The framework would be falsified if a tree-level dipole coupling were observed in a regime where these premises hold, or if environment-controlled variations predicted by the effective theory fail to track gradients of the time-density ρ_t or coherence ratio R_{coh}. Standard QED loop corrections to $g - 2$ lie outside the scope of this test and are not modified by the present analysis. This conclusion follows directly from Theorem 1 and Corollary 1, together with the standard Foldy–Wouthuysen reduction.

Acknowledgments

Thanks to colleagues whose feedback sharpened two key points emphasized here: (i) distinguishing pure-gauge gradients from genuine connections, and (ii) being explicit that $g = 2$ at tree level is the Dirac result—the SIT contribution is geometric and predictive.

References

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