Files Outline

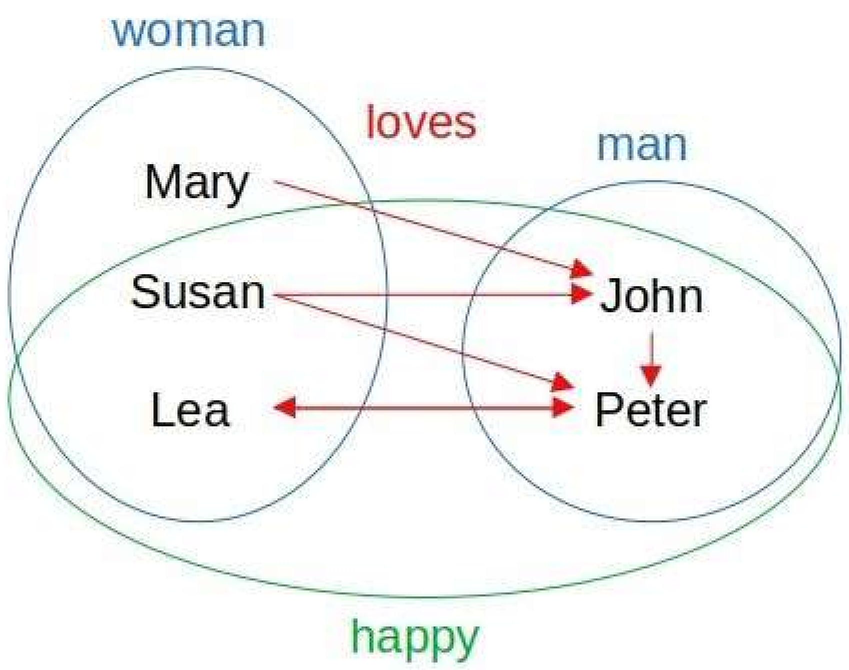
Automated Model Generation, Model Checking and Theorem Proving for Linguistic Applications

Natalie Clarius
University of Tübingen

pyPL

https://github.com/nclarius/pyPL

Model checking



Every woman loves a man.

$$\forall x(Woman(x) \rightarrow \exists y(Man(y) \wedge Loves(x, y)))$$

True

Every man loves a woman.

$$\forall x(Man(x) \rightarrow \exists y(Woman(y) \wedge Loves(x, y)))$$

False

Theorem proving

Inference

$$\underbrace{\psi_1, \ldots, \psi_n}_{\text{premises}} \models \underbrace{\phi}_{\text{conclusion}}$$

$\Longleftrightarrow$ There exists no structure in which all premises are true but the conclusion is false

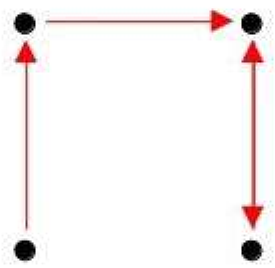
Analytic tableaus

- Refutation calculus:
  Assume that the premises are true but the conclusion is false, and derive a contradiction
- Systematically analyze in a tree structure what must be the case if the assumptions are to hold
- Each branch stands for one way of making the assumptions true;
  formulas on one branch are read as simulateneously true
- If both *P* and *not P* appears on a branch, then this way of trying to invalidate the inference fails:
  The branch is closed $\times$
- If no contradiction arises, then this branch enables the extraction a counterexample:
  The branch is open $\circ$
- If all branches of the tree are closed, there is no counterexample and the inference is valid;
  if at least one branch of the tree is open, there is a counterexample and the inference is invalid

Everyone heard someone.

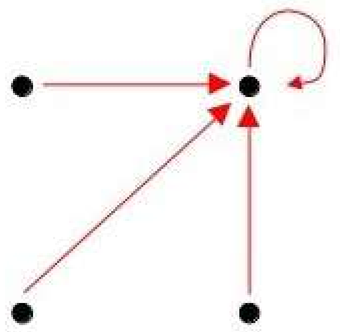
(1) For everyone there is someone they heard, but different people may have heard different persons

$$\forall x \exists y Heard(x, y)$$



(2) There is a common person that everyone heard

$$\exists y \forall x Heard(x, y)$$



$$\exists y \forall x Heard(x, y) \models \forall x \exists y Heard(x, y)$$

$$\forall x \exists y Heard(x, y) \not\models \exists y \forall x Heard(x, y)$$

Conventional tableaux

$$
\begin{array}{c}
\forall v \phi(v) \\
\phi(c_1) \\
\vdots \\
\phi(c_n) \\
\\
c_i \text{ arbitrary}
\end{array}
\qquad\qquad
\begin{array}{c}
\exists v \phi(v) \\
\phi(c_1) \\
\vdots \\
\phi(c_n) \\
\\
c_i \text{ new}
\end{array}
$$

- try to find contradictions (closed branches)
- instantiate every existential claim with a different individual to preserve generality

Modified tableaux

$$
\begin{array}{c}
\forall v \phi(v) \\
\phi(c_1) \\
\vdots \\
\phi(c_n) \\
\\
c_i \text{ old}
\end{array}
\qquad\qquad
\begin{array}{ccc}
 & \exists v \phi(v) & \\
\swarrow & \downarrow & \searrow \\
\phi(c_1) & \ldots & \phi(c_n) \\
\\
 & c_i \text{ arbitrary} &
\end{array}
$$

- try to find models (open branches)
- try to identify an existential witness with an already known individual to preserve minimaility; if that fails, try a different one until a suitable structure is found

Model generation

A student is reading a book.

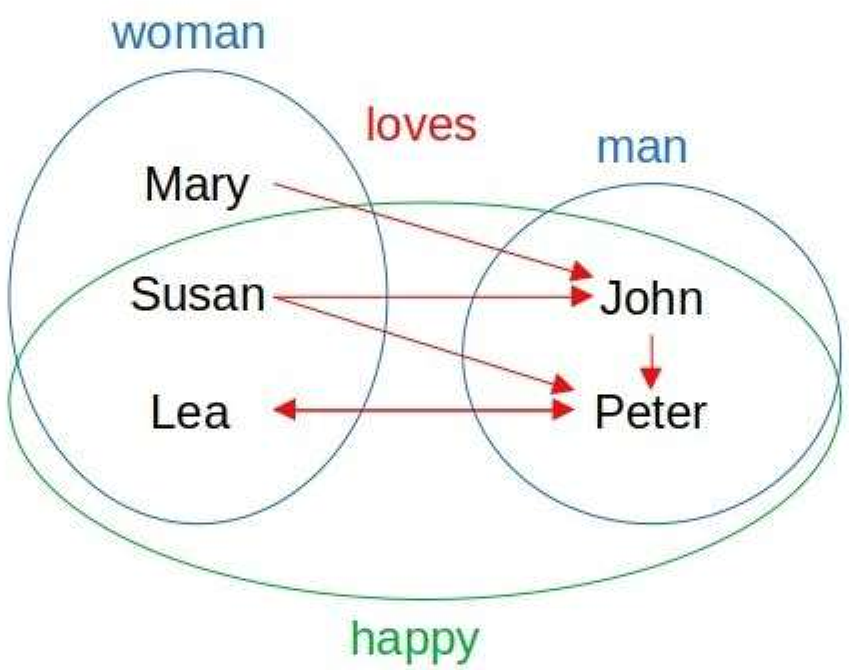
- the student $\neq$ the book   (students are not books)

There are two birds in the garden. A sparrow is chirping and a blackbird is taking a bath in the pond.

- bird#1 $\neq$ bird#2   ("two" = $\exists x \exists y (x \neq y)$)
- the sparrow $\neq$ the blackbird (sparrows are not blackbirds and blackbirds are not sparrows)
- bird#1 = the sparrow & bird#2 = the blackbird

Special phenomena

Generalized quantifiers



Most women are happy.

$$|\text{woman} \cap \text{happy}| > |\text{woman} - \text{happy}|$$

True

More women than men are happy.

$$|\text{woman} \cap \text{happy}| > |\text{man} \cap \text{happy}|$$

False

Modality

A comet might hit and destroy earth.

(1) There exists a comet which in some hypothetical situation is going to hit earth

$$\exists x \Diamond Comet(x)$$

(2) There is a hypothetical situation in which there is a comet that is going to hit earth

$$\Diamond \exists x Comet(x)$$

possible worlds

Intensional contexts

Joe Biden is a democrat and not a republican. Donald Trump is a republican and not a democrat.

Joe Biden is the president elect of the U.S.

Mary believes that Donald Trump is the president elect of the U.S.

Mary believes that Joe Biden is a democrat.

Mary does not believe that the president elect is a democrat.

$biden = president$: True
$Believe(mary, {}^\wedge Democrat(biden))$: True
$Believe(mary, {}^\wedge Demcrat(president))$: False

	real world	Mary's world
$biden$	Joe Biden	Joe Biden
$Democrat(biden)$	True	True
$president$	**Joe Biden**	**Donald Trump**
$Democrat(president)$	True	False

Foundations and limitations

Conventional tableaus:

Complete for validity (R. Smullyan 1965)

- If an inference is valid, the conventional tableau method will find a proof
- If an inference is invalid, the conventional tableau method will sometimes find a refutation

Modified tableaus:

Complete for finite satisfiability (G. Boolos 1984)

- If a set of formulas has a finite model, the modified tableau method will find one
- If an inference is invalid with a finite countermodel, the modified method will find a refutation

=> Conventional tableaus **+** modified tableaus:

- All valid inferences and all invalid inferences with finite countermodels can be detected
- Only some invalid inferences that only have infinite countermodels can not be detected

Undecidability of first-order logic (A. Church & A. Turing 1936)

- There is no algorithm that can detect for every first-order inference whether it is valid or invalid

Every tupperware box has a fitting lid.

There is no lid that fits on all tupperware boxes.

Tupperware boxes are not lids.

Tupperware boxes exist.

2^{24} possible minimal structures, out of which only
2^{5} (every ~100.000th branch) are models of the theory

---

infinite domains

Outlook

Extensions

- efficiency
- more world knowledge
- lambda calculus and e-t type theory
- other modal logics, tense logic, full intuitionistic logic, fuzzy logics
- other frameworks, e.g. DRT
- other calculi, e.g. ND

Didactic use

- verification of inferences
- theory via implementation
- programming assignments