# Invertable Matrix Theorem

10 Statements are either all true or all false

(1) A is invertible

(2) The equation $A\vec{x} = \vec{b}$ has exactly one solution for each $\vec{b} \in \mathbb{R}^n$

(3) The null space of A contains only the zero vector $\mathcal{N}(A) = \{\vec{0}\}$

(4) The equation $A\vec{x} = \vec{0}$ has only the trivial solution $\vec{x} = \vec{0}$

(5) The columns of A form a basis for $\mathbb{R}^n$:
- The columns of A are linearly independent
- The columns of A span $\mathbb{R}^n$; $\mathcal{C}(A) = \mathbb{R}^n$

(6) The rank of the matrix A is n

(7) The RREF of A is the $n \times n$ identity matrix $\mathbb{1}_n$

(8) The transpose matrix $A^\top$ is invertible

(9) The rows of A form a basis for $\mathbb{R}^n$:
- The rows of A are linearly independent
- The rows of A span $\mathbb{R}^n$; $\mathcal{R}(A) = \mathbb{R}^n$

(10) The determinant of A is nonzero $\det(A) \neq 0$

NO BULLSHIT
guide to
LINEAR ALGEBRA

by Ivan Savov
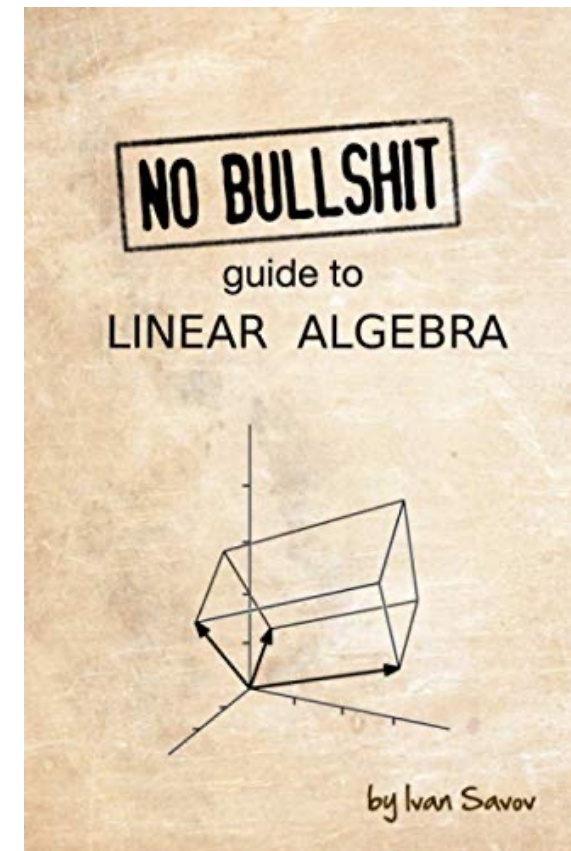
The book (Chapter 5) does an excellent job of proving statements (1) to (10) are equivalent.

You should review the proofs if you are interested in more mathematical research (theory, computation).

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