# 22.62 MHD Theory of Tokamaks

## Final Summary Paper

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May 19, 2021

In this paper, Snyder et al. (2002) [4] answer the question - What limits do edge localized modes (ELMs) place on pedestal pressure gradient and current density for H-mode tokamak operation? In order to do so, the authors numerically probe the stability of peeling-ballooning modes, and propose a model where unstable low-to-intermediate toroidal wavenumber $n$ modes provide the trigger to ELMs. The predictions of the numerical code are then benchmarked against experimental data, which shows good agreement.

Edge localized modes (ELMs) are tokamak instabilities driven by a combination of pressure gradients $p'$ and large edge currents $J$. ELMs can be undesirable because they can lead to density and pressure losses from the plasma, and they can deposit large bursts of energy onto the divertor plates [4][2]. They are commonly observed in H-mode operation, which is characterized by a transport barrier or pedestal, which results is steep edge pressure gradients and currents (large gradients in pressure drive bootstrap currents at the plasma edge). The presence of large pressure gradients and currents at the plasma edge makes the plasma susceptible to pressure-driven ballooning modes and current-driven peeling modes. Thus, peeling-ballooning modes place limits on edge pressure gradients and currents, which, in effect, limits the height of the pedestal for H-mode operation. Experimental data suggests that ballooning theory alone cannot account for the pressure gradients observed in H-mode operation [3], and theoretical treatment of edge ballooning modes suggests that the pressure limit scales as $n^{-2/3}$ rather than as $n^{-1}$, as predicted by ideal ballooning theory [1]. The pressure limits imposed by ideal ballooning theory are observed to be exceeded by factors of 2-3 in experiments [3], which suggests that the effect of current gradients and peeling modes must be incorporated in the study of ELMs.

Peeling modes are high m external kink modes driven by large edge currents or edge current gradients. Presence of a finite edge currents $J(a)$ results in stability windows in $q_a$ space ($q$ is the tokamak safety factor). In particular, in can be shown that in the straight tokamak limit with circular cross-section, the instability window for high m peeling modes is given by [2]:

$$\begin{aligned} m - \frac{J_a}{\bar{J}} < nq_a < m \qquad & J_a \neq 0 \\ m - \exp\left(\frac{2m\bar{J}}{aJ_a'}\right) < nq_a < m \qquad & J_a = 0 \end{aligned} \tag{1}$$

Here, $\bar{J}$ is the average current density and $m$, $n$ are the poloidal and toroidal wavenumbers respectively. Increasing the edge current density $J(a)$ or the current denisty gradient $J'(a)$ causes the instability bands to become wider. A necessary (but not sufficient) stability criterion for the general tokamak case derived from the ideal MHD energy principle is [1]:

$$\sqrt{1-4D_M} > 1 + \frac{2}{2\pi q'} \oint \frac{J_\parallel B}{R^2 B_p^3} dl \tag{2}$$

The stability of peeling modes depends on the edge current $J_a$, magnetic shear $\propto q'$ and Mercier coefficient $D_m \propto p'$ . We see that increasing edge current destabilizes peeling modes, while the magnetic shear and pressure gradient improves stability.

Ballooning modes are pressure-driven modes which exploit regions of poor curvature in a tokamak. These modes are typically localized to rational surfaces which minimize the stabilizing effect of line-bending ($\mathbf{k} \cdot \mathbf{B} = 0$). Ballooning modes set limits on the highest beta achievable in a tokamak (e.g. Skyles limit) [2]. A common way to represent the stability to ballooning modes is the $s-\alpha$ diagram, where $s=(r/q)q'$ is the magnetic shear and $\alpha$ is a term proportional to the pressure gradient $p'$ [2]. There are two stability regions in the $s-\alpha$ diagram – one at low $\alpha$ and high $s$, and the other at high $\alpha$ and low $s$. Thus, for large values of pressure gradient (such as in the pedestal), ballooning modes can be stabilized if the shear is reduced.

Thus, we see that peeling modes are stable for large shear and pressure gradients and unstable for large current density gradients, while ballooning modes at large pressure gradients are stable if shear ( $\sim 1/J$) is small. Thus, the effect of current is two-fold – (1) increasing the current destabilizes peeling modes, and (2) increasing the current stabilizes high-n ballooning modes by decreasing magnetic shear.

For large n, peeling-ballooning modes are uncoupled, and have access to the second stability region. For finite n, the peeling-ballooning modes couple and cut-off access to the second stability region. The result is that high-n ballooning modes are stabilized by shear, while intermediate-to-low n coupled peeling-ballooning modes become the limiting instability. The stability of these modes depends on the pedestal current and pressure profiles, as well as tokamak cross-section. Effects of shaping and finite aspect ratio are quantified using the magnetic well factor $d_m = D_m s^2/\alpha$. Increase in the magnetic well factor increases access to the second stability region and stabilizes high n ballooning modes. Improved shaping allows us to reach stability at higher values of pedestal current and pressure gradients, and lower n modes, by decoupling the peeling and ballooning modes.

The stability of peeling-ballooning modes at different $n$ in the presence of large bootstrap currents is probed numerically using a code called ELITE. Stability thresholds are calculated using ELITE for low-to-intermediate and high n modes, and the results were benchmarked against GATO (low n mode ELM code) and experimental data of DIII-D, JT-60U and C-Mod. The presence of a large bootstrap current is found to stabilize high $n > 15$ modes, which exceed the high-n pressure limit imposed by pure ballooning modes in the absence of a current, which is consistent with experimental data of the DIII-D discharge. Comparison with DIII-D data shows that unstable intermediate $n$ modes provide the trigger for the ELM. The ELM depth is observed to be in good agreement with the mode depth predicted by ELITE. The ELM size depends on the radial depth of the most unstable mode. This shows that low-to-intermediate n mode peeling-ballooning modes which have no access to the second stability region are the limiting ELM triggering instability.

For a given cross-section and pedestal height, stability boundaries were calculated in $J$ and $p'$ space, and as a function of the magnetic well factor (shaping). Thus, for a given totamak, the marginal stability boundaries can be determined as a function of pedestal current and pressure gradient. Improving the shaping is observed to decouple peeling and ballooning modes and increase the stability region in $J-p'$ space. Finally, qualitative models for Type I and III ELM cycles have have also been proposed. Low density Type III ELMs are expected to occur at low density and low input power, where the current exceeds the peeling limit but the pressure does not exceed the ballooning limit, giving rise to small ELMs whose frequency decreases with input power. These are triggered by peeling modes with small radial depths. Large type I ELMs are expected to occur at high power and low density. These are triggered by low n peeling-ballooning modes with large radial depths. Finally, we expect to see small ELMs at high power and low current, where the peeling current limit is not exceeded. These ELMs are triggered by the high n-ballooning modes and intermediate n modes, which are most unstable at low current, and have narrow radial structures.

## References

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