Two Channels, Three Readouts: A Typing Refinement of Structural Spin

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Abstract

Structural spin is the non-gradient layer S in a flow $\dot{x} = -\nabla V + S$. Its projection-relative signature has two formally independent components: a local differential channel $d(S^\flat)$, whose half is operational spin, and a global cohomological channel $[\eta_S] \in H^1_{\mathrm{dR}}(M)$, whose periods provide winding witnesses. This note separates that two-channel structure from three scalar readout families—period, path norm/amplitude, and vorticity flux—and from a fourth object, the dynamic residual used in drift or cost ledgers.

The distinction is load-bearing. A non-zero period forces a positive path norm when both are read from the same declared one-form; this is the triangle inequality, not a lower bound on vorticity. Conversely, neither a path norm nor a vorticity flux determines the cohomology class. Explicit local-only and topological-only examples show that the two channels are logically independent over the class of all declared signature data, while an equal-norm collision on a flat torus shows that an amplitude-only monitor is incomplete for winding. On the compact admissible branch, the topological-only sector remains premise-gated by the harmonic obstruction space W_V. A registered finite spanning family of harmonic-evaluation covectors reduces this gate exactly to the zero test $Ea = 0$, so algebraic independence is not confused with unrestricted realizability.

The resulting typing principle requires every bridge from structural spin to a measured rotation index, dynamic residual, or maintenance cost to declare its probe and observation map. Applied to cortical rotating waves, it yields a narrow empirical hypothesis: winding can collapse while 2–8 Hz amplitude remains above a calibrated detection floor and within a predeclared equivalence margin. The dual local-only arm uses an antisymmetric-Jacobian score defined without explicitly consuming the registered amplitude functional, a positive instrument/effect floor, and an exchangeability-calibrated rank threshold. The stronger *survives-but-dead* verdict additionally requires both an independent viability/continuation control and a predeclared identity-witness bridge.

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1 Purpose and relation to the preceding work

The preceding structural-spin paper established a faithful-not-full forgetful separation over a gradient skeleton, a cohomological witness for topological identity transfer, the necessity/blindness limits of that witness, and a compact-branch obstruction to eliminating operational spin while preserving an admissible non-zero class [1]. It also stated the central decomposition: one non-gradient layer has a local vorticity channel and a global cohomological channel.

The present note does not replace those results. It resolves a narrower typing ambiguity created when the phrases *local channel*, *spin magnitude*, *vorticity flux*, and *residual spin* are allowed to stand for one another. They do not have the same type. The correction can be stated in one line:

$$\boxed{\begin{array}{c} \text{one carrier} \;\longrightarrow\; \text{two structural channels} \\ \text{three readout families} \;\longrightarrow\; \text{an explicitly bridged dynamic ledger} \end{array}}$$

This separation sharpens the spin-channel bridge [3], the physical operational-spin construction [2], and the consolidated NC2.5 programme statement [4]. It also isolates exactly what a rotating-wave experiment can test without importing the identity semantics of the abstract theory.

2 Carrier and structural signature

Let (M, g) be an oriented Riemannian manifold, let ∇ denote its Levi-Civita connection, and let

$$F = -\nabla V + S, \qquad \omega_S := S^\flat.$$

The metric is fixed; the musical isomorphism and all norm statements are relative to g. A splitting $F = -\nabla V + S$ supplies a representative, but the signature below is invariant under its gauge once the selector is fixed: the substitution $(V, S) \mapsto (V + u, S + \nabla u)$ leaves F unchanged while replacing ω_S by $\omega_S + du$. Throughout, the exterior derivative of a 1-form uses the convention

$$d\omega(X, Y) = X\omega(Y) - Y\omega(X) - \omega([X, Y]),$$

so that $d(-y\,dx + x\,dy) = 2\,dx \wedge dy$; the factor $\frac{1}{2}$ in Definition 1 is stated relative to this convention and changes under the alternating normalization. On a compact oriented boundaryless manifold, Hodge theory [7] gives

$$\omega_S = df + \delta\beta + h.$$

On the alternative closed-component branch, assume a fixed declared linear projection

$$P_{\rm cl} : \Omega^1(M) \longrightarrow Z^1(M), \qquad Z^1(M) := \ker\Big(d : \Omega^1(M) \to \Omega^2(M)\Big),$$

and set $\eta_S := P_{\rm cl}(\omega_S)$. Require $P_{\rm cl} \circ P_{\rm cl} = P_{\rm cl}$, and require the map to preserve cohomology on closed inputs: for every closed $\alpha \in \Omega^1(M)$, the form $P_{\rm cl}(\alpha) - \alpha$ is exact. (The harmonic projection satisfies both; note that it is *not* the identity on $Z^1(M)$, so idempotence may not be strengthened to $P_{\rm cl}|_{Z^1} = \mathrm{id}$.) These conditions constrain $P_{\rm cl}$ only on closed inputs. They nevertheless make the splitting gauge harmless: linearity and closed-input cohomology preservation give $[P_{\rm cl}(du)] = [du] = 0$, while $d(\omega_S + du) = d\omega_S$. Thus the signature below is an invariant of the declared pair $(F, P_{\rm cl})$, even though its representative η_S may change. The selector remains part of the type: when ω_S is not closed, different admissible selectors may assign different classes to the same F, and those classes may not be compared. In the compact case one may take $\eta_S = h$, the harmonic projection, which satisfies the requirement: for closed $\alpha = df + \delta\beta + h$ one has $d\delta\beta = 0$, since $d\alpha = 0$ and $ddf = dh = 0$, whence $\|\delta\beta\|^2 = \langle \beta, d\delta\beta \rangle = 0$, $\delta\beta = 0$ and $\alpha - h = df$ is exact.

De Rham cohomology is used throughout in the sense of [6].

Definition 1 (Structural-spin signature)**.** On the declared-selector branch, the *structural-spin signature* is

$$\Sigma_{\mathrm{spin}}(F;P_{\mathrm{cl}}) := ([\eta_S], d\omega_S) \in H^1_{\mathrm{dR}}(M) \times \Omega^2(M).$$

On the compact Hodge branch the harmonic projection is canonically fixed by g; there we suppress it and write $\Sigma_{\mathrm{spin}}(F)$. Selector arguments may be suppressed nowhere else. The *topological channel* is $[\eta_S]$. The *local differential channel* is $d\omega_S$, with operational spin

$$\Omega_{\mathrm{op}} := \frac{1}{2} d\omega_S.$$

Tensorially, let $K_{\mathrm{op}} \in \mathrm{End}(TM)$ be the skew-adjoint endomorphism defined by

$$g(K_{\mathrm{op}}X, Y) = \Omega_{\mathrm{op}}(X,Y) = \frac{1}{2} d\omega_S(X,Y).$$

If $J_F(X) := \nabla_X F$ and J_F^{*g} is its metric adjoint, then

$$K_{\mathrm{op}} = \frac{1}{2}(J_F - J_F^{*g}).$$

Indeed, metric compatibility gives $(\nabla_X \omega_S)(Y) = g(\nabla_X S, Y)$ and torsion-freeness gives $d\omega_S(X,Y) = (\nabla_X\omega_S)(Y) - (\nabla_Y\omega_S)(X)$, so that $d\omega_S(X,Y) = g((J_S - J_S^{*g})X, Y)$ for $J_S(X) := \nabla_X S$. Torsion-freeness also makes $\nabla^2 V$ symmetric, so the gradient layer is self-adjoint and contributes nothing to the skew part: $J_F - J_F^{*g} = J_S - J_S^{*g}$. The local differential channel is therefore an invariant of F alone. The topological channel is also splitting-gauge invariant for fixed P_{cl}, but remains selector-relative; hence the full minimal type is (F, P_{cl}), or F relative to the fixed metric on the compact Hodge branch. Only in an orthonormal frame does this become the matrix formula $K_{\mathrm{op}} = \frac{1}{2}(J_F - J_F^{\top})$. The physical construction in [2] applies the same antisymmetrization type, $\mathrm{skew}(A) := \frac{1}{2}(A - A^{*g})$, to a directed-change field $J_t = \partial_t u_t$: $\Omega_t = \mathrm{skew}(\nabla J_t)$. The fields F and J_t are not thereby identified; what is shared is the operation and its tensor type.

Remark 2 (Premise-minimal scope)**.** Architectural spin can be asserted more weakly as an obstruction to a global scalar-gradient realization. Definition 1 requires the additional representative/regularity branch. Without it, neither component of the displayed pair may be silently manufactured.

3 Formal independence of the two channels

Proposition 3 (Channel independence)**.** *The two vanishing conditions are logically independent over the class of all declared signature data—quadruples $(M, g, F, P_{\mathrm{cl}})$ on the selector branch and triples (M, g, F) on the compact Hodge branch: neither*

$$d\omega_S \neq 0 \implies [\eta_S] \neq 0 \qquad \textit{nor} \qquad [\eta_S] \neq 0 \implies d\omega_S \neq 0$$

holds universally over that class.

Proof. Work on the single flat torus $M = \mathbb{R}^2/(2\pi\mathbb{Z})^2$ with its standard metric and canonical Hodge projection. Put $V = 0$, so $F = S$, for both witnesses. For the local-only direction take $S_{\mathrm{L}} = \sqrt{2}\cos y\, \partial_x$. Then

$$\omega_{\mathrm{L}} = \sqrt{2}\cos y\, dx, \qquad d\omega_{\mathrm{L}} = \sqrt{2}\sin y\, dx \wedge dy \neq 0,$$

while the harmonic projection of ω_{L} is zero because its coefficient has zero torus mean. Thus $[\eta_{\mathrm{L}}] = 0$.

For the topological-only direction on the same (M, g), take $S_{\mathrm{T}} = \partial_x$. Then

$$\omega_{\mathrm{T}} = dx, \qquad d\omega_{\mathrm{T}} = 0, \qquad \oint_{\{y=0\}} dx = 2\pi.$$

Since dx is harmonic, $[\eta_{\mathrm{T}}] = [dx] \neq 0$, while the local differential channel vanishes. Both vector fields are non-gradient and divergence-free; divergence-freeness is an additional property of these witnesses, not part of the definition of non-gradient. □

Corollary 4 (No channel substitution)**.** *No inference from the non-vanishing of one channel to the non-vanishing of the other is licensed by the signature alone: a local vorticity detector is not a complete topological-witness detector, and a winding detector is not a complete local-vorticity detector. In particular, the phrase* two channels *refers to* $([\eta_S], d\omega_S)$*, not to winding and amplitude.*

Remark 5 (Scope of the independence claim)**.** The proof realizes both one-channel directions on one fixed compact (M, g), on the canonical Hodge branch. The same torus also realizes the jointly non-zero pattern with $\omega = (1 + \sqrt{2} \cos y)\, dx$. A zero–zero form such as $d(\cos x)$ is exact and therefore gradient; it lies outside the declared non-gradient carrier class. Hence all three admissible non-zero sign patterns occur on this one compact manifold, without a claim that a fourth non-gradient zero–zero pattern exists.

4 Three readout families

Structural channels are not yet scalar measurements. A measurement requires a probe.

Definition 6 (Typed readouts)**.** Let $\gamma : [0, T] \to M$ be a piecewise C^1 closed curve. Let $\mathcal{P}$ be an oriented compact piecewise C^1 2-chain (or embedded probe surface, possibly with boundary) on which the pullback of $d\omega_S$ is integrable. For a declared one-form α, define

$$W_\gamma(\alpha) := \oint_\gamma \alpha, \qquad \text{period/winding readout,} \tag{1}$$

$$A_\gamma(\alpha) := \int_0^T \left|\alpha_{\gamma(t)}(\dot\gamma(t))\right| dt, \qquad \text{path-amplitude/total-variation readout,} \tag{2}$$

$$Q_{\mathcal{P}}(\omega_S) := \int_{\mathcal{P}} d\omega_S, \qquad \text{oriented vorticity-flux readout.} \tag{3}$$

Here W_γ and A_γ are functionals of α, and $Q_{\mathcal{P}}$ is a functional of ω_S; their displayed evaluated values are real scalars. When $\alpha = \eta_S$ is closed, $W_\gamma(\eta_S)$ depends only on the homology class of γ. The readout A_γ is invariant under orientation-preserving reparametrization, by the change of variables $|\alpha(\dot\gamma \circ \varphi)\varphi'|\, ds = |\alpha(\dot\gamma)|\, dt$, so it depends only on the traversed path and not on the chosen $[0, T]$; it is a *seminorm* in α, since $A_\gamma(\alpha) = 0$ whenever α annihilates $\dot\gamma$ along γ, without $\alpha = 0$. All probes and geometries remain declared. If $\mathcal{P}$ is a closed 2-cycle and ω_S is globally defined on it, then $Q_{\mathcal{P}}(\omega_S) = 0$ by Stokes. Throughout this note ω_S is globally defined, so a non-zero flux readout is used only for a chain with $\partial\mathcal{P} \neq 0$, where

$$Q_{\mathcal{P}}(\omega_S) = W_{\partial\mathcal{P}}(\omega_S).$$

Here W is extended additively over the components of the boundary cycle. Patchwise connections, distributional singularities and removed-core fluxes require a separate carrier definition and are outside the present scope. Stokes is the one determinate relation holding between two of these readouts without further model data.

Proposition 7 (The norm inequality is not a vorticity inequality)**.** *For every declared pair* (γ, α)*,*

$$A_\gamma(\alpha) \geq |W_\gamma(\alpha)|.$$

If $\alpha = \eta_S$*, a non-zero topological period therefore forces a positive path norm of that representative. It does not force* $d\omega_S \neq 0$*, and it supplies no lower bound on* $|Q_{\mathcal{P}}(\omega_S)|$*.*

Proof. The displayed inequality is

$$\int_0^T |\alpha(\dot\gamma)|\, dt \geq \left| \int_0^T \alpha(\dot\gamma)\, dt \right|,$$

the ordinary triangle inequality. The topological-only member of Proposition 3 has non-zero period and positive path norm but $d\omega_S = 0$, hence every vorticity-flux readout is zero. □

Remark 8 (Instrument compatibility)**.** If an experiment measures amplitude from a signal other than the one-form used for the period, Proposition 7 does not apply until a reconstruction map between those observables is declared. The same name *rotation* is not such a map.

Proposition 9 (An amplitude-only collision)**.** *A monitor reading only the path amplitude of the carrier* ω_S *cannot recover the topological channel, even on a fixed probe manifold and loop, and even on the compact Hodge branch: two divergence-free, non-gradient carriers with equal amplitude readouts can have topological classes* 0 *and* $[dx] \neq 0$*.*

Proof. Take the flat torus $M = (\mathbb{R}/2\pi\mathbb{Z})^2$, with coordinates (x, y), and the oriented loop

$$\gamma(t) = (t, \pi/4), \qquad 0 \leq t \leq 2\pi.$$

Let

$$\omega_1 = dx, \qquad \omega_0 = \sqrt{2}\cos y\, dx.$$

Along γ, both forms evaluate to 1 on $\dot\gamma$, hence

$$A_\gamma(\omega_1) = 2\pi = A_\gamma(\omega_0).$$

The harmonic 1-forms on the flat torus are spanned by dx, dy. The harmonic projection of ω_1 is dx, whereas that of ω_0 is zero: its dx-coefficient has zero torus average and its dy-coefficient vanishes. Thus

$$\eta_1 = dx, \qquad \eta_0 = 0, \qquad W_\gamma(\eta_1) = 2\pi, \qquad W_\gamma(\eta_0) = 0.$$

Both associated fields are divergence-free. Moreover

$$d\omega_0 = \sqrt{2}\sin y\, dx \wedge dy \neq 0,$$

and on γ its coefficient equals 1; consequently ω_0 is genuinely non-gradient and carries a non-zero local differential channel despite its zero topological channel. The collision therefore remains entirely inside the declared non-gradient carrier class: an amplitude-only monitor assigns the same value to opposite topological witness states. □

Remark 10 (Why the readouts must be typed to their sources)**.** The collision is between $A_\gamma(\omega_i)$, read from each carrier, and $W_\gamma(\eta_i)$, read from its harmonic component. It is not a counterexample to Proposition 7, which compares two readouts of the *same* declared one-form: for each η_i separately, $A_\gamma(\eta_i) \geq |W_\gamma(\eta_i)|$. The two statements are compatible precisely because their sources differ, which is the typing point of Section 5.

5 The dynamic ledger and the typed bridge

The residual variable used in a drift law is a fourth object. Write it as $\Omega_t^{\mathrm{res}} \in \mathcal{R}_t$, where $\mathcal{R}_t$ is the declared residual space. A ledger statement has, schematically, the form

$$D_T \geq c_1 \int_0^T \|\Omega_t^{\mathrm{res}}\|_{\mathcal{R}_t}\, dt,$$

where D_T is the declared accumulated drift or maintenance budget over $[0, T]$ and $c_1 > 0$ is a declared calibration constant with units

$$[c_1] = \frac{[D_T]}{[\|\Omega^{\mathrm{res}}\|]\,[\mathrm{time}]};$$

neither its value nor its units are fixed by the structural signature. Nothing in Definition 1 makes Ω^{res} equal to $d\omega_S$, its flux, or a path norm.

Convention 11 (Typed bridge principle)**.** Let a structural-spin model use any of the following in a measurement, drift, or cost law:

$$[\eta_S], \quad d\omega_S, \quad W_\gamma(\alpha), \quad A_\gamma(\alpha), \quad Q_{\mathcal{P}}(\omega_S), \quad \Omega^{\mathrm{res}}.$$

Then:

(i) the first two are the structural channels;

(ii) $W_\gamma(\alpha), A_\gamma(\alpha)$, and $Q_{\mathcal{P}}(\omega_S)$ are evaluated probe-dependent scalar readouts, not further spin objects;

(iii) the period–norm inequality of Proposition 7 is licensed when both readouts use the same declared one-form and locus; that proposition licenses no inequality between readouts of *different* declared one-forms, and any such relation requires its own derivation;

(iv) any relation between $d\omega_S$ or $Q_{\mathcal{P}}(\omega_S)$ and Ω^{res} requires an explicit observation map, regulated estimator, or sealed dependence;

(v) any scalar cost law must name its readout mode and calibration.

The readouts are related to one another only by Stokes, $Q_{\mathcal{P}}(\omega_S) = W_{\partial\mathcal{P}}(\omega_S)$, and by Proposition 7; no equality between a structural channel and the dynamic residual follows from the structural signature alone.

Remark 12 (Justification)**.** Items (i) and (ii) record type membership; the codomains already separate the objects:

$$[\eta_S] \in H^1_{\mathrm{dR}}(M), \quad d\omega_S \in \Omega^2(M), \quad W_\gamma(\alpha), A_\gamma(\alpha), Q_{\mathcal{P}}(\omega_S) \in \mathbb{R}, \quad \Omega_t^{\mathrm{res}} \in \mathcal{R}_t.$$

The scalar readouts arise only after evaluation on a curve, normed locus, or surface. Item (iii) is a restatement of Proposition 7; items (iv) and (v) are declared modelling requirements rather than propositions. Proposition 7 gives the sole universal period–norm relation used here, and Proposition 3 rules out either structural non-vanishing implication. A map such as

$$\mathcal{O}_t : \Omega^2(M) \longrightarrow \mathcal{R}_t \quad \text{or} \quad \mathcal{B}_t : \mathbb{R} \longrightarrow \mathcal{R}_t, \qquad \Omega_t^{\mathrm{res}} = \mathcal{B}_t(Q_{\mathcal{P}}(\omega_S))$$

is therefore additional model data. The same is true of a cost functional

$$\mathcal{C} = c_W |W_\gamma(\eta_S)| + c_A A_\gamma(\alpha) + c_Q |Q_{\mathcal{P}}(\omega_S)|,$$

whose coefficients, units, probes, and active terms must be declared.

Corollary 13 (Correct reading of the boundary norm law)**.** *Let σ_H and Λ_{loc} be the boundary period and the compatible discrete path norm of [1]. The implication*

$$\sigma_H \neq 0 \quad \Longrightarrow \quad \Lambda_{\mathrm{loc}} > 0$$

is valid when σ_H is a period and Λ_{loc} is the compatible path norm. It is not an identity between topological spin and operational spin, and it does not identify Λ_{loc} with Ω^{res}.

6 Compact admissibility and the topological-only sector

Formal independence does not imply that every pair of channel values is realizable under every admissibility package. The compact branch of [1] makes this precise.

Let M be compact and boundaryless and $V \in C^\infty(M)$. Write $\mathcal{H}^1(M)$ for the harmonic 1-forms and $\Phi : \mathcal{H}^1(M) \xrightarrow{\sim} H^1_{\mathrm{dR}}(M)$ for the Hodge isomorphism. Define

$$L : \mathcal{H}^1(M) \to C^\infty(M), \qquad L(h) = h(\nabla V),$$

and

$$W_V := \Phi(\ker L) \subseteq H^1_{\mathrm{dR}}(M).$$

Proposition 14 (Premise gate for topological-only spin)**.** *Say that S* carries *the class c if $[\eta_S] = c$. A class $c \neq 0$ is carried by a divergence-free, operational-spin-free field satisfying $\langle S, \nabla V\rangle = 0$ pointwise if and only if $c \in W_V$. (The equivalence also holds for $c = 0$; the hypothesis $c \neq 0$ serves only to exclude the trivial witness $S = 0$.)*

Proof. Divergence-free and operational-spin-free mean $\delta\omega_S = 0$ and $d\omega_S = 0$, so $\Delta\omega_S = (d\delta + \delta d)\omega_S = 0$ and ω_S is harmonic; this step uses neither compactness nor boundarylessness. Both are used next, for uniqueness: if a harmonic h is exact, $h = d\alpha$, then $\|h\|^2_{L^2} = \langle d\alpha, h\rangle = \langle \alpha, \delta h\rangle = 0$ by integration by parts on a compact boundaryless M, so $\mathcal{H}^1(M) \cap \operatorname{im} d = 0$ and each class has exactly one harmonic representative. Hence $\omega_S = h_c$, and since ω_S is already harmonic, $\eta_S = h = \omega_S$ on this branch, so S carries $[\omega_S]$. Pointwise admissibility is precisely $h_c(\nabla V) = 0$, equivalently $h_c \in \ker L$.

Conversely, let $c \in W_V$, so $c = \Phi(h)$ with $h \in \ker L$; injectivity of Φ forces $h = h_c$. Put $S := h_c^\sharp$. Then $\omega_S = h_c$ is harmonic, so $d\omega_S = 0$ (operational-spin-free) and $\delta\omega_S = 0$ (divergence-free); moreover $\langle S, \nabla V\rangle = h_c(\nabla V) = L(h_c) = 0$ pointwise, and $[\eta_S] = [h_c] = c$. For $c \neq 0$ we get $h_c \neq 0$, so $S \not\equiv 0$. □

Corollary 15 (Finite exact W_V arm)**.** *For the fixed Riemannian metric and Hodge identification above, let $h_1, \dots, h_b$ be a registered harmonic basis and define*

$$F_V(p) \in \mathcal{H}^1(M)^*, \qquad F_V(p)(h) := h_p(\nabla V_p).$$

Let $p_1, \dots, p_m \in M$ satisfy the spanning-certificate premise

$$\operatorname{span}\{F_V(p_j) : 1 \le j \le m\} = \operatorname{span}\{F_V(p) : p \in M\}.$$

Set $E_{ji} := h_i(\nabla V)(p_j)$. For

$$c = \Phi\left(\sum_{i=1}^{b} a_i h_i\right), \qquad a = (a_1, \dots, a_b)^\top \neq 0,$$

one has

$$c \in W_V \quad \Longleftrightarrow \quad Ea = 0.$$

Consequently

$$\rho_V^{(\mathbf{h},P)}(c) := \|Ea\|_\infty, \qquad \mathbf{h} = (h_1, \dots, h_b), \quad P = (p_1, \dots, p_m),$$

written $\rho_V(c)$ whenever the registration $(\mathbf{h}, P)$ is fixed, is an executable observable with exact threshold zero when the class coordinates and matrix entries are exact symbolic or rational data. A basis change $\mathbf{h}' = \mathbf{h}T$ gives $E' = ET$ and $a' = T^{-1}a$, hence $E'a' = Ea$: for a fixed ordered sample tuple the whole residual vector, and therefore ρ_V, is basis-independent. Its zero set is also invariant under non-zero row rescaling and replacement of P by another certified spanning sample family. The positive magnitude does change under row rescaling and sample choice (enlarging P can only increase it) and is positively homogeneous of degree one in c, so it induces no meaningful comparison between classes. Numerical harmonic solvers require a separately derived error bound or interval certificate and are outside this exact arm. Under the stated certificates, a declared membership with $\rho_V(c) > 0$, or a declared non-membership with $\rho_V(c) = 0$, falsifies the registered claim. The spanning-certificate premise is load-bearing in exactly one direction: $Ea \neq 0$ exhibits a point at which $h_a(\nabla V) \neq 0$ and so excludes membership without any span premise, whereas $Ea = 0$ bounds $h_a(\nabla V)$ only on the sampled points. Accordingly, without that premise a non-zero sampled residual still excludes membership, but a zero sampled residual is only non-rejection on the sampled points.

Proof. For $h_a = \sum_i a_i h_i$, the vector Ea is $(F_V(p_1)(h_a), \dots, F_V(p_m)(h_a))^\top$. Its vanishing annihilates the registered span, which equals the span of every $F_V(p)$; hence $h_a(\nabla V) = 0$ pointwise. The reverse implication is immediate. Proposition 14 completes the equivalence. □

Example 16 (Registered flat-torus instance)**.** Take $M = (\mathbb{R}/2\pi\mathbb{Z})^2$ with $g = dx^2 + dy^2$, coordinates (x, y), $V = \cos x$, and harmonic basis (dx, dy). Then

$$L(a\,dx + b\,dy) = -a \sin x.$$

In the dual basis $F_V(x, y) = (-\sin x,\, 0)$ for every $(x, y) \in M$, so the whole evaluation family lies on a single line and one point discharges the spanning certificate: $p_1 = (\pi/2, 0)$ gives the spanning row $E = (-1, 0)$. (This is a feature of $\partial_y V \equiv 0$, not a general sufficiency of one sample.) Thus $W_V = \operatorname{span}\{[dy]\}$, consistently with $\dim W_V = b_1 - 1 = 1$, and every non-zero class $c_{a,b} = [a\,dx + b\,dy]$ passes the exact gate if and only if $a = 0$. This is the registered exact-arithmetic instance used by the companion code; the code does not authenticate the harmonic-basis or spanning certificates themselves.

Thus the topological-only sector is real but constrained. With $F_V(p)$ as in Corollary 15, the preceding work proves

$$\dim W_V = b_1(M) - \dim_{\mathbb{R}} \operatorname{span}\{F_V(p) : p \in M\},$$

since $\ker L$ is the annihilator in $\mathcal{H}^1(M)$ of that span, which is finite-dimensional as a subspace of the b-dimensional dual $\mathcal{H}^1(M)^*$ despite its infinite index set.

The standing hypothesis of Proposition 14 and Corollary 15 is $V \in C^\infty(M)$. The genericity claim has the following corrected C^k form. For $k \geq 1$ and $V \in C^k(M)$, let

$$L_V : \mathcal{H}^1(M) \longrightarrow C^{k-1}(M), \qquad L_V(h) = h(\nabla V), \qquad W(V) := \Phi(\ker L_V).$$

Then

$$\{V \in C^k(M) : W(V) = \{0\}\}$$

is open and dense in $C^k(M)$. Here is the short proof, included to avoid relying on the predecessor wording that placed a C^∞ subset inside the C^k topology. If $b_1(M) = 0$ the claim is immediate. Otherwise fix a harmonic basis $h_1, \dots, h_b$. Injectivity of L_V is equivalent to the existence of points $p_1, \dots, p_b$ for which

$$\det\bigl[h_i(\nabla V)(p_j)\bigr]_{i,j=1}^{b} \neq 0.$$

This determinant depends continuously on the first jet of V, proving openness. For density, the evaluation covectors $h \mapsto h_p(v)$, with $p \in M$ and $v \in T_pM$, span $\mathcal{H}^1(M)^*$: a harmonic form annihilating all of them is zero. Choose distinct p_j and $v_j \in T_{p_j}M$ whose evaluation covectors form a basis, and smooth bump functions ψ_j with disjoint supports and $\nabla\psi_j(p_j) = v_j$. For $V_s = V_0 + \sum_j s_j\psi_j$, the displayed determinant is a polynomial in $s = (s_1, \dots, s_b)$ with non-zero top-degree coefficient. Arbitrarily small generic s therefore makes it non-zero, proving density in every C^k topology with $k \geq 1$.

This is a realizability obstruction on a specified compact admissible class, not a collapse of the two structural types. Note its direction: for generic V the compact admissible topological-only sector is *empty*, so positive instances such as Example 16 must be exhibited by construction and never expected generically.

7 Identity, cortical waves, and the dissociation test

The topological channel carries identity only on the declared identity-transfer domain of [1]. It does not follow that every non-zero winding in nature carries that semantic role. The correct empirical move is to transport the invariant type first and test the proposed role separately.

For each condition $c \in \{\text{pre}, \text{post}\}$, let U_c be the registered cortical domain and let

$$z_B^{(c)} := \mathcal{H}_B(x^{(c)}), \qquad a^{(c)} := |z_B^{(c)}|, \qquad \phi_c := z_B^{(c)}/|z_B^{(c)}|$$

be the registered band-limited analytic signal, its amplitude and its phase on the phase domain $D_c := \{a^{(c)} > a_{\min}\}$, where the pointwise floor $a_{\min} > 0$ is fixed before inspection. Write $d\vartheta$ for the angular 1-form normalized by $\oint_{S^1} d\vartheta = 2\pi$. For a piecewise C^1 loop $\gamma_c \subset D_c$,

$$n_{\gamma_c} := \frac{1}{2\pi}\oint_{\gamma_c} \phi_c^*(d\vartheta) \in \mathbb{Z}.$$

This is the same invariant type—degree/winding around a puncture—as the topological readout above. It is not literally the same physical carrier or the same identity certificate.

For an executable discrete certificate, sample a complete oriented loop at $p_0, \dots, p_{N-1}$, obtaining noisy observed values $z_i = z_B(p_i)$. Independently register one cyclically glued, piecewise-C^1, non-vanishing reference reconstruction ζ on the oriented edges $e_i : p_i \to p_{i+1}$. The observed values need not be exact endpoint values of ζ. Let $\widetilde{\vartheta}_i^*$ be the chosen continuous phase lift of $\zeta|_{e_i}$, write

$$\delta_i^* := \widetilde{\vartheta}_i^*(1) - \widetilde{\vartheta}_i^*(0), \qquad \theta_i := \arg z_i,$$

and set cyclically

$$\Delta_i := \operatorname{Arg}\exp\bigl(\mathrm{i}(\theta_{i+1} - \theta_i)\bigr) \in (-\pi, \pi], \qquad \widehat{n} := \frac{1}{2\pi}\sum_{i=0}^{N-1} \Delta_i.$$

Register a within-condition reference-reconstruction/edge-lift certificate, an independently justified mesh/regularity bound b_ϕ, a certified per-edge *linear, branch-coupled* observation error $\varepsilon_{\text{step}} \geq 0$, an ambiguity margin $0 < \kappa < \pi$, and an integer tolerance $0 \le \varepsilon_{\mathbb{Z}} < 1/2$. The certificate records every real lift increment δ_i^*, pairs it with the observed principal increment on the declared branch, and either defines ζ as the operational phase target or supplies a within-condition degree-preserving bridge from ζ to $\phi|_\gamma$. Thus the next absolute value is on $\mathbb{R}$, not a circular distance:

$$|\delta_i^*| \le b_\phi, \qquad |\Delta_i - \delta_i^*| \le \varepsilon_{\text{step}} \quad \text{for every } i.$$

The numerical amplitude ledger records

$$a_{\text{obs,min}} := \min_i |z_i|, \qquad a_{\text{ref,min}} := \inf_{t \in S^1} |\zeta(t)|.$$

A winding value is certified only if both recorded minima strictly exceed the predeclared positive floor $a_{\min}$, the certificates cover every edge, and

$$b_\phi + \varepsilon_{\text{step}} < \pi - \kappa, \qquad N\varepsilon_{\text{step}} < \pi, \qquad \max_i |\Delta_i| \le b_\phi + \varepsilon_{\text{step}}, \qquad |\widehat{n} - \text{round}(\widehat{n})| \le \varepsilon_{\mathbb{Z}}.$$

The reference reconstruction satisfies $n_{\text{ref}} = (2\pi)^{-1} \sum_i \delta_i^* \in \mathbb{Z}$, while

$$|\widehat{n} - n_{\text{ref}}| \le \frac{N\varepsilon_{\text{step}}}{2\pi} < \frac{1}{2}.$$

Consequently $n_{\text{ref}} = \text{round}(\widehat{n})$; when the declared degree-preserving bridge is present this is also n_γ. The numerical reference steps, their branch coupling to the observations, the true-step bound and the aggregate error gate are all load-bearing: an integer residual by itself cannot exclude a reference/observation full-turn mismatch. Failure of any certificate gives non-applicability, not zero winding.

Cross-condition comparison uses an orientation-preserving registered correspondence $T_j : \gamma_j^{\text{pre}} \to \gamma_j^{\text{post}}$, together with endpoint-specific annular phase-domain and singularity-tracking certificates. It does *not* assume a homotopy from ϕ_{pre} to ϕ_{post} through non-vanishing phase maps: such a homotopy would force equality of the two degrees. If a time-resolved physical interpolation is studied, a degree change instead requires a zero-amplitude or singularity crossing in the intervening world tube; that crossing is a candidate mechanism to locate, not a premise to exclude.

The Science study of brain-wide rotating waves [5] analyzes rotating phase waves primarily in the 2–8 Hz band, reports brain-wide coordination and behavioural recruitment, and finds that disrupting circular somatosensory wiring reduces the waves. It treats rotation and oscillation amplitude together; it does not introduce the cohomological identity semantics or the typed separation above.

For an empirical test, predeclare a spatial locus $\mathcal{U}$, time window, calibration, band $B = [2, 8]$ Hz, and scalar amplitude functional $\mathcal{A}_{\mathcal{U},B}$ applied to the same registered analytic-signal magnitude:

$$A^{(c)}_{\mathcal{U},B} := \mathcal{A}_{\mathcal{U},B}\left(a^{(c)}\right) \in \mathbb{R}_{\ge 0}.$$

A different amplitude source requires an explicit observation bridge and a validation that the two estimators do not force winding and amplitude to co-vary. Predeclare a calibrated detection floor $A_{\min} > 0$ and an observed-record equivalence margin

$$0 \le \delta_A < A_{\min}.$$

The instrument calibration must resolve differences at that margin; if its uncertainty is larger, the arm is non-applicable rather than licensed to widen δ_A. This criterion concerns the registered observed records. It is not by itself a confidence statement about equality of latent population amplitudes.

Hypothesis 17 (Empirical dissociation criterion). *Let $\{(\gamma_j^{\text{pre}}, \gamma_j^{\text{post}}, T_j)\}$ be a preregistered endpoint loop family, and let the evaluated loop be named in advance or selected under a preregistered multiplicity plan. Require the complete observed/reference reconstruction, branch-coupled edge-lift and degree-preserving discrete certificate above in both conditions, using the same registered analytic-signal, loop, calibration and sampling pipeline, and require endpoint annular phase-domain and singularity-tracking certificates. Under those premises, a loop-local empirical realization of the separation has, for the evaluated loop,*

$$|n_{\gamma_j}^{\text{pre}}| > 0, \qquad n_{\gamma_j}^{\text{post}} = 0,$$
$$\min\left\{A_{\mathcal{U},B}^{(\text{pre})}, A_{\mathcal{U},B}^{(\text{post})}\right\} \geq A_{\min},$$
$$\left|A_{\mathcal{U},B}^{(\text{post})} - A_{\mathcal{U},B}^{(\text{pre})}\right| \leq \delta_A.$$

This is observed-record $\mathcal{U}$-amplitude-preserved loss of winding on that loop. It is loop-local in winding and $\mathcal{U}$-local in amplitude; only when $\mathcal{U}$ is a registered neighbourhood of γ_j is it a loop-local amplitude/winding dissociation. A singularity lying on an endpoint loop makes its winding undefined; recording that case as zero would manufacture the dissociation from an artefact.

Collapse on one loop bounds nothing on the remaining periods. If the classes $\{[\gamma_j]\}$ generate a registered monitored subspace $H_{\text{mon}} \subseteq H_1(U_{\text{post}}; \mathbb{R})$ and every post-condition generator has zero winding, then the post-condition cohomology class vanishes only on H_{mon}. Full triviality in $H^1_{\text{dR}}(U_{\text{post}})$ follows only when $H_{\text{mon}} = H_1(U_{\text{post}}; \mathbb{R})$; at least one pre-condition generator must have non-zero winding for a sector collapse. The stronger survives-but-dead *description additionally requires (i) an independently declared viability or functional-continuation variable to be preserved and (ii) a predeclared bridge validating the monitored winding as an identity-transfer witness in the target system. Neither premise is supplied by the amplitude/winding dissociation alone.*

For the dual local-only arm, register one loop γ, a neighbouring region $\mathcal{U}_\gamma$, a spatiotemporal window, an oriented orthonormal frame and units, the derivative/smoothing/mask pipeline, a same-carrier observation bridge, and a multiplicity plan. At $N \geq 1$ uniformly weighted registered samples, let J_i be the Jacobian of the same local vector-field estimator and define the coherent signed-spin score

$$r_i := \frac{1}{2}\big((J_i)_{21} - (J_i)_{12}\big), \qquad R_{\text{loc}} := \left|\frac{1}{N}\sum_{i=1}^{N} r_i\right|.$$

This estimator does not explicitly consume the registered oscillation- amplitude functional. Its mask, derivative scale and reliability may still depend on signal quality, so empirical amplitude invariance would require a separate validation. Oppositely oriented subregions can cancel, so it tests this registered mean readout rather than the existence of $d\omega_S \neq 0$ somewhere. Since $R_{\text{loc}} = |\bar{r}|$ is folded, the procedure is a two-sided test of the signed mean and is silent on handedness. Run the identical full pipeline—including sample count, window, mask, frame, smoothing, and nuisance fitting—on m preregistered no-rotation calibration units, obtaining $R_1^{(0)}, \ldots, R_m^{(0)}$. Because the reference distribution is built at the level of whole units, spatial autocorrelation among the N within-unit samples does not invalidate the procedure; the effective null sample size is m, not N. Before the test, fix

$$0 < \alpha_{\text{loc}} < 1, \qquad R_{\min} > 0, \qquad \frac{1}{m+1} \leq \alpha_{\text{loc}},$$

where $R_{\min}$ is justified by instrument resolution or a declared minimum practical effect. The last condition is necessary and sufficient for the rank grid to contain a value at most α_{loc}, since its minimum is $1/(m+1)$; it does not guarantee rejection in realized data, where ties or a non-extreme observed score can keep p_{loc} larger. Now assume the registered *design* premise—logically independent of the sharp no-rotation null, and not testable from these data—that the experimental unit and the m calibration units are exchangeable draws from a common data-generating distribution, matched in signal-to-noise, coverage, mask geometry and nuisance structure, so that under the sharp null the vector $(R_{\mathrm{loc}}, R_1^{(0)}, \ldots, R_m^{(0)})$ is permutation-invariant. Running the identical pipeline controls pipeline heterogeneity, not unit heterogeneity; unlike a within-unit randomization null, this between-unit reference distribution is assumed rather than constructed, so the validity of the arm is exactly as good as that matching. Under that premise, use the conservative rank value

$$p_{\mathrm{loc}} := \frac{1 + \#\{k : R_k^{(0)} \geq R_{\mathrm{loc}}\}}{m+1}.$$

Hypothesis 18 (Calibrated local-only criterion)**.** *A loop/region-local empirical realization of the local-only sector is declared exactly when a winding estimate satisfies the phase-domain, registered-loop, observed/reference reconstruction, branch-coupled edge-lift, aggregate-error, and sub-π no-alias gates and obeys*

$$n_\gamma = 0, \qquad R_{\mathrm{loc}} \geq R_{\min}, \qquad p_{\mathrm{loc}} \leq \alpha_{\mathrm{loc}}.$$

The use of $\geq$ in the null exceedance count makes the rank test conservative under ties. Here $R_{\min}$ is a practical decision floor; the displayed rule alone is not a confidence-bound claim that the latent effect exceeds $R_{\min}$. Invalid winding, same-carrier bridge, provenance, calibration, cross-unit matching, joint-exchangeability, or multiplicity premises are non-applicability, not negative evidence. When the premises are valid but any displayed gate fails, the verdict is no local-only detection *for the registered loop/region/window. A family-wide statement additionally requires generator completeness and its preregistered multiplicity correction. Neither outcome refutes Proposition 3.*

Falsifiable population formulation. The two hypotheses above are unit-level declaration rules. To attach a falsifiable population claim to either arm $q \in \{\mathrm{wind}, \mathrm{loc}\}$, preregister a source-population law ν_q, an outcome-independent eligibility event E_q with $\Pr_{\nu_q}(\mathsf{E}_q = 1) > 0$, and the eligible-unit law

$$\mu_q := \nu_q(\,\cdot \mid \mathsf{E}_q = 1).$$

Also predeclare a minimum true-realization prevalence $\pi_q \in (0,1)$, a detection-sensitivity lower bound $s_q \in (0,1]$, and a test level $\alpha_q \in (0,1)$. With R_q denoting a true realization and D_q its registered detection, the conjecture is explicitly conditional on eligibility:

$$\Pr_{\nu_q}(\mathsf{R}_q \mid \mathsf{E}_q = 1) = \Pr_{\mu_q}(\mathsf{R}_q) \geq \pi_q, \qquad \Pr_{\nu_q}(\mathsf{D}_q \mid \mathsf{R}_q, \mathsf{E}_q = 1) = \Pr_{\mu_q}(\mathsf{D}_q \mid \mathsf{R}_q) \geq s_q.$$

For N_q independent draws from μ_q and K_q detections, it is disconfirmed at the registered eligible-unit level when

$$\sum_{r=0}^{K_q} \binom{N_q}{r} (\pi_q s_q)^r (1 - \pi_q s_q)^{N_q - r} \leq \alpha_q.$$

In particular, zero detections require $(1 - \pi_q s_q)^{N_q} \leq \alpha_q$. Clustered or dependent units require a preregistered dependence-valid replacement bound. Non-applicable units are reported but are not negative observations, and post-outcome exclusion or sampling from a law other than the declared μ_q invalidates the arm. This probabilistic claim is distinct from the formal existence of the mathematical channel sectors.

The formal theory predicts that this sector is not type-forbidden. Whether a particular cortex, task, and estimator class realizes it remains an empirical question.

8 Falsification surface and limits

1. Proposition 3 is established by two explicit fields on one flat compact torus under the canonical Hodge projection, so the route to falsification is an arithmetic, Hodge-projection or hypothesis error in those constructions. The zero–zero exact form noted afterwards is not a non-gradient carrier witness.

2. Proposition 14 is executable through Corollary 15: after the compactness, harmonic-basis, class-coordinate, evaluation-span, and exact-arithmetic (symbolic or rational) certificates are supplied, the observable is $\rho_V(c) = \|Ea\|_\infty$ and the threshold is exactly zero. A missing span certificate makes a sampled zero non-applicable as a membership certificate; a floating-point pipeline places the instance outside this arm rather than merely degrading it.

3. Proposition 7 concerns the same one-form on the same loop. Applying it across unmatched instruments is non-applicability, not a theorem failure.

4. The empirical bridge is weakened if winding cannot be estimated robustly under registered-loop transport and singularity tracking. It is *non-applicable*, not merely weakened, if the phase-domain, registered-loop, observed/reference reconstruction and branch-coupled edge-lift coupling, aggregate-error or sub-π no-alias gates fail, or if amplitude and winding are algorithmically forced to co-vary by the estimator: the latter is a same-carrier bridge failure.

5. Hypotheses 17 and 18 are unit-level declaration rules. The falsifiable object is the registered eligible-unit prevalence claim $\Pr_{\mu_q}(\mathsf{R}_q) \geq \pi_q$, together with its declared sensitivity, eligibility-conditioning and sampling premises. Its falsifier measures the registered detection count K_q and uses the displayed lower-tail threshold; it does not treat finite non-observation as the logical negation of existence. Invalid phase, provenance, unit matching, eligibility law or exchangeability premises are non-applicability.

6. Failure to observe either dissociation disconfirms the registered population claim only when its prevalence/sensitivity/power bound crosses the preregistered level. Otherwise it is no detection, and in no case does it refute Proposition 3.

7. The genericity statement proved in Section 6 is itself a limit on the programme: the compact admissible topological-only sector is empty for generic V, so Hypothesis 17 hunts a non-generic sector on that branch.

8. A preserved amplitude without an independent continuation variable is not evidence of survival, and winding loss without a validated identity-witness bridge is not evidence of identity death. A zero winding on a probe domain with trivial H^1 is not evidence that every form of identity is absent.

9 Conclusion

Structural spin is one non-gradient layer with two formal channels:

$$\Sigma_{\text{spin}}(F; P_{\text{cl}}) = ([\eta_S], d\omega_S),$$

with the selector suppressed only on the compact Hodge branch. They are logically independent over the class of all declared signature data, and jointly constrained—generically to the point of collapsing the topological-only sector—under the compact admissibility package. Period, path norm/amplitude, and vorticity flux are three ways to read a declared one-form drawn from that layer, once probes are chosen. The dynamic residual is a further ledger variable. The norm inequality connects a period to a compatible path norm and nothing more. The compact topological-only premise gate is executable, conditional on the registered basis, evaluation-span and exact-arithmetic certificates, through the exact residual Ea. The dual empirical arm returns a three-valued verdict at its declared resolution—non-applicability, no detection, detection—with certified zero winding requiring a positive phase-domain floor, an independent sub-π true-step bound, a numerical reference edge-lift ledger with branch-coupled observation errors, an aggregate error below half a turn, an integer tolerance, a positive observed/reference amplitude floor, and complete loop registration. Once these types are kept separate, the central experimental bet becomes both stronger and cleaner: the amplitude statistic from the same registered analytic signal can remain above its detection floor and within its observed-record equivalence margin while winding is lost on the registered loop—or, under the generator conditions of Hypothesis 17, on the monitored homology subspace. Full cohomology-class collapse requires generators of all H_1; population disconfirmation requires the separately registered eligible-unit prevalence/sensitivity bound. Identity death still requires the independent transfer and viability premises that give the winding witness its meaning.

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