




1 AQCNES: A Quasi-Continuum Non-Equilibrium Solver

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7 Summary

8 The behavior of macroscopic structures is determined by fast atomic interactions at the
9 nanoscales. Current atomic simulation techniques, such as molecular dynamics (MD), are
10 limited to a millions of atoms and hence a few micrometers of domain length. Moreover, finite-
11 temperature vibrational frequencies of around tens of terahertz restrict the time step of MD to
12 femtoseconds, precluding the simulation of problems of engineering interest. Consequently, there
13 has been a significant focus in recent decades on developing multiscale modeling techniques to
14 extend atomistic accuracy to larger length scales and longer time frames. AQCNES is a scalable
15 C++-based framework that integrates the spatial-upscaling technique of the quasicontinuum
16 method with the statistical-mechanics-based temporal upscaling technique known as Gaussian
17 phase packets. This enables computationally efficient and robust simulations of large atomistic
18 ensembles at finite temperature.

19 Statement of need

20 AQCNES is a software that can predict long-term behavior of large atomic ensembles using
21 spatio-temporal upscaling of classical atomistic calculations. It enables the calculation of
22 material properties at finite (non-zero) temperature from atomic scales, offering promising
23 applications in solid-state material science across scales. Current atomic simulation techniques
24 describe the entire ensemble as a collection of particles, each having a position and velocity
25 in the three-dimensional space. This fully refined spatial representation restricts the possible
26 dimensions of the ensemble to microscopic scales. Moreover, case studies of material defects
27 using atomistic simulations are often limited to Molecular Statics (MS) or use unrealistically
28 high loading rates ([Homer et al., 2022](#); [Shenoy, 2005](#)). This is unavoidable because finite-
29 temperature MD simulations need long equilibration times and expensive post-processing
30 techniques ([Frenkel & Ladd, 1984](#)) to extract relevant thermodynamic information. However,
31 physically relevant material behavior is observed at finite temperature. Therefore, research
32 in the past decades has focused on multiscale modeling techniques to advance atomistic
33 simulations to larger length scales and longer time scales.

34 To bridge across length scales, AQCNES uses a spatial coarse-graining technique known as the
35 quasicontinuum (QC) method ([Ronald E. Miller & Tadmor, 2002](#); [Tadmor, 1996](#)). QC exploits
36 the long-range order in crystalline materials to explicitly model a few representative atoms in the
37 domain and uses techniques from continuum-level finite-element modeling to obtain all other
38 atomic degrees of freedom as a function of those of the representative atoms. Fully atomistic
39 resolution is retained in the vicinity of material defects where the long-range order is broken. A
40 technique known as Gaussian Phase Packets (GPP) ([Gupta et al., 2021](#)) is used to upscale in
41 time. Instead of the instantaneous phase-space coordinates (position and velocity), statistical
42 averages and variances of these coordinates are tracked in GPP, thus separating the slow

43 mean atomic motion from fast atomic vibrations. The governing equations of those statistical
44 parameters can be shown to yield configurations which minimize a thermalized Helmholtz
45 free-energy at the temperature of interest. Thus a thermodynamically relaxed structure of the
46 ensemble is obtained after a quasi-static minimization. Hence, this approach is more robust
47 and computationally efficient (Saxena et al., 2022) than tracking individual atomic trajectories.
48 Moreover, it makes the simulation of realistic and quasi-static loading scenarios possible, in
49 contrast to the unrealistically high strain-rates accessible by MD simulations (Vu-Bac et al.,
50 2014; Zhao et al., 2010).

51 Most of the research groups working in the broad field of upscaling atomistic simulations have
52 their proprietary codes. Two available open-source codes are QuasiContinuum (Ron Earle Miller
53 & Tadmor, 2012) based on the QC method and MultiBench (Ronald E. Miller & Tadmor, 2009),
54 which is an implementation of fourteen popular spatial upscaling methods. However, both
55 are limited to zero-temperature simulations for crystalline solids in two dimensions. The MXE
56 package in LAMMPS (Mendez & Ponga, 2021) is an implementation of the temporal upscaling
57 technique max-ent for a fully resolved atomic ensemble with no spatial coarse-graining. To the
58 best of the authors' knowledge, there is no such open-source atomistic simulation software that
59 combines the aforementioned spatial and temporal upscaling techniques. AQCNESS is capable of
60 simulating amorphous materials at atomistic resolution and crystalline materials with spatial
61 coarse-graining, while seamlessly transitioning to atomistic resolution at both zero and non-zero
62 temperatures. It can also simulate large atomic rearrangements induced by severe deformations
63 in multi-resolution domains by using an updated Lagrangian formulation (Gupta et al., 2021).
64 This formulation uses the relaxed state after every load step as the new reference configuration.
65 An adaptive neighborhood calculation strategy (similar to Amelang (2016)) is adopted, where
66 the neighborhoods are regenerated if the maximum relative displacement of a neighbor with
67 respect to a sampling atom exceeds a given buffer radius. This unique combination of features
68 positions AQCNESS as a versatile and powerful tool in the realm of atomistic simulations.

69 Detailed documentation of the API can be found on the AQCNESS website, and simple examples
70 to get started can be found here. Importantly, the use of AQCNESS is not limited to these
71 examples and the ones listed in the next section.

72 Example applications

73 The following gives a summary of the mechanics and material science applications where
74 AQCNESS has already been used:

- 75 ■ **Surface Elasticity:** Surfaces in solids are the simplest extended defects and contribute
76 to excess energy, leading to an inherent stress associated with them. Their presence
77 also changes the elastic moduli of the solid as compared to those of the bulk solid.
78 AQCNESS was used by Saxena et al. (2022) to perform a detailed case study of three
79 different crystallographically oriented surfaces for FCC and BCC metals as a function of
80 temperature. The results were also compared against those obtained from state-of-the-art
81 thermodynamic integration techniques (Freitas et al., 2016), suggesting a convincing
82 accuracy. The comparison of computational times taken by MD and AQCNESS (see Table 5
83 in Saxena et al. (2022)) shows that there is an approximately fifty-fold computational
84 speedup and a reduction in the computational resources needed for the same simulation.
- 85 ■ **Grain Boundaries:** Grain boundaries (GBs) are regions of crystallographic mismatch
86 between two differently oriented grains. They significantly influence the mechanical and
87 thermal properties of polycrystalline materials. Hence, investigating GB properties via
88 atomic simulations is of scientific interest in the material science community. AQCNESS
89 has been used to find relaxed energies of [001] and [011] symmetric-tilt GBs (Spínola et
90 al., 2024) in copper as a function of temperature for a range of tilt angles. Different
91 metastable states have been explored for each temperature and tilt angle. In addition,
92 the lowest-energy metastable state was subjected to a quasi-static displacement-driven

93 shear to obtain the shear coupling factor of all grain boundaries. AQCNES could also
94 identify the Helmholtz free energy of bicrystals, for which the standard thermodynamic
95 integration techniques failed due to hops of the system from one metastable state to
96 another.

97 ■ **Nanoindentation:** Nanoindentation is a widely used technique to probe the mechanical
98 properties of materials and nanostructures. AQCNES has been used to simulate the three-
99 dimensional thermo-mechanically-coupled nanoindentation of copper (Gupta et al., 2021).
100 Two layers of spatial coarse graining were used to simulate a cube of side length $0.077\mu\text{m}$
101 with 0.2 million representative atoms. The complicated microstructure of prismatic
102 dislocation loops below the nanoindenter could be observed in the finite-temperature
103 simulations at 300 and 600 K. The temperature dependence of the critical indenter force
104 before dislocation nucleation could also be captured.

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