

The University of Sheffield
ACS6101 Foundations of Control Systems
Week 5 Assignment

Paulo Roberto Loma Marconi
prlomarconi1@sheffield.ac.uk

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1 Question 1

The aim of this task is to design a digital controlled system with some requirements, small steady state error, small settling time, minimum input action, and minimum overshoot.

The plant to be studied is,

$$G_p(s) = \frac{0.04(s + 1)}{s^2 + 0.2s + 0.04} \quad (1)$$

and the digital controller should have the form,

$$D(z) = K \frac{z - A}{z - B} \quad (2)$$

Fig.1 shows that the plant is very slow with a big overshoot. A phase margin (PM) of 60° can be the unique requirement. As long as the compensated phase margin is around that value, the settling time and overshoot should be minimized as much as possible.

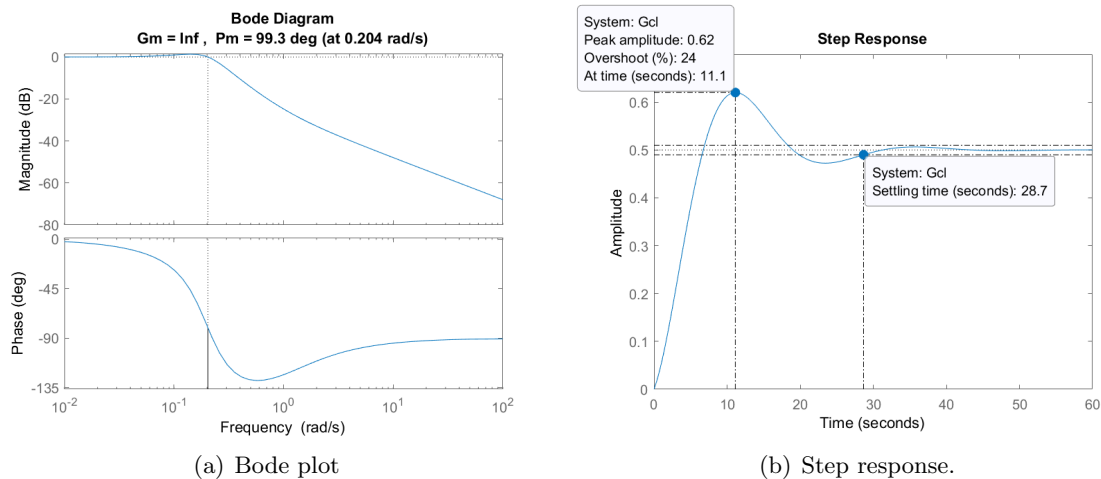


Figure 1: Evaluation of the plant G_p

1.1 Phase-Lead compensator

The selected compensator can be written as follows,

$$G_c = K_c \frac{s + z}{s + p}, \quad |z| \leq |p| \quad (3)$$

Step 1. Calculate a gain K that satisfies the desired phase margin $PM_d = 60^\circ$. Applying the angle condition for a PM_d over G_p ,

$$\begin{aligned}\angle G_p(j\omega'_c) &= PM_d - 180^\circ \\ \angle G_p(j\omega'_c) &= 60^\circ - 180^\circ \\ \angle G_p(j\omega'_c) &= -120^\circ\end{aligned}\quad (4)$$

where ω'_c is the new crossover frequency for the desired phase margin $PM_d = 60^\circ$.

Using the bode plot of G_p , Fig. 2a, the logarithm gain at -120° is -8.02 dB, so the gain K can be calculated as follows,

$$\begin{aligned}20 \log_{10} K &= |-8.02| \\ K &= 2.52\end{aligned}\quad (5)$$

Therefore, the uncompensated G_p that satisfies the desired phase margin is,

$$\begin{aligned}G_{p1} &= K G_p \\ G_{p1} &= 2.52 \frac{0.04(s+1)}{s^2 + 0.2s + 0.04}\end{aligned}\quad (6)$$

and Fig. 2b shows that the new uncompensated plant G_{p1} satisfies the desired phase margin.

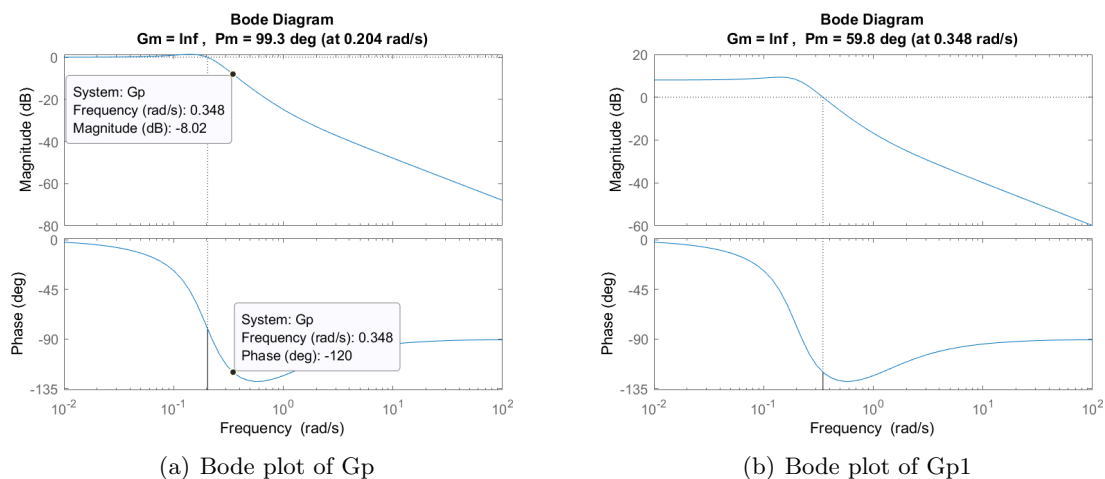


Figure 2: Evaluation of the plant G_p and G_{p1}

The following Matlab scripts simulates the previous results.

```
%% =====
% ---- ACS6101 Assignment week 5
% ---- Registration number: 180123717
% ---- Name: Paulo Roberto Loma Marconi
% ---- 03/11/2018
%% === Question 1 =====
clear; clc; close all; % clean previous data in order to avoid erros.
%% plant Gp
s = tf('s');
Gp = 0.04*(s+1)/(s^2+0.2*s+0.04); % uncompensated plant

fig = figure(1);
margin(Gp); % calculates the phase margin and gain margin at their frequencies
[Gm,Pm,Wcg,Wcp] = margin(Gp);
Gcl = feedback(Gp,1); % closed-loop of the uncompensated plant
```

```

saveas(fig,'Q1_Gp_margin.png');

fig = figure(2);
step(Gc1); % step response to the closed-loop system
stepinfo(Gc1) % system performance values
saveas(fig,'Q1_Gp_step.png');
%% Design requirements
PO = 10; % percentage overshoot
zeta = log(100/PO)/sqrt(pi^2+ (log(100/PO))^2 ); % damping ratio
PM_d = round(100*zeta)+1; % desired PM
%% Obtaining the gain K that meets the desired PM_d
K = 10^(8.03/20); % the gain 8.03 obtained from the bode plot
Gp1 = K*Gp; % new uncompensated plant

fig = figure(11);
margin(Gp1);
saveas(fig,'Q1_Gp1_K_margin.png');

```

Step 2. The digital uncompensated plant G_{z1} of G_{p1} can be calculated using a zero-order holder with a sampling time $T_s = 0.01$.

$$G_{z1} = 1.01 \cdot 10^{-3} \frac{z - 0.99}{z^2 - 1.99z + 0.99} \quad (7)$$

In Fig. 3 it can be seen that the continuous and discrete plant are almost similar. Also, the settling time has been reduced but the steady state error is too big.

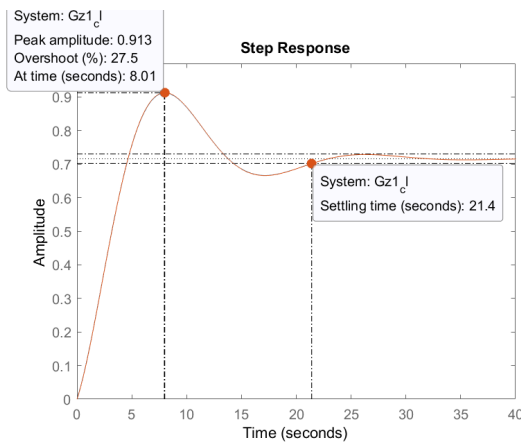


Figure 3: Step response of G_{z1} and G_{p1}

The following Matlab scripts simulates the previous results.

```

%% Digital uncompensated system with the new gain K
Ts = 0.01; % sampling time
Gz1 = c2d(Gp1,Ts,'zoh'); % convert the continuous time plant Gp2 to the
% discrete time domain with the zero order holder
Gp1_cl = feedback(Gp1,1); % closed-loop system in continuous time
Gz1_cl = feedback(Gz1,1); % closed-loop system in discrete time

fig = figure(3);
step(Gp1_cl,Gz1_cl); % step response of both continuous and discrete systems
saveas(fig,'Q1_Gpz_K_step.png');
stepinfo(Gz1_cl) % system performance values

```

Step 3. With the desired phase margin, the value of β can be calculated as follows,

$$PM_{act} - PM_d + \theta = \arctan \frac{\beta - 1}{2\sqrt{\beta}} \quad (8)$$

where PM_{act} is the actual phase margin of the uncompensated plant G_{p1} , and θ is a factor of correction.

After some operations,

$$\beta^2 - \beta [2 + 4 (\tan(PM_d - PM_{act} + \theta))] + 1 = 0$$

if $\theta = 6^\circ$, $PM_d = 60^\circ$, and $PM_{act} = 61.25^\circ$ obtained from Fig. 4. The β will be,

$$\beta = 1.18$$

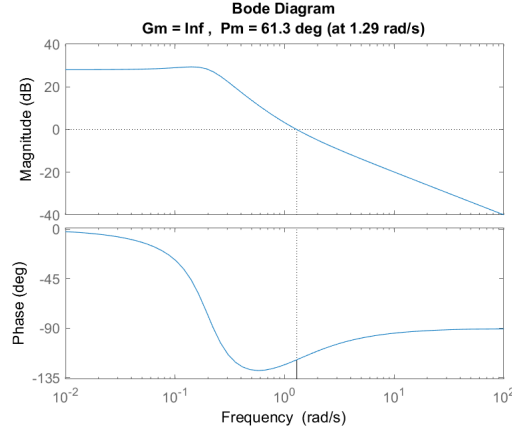


Figure 4: Bode plot of Gp2

Step 4. Now, the crossover over frequency ω_c needs to be calculated using the following gain condition formula,

$$|G_{p1}(j\omega_c)| = \frac{1}{\sqrt{\beta}} \quad (9)$$

if we use the peak magnitude M_{pc} relation,

$$M_{pc} = \frac{1}{\sqrt{\beta}} \quad (10)$$

and using `getGainCrossover(Gp2,Mpc)` command on Matlab,

$$\omega_c = 1.37 \text{ rad/sec}$$

Step 5. Determining the zero of the controller,

$$\begin{aligned} \omega_c &= \sqrt{\beta z^2} \\ z &= 1.26 \end{aligned} \quad (11)$$

therefore, the compensator in continuous time can be written as follows,

$$\begin{aligned} G_c &= \beta \frac{s + z}{s + \beta z} \\ G_c &= 1.18 \frac{s + 1.26}{s + 1.49} \end{aligned} \quad (12)$$

and in discrete time,

$$G_c z = 1.18 \frac{z - 0.99}{z - 0.98}$$

The open-loop compensated in continuous and discrete time are,

$$G_{ol} = G_c G_{p1} \quad (13)$$

$$G_{ol} = 1.18 \frac{s + 1.26}{s + 1.49} \quad 2.52 \frac{0.04(s + 1)}{s^2 + 0.2s + 0.04} \quad (14)$$

$$G_{olz} = 0.01 \frac{z - 0.99}{z - 0.98} \quad \frac{z - 0.98}{z^2 - 1.99 + 0.99} \quad (15)$$

Fig. 5 shows that the compensated system using a Phase-Lead controller achieves with success the desired phase margin of 60° , and has a small settling time $t_s = 4.84 \text{ sec}$.

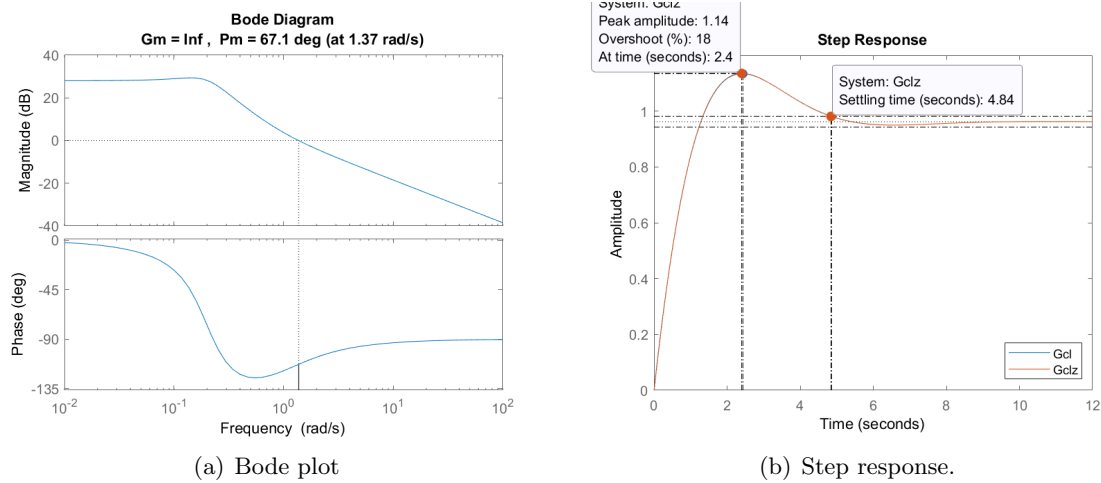


Figure 5: Evaluation of Phase-Lead compensated system in continuous and discrete time.

The following Matlab script simulates and evaluates the previous design.

```
%% Designing the Phase-lead digital controller
% introducing a new gain 10 times faster in order to obtain a fast
% response to the step input
Gp2 = K*10*Gp;

fig = figure(4);
margin(Gp2); % checking the desired phase margin PM_d
[Gm1,PM_act,Wcg1,Wcp1] = margin(Gp2);
saveas(fig,'Q1_Gp2_margin.png');

% step 1) obtaining beta with the actual PM and the desired PM
theta = 6; % correction factor
beta = roots([1 -(2+4*(tand(PM_d-PM_act+theta))^2) 1]);
% beta = (1+sind(PM_d-PM_act+theta))/(1-sind(PM_d-PM_act+theta));
% step 2) calculating the new crossover frequency
Mpc = 1/sqrt(beta(1)); % find compensator peak magnitude.
omega_c = getGainCrossover(Gp2,Mpc); % The new gain crossover frequency wc
% step 3) determining the zero and the pole of the controller
zc = omega_c/sqrt(beta(1)); % zero of the controller
pc = beta(1)*zc; % pole of the controller
% step 4) controller in continuous and discrete time
Gc = beta(1)*(s+zc)/(s+pc); % phase-lead controller in continuous time
Gcz = c2d(Gc,Ts,'zoh'); % phase-lead controller in discrete time
```

```

%% Evaluating the phase-lead controller
Gol = Gc*Gp2; % open-loop compensated in continuous time
Gcl = feedback(Gol,1); % closed-loop in continuous time
Gp2z = c2d(Gp2,Ts,'zoh'); % plant in discrete time
Golz = Gcz*Gp2z; % open-loop compensated in discrete time
Gclz = feedback(Golz,1); % closed-loop in discrete time

fig = figure(5);
margin(Gol);
saveas(fig,'Q1_lead_margin.png');

fig = figure(6);
step(Gcl,Gclz);
saveas(fig,'Q1_lead_step.png');

```

2 Question 2

The deadbeat controller approach finds an input signal in order to bring the output to the steady-state in the smallest number of time steps [1]. It has some characteristics:

- Zero steady-state error.
- Minimum rise time.
- Minimum settling time
- Less than 2% overshoot/undershoot.
- Very high control signal output.
- Can not be used in continuous time.

As long as the plant $G(z)$ has all zeros and poles inside the unit circle (minimum phase), the plant can be written as,

$$G(z) = \frac{z^d B(z)}{A(z)} = \frac{z^d (b_0 + b_1 z^{-1} + \dots + b_m z^{-m})}{1 + a_1 z^{-1} + \dots + a_n z^{-n}}, \quad d = n - m > 0$$

with an step input $r(k)$, the deadbeat response requirement implies,

$$M(z) = z^{-d}$$

The digital deadbeat will have the following transfer function,

$$D(z) = \frac{A(z)}{z^{-d} B(z)} \frac{z^{-d}}{1 - z^{-d}} \quad (16)$$

and the closed-loop system is,

$$Y(z) = z^{-d} U(z)$$

which means,

$$y(k) = u(k - d)$$

So when $u(k)$ is a step input, the output will be d-step delay of the same signal.

The aim of this question is to design a digital deadbeat controller for the plant,

$$G_p(s) = \frac{0.04(s+1)}{s^2 + 0.2s + 0.04}$$

$$G_p(z) = \frac{5.47 \times 10^{-2}(z - 0.34)}{z^2 - 1.78z + 0.82}$$

with sampling time $T_s = 1$.

After using the following code in Matlab.

```

%% =====
% ---- ACS6101 Assignment week 5
% ---- Registration number: 180123717
% ---- Name: Paulo Roberto Loma Marconi
% ---- 03/11/2018
%% === Question 2 =====
clear; clc; close all;
s = tf('s');
Gp = 0.04*(s+1)/(s^2+0.2*s+0.04); % uncompensated plant
Ts = 1; % sampling time
Gz = c2d(Gp,Ts,'zoh'); % System transfer function in discrete time
% Minimum phase case
z=zpk('z',Ts);
Mz = 1/z; % closed-loop because the relative order d = 1
Dz = Mz/(Gz*(1-Mz)); % deadbeat controller
Dz = minreal(Dz); % cancel common factors
fig = figure(1);
step(Dz*Gz/(1+Dz*Gz)) % step response of closed-loop
saveas(fig,'Q2_deadbeat_step.png');
fig = figure(2);
step(Dz/(1+Dz*Gz)) % control signal
saveas(fig,'Q2_deadbeat_control.png');

```

The closed-loop discrete system is,

$$G_{dz} = \frac{(z-1)(z-0.3392)^2(z^2-1.783z+0.8187)^2}{z(z-0.3392)^2(z-1)(z^2-1.783z+0.8187)^2}$$

where the controller is,

$$G_{cz} = \frac{18.288(z-1)(z-0.3392)(z^2-1.783z+0.8187)^2}{z(z-0.3392)^2(z-1)(z^2-1.783z+0.8187)}$$

Fig. 6 shows the output of the deadbeat controller and the closed-loop system response to a unit step input. It shows clearly that the steady-state error reaches zero value at the first sampled time.

In addition, the control signal is very high when the sampling time is increased, Fig. 7.

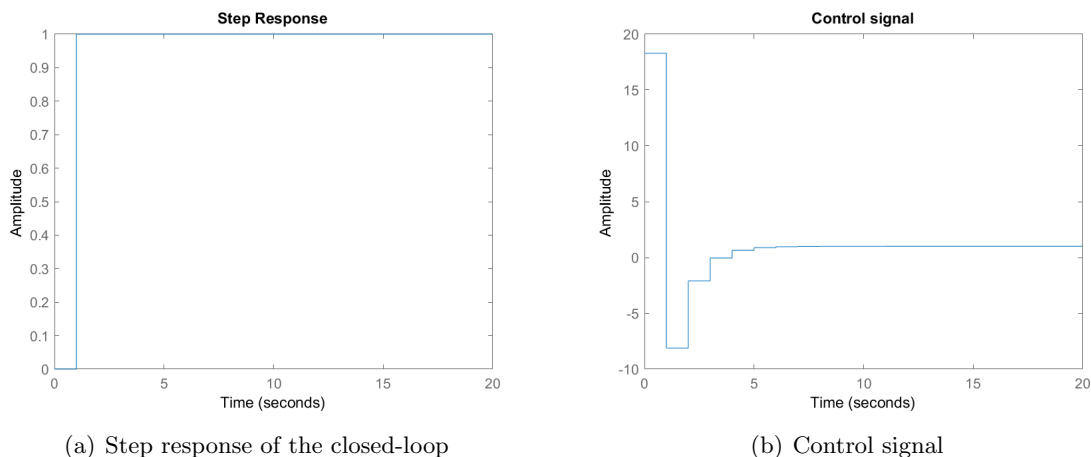


Figure 6: Performance of the deadbeat controller system for $T_s = 1$

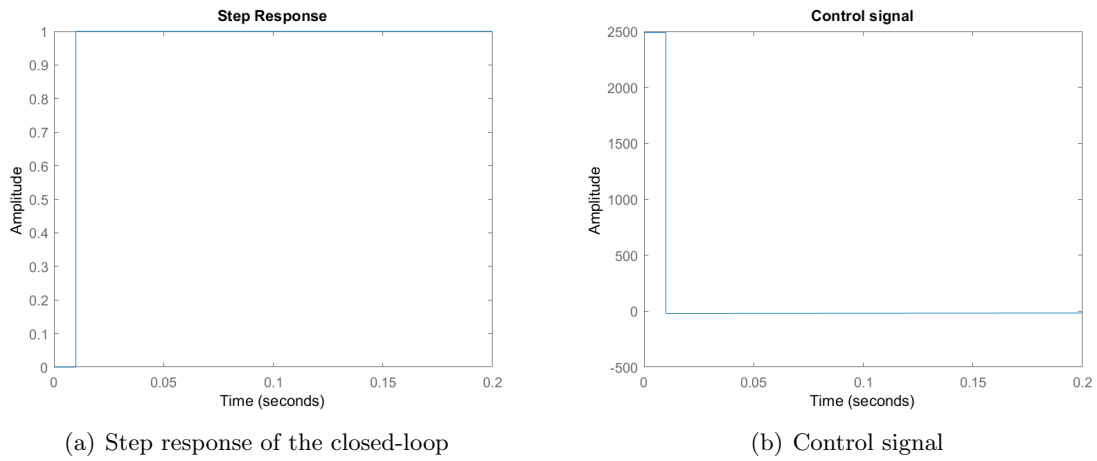


Figure 7: Performance of the deadbeat controller system for $T_s = 0.01$

References

- [1] Louis C. Westphal. *Handbook of Control Systems Engineering*. Springer Us, Dec. 6, 2012. URL: https://www.ebook.de/de/product/25178612/louis_c_westphal_handbook_of_control_systems_engineering.html (cit. on p. 6).