A Supplementary File for "Speeding up Local Search for the Indicator-based Subset Selection Problem by a Candidate List Strategy"

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S.1 Analysis of the relationship between the improvement of quality indicator values and $\texttt{dist}^{\mathrm{s}}$

Below, this section investigates the relationship between the improvement of quality indicator values and $\texttt{dist}^{\mathrm{s}}$ for conventional LS on the ISSPs using the seven quality indicators. We used the same experimental setup as in Section III, where we set d to 4, n to 5 000, and $k = 100$. The improvement of quality indicator values $\texttt{improvement}$ was calculated as follows:

$$\texttt{improvement} = \frac{\mathcal{I}(S^{\mathrm{prev}}) - \mathcal{I}(S^{\mathrm{new}})}{\mathcal{I}(S^{\mathrm{init}}) - \mathcal{I}(S^{\mathrm{last}})}$$

where $\mathcal{I}(S^{\mathrm{init}})$ is the quality indicator value of the initial subset, $\mathcal{I}(S^{\mathrm{last}})$ is that of the last subset (i.e., the best-so-far subset found by LS during the search), $\mathcal{I}(S^{\mathrm{prev}})$ is that of the previous subset, and $\mathcal{I}(S^{\mathrm{new}})$ is that of the new subset after the successful swap. The improvement value was normalized by $\mathcal{I}(S^{\mathrm{init}})$ and $\mathcal{I}(S^{\mathrm{last}})$.

Fig. S.1 shows the distribution of the improvement of quality indicator values with respect to $\texttt{dist}^{\mathrm{s}}$. Note that the y-axis in Fig. S.1 is on a logarithmic scale. The colors of the points indicate the number of function evaluations. The dark and light points mean that they were on the early and later stages of the search, respectively.

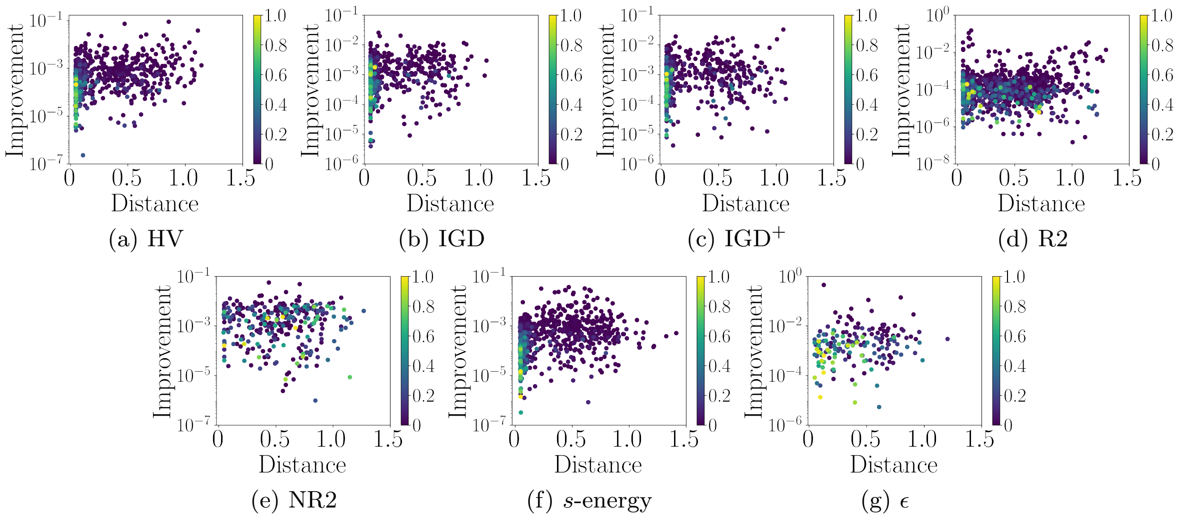
As seen from Fig. S.1, at the early stage of the search, a large improvement can be observed regardless of $\texttt{dist}^{\mathrm{s}}$. This is partially consistent with the observation in Section III. Since the diversity of the initial subset is poor, it is relatively easy to improve the quality of subset. We can see that the improvement at the later stage is slightly smaller than that at the early stage.

S.2 Analysis of the relationship between the success rate of swapping and $\texttt{dist}^{\mathrm{s}}$

This section investigates the relationship between the success rate of swapping and $\texttt{dist}^{\mathrm{s}}$ for conventional LS on the ISSPs using the seven quality indicators. We used the same experimental setup as in Section III.

Fig. S.2 shows the distribution of the success rate, where the x-axis represents the number of subset evaluations, the y-axis represents $\texttt{dist}^{\mathrm{s}}$, and the z-axis represents the success rate. Let $\texttt{num}^{\mathrm{local}}$ be the number of subset evaluations to find a local optimum. We calculated the rate of successful swaps for every $\texttt{num}^{\mathrm{local}}/10$ subset evaluations and 0.15 distance. The x-axis in Fig. S.2 is normalized divided by $\texttt{num}^{\mathrm{local}}$. For the sake of clarity, different colors are used to distinguish adjacent bars.

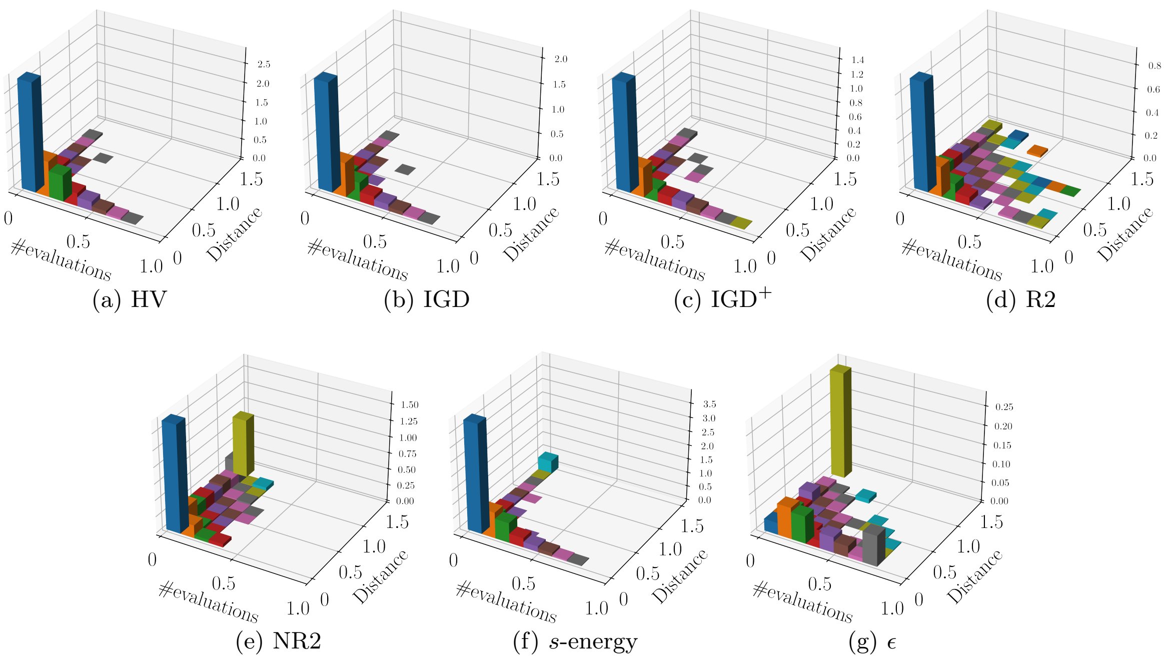
As seen from Fig. S.2, for HV, IGD, IGD^{+}, and R2, the success rate of two points close to each other is high at the early stage of the search. This observation suggests the rationale for the nearest neighbor list described in Section IV-B. In contrast, as shown in Fig. S.2, for NR2 and s-energy, the success rate of two points *far away from* each other is high at the early stage of the search. Here, swapping two points far away from each other hardly occurs when using the nearest neighbor list. This may be the reason why LS-N does not work well for the ISSP using NR2 and s-energy.



(a) HV (b) IGD (c) IGD$^+$ (d) R2

(e) NR2 (f) s-energy (g) ϵ

Fig. S.1: Distributions of `improvement` and `dist`^{s}.



(a) HV (b) IGD (c) IGD$^+$ (d) R2

(e) NR2 (f) s-energy (g) ϵ

Fig. S.2: Distribution of the success rate of swapping with respect to the distance between two points and the number of evaluations.

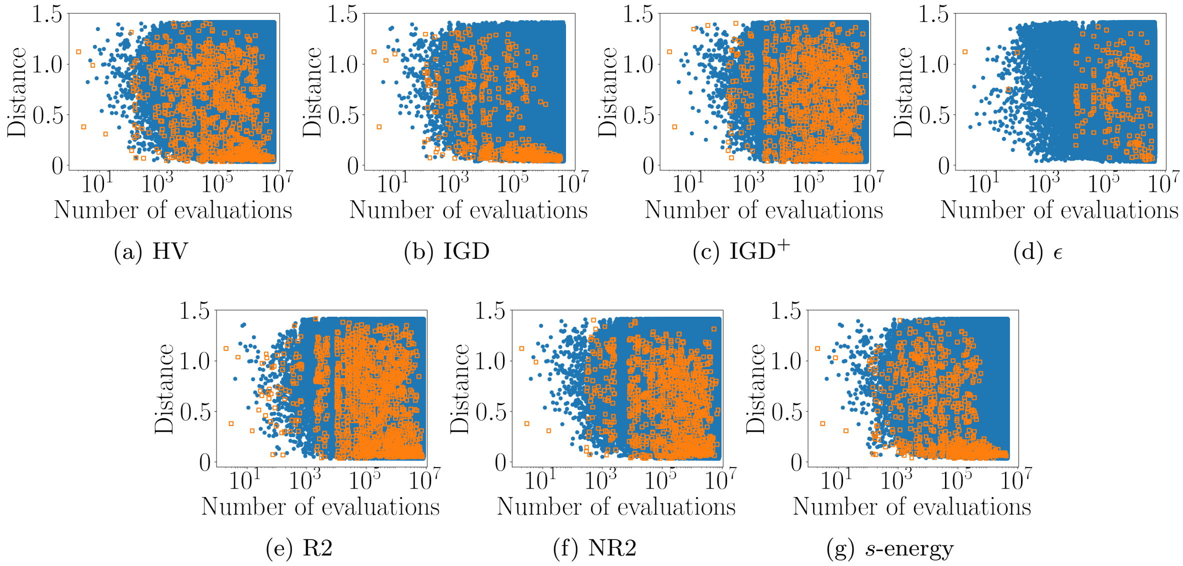


Fig. S.3: Distribution of $\texttt{dist}^{\mathrm{s}}$ and $\texttt{dist}^{\mathrm{us}}$ values on the ISSP with the nonconvex Pareto front in a representative run of LS. □ indicates a successful swap, and • indicates an unsuccessful swap.

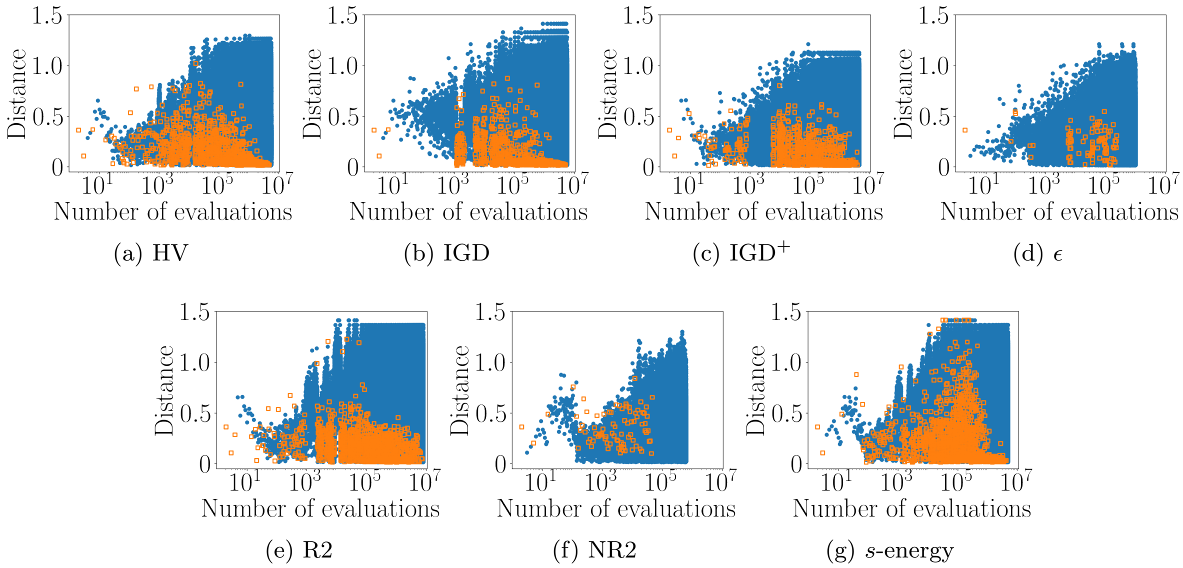
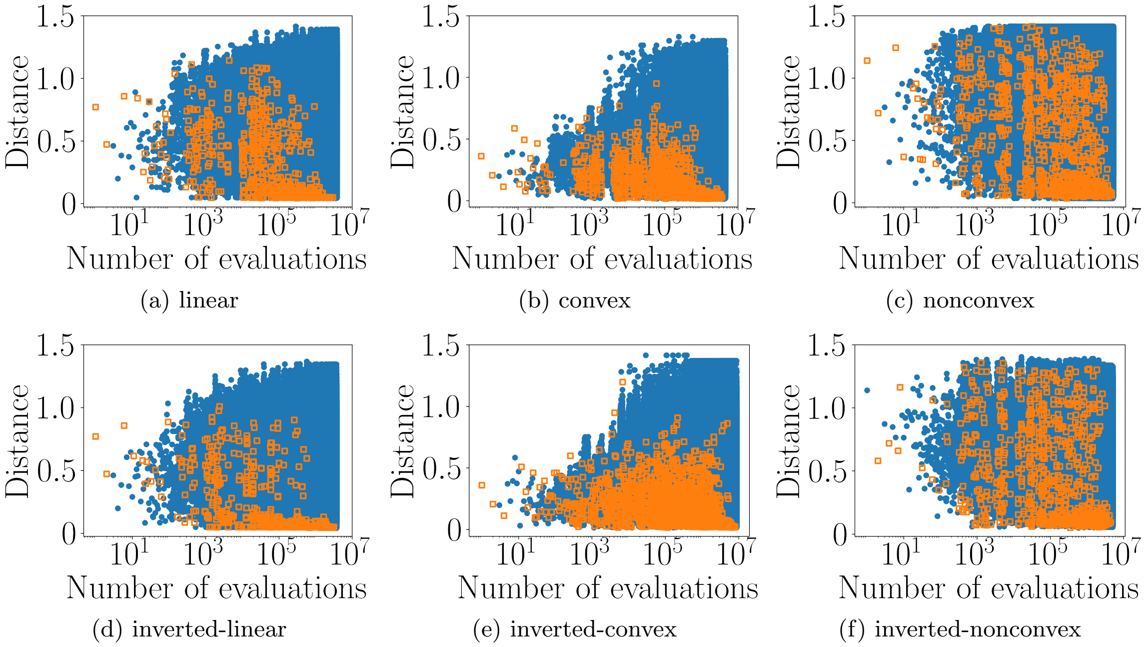


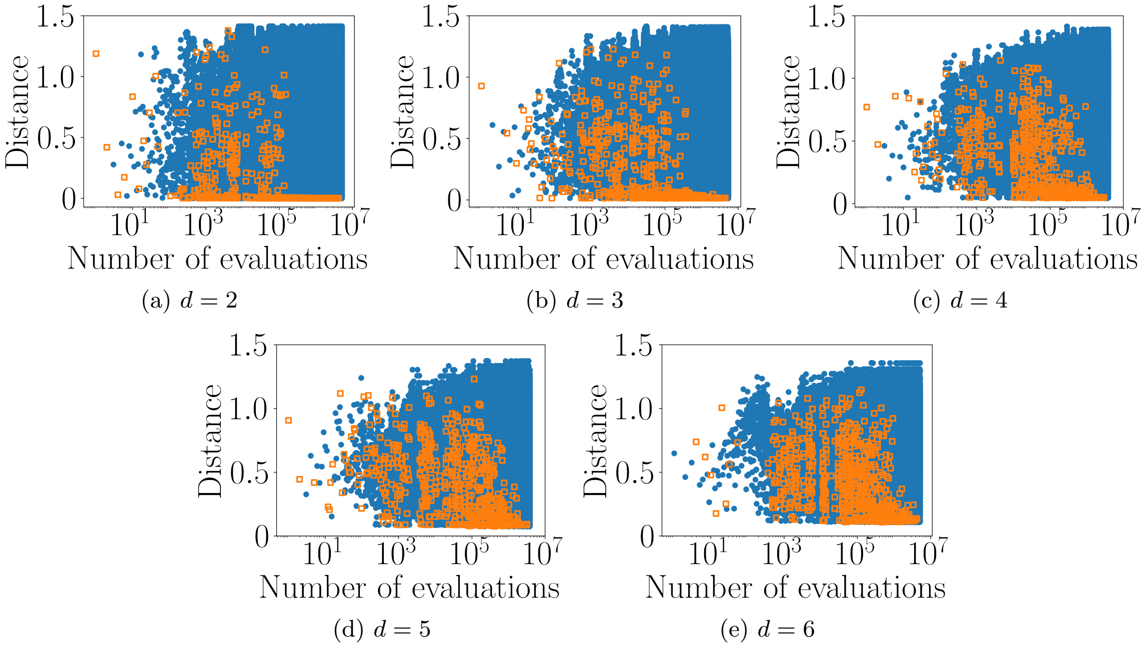
Fig. S.4: Distribution of $\texttt{dist}^{\mathrm{s}}$ and $\texttt{dist}^{\mathrm{us}}$ values on the ISSP with the convex Pareto front in a representative run of LS. □ indicates a successful swap, and • indicates an unsuccessful swap.



(a) linear (b) convex (c) nonconvex

(d) inverted-linear (e) inverted-convex (f) inverted-nonconvex

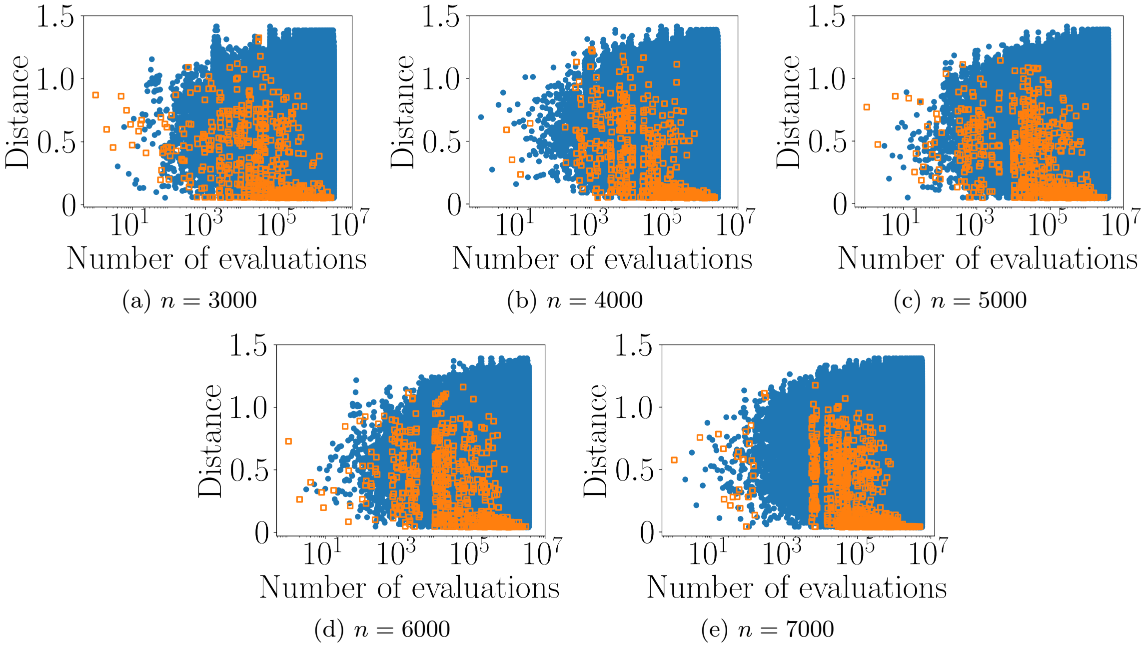
Fig. S.5: Distribution of $\texttt{dist}^{\mathrm{s}}$ and $\texttt{dist}^{\mathrm{us}}$ values in a representative run of LS on the ISSPs for the six PF shapes, where $\square$ indicates a successful swap, and $\bullet$ indicates an unsuccessful swap.



(a) $d = 2$ (b) $d = 3$ (c) $d = 4$

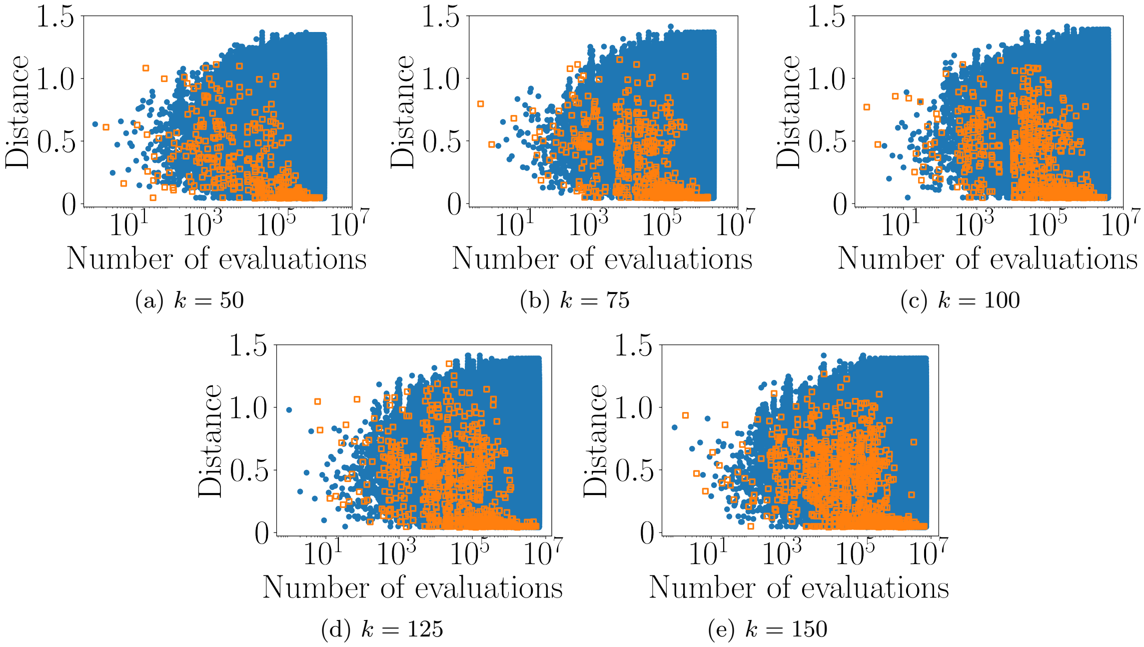
(d) $d = 5$ (e) $d = 6$

Fig. S.6: Distribution of $\texttt{dist}^{\mathrm{s}}$ and $\texttt{dist}^{\mathrm{us}}$ values in a representative run of LS on the ISSPs for $d \in \{2, 3, 4, 5, 6\}$, where $\square$ indicates a successful swap, and $\bullet$ indicates an unsuccessful swap.



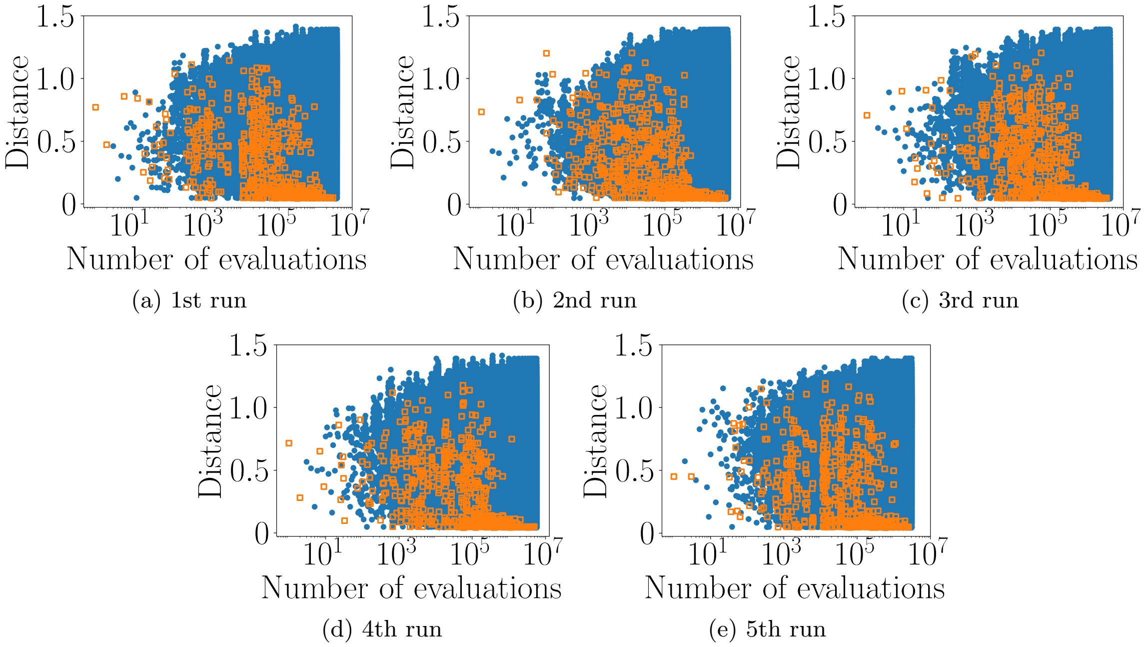
(a) $n = 3000$ (b) $n = 4000$ (c) $n = 5000$ (d) $n = 6000$ (e) $n = 7000$

Fig. S.7: Distribution of $\texttt{dist}^{\mathrm{s}}$ and $\texttt{dist}^{\mathrm{us}}$ values in a representative run of LS on the ISSPs for $n \in \{3\,000, 4\,000, 5\,000, 6\,000, 7\,000\}$, where □ indicates a successful swap, and • indicates an unsuccessful swap.



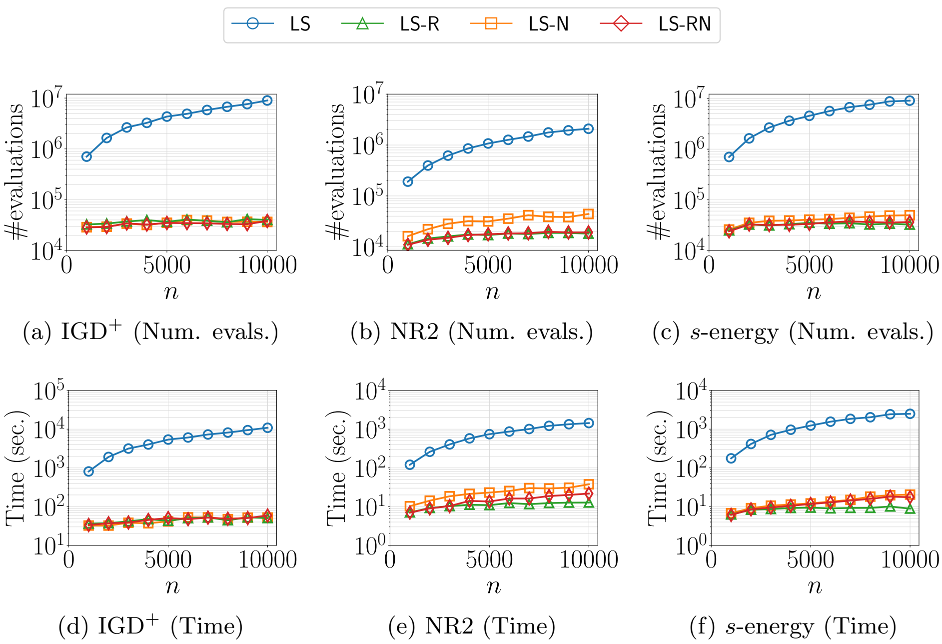
(a) $k = 50$ (b) $k = 75$ (c) $k = 100$ (d) $k = 125$ (e) $k = 150$

Fig. S.8: Distribution of $\texttt{dist}^{\mathrm{s}}$ and $\texttt{dist}^{\mathrm{us}}$ values in a representative run of LS on the ISSPs for $k \in \{50, 75, 100, 125, 150\}$, where □ indicates a successful swap, and • indicates an unsuccessful swap.



(a) 1st run (b) 2nd run (c) 3rd run (d) 4th run (e) 5th run

Fig. S.9: Distribution of $\texttt{dist}^{\mathrm{s}}$ and $\texttt{dist}^{\mathrm{us}}$ values in the five runs of LS on the ISSP using the HV, where □ indicates a successful swap, and • indicates an unsuccessful swap.



(a) IGD$^+$ (Num. evals.) (b) NR2 (Num. evals.) (c) s-energy (Num. evals.)

(d) IGD$^+$ (Time) (e) NR2 (Time) (f) s-energy (Time)

Fig. S.10: Results of the four LS methods on the ISSP with the linear PF using IGD$^+$, NR2, and s-energy.

Table S.1: Comparison of the four LS methods on the ISSP instances with the linear PF and HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy. The mean, [standard deviation], and (relative error based on LS) of the quality indicator values of the subsets found by each LS method are shown. The symbols + , - , and $\approx$ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to conventional LS.

(a) HV

n	LS	LS-N	LS-R	LS-RN
1K	1.37e+00 [9.46e−05]	1.37e+00 [8.39e−05] - (6.42e−03%)	1.37e+00 [1.53e−04] - (7.56e−02%)	1.37e+00 [1.05e−04] - (8.36e−03%)
2K	1.37e+00 [6.79e−05]	1.37e+00 [6.14e−05] - (3.57e−03%)	1.37e+00 [1.69e−04] - (9.92e−02%)	1.37e+00 [9.41e−05] - (5.14e−03%)
3K	1.37e+00 [9.37e−05]	1.37e+00 [8.87e−05] - (6.44e−03%)	1.37e+00 [1.67e−04] - (1.08e−01%)	1.37e+00 [8.12e−05] - (6.96e−03%)
4K	1.37e+00 [7.64e−05]	1.37e+00 [9.66e−05] - (6.10e−03%)	1.37e+00 [2.16e−04] - (1.20e−01%)	1.37e+00 [7.39e−05] - (6.38e−03%)
5K	1.37e+00 [6.64e−05]	1.37e+00 [7.13e−05] - (7.72e−03%)	1.37e+00 [1.27e−04] - (1.25e−01%)	1.37e+00 [9.38e−05] - (6.19e−03%)
6K	1.37e+00 [7.11e−05]	1.37e+00 [8.46e−05] - (7.19e−03%)	1.37e+00 [1.59e−04] - (1.36e−01%)	1.37e+00 [9.44e−05] - (7.54e−03%)
7K	1.37e+00 [6.29e−05]	1.37e+00 [7.85e−05] - (5.78e−03%)	1.37e+00 [1.94e−04] - (1.38e−01%)	1.37e+00 [7.26e−05] - (7.17e−03%)
8K	1.37e+00 [7.94e−05]	1.37e+00 [6.49e−05] - (6.00e−03%)	1.37e+00 [2.07e−04] - (1.41e−01%)	1.37e+00 [7.18e−05] - (8.97e−03%)
9K	1.37e+00 [5.98e−05]	1.37e+00 [9.50e−05] - (6.95e−03%)	1.37e+00 [2.13e−04] - (1.40e−01%)	1.37e+00 [5.98e−05] - (9.03e−03%)
10K	1.37e+00 [5.74e−05]	1.37e+00 [7.72e−05] - (8.56e−03%)	1.37e+00 [1.97e−04] - (1.40e−01%)	1.37e+00 [8.77e−05] - (8.53e−03%)

(b) IGD

n	LS	LS-N	LS-R	LS-RN
1K	8.07e−02 [1.61e−04]	8.10e−02 [2.48e−04] - (4.25e−01%)	8.24e−02 [3.06e−04] - (2.14e+00%)	8.08e−02 [1.77e−04] - (1.52e−01%)
2K	8.06e−02 [1.01e−04]	8.09e−02 [1.70e−04] - (3.24e−01%)	8.31e−02 [2.92e−04] - (3.06e+00%)	8.08e−02 [1.29e−04] - (2.02e−01%)
3K	7.98e−02 [1.39e−04]	8.00e−02 [1.68e−04] - (2.33e−01%)	8.31e−02 [2.78e−04] - (4.09e+00%)	8.00e−02 [1.50e−04] - (2.91e−01%)
4K	7.96e−02 [1.26e−04]	7.98e−02 [1.78e−04] - (2.99e−01%)	8.30e−02 [2.45e−04] - (4.31e+00%)	7.97e−02 [1.38e−04] - (1.99e−01%)
5K	7.94e−02 [1.16e−04]	7.96e−02 [1.71e−04] - (3.66e−01%)	8.28e−02 [2.14e−04] - (4.34e+00%)	7.96e−02 [1.39e−04] - (2.53e−01%)
6K	7.92e−02 [1.15e−04]	7.95e−02 [1.66e−04] - (3.74e−01%)	8.28e−02 [2.65e−04] - (4.51e+00%)	7.94e−02 [1.39e−04] - (2.04e−01%)
7K	7.92e−02 [1.02e−04]	7.94e−02 [1.48e−04] - (2.84e−01%)	8.27e−02 [2.60e−04] - (4.47e+00%)	7.93e−02 [1.23e−04] - (1.83e−01%)
8K	7.91e−02 [9.22e−05]	7.94e−02 [1.79e−04] - (3.30e−01%)	8.28e−02 [2.43e−04] - (4.59e+00%)	7.93e−02 [1.13e−04] - (1.91e−01%)
9K	7.90e−02 [1.02e−04]	7.93e−02 [1.51e−04] - (3.67e−01%)	8.27e−02 [2.73e−04] - (4.72e+00%)	7.92e−02 [1.11e−04] - (2.89e−01%)
10K	7.90e−02 [1.05e−04]	7.93e−02 [1.98e−04] - (3.70e−01%)	8.28e−02 [2.65e−04] - (4.87e+00%)	7.92e−02 [1.04e−04] - (2.52e−01%)

(c) IGD$^+$

n	LS	LS-N	LS-R	LS-RN
1K	5.64e−02 [8.22e−05]	5.66e−02 [1.22e−04] - (2.48e−01%)	5.73e−02 [1.47e−04] - (1.59e+00%)	5.65e−02 [9.30e−05] - (1.46e−01%)
2K	5.45e−02 [7.91e−05]	5.46e−02 [1.29e−04] - (3.04e−01%)	5.61e−02 [1.89e−04] - (3.03e+00%)	5.47e−02 [9.22e−05] - (3.32e−01%)
3K	5.35e−02 [7.35e−05]	5.37e−02 [1.35e−04] - (3.70e−01%)	5.56e−02 [2.11e−04] - (4.04e+00%)	5.36e−02 [9.94e−05] - (3.18e−01%)
4K	5.31e−02 [7.54e−05]	5.33e−02 [1.03e−04] - (3.89e−01%)	5.53e−02 [2.07e−04] - (4.13e+00%)	5.32e−02 [9.49e−05] - (3.15e−01%)
5K	5.28e−02 [6.97e−05]	5.31e−02 [1.33e−04] - (5.51e−01%)	5.52e−02 [1.75e−04] - (4.55e+00%)	5.30e−02 [9.02e−05] - (3.40e−01%)
6K	5.29e−02 [5.02e−05]	5.31e−02 [1.16e−04] - (4.61e−01%)	5.52e−02 [1.71e−04] - (4.46e+00%)	5.30e−02 [8.37e−05] - (2.78e−01%)
7K	5.26e−02 [6.24e−05]	5.29e−02 [1.70e−04] - (5.53e−01%)	5.52e−02 [2.76e−04] - (4.99e+00%)	5.28e−02 [1.01e−04] - (3.93e−01%)
8K	5.28e−02 [6.44e−05]	5.30e−02 [1.07e−04] - (4.60e−01%)	5.52e−02 [1.75e−04] - (4.59e+00%)	5.29e−02 [7.46e−05] - (3.40e−01%)
9K	5.27e−02 [8.62e−05]	5.29e−02 [1.04e−04] - (4.02e−01%)	5.51e−02 [2.20e−04] - (4.64e+00%)	5.29e−02 [9.70e−05] - (4.16e−01%)
10K	5.27e−02 [8.29e−05]	5.29e−02 [1.15e−04] - (4.61e−01%)	5.52e−02 [2.17e−04] - (4.91e+00%)	5.28e−02 [1.09e−04] - (3.38e−01%)

(d) R2

n	LS	LS-N	LS-R	LS-RN
1K	2.17e−02 [5.46e−06]	2.17e−02 [1.27e−05] - (1.19e−01%)	2.18e−02 [1.80e−05] - (4.96e−01%)	2.17e−02 [8.51e−06] - (1.08e−01%)
2K	2.15e−02 [7.79e−06]	2.16e−02 [2.21e−05] - (3.12e−01%)	2.17e−02 [1.36e−04] - (1.00e+00%)	2.16e−02 [2.18e−05] - (2.95e−01%)
3K	2.13e−02 [7.72e−06]	2.14e−02 [2.71e−05] - (3.52e−01%)	2.18e−02 [3.37e−04] - (1.98e+00%)	2.14e−02 [2.33e−05] - (3.55e−01%)
4K	2.13e−02 [9.42e−06]	2.14e−02 [3.25e−05] - (4.63e−01%)	2.19e−02 [4.28e−04] - (2.78e+00%)	2.14e−02 [2.64e−05] - (4.31e−01%)
5K	2.13e−02 [1.27e−05]	2.14e−02 [3.13e−05] - (5.38e−01%)	2.19e−02 [3.85e−04] - (2.76e+00%)	2.14e−02 [2.62e−05] - (4.79e−01%)
6K	2.12e−02 [1.17e−05]	2.14e−02 [4.42e−05] - (5.99e−01%)	2.19e−02 [3.46e−04] - (3.32e+00%)	2.13e−02 [3.73e−05] - (5.62e−01%)
7K	2.12e−02 [1.19e−05]	2.13e−02 [3.84e−05] - (6.77e−01%)	2.22e−02 [4.09e−04] - (4.47e+00%)	2.13e−02 [3.19e−05] - (6.22e−01%)
8K	2.12e−02 [1.15e−05]	2.13e−02 [3.53e−05] - (6.88e−01%)	2.20e−02 [4.75e−04] - (3.85e+00%)	2.14e−02 [4.18e−05] - (7.65e−01%)
9K	2.12e−02 [8.10e−06]	2.13e−02 [4.17e−05] - (7.43e−01%)	2.21e−02 [4.33e−04] - (4.46e+00%)	2.13e−02 [3.61e−05] - (6.23e−01%)
10K	2.12e−02 [1.12e−05]	2.13e−02 [5.15e−05] - (8.97e−01%)	2.21e−02 [3.80e−04] - (4.57e+00%)	2.13e−02 [3.75e−05] - (8.18e−01%)

(e) NR2

n	LS	LS-N	LS-R	LS-RN
1K	2.88e+01 [3.61e−15]	2.88e+01 [1.02e−02] - (2.18e−02%)	2.88e+01 [7.08e−03] - (2.72e−02%)	2.88e+01 [4.36e−03] - (4.74e−03%)
2K	2.89e+01 [7.22e−15]	2.89e+01 [8.88e−03] - (3.33e−02%)	2.89e+01 [1.92e−02] - (1.39e−01%)	2.89e+01 [1.94e−03] - (3.49e−03%)
3K	2.90e+01 [1.44e−14]	2.89e+01 [1.29e−02] - (5.51e−02%)	2.89e+01 [2.11e−02] - (1.80e−01%)	2.89e+01 [5.86e−03] - (7.98e−03%)
4K	2.90e+01 [7.22e−15]	2.89e+01 [1.40e−02] - (7.35e−02%)	2.89e+01 [1.74e−02] - (2.03e−01%)	2.90e+01 [6.02e−03] - (8.98e−03%)
5K	2.90e+01 [7.22e−15]	2.90e+01 [1.78e−02] - (9.00e−02%)	2.89e+01 [2.13e−02] - (3.12e−01%)	2.90e+01 [5.36e−03] - (9.17e−03%)
6K	2.90e+01 [3.61e−15]	2.90e+01 [2.00e−02] - (1.22e−01%)	2.89e+01 [2.02e−02] - (3.09e−01%)	2.90e+01 [4.59e−03] - (1.06e−02%)
7K	2.90e+01 [7.22e−15]	2.90e+01 [2.04e−02] - (9.55e−02%)	2.89e+01 [1.58e−02] - (3.18e−01%)	2.90e+01 [4.94e−03] - (1.07e−02%)
8K	2.90e+01 [0.00e+00]	2.90e+01 [1.93e−02] - (1.34e−01%)	2.89e+01 [1.62e−02] - (3.36e−01%)	2.90e+01 [5.54e−03] - (1.14e−02%)
9K	2.90e+01 [1.44e−14]	2.90e+01 [1.37e−02] - (7.86e−02%)	2.89e+01 [1.92e−02] - (3.17e−01%)	2.90e+01 [2.90e−03] - (5.86e−03%)
10K	2.90e+01 [0.00e+00]	2.90e+01 [1.62e−02] - (1.22e−01%)	2.89e+01 [2.01e−02] - (3.57e−01%)	2.90e+01 [3.96e−03] - (9.47e−03%)

(f) ϵ

n	LS	LS-N	LS-R	LS-RN
1K	8.22e−02 [2.66e−02]	1.04e−01 [2.57e−02] - (2.71e+01%)	8.11e−02 [2.54e−02] + (−1.31e+00%)	8.07e−02 [2.50e−02] + (−1.90e+00%)
2K	9.30e−02 [6.63e−03]	9.78e−02 [7.94e−03] - (5.24e+00%)	9.48e−02 [3.04e−03] $\approx$ (1.94e+00%)	9.58e−02 [8.42e−04] $\approx$ (3.05e+00%)
3K	8.67e−02 [1.24e−02]	9.30e−02 [1.67e−02] - (7.34e+00%)	8.57e−02 [1.03e−02] $\approx$ (−1.09e+00%)	8.43e−02 [7.40e−03] $\approx$ (−2.71e+00%)
4K	7.87e−02 [1.07e−02]	8.74e−02 [1.70e−02] - (1.11e+01%)	8.50e−02 [1.54e−02] - (7.97e+00%)	7.88e−02 [1.07e−02] $\approx$ (1.94e−01%)
5K	7.28e−02 [9.71e−03]	8.50e−02 [1.57e−02] - (1.68e+01%)	7.97e−02 [9.68e−03] - (9.57e+00%)	7.72e−02 [1.07e−02] - (6.14e+00%)
6K	7.36e−02 [1.23e−02]	9.81e−02 [9.06e−03] - (3.33e+01%)	8.55e−02 [1.11e−02] - (1.61e+01%)	8.48e−02 [1.15e−02] - (1.52e+01%)
7K	7.74e−02 [1.37e−02]	9.29e−02 [5.38e−03] - (1.99e+01%)	9.01e−02 [4.15e−03] - (1.63e+01%)	8.39e−02 [8.67e−03] $\approx$ (8.36e+00%)
8K	7.75e−02 [1.23e−02]	8.84e−02 [6.58e−04] - (1.41e+01%)	8.77e−02 [1.86e−03] - (1.31e+01%)	8.46e−02 [4.98e−03] $\approx$ (9.14e+00%)
9K	7.25e−02 [1.03e−02]	8.78e−02 [7.95e−03] - (2.12e+01%)	8.79e−02 [5.95e−03] - (2.13e+01%)	8.23e−02 [4.91e−03] - (1.36e+01%)
10K	7.44e−02 [1.01e−02]	9.13e−02 [1.13e−02] - (2.27e+01%)	8.60e−02 [6.57e−03] - (1.56e+01%)	8.27e−02 [3.42e−03] - (1.11e+01%)

(g) s–energy

n	LS	LS-N	LS-R	LS-RN
1K	1.37e+05 [6.03e+02]	1.37e+05 [9.03e+02] $\approx$ (2.50e−01%)	1.51e+05 [2.31e+03] - (1.02e+01%)	1.37e+05 [8.21e+02] - (4.51e−01%)
2K	1.35e+05 [5.11e+02]	1.36e+05 [7.50e+02] - (3.76e−01%)	1.52e+05 [2.53e+03] - (1.26e+01%)	1.36e+05 [7.92e+02] - (4.05e−01%)
3K	1.34e+05 [7.79e+02]	1.35e+05 [7.22e+02] - (5.85e−01%)	1.55e+05 [2.59e+03] - (1.56e+01%)	1.35e+05 [6.45e+02] - (5.53e−01%)
4K	1.33e+05 [5.32e+02]	1.34e+05 [6.48e+02] - (5.80e−01%)	1.56e+05 [3.23e+03] - (1.72e+01%)	1.34e+05 [7.41e+02] - (5.86e−01%)
5K	1.33e+05 [2.96e+02]	1.34e+05 [5.56e+02] - (5.66e−01%)	1.58e+05 [3.33e+03] - (1.86e+01%)	1.34e+05 [5.11e+02] - (5.39e−01%)
6K	1.33e+05 [3.92e+02]	1.34e+05 [5.41e+02] - (7.89e−01%)	1.58e+05 [2.83e+03] - (1.92e+01%)	1.33e+05 [5.65e+02] - (5.23e−01%)
7K	1.32e+05 [5.13e+02]	1.33e+05 [6.96e+02] - (7.97e−01%)	1.59e+05 [3.07e+03] - (2.05e+01%)	1.33e+05 [4.54e+02] - (5.39e−01%)
8K	1.32e+05 [3.90e+02]	1.33e+05 [4.70e+02] - (5.98e−01%)	1.59e+05 [2.69e+03] - (2.02e+01%)	1.33e+05 [6.24e+02] - (3.72e−01%)
9K	1.32e+05 [3.86e+02]	1.33e+05 [5.81e+02] - (7.93e−01%)	1.58e+05 [2.74e+03] - (2.00e+01%)	1.32e+05 [7.19e+02] - (5.86e−01%)
10K	1.32e+05 [3.38e+02]	1.33e+05 [5.51e+02] - (7.39e−01%)	1.59e+05 [2.93e+03] - (2.08e+01%)	1.32e+05 [6.26e+02] - (5.52e−01%)

Table S.2: Comparison of LS-N, LS-R, and LS-RN on the ISSP instances with the linear PF and HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy. The symbols + , - , and $\approx$ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to LS-RN.

(a) HV

n	LS-N	LS-R
1K	$\approx$	-
2K	$\approx$	-
3K	$\approx$	-
4K	$\approx$	-
5K	$\approx$	-
6K	$\approx$	-
7K	$\approx$	-
8K	+	-
9K	$\approx$	-
10K	$\approx$	-

(b) IGD

n	LS-N	LS-R
1K	-	-
2K	-	-
3K	$\approx$	-
4K	$\approx$	-
5K	-	-
6K	-	-
7K	-	-
8K	-	-
9K	$\approx$	-
10K	-	-

(c) IGD$^+$

n	LS-N	LS-R
1K	$\approx$	-
2K	$\approx$	-
3K	$\approx$	-
4K	$\approx$	-
5K	-	-
6K	-	-
7K	$\approx$	-
8K	-	-
9K	$\approx$	-
10K	-	-

(d) R2

n	LS-N	LS-R
1K	$\approx$	-
2K	$\approx$	-
3K	$\approx$	-
4K	$\approx$	-
5K	$\approx$	-
6K	$\approx$	-
7K	$\approx$	-
8K	$\approx$	-
9K	-	-
10K	$\approx$	-

(e) NR2

n	LS-N	LS-R
1K	-	-
2K	-	-
3K	-	-
4K	-	-
5K	-	-
6K	-	-
7K	-	-
8K	-	-
9K	-	-
10K	-	-

(f) ϵ

n	LS-N	LS-R
1K	-	$\approx$
2K	-	$\approx$
3K	-	$\approx$
4K	-	-
5K	$\approx$	-
6K	-	$\approx$
7K	-	-
8K	-	-
9K	-	-
10K	-	-

(g) s–energy

n	LS-N	LS-R
1K	$\approx$	-
2K	$\approx$	-
3K	$\approx$	-
4K	$\approx$	-
5K	$\approx$	-
6K	-	-
7K	-	-
8K	-	-
9K	$\approx$	-
10K	-	-

Table S.3: Comparison of the four LS methods on the ISSP instances with HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy, the linear PF, $n = 5\,000$, $k = 100$, and $d \in \{2, 3, 4, 5, 6\}$. The mean, [standard deviation], and (relative error based on LS) of the quality indicator values of the subsets found by each LS method are shown. The symbols + , - , and $\approx$ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to conventional LS.

(a) HV

d	LS	LS-N	LS-R	LS-RN
2	7.05e-01 [5.50e-06]	7.05e-01 [4.69e-05] - (1.41e-02%)	7.05e-01 [3.15e-05] - (3.45e-02%)	7.05e-01 [7.12e-06] - (5.33e-04%)
3	1.12e+00 [7.68e-05]	1.12e+00 [1.17e-04] - (1.61e-02%)	1.12e+00 [1.76e-04] - (1.55e-01%)	1.12e+00 [8.10e-05] - (9.22e-03%)
4	1.37e+00 [6.64e-05]	1.37e+00 [7.13e-05] - (7.72e-03%)	1.37e+00 [1.27e-04] - (1.25e-01%)	1.37e+00 [9.38e-05] - (6.19e-03%)
5	1.57e+00 [4.70e-05]	1.57e+00 [6.46e-05] - (2.86e-03%)	1.57e+00 [1.64e-04] - (7.98e-02%)	1.57e+00 [5.16e-05] - (3.46e-03%)
6	1.75e+00 [3.97e-05]	1.75e+00 [3.58e-05] - (2.73e-03%)	1.75e+00 [1.17e-04] - (5.08e-02%)	1.75e+00 [4.17e-05] - (4.27e-03%)

(b) IGD

d	LS	LS-N	LS-R	LS-RN
2	3.37e-03 [1.50e-05]	3.73e-03 [1.11e-04] - (1.09e+01%)	3.60e-03 [2.23e-05] - (7.04e+00%)	3.39e-03 [1.52e-05] - (6.69e-01%)
3	3.67e-02 [5.41e-05]	3.71e-02 [1.68e-04] - (1.05e+00%)	3.85e-02 [1.42e-04] - (4.71e+00%)	3.68e-02 [6.97e-05] - (2.73e-01%)
4	7.94e-02 [1.16e-04]	7.96e-02 [1.71e-04] - (3.66e-01%)	8.28e-02 [2.14e-04] - (4.34e+00%)	7.96e-02 [1.39e-04] - (2.53e-01%)
5	1.20e-01 [1.31e-04]	1.20e-01 [1.77e-04] - (2.20e-01%)	1.24e-01 [4.08e-04] - (3.23e+00%)	1.20e-01 [1.82e-04] - (2.26e-01%)
6	1.56e-01 [2.12e-04]	1.56e-01 [2.39e-04] - (2.09e-01%)	1.60e-01 [3.89e-04] - (2.64e+00%)	1.57e-01 [2.29e-04] - (3.68e-01%)

(c) IGD$^+$

d	LS	LS-N	LS-R	LS-RN
2	2.38e-03 [8.53e-06]	2.64e-03 [8.19e-05] - (1.08e+01%)	2.55e-03 [1.56e-05] - (7.12e+00%)	2.40e-03 [1.01e-05] - (7.18e-01%)
3	2.52e-02 [5.58e-05]	2.55e-02 [1.09e-04] - (1.09e+00%)	2.64e-02 [1.06e-04] - (4.93e+00%)	2.53e-02 [4.89e-05] - (2.69e-01%)
4	5.28e-02 [6.97e-05]	5.31e-02 [1.33e-04] - (5.51e-01%)	5.52e-02 [1.75e-04] - (4.55e+00%)	5.30e-02 [9.02e-05] - (3.40e-01%)
5	7.79e-02 [1.38e-04]	7.84e-02 [2.00e-04] - (5.51e-01%)	8.08e-02 [3.05e-04] - (3.62e+00%)	7.85e-02 [2.34e-04] - (7.33e-01%)
6	9.94e-02 [1.13e-04]	1.00e-01 [2.35e-04] - (7.47e-01%)	1.03e-01 [6.25e-04] - (3.67e+00%)	1.00e-01 [2.28e-04] - (9.79e-01%)

(d) R2

d	LS	LS-N	LS-R	LS-RN
2	1.67e-01 [3.30e-06]	1.68e-01 [5.45e-05] - (8.45e-02%)	1.68e-01 [8.69e-06] - (4.45e-02%)	1.67e-01 [4.90e-06] - (7.43e-03%)
3	5.87e-02 [1.36e-05]	5.87e-02 [2.52e-05] - (1.43e-01%)	5.90e-02 [9.07e-05] - (6.43e-01%)	5.87e-02 [1.34e-05] - (6.64e-02%)
4	2.13e-02 [1.27e-05]	2.14e-02 [3.13e-05] - (5.38e-01%)	2.19e-02 [3.85e-04] - (2.76e+00%)	2.14e-02 [2.62e-05] - (4.79e-01%)
5	4.03e-03 [3.68e-07]	4.19e-03 [2.21e-05] - (3.81e+00%)	4.61e-03 [6.71e-04] - (1.42e+01%)	4.10e-03 [1.33e-05] - (1.53e+00%)
6	6.42e-04 [4.41e-19]	6.44e-04 [2.43e-06] - (2.26e-01%)	1.04e-03 [2.72e-04] - (6.21e+01%)	6.46e-04 [2.97e-06] - (6.78e-01%)

(e) NR2

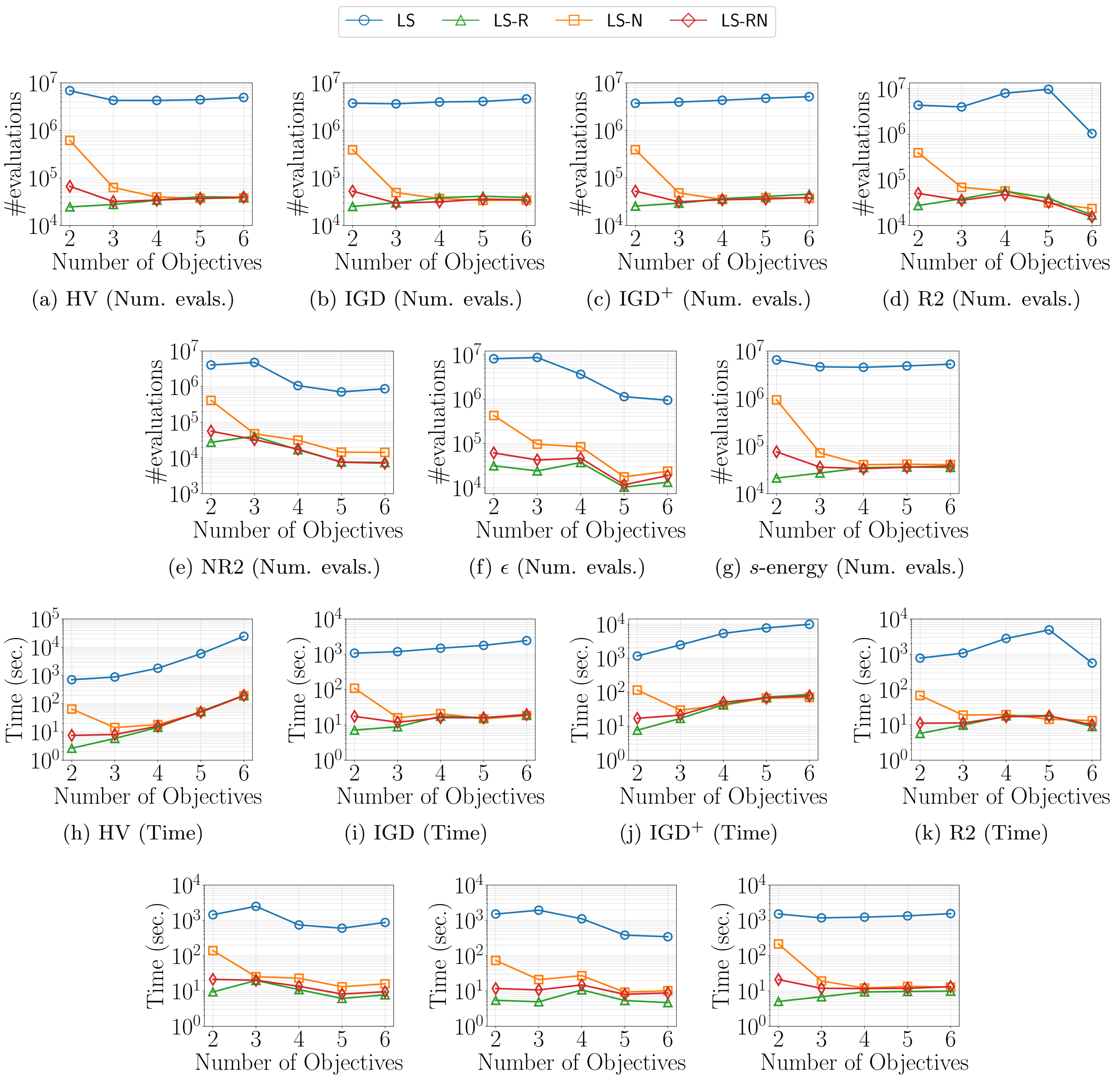
d	LS	LS-N	LS-R	LS-RN
2	1.41e+00 [4.14e-05]	1.41e+00 [4.26e-04] - (8.88e-02%)	1.41e+00 [9.80e-05] - (6.63e-02%)	1.41e+00 [5.21e-05] - (8.99e-03%)
3	6.59e+00 [1.12e-03]	6.57e+00 [4.89e-03] - (2.68e-01%)	6.55e+00 [3.90e-03] - (5.36e-01%)	6.58e+00 [2.09e-03] - (9.81e-02%)
4	2.90e+01 [7.22e-15]	2.90e+01 [1.78e-02] - (9.00e-02%)	2.89e+01 [2.13e-02] - (3.12e-01%)	2.90e+01 [5.36e-03] - (9.17e-03%)
5	1.28e+02 [4.33e-14]	1.28e+02 [4.33e-14] $\approx$ (0.00e+00%)	1.28e+02 [1.01e-01] - (6.20e-02%)	1.28e+02 [3.69e-02] $\approx$ (5.18e-03%)
6	5.80e+02 [1.16e-13]	5.80e+02 [1.16e-13] $\approx$ (0.00e+00%)	5.80e+02 [6.01e-01] $\approx$ (1.86e-02%)	5.80e+02 [1.16e-13] $\approx$ (0.00e+00%)

(f) ϵ

d	LS	LS-N	LS-R	LS-RN
2	5.95e-03 [1.98e-04]	1.22e-02 [2.38e-03] - (1.04e+02%)	6.75e-03 [1.92e-04] - (1.35e+01%)	6.48e-03 [2.29e-04] - (8.88e+00%)
3	4.29e-02 [2.17e-03]	5.85e-02 [1.04e-02] - (3.64e+01%)	5.05e-02 [3.18e-03] - (1.78e+01%)	4.74e-02 [4.19e-03] - (1.04e+01%)
4	7.28e-02 [9.71e-03]	8.50e-02 [1.57e-02] - (1.68e+01%)	7.97e-02 [9.68e-03] - (9.57e+00%)	7.72e-02 [1.07e-02] - (6.14e+00%)
5	1.18e-01 [1.13e-02]	1.26e-01 [1.21e-02] - (6.99e+00%)	1.24e-01 [8.54e-03] $\approx$ (4.90e+00%)	1.23e-01 [9.08e-03] $\approx$ (4.14e+00%)
6	1.25e-01 [2.69e-02]	1.31e-01 [2.74e-02] $\approx$ (4.38e+00%)	1.26e-01 [2.40e-02] $\approx$ (9.87e-01%)	1.14e-01 [2.43e-02] $\approx$ (-9.14e+00%)

(g) s-energy

d	LS	LS-N	LS-R	LS-RN
2	5.81e+04 [1.48e+01]	5.83e+04 [1.21e+02] - (3.94e-01%)	5.92e+04 [1.48e+02] - (1.96e+00%)	5.81e+04 [1.56e+01] - (1.24e-02%)
3	9.11e+04 [1.64e+02]	9.14e+04 [1.92e+02] - (3.46e-01%)	9.91e+04 [8.38e+02] - (8.77e+00%)	9.13e+04 [2.16e+02] - (1.99e-01%)
4	1.33e+05 [2.96e+02]	1.34e+05 [5.56e+02] - (5.66e-01%)	1.58e+05 [3.33e+03] - (1.86e+01%)	1.34e+05 [5.11e+02] - (5.39e-01%)
5	1.80e+05 [1.44e+03]	1.83e+05 [2.43e+03] - (1.44e+00%)	2.37e+05 [5.73e+03] - (3.16e+01%)	1.85e+05 [2.91e+03] - (2.58e+00%)
6	2.23e+05 [1.20e+03]	2.29e+05 [3.49e+03] - (2.84e+00%)	3.54e+05 [1.32e+04] - (5.85e+01%)	2.31e+05 [4.16e+03] - (3.67e+00%)



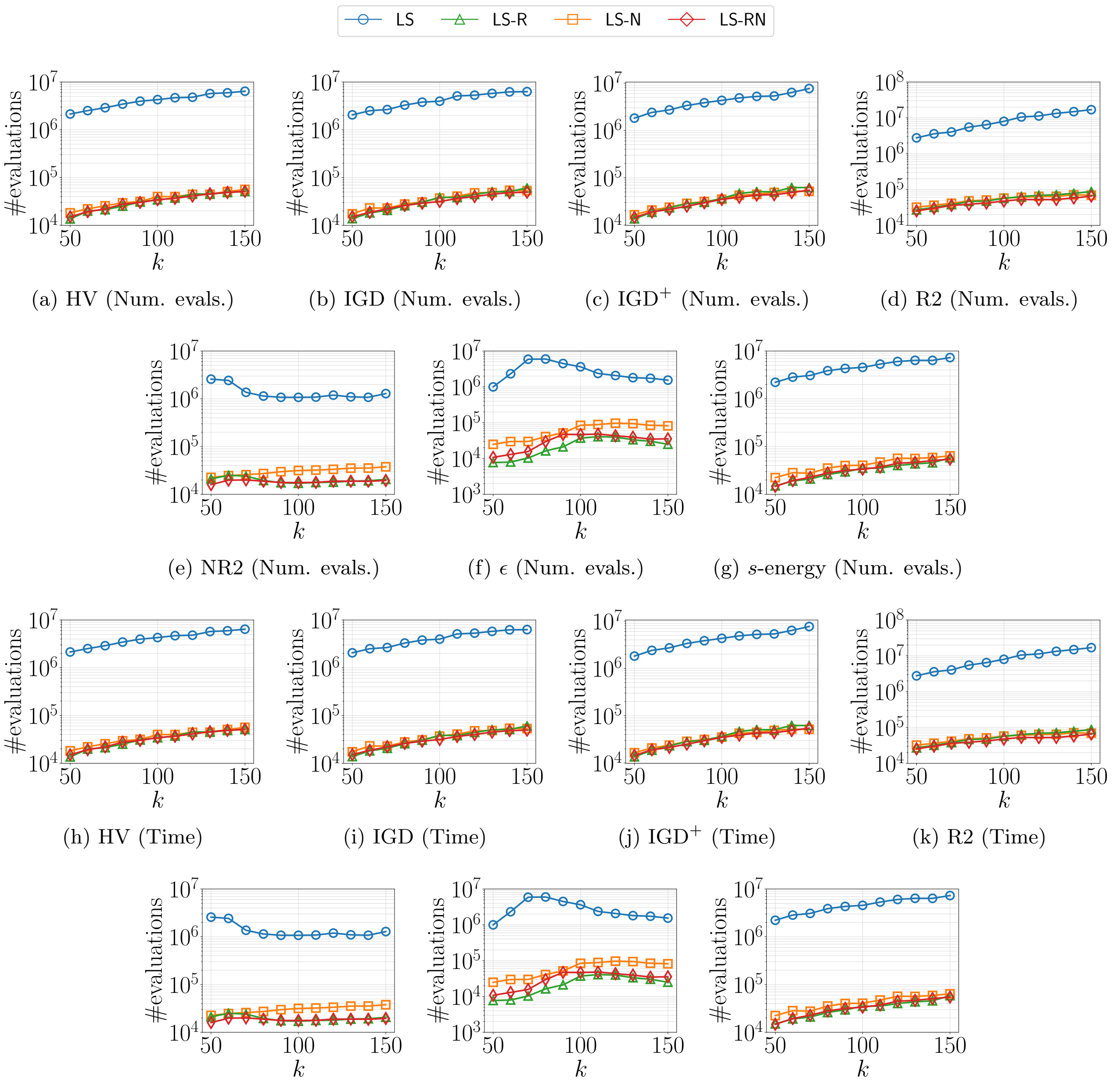
(a) HV (Num. evals.) (b) IGD (Num. evals.) (c) IGD$^+$ (Num. evals.) (d) R2 (Num. evals.)

(e) NR2 (Num. evals.) (f) ϵ (Num. evals.) (g) s-energy (Num. evals.)

(h) HV (Time) (i) IGD (Time) (j) IGD$^+$ (Time) (k) R2 (Time)

(l) NR2 (Time) (m) ϵ (Time) (n) s-energy (Time)

Fig. S.11: Results of the four LS methods on the ISSP with the linear PF using HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy.



(a) HV (Num. evals.) (b) IGD (Num. evals.) (c) IGD$^+$ (Num. evals.) (d) R2 (Num. evals.)

(e) NR2 (Num. evals.) (f) ϵ (Num. evals.) (g) s-energy (Num. evals.)

(h) HV (Time) (i) IGD (Time) (j) IGD$^+$ (Time) (k) R2 (Time)

(l) NR2 (Time) (m) ϵ (Time) (n) s-energy (Time)

Fig. S.12: Results of the four LS methods on the ISSP with the linear PF using HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy.

Table S.4: Comparison of the four LS methods on the ISSP instances with HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy, the linear PF, $n = 5\,000$, $d = 4$, and $k \in \{50, 60, \dots, 150\}$. The mean, [standard deviation], and (relative error based on LS) of the quality indicator values of the subsets found by each LS method are shown. The symbols + , - , and $\approx$ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to conventional LS.

(a) HV

k	LS	LS-N	LS-R	LS-RN
50	1.35e+00 [1.87e−04]	1.35e+00 [1.91e−04] - (1.49e−02%)	1.35e+00 [4.97e−04] - (2.65e−01%)	1.35e+00 [2.39e−04] - (2.29e−02%)
60	1.36e+00 [1.07e−04]	1.36e+00 [1.60e−04] - (8.94e−03%)	1.36e+00 [3.47e−04] - (2.08e−01%)	1.36e+00 [1.38e−04] - (1.45e−02%)
70	1.36e+00 [9.22e−05]	1.36e+00 [1.14e−04] - (5.63e−03%)	1.36e+00 [3.06e−04] - (1.75e−01%)	1.36e+00 [1.12e−04] - (7.17e−03%)
80	1.37e+00 [9.71e−05]	1.37e+00 [9.60e−05] - (5.83e−03%)	1.37e+00 [2.13e−04] - (1.63e−01%)	1.37e+00 [9.66e−05] - (9.19e−03%)
90	1.37e+00 [6.46e−05]	1.37e+00 [9.42e−05] - (5.50e−03%)	1.37e+00 [2.34e−04] - (1.40e−01%)	1.37e+00 [7.67e−05] - (9.13e−03%)
100	1.37e+00 [6.64e−05]	1.37e+00 [7.13e−05] - (7.72e−03%)	1.37e+00 [1.27e−04] - (1.25e−01%)	1.37e+00 [9.38e−05] - (6.19e−03%)
110	1.38e+00 [6.76e−05]	1.38e+00 [7.82e−05] - (7.04e−03%)	1.37e+00 [1.25e−04] - (1.15e−01%)	1.38e+00 [7.59e−05] - (4.20e−03%)
120	1.38e+00 [5.00e−05]	1.38e+00 [6.39e−05] $\approx$ (2.57e−03%)	1.38e+00 [1.54e−04] - (1.01e−01%)	1.38e+00 [7.77e−05] - (3.87e−03%)
130	1.38e+00 [6.53e−05]	1.38e+00 [6.63e−05] - (5.89e−03%)	1.38e+00 [1.03e−04] - (1.01e−01%)	1.38e+00 [8.33e−05] - (5.37e−03%)
140	1.38e+00 [5.11e−05]	1.38e+00 [5.42e−05] - (3.77e−03%)	1.38e+00 [1.06e−04] - (9.21e−02%)	1.38e+00 [4.79e−05] - (4.28e−03%)
150	1.38e+00 [5.23e−05]	1.38e+00 [6.06e−05] - (4.29e−03%)	1.38e+00 [1.22e−04] - (8.68e−02%)	1.38e+00 [4.54e−05] - (4.05e−03%)

(b) IGD

k	LS	LS-N	LS-R	LS-RN
50	1.01e−01 [2.11e−04]	1.02e−01 [1.97e−04] - (3.16e−01%)	1.06e−01 [5.71e−04] - (4.81e+00%)	1.01e−01 [2.25e−04] - (1.86e−01%)
60	9.48e−02 [1.24e−04]	9.52e−02 [2.89e−04] - (4.18e−01%)	9.92e−02 [4.07e−04] - (4.64e+00%)	9.51e−02 [1.95e−04] - (2.38e−01%)
70	9.01e−02 [1.30e−04]	9.02e−02 [1.75e−04] - (1.94e−01%)	9.40e−02 [3.61e−04] - (4.35e+00%)	9.02e−02 [1.48e−04] - (1.29e−01%)
80	8.59e−02 [1.41e−04]	8.62e−02 [2.13e−04] - (3.64e−01%)	8.97e−02 [3.68e−04] - (4.48e+00%)	8.61e−02 [1.60e−04] - (2.15e−01%)
90	8.24e−02 [1.59e−04]	8.27e−02 [1.60e−04] - (3.60e−01%)	8.60e−02 [3.13e−04] - (4.39e+00%)	8.25e−02 [1.23e−04] - (2.04e−01%)
100	7.94e−02 [1.16e−04]	7.96e−02 [1.71e−04] - (3.66e−01%)	8.28e−02 [2.14e−04] - (4.34e+00%)	7.96e−02 [1.39e−04] - (2.53e−01%)
110	7.67e−02 [1.53e−04]	7.69e−02 [1.79e−04] - (3.19e−01%)	8.00e−02 [2.42e−04] - (4.37e+00%)	7.69e−02 [1.20e−04] - (2.81e−01%)
120	7.43e−02 [1.39e−04]	7.46e−02 [2.19e−04] - (3.87e−01%)	7.76e−02 [1.99e−04] - (4.43e+00%)	7.45e−02 [2.20e−04] - (3.56e−01%)
130	7.20e−02 [1.89e−04]	7.24e−02 [1.57e−04] - (5.64e−01%)	7.53e−02 [2.39e−04] - (4.54e+00%)	7.24e−02 [1.83e−04] - (4.89e−01%)
140	7.02e−02 [1.43e−04]	7.05e−02 [1.98e−04] - (4.59e−01%)	7.34e−02 [2.26e−04] - (4.54e+00%)	7.05e−02 [1.57e−04] - (4.90e−01%)
150	6.85e−02 [9.68e−05]	6.89e−02 [1.64e−04] - (4.86e−01%)	7.15e−02 [1.90e−04] - (4.28e+00%)	6.88e−02 [1.36e−04] - (3.98e−01%)

(c) IGD$^+$

k	LS	LS-N	LS-R	LS-RN
50	6.80e−02 [1.10e−04]	6.81e−02 [1.43e−04] - (1.28e−01%)	7.07e−02 [3.32e−04] - (3.91e+00%)	6.82e−02 [1.27e−04] - (2.17e−01%)
60	6.39e−02 [8.46e−05]	6.41e−02 [1.12e−04] - (2.50e−01%)	6.61e−02 [2.76e−04] - (3.45e+00%)	6.40e−02 [1.08e−04] - (1.59e−01%)
70	6.05e−02 [6.22e−05]	6.07e−02 [9.17e−05] - (2.38e−01%)	6.27e−02 [2.49e−04] - (3.62e+00%)	6.06e−02 [9.38e−05] - (1.27e−01%)
80	5.76e−02 [7.09e−05]	5.78e−02 [1.28e−04] - (3.01e−01%)	5.98e−02 [1.88e−04] - (3.83e+00%)	5.77e−02 [7.87e−05] - (1.98e−01%)
90	5.50e−02 [7.79e−05]	5.53e−02 [1.37e−04] - (4.19e−01%)	5.75e−02 [2.41e−04] - (4.40e+00%)	5.52e−02 [1.07e−04] - (2.51e−01%)
100	5.28e−02 [6.97e−05]	5.31e−02 [1.33e−04] - (5.51e−01%)	5.52e−02 [1.75e−04] - (4.55e+00%)	5.30e−02 [9.02e−05] - (3.40e−01%)
110	5.09e−02 [7.74e−05]	5.12e−02 [1.42e−04] - (5.37e−01%)	5.32e−02 [2.17e−04] - (4.55e+00%)	5.11e−02 [8.70e−05] - (3.41e−01%)
120	4.92e−02 [6.91e−05]	4.95e−02 [1.35e−04] - (5.76e−01%)	5.15e−02 [2.25e−04] - (4.72e+00%)	4.94e−02 [9.41e−05] - (3.76e−01%)
130	4.78e−02 [8.68e−05]	4.80e−02 [9.22e−05] - (5.83e−01%)	5.01e−02 [1.79e−04] - (4.88e+00%)	4.80e−02 [7.64e−05] - (5.15e−01%)
140	4.64e−02 [7.17e−05]	4.67e−02 [1.06e−04] - (5.99e−01%)	4.87e−02 [1.90e−04] - (4.90e+00%)	4.67e−02 [9.11e−05] - (6.99e−01%)
150	4.51e−02 [8.69e−05]	4.54e−02 [1.05e−04] - (7.24e−01%)	4.73e−02 [1.99e−04] - (4.83e+00%)	4.54e−02 [8.49e−05] - (7.15e−01%)

(d) R2

k	LS	LS-N	LS-R	LS-RN
50	2.25e−02 [1.84e−05]	2.26e−02 [6.41e−05] - (6.28e−01%)	2.40e−02 [5.71e−04] - (6.44e+00%)	2.27e−02 [6.82e−05] - (7.92e−01%)
60	2.21e−02 [2.09e−05]	2.23e−02 [4.10e−05] - (5.74e−01%)	2.32e−02 [6.12e−04] - (4.57e+00%)	2.23e−02 [6.52e−05] - (7.55e−01%)
70	2.19e−02 [1.26e−05]	2.20e−02 [6.05e−05] - (5.33e−01%)	2.27e−02 [5.36e−04] - (3.76e+00%)	2.20e−02 [5.50e−05] - (6.60e−01%)
80	2.16e−02 [1.18e−05]	2.18e−02 [3.56e−05] - (5.33e−01%)	2.23e−02 [4.26e−04] - (3.20e+00%)	2.18e−02 [3.68e−05] - (6.22e−01%)
90	2.15e−02 [1.16e−05]	2.16e−02 [3.07e−05] - (5.09e−01%)	2.22e−02 [4.34e−04] - (3.39e+00%)	2.16e−02 [3.57e−05] - (5.41e−01%)
100	2.13e−02 [1.27e−05]	2.14e−02 [3.13e−05] - (5.38e−01%)	2.19e−02 [3.85e−04] - (2.76e+00%)	2.14e−02 [2.62e−05] - (4.79e−01%)
110	2.12e−02 [1.09e−05]	2.13e−02 [2.75e−05] - (5.12e−01%)	2.17e−02 [3.94e−04] - (2.40e+00%)	2.12e−02 [3.01e−05] - (4.27e−01%)
120	2.10e−02 [1.19e−05]	2.12e−02 [3.26e−05] - (5.14e−01%)	2.15e−02 [3.92e−04] - (2.36e+00%)	2.11e−02 [1.91e−05] - (3.95e−01%)
130	2.09e−02 [7.93e−06]	2.10e−02 [3.46e−05] - (5.01e−01%)	2.14e−02 [3.37e−04] - (2.06e+00%)	2.10e−02 [2.69e−05] - (4.45e−01%)
140	2.08e−02 [8.14e−06]	2.09e−02 [3.10e−05] - (5.18e−01%)	2.12e−02 [2.07e−04] - (1.59e+00%)	2.09e−02 [2.24e−05] - (3.69e−01%)
150	2.07e−02 [7.59e−06]	2.09e−02 [2.83e−05] - (5.49e−01%)	2.11e−02 [2.09e−04] - (1.51e+00%)	2.08e−02 [2.26e−05] - (3.67e−01%)

(e) NR2

k	LS	LS-N	LS-R	LS-RN
50	2.89e+01 [1.05e−02]	2.87e+01 [5.82e−02] - (6.84e−01%)	2.85e+01 [5.40e−02] - (1.32e+00%)	2.88e+01 [3.06e−02] - (3.62e−01%)
60	2.90e+01 [4.93e−03]	2.88e+01 [4.97e−02] - (5.33e−01%)	2.87e+01 [4.41e−02] - (9.30e−01%)	2.89e+01 [2.19e−02] - (1.71e−01%)
70	2.90e+01 [7.22e−15]	2.89e+01 [2.73e−02] - (2.96e−01%)	2.88e+01 [3.17e−02] - (5.56e−01%)	2.90e+01 [1.50e−02] - (7.30e−02%)
80	2.90e+01 [7.22e−15]	2.89e+01 [2.49e−02] - (1.85e−01%)	2.89e+01 [2.31e−02] - (4.05e−01%)	2.90e+01 [5.14e−03] - (1.93e−02%)
90	2.90e+01 [7.22e−15]	2.90e+01 [1.76e−02] - (1.40e−01%)	2.89e+01 [2.38e−02] - (3.36e−01%)	2.90e+01 [3.06e−03] - (6.06e−03%)
100	2.90e+01 [7.22e−15]	2.90e+01 [1.78e−02] - (9.00e−02%)	2.89e+01 [2.13e−02] - (3.12e−01%)	2.90e+01 [5.36e−03] - (9.17e−03%)
110	2.90e+01 [7.22e−15]	2.90e+01 [1.40e−02] - (6.19e−02%)	2.89e+01 [2.07e−02] - (2.67e−01%)	2.90e+01 [1.81e−03] - (2.07e−03%)
120	2.90e+01 [7.22e−15]	2.90e+01 [9.14e−03] - (3.70e−02%)	2.89e+01 [1.63e−02] - (2.20e−01%)	2.90e+01 [2.37e−03] - (3.32e−03%)
130	2.90e+01 [7.22e−15]	2.90e+01 [6.57e−03] - (2.93e−02%)	2.89e+01 [1.79e−02] - (2.07e−01%)	2.90e+01 [2.41e−03] - (2.81e−03%)
140	2.90e+01 [7.22e−15]	2.90e+01 [6.16e−03] - (2.06e−02%)	2.89e+01 [1.57e−02] - (1.84e−01%)	2.90e+01 [1.43e−03] $\approx$ (1.20e−03%)
150	2.90e+01 [7.22e−15]	2.90e+01 [6.89e−03] - (1.78e−02%)	2.89e+01 [1.48e−02] - (1.73e−01%)	2.90e+01 [7.22e−15] $\approx$ (0.00e+00%)

(f) ϵ

k	LS	LS-N	LS-R	LS-RN
50	1.03e−01 [2.10e−03]	1.10e−01 [1.35e−02] - (7.37e+00%)	1.09e−01 [1.12e−02] - (5.72e+00%)	1.06e−01 [9.33e−03] - (3.42e+00%)
60	9.55e−02 [9.69e−03]	1.07e−01 [1.02e−02] - (1.17e+01%)	1.05e−01 [6.12e−03] - (9.55e+00%)	1.01e−01 [4.82e−03] - (5.96e+00%)
70	8.34e−02 [1.40e−02]	1.04e−01 [6.27e−03] - (2.46e+01%)	1.02e−01 [2.99e−03] - (2.21e+01%)	9.85e−02 [7.12e−03] - (1.80e+01%)
80	7.72e−02 [1.24e−02]	1.01e−01 [1.42e−02] - (3.10e+01%)	9.72e−02 [8.19e−03] - (2.59e+01%)	8.97e−02 [1.03e−02] - (1.62e+01%)
90	7.45e−02 [1.12e−02]	9.63e−02 [1.71e−02] - (2.93e+01%)	9.12e−02 [1.18e−02] - (2.25e+01%)	8.16e−02 [1.16e−02] - (9.55e+00%)
100	7.28e−02 [9.71e−03]	8.50e−02 [1.57e−02] - (1.68e+01%)	7.97e−02 [9.68e−03] - (9.57e+00%)	7.72e−02 [1.07e−02] - (6.14e+00%)
110	7.28e−02 [1.01e−02]	7.90e−02 [1.41e−02] - (8.48e+00%)	7.37e−02 [5.51e−03] - (1.17e+00%)	7.13e−02 [2.63e−03] + (−2.03e+00%)
120	7.04e−02 [6.05e−03]	7.64e−02 [1.32e−02] - (8.44e+00%)	7.02e−02 [1.01e−03] + (−3.10e−01%)	7.14e−02 [5.83e−03] - (1.45e+00%)
130	7.13e−02 [8.39e−03]	7.34e−02 [9.81e−03] - (2.88e+00%)	6.97e−02 [4.00e−04] + (−2.33e+00%)	6.94e−02 [2.77e−04] $\approx$ (−2.75e+00%)
140	6.91e−02 [3.92e−04]	7.36e−02 [9.75e−03] - (6.54e+00%)	6.96e−02 [8.13e−04] - (8.02e−01%)	6.93e−02 [2.72e−04] - (2.94e−01%)
150	6.90e−02 [5.11e−04]	7.51e−02 [1.23e−02] - (8.76e+00%)	7.05e−02 [5.66e−03] - (2.15e+00%)	6.93e−02 [7.30e−04] $\approx$ (4.18e−01%)

(g) s-energy

k	LS	LS-N	LS-R	LS-RN
50	2.11e+04 [5.85e+01]	2.12e+04 [1.38e+02] - (3.51e−01%)	2.72e+04 [6.58e+02] - (2.87e+01%)	2.12e+04 [1.28e+02] - (4.43e−01%)
60	3.42e+04 [1.53e+02]	3.45e+04 [2.80e+02] - (1.09e+00%)	4.39e+04 [1.47e+03] - (2.83e+01%)	3.46e+04 [2.37e+02] - (1.12e+00%)
70	5.22e+04 [2.35e+02]	5.26e+04 [4.21e+02] - (7.89e−01%)	6.48e+04 [1.30e+03] - (2.41e+01%)	5.24e+04 [2.81e+02] - (4.54e−01%)
80	7.39e+04 [3.50e+02]	7.42e+04 [4.51e+02] - (5.07e−01%)	8.98e+04 [2.12e+03] - (2.16e+01%)	7.44e+04 [4.47e+02] - (6.63e−01%)
90	1.01e+05 [3.69e+02]	1.01e+05 [4.46e+02] - (8.53e−01%)	1.21e+05 [2.44e+03] - (2.01e+01%)	1.01e+05 [3.28e+02] - (6.31e−01%)
100	1.33e+05 [2.96e+02]	1.34e+05 [5.56e+02] - (5.66e−01%)	1.58e+05 [3.33e+03] - (1.86e+01%)	1.34e+05 [5.11e+02] - (5.39e−01%)
110	1.70e+05 [4.23e+02]	1.70e+05 [6.95e+02] - (4.81e−01%)	2.00e+05 [3.20e+03] - (1.82e+01%)	1.71e+05 [6.32e+02] - (5.83e−01%)
120	2.11e+05 [9.01e+02]	2.13e+05 [1.15e+03] - (8.61e−01%)	2.48e+05 [3.23e+03] - (1.73e+01%)	2.13e+05 [1.26e+03] - (8.52e−01%)
130	2.61e+05 [1.11e+03]	2.62e+05 [1.22e+03] - (4.91e−01%)	3.02e+05 [3.27e+03] - (1.60e+01%)	2.62e+05 [1.06e+03] - (3.47e−01%)
140	3.15e+05 [9.93e+02]	3.17e+05 [1.31e+03] - (4.37e−01%)	3.61e+05 [3.98e+03] - (1.46e+01%)	3.17e+05 [1.38e+03] - (4.15e−01%)
150	3.76e+05 [1.23e+03]	3.77e+05 [1.28e+03] - (3.06e−01%)	4.27e+05 [4.42e+03] - (1.35e+01%)	3.78e+05 [1.43e+03] - (4.44e−01%)

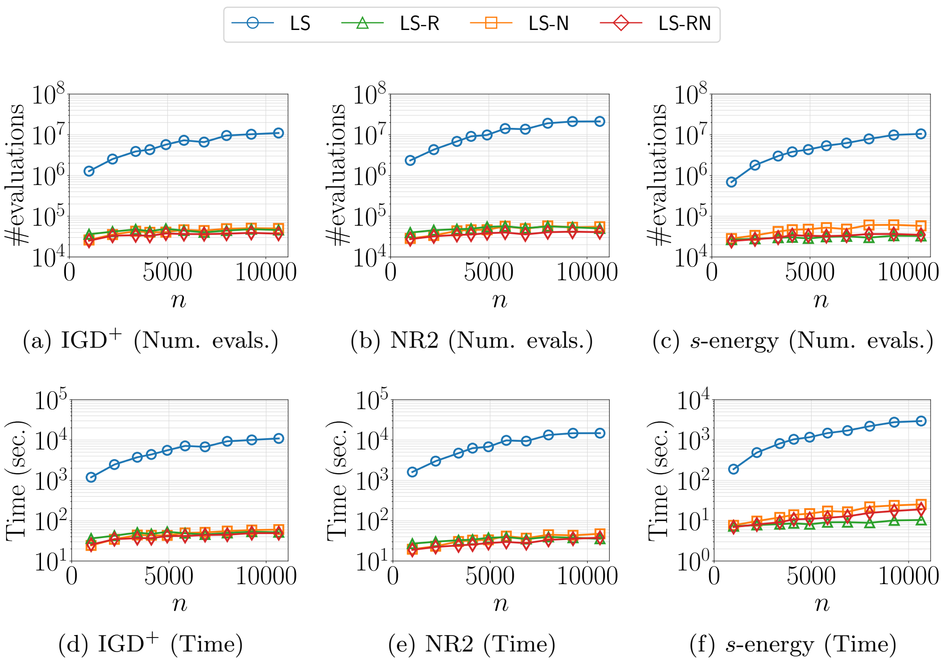


Fig. S.13: Results of the four LS methods on the ISSP with the discontinuous PF using IGD$^+$, NR2, and s-energy.

Table S.5: Comparison of the four LS methods on the ISSP instances with the discontinuous PF and HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy. The mean, [standard deviation], and (relative error based on LS) of the quality indicator values of the subsets found by each LS method are shown. The symbols + , - , and ≈ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to conventional LS.

(a) HV

n	LS	LS-N	LS-R	LS-RN
1K	5.90e−01 [3.99e−05]	5.83e−01 [2.78e−03] - (1.09e+00%)	5.88e−01 [5.03e−04] - (3.55e−01%)	5.89e−01 [1.55e−04] - (7.39e−02%)
2K	5.94e−01 [1.23e−04]	5.89e−01 [1.74e−03] - (8.84e−01%)	5.91e−01 [5.30e−04] - (5.25e−01%)	5.93e−01 [1.78e−04] - (8.98e−02%)
3K	5.94e−01 [9.26e−05]	5.89e−01 [2.25e−03] - (8.53e−01%)	5.91e−01 [5.11e−04] - (5.61e−01%)	5.94e−01 [1.70e−04] - (7.23e−02%)
4K	5.94e−01 [8.91e−05]	5.86e−01 [3.80e−03] - (1.29e+00%)	5.90e−01 [5.32e−04] - (6.72e−01%)	5.93e−01 [1.73e−04] - (7.77e−02%)
5K	5.95e−01 [9.49e−05]	5.89e−01 [1.52e−03] - (9.63e−01%)	5.91e−01 [5.50e−04] - (6.36e−01%)	5.94e−01 [1.65e−04] - (7.10e−02%)
6K	5.94e−01 [9.41e−05]	5.87e−01 [2.34e−03] - (1.20e+00%)	5.90e−01 [6.53e−04] - (7.16e−01%)	5.94e−01 [1.67e−04] - (8.93e−02%)
7K	5.95e−01 [8.05e−05]	5.89e−01 [1.99e−03] - (1.05e+00%)	5.91e−01 [5.62e−04] - (6.77e−01%)	5.94e−01 [1.65e−04] - (7.63e−02%)
8K	5.94e−01 [6.72e−05]	5.87e−01 [2.59e−03] - (1.27e+00%)	5.90e−01 [4.52e−04] - (7.46e−01%)	5.94e−01 [2.18e−04] - (8.06e−02%)
9K	5.95e−01 [9.44e−05]	5.89e−01 [3.10e−03] - (1.02e+00%)	5.91e−01 [4.61e−04] - (7.23e−01%)	5.94e−01 [1.56e−04] - (8.16e−02%)
10K	5.95e−01 [1.16e−04]	5.90e−01 [1.64e−03] - (7.39e−01%)	5.91e−01 [4.39e−04] - (7.13e−01%)	5.94e−01 [1.47e−04] - (6.88e−02%)

(b) IGD

n	LS	LS-N	LS-R	LS-RN
1K	7.58e−02 [8.19e−05]	7.70e−02 [8.03e−04] - (1.54e+00%)	7.74e−02 [3.07e−04] - (2.18e+00%)	7.60e−02 [1.38e−04] - (2.39e−01%)
2K	7.27e−02 [7.53e−05]	7.38e−02 [7.26e−04] - (1.43e+00%)	7.57e−02 [3.03e−04] - (4.13e+00%)	7.29e−02 [1.02e−04] - (2.07e−01%)
3K	7.24e−02 [4.94e−05]	7.38e−02 [1.05e−03] - (2.04e+00%)	7.56e−02 [3.23e−04] - (4.42e+00%)	7.25e−02 [1.17e−04] - (2.26e−01%)
4K	7.24e−02 [9.64e−05]	7.40e−02 [9.06e−04] - (2.24e+00%)	7.60e−02 [4.07e−04] - (4.94e+00%)	7.26e−02 [1.29e−04] - (3.19e−01%)
5K	7.18e−02 [9.34e−05]	7.30e−02 [7.18e−04] - (1.70e+00%)	7.55e−02 [3.67e−04] - (5.20e+00%)	7.19e−02 [1.50e−04] - (2.41e−01%)
6K	7.18e−02 [1.20e−04]	7.34e−02 [8.33e−04] - (2.30e+00%)	7.59e−02 [3.65e−04] - (5.79e+00%)	7.21e−02 [1.31e−04] - (3.86e−01%)
7K	7.12e−02 [1.00e−04]	7.26e−02 [7.93e−04] - (2.08e+00%)	7.52e−02 [3.37e−04] - (5.75e+00%)	7.14e−02 [1.31e−04] - (3.18e−01%)
8K	7.12e−02 [9.31e−05]	7.27e−02 [6.22e−04] - (2.07e+00%)	7.59e−02 [4.56e−04] - (6.62e+00%)	7.14e−02 [1.26e−04] - (3.41e−01%)
9K	7.12e−02 [8.05e−05]	7.27e−02 [7.66e−04] - (2.06e+00%)	7.50e−02 [2.84e−04] - (5.36e+00%)	7.14e−02 [9.89e−05] - (3.00e−01%)
10K	7.10e−02 [8.25e−05]	7.24e−02 [9.21e−04] - (2.00e+00%)	7.47e−02 [2.81e−04] - (5.22e+00%)	7.11e−02 [8.86e−05] - (1.34e−01%)

(c) IGD$^+$

n	LS	LS-N	LS-R	LS-RN
1K	3.77e−02 [2.24e−05]	3.85e−02 [3.88e−04] - (2.30e+00%)	3.85e−02 [1.79e−04] - (2.08e+00%)	3.79e−02 [5.78e−05] - (4.93e−01%)
2K	3.77e−02 [3.67e−05]	3.82e−02 [2.33e−04] - (1.42e+00%)	3.87e−02 [1.52e−04] - (2.74e+00%)	3.78e−02 [5.63e−05] - (3.44e−01%)
3K	3.71e−02 [5.73e−05]	3.78e−02 [3.97e−04] - (2.01e+00%)	3.84e−02 [1.31e−04] - (3.51e+00%)	3.72e−02 [8.70e−05] - (4.04e−01%)
4K	3.64e−02 [8.62e−05]	3.76e−02 [4.93e−04] - (3.04e+00%)	3.81e−02 [1.77e−04] - (4.46e+00%)	3.66e−02 [8.19e−05] - (5.45e−01%)
5K	3.70e−02 [5.66e−05]	3.76e−02 [2.58e−04] - (1.72e+00%)	3.84e−02 [1.43e−04] - (3.85e+00%)	3.71e−02 [6.59e−05] - (3.46e−01%)
6K	3.65e−02 [4.53e−05]	3.74e−02 [3.40e−04] - (2.41e+00%)	3.80e−02 [1.58e−04] - (4.09e+00%)	3.67e−02 [7.33e−05] - (4.66e−01%)
7K	3.69e−02 [4.12e−05]	3.76e−02 [3.25e−04] - (1.76e+00%)	3.84e−02 [1.57e−04] - (4.11e+00%)	3.71e−02 [6.37e−05] - (3.81e−01%)
8K	3.65e−02 [4.42e−05]	3.73e−02 [3.16e−04] - (2.21e+00%)	3.81e−02 [1.40e−04] - (4.34e+00%)	3.66e−02 [7.48e−05] - (3.68e−01%)
9K	3.68e−02 [4.52e−05]	3.74e−02 [3.38e−04] - (1.89e+00%)	3.85e−02 [1.10e−04] - (4.64e+00%)	3.69e−02 [7.46e−05] - (4.61e−01%)
10K	3.64e−02 [5.41e−05]	3.71e−02 [3.35e−04] - (1.87e+00%)	3.82e−02 [1.72e−04] - (4.87e+00%)	3.66e−02 [6.56e−05] - (4.63e−01%)

(d) R2

n	LS	LS-N	LS-R	LS-RN
1K	9.40e−02 [8.31e−06]	9.42e−02 [8.06e−05] - (2.29e−01%)	9.41e−02 [2.97e−05] - (1.67e−01%)	9.40e−02 [1.83e−05] - (5.19e−02%)
2K	9.33e−02 [1.54e−05]	9.37e−02 [9.50e−05] - (4.01e−01%)	9.36e−02 [1.34e−04] - (3.77e−01%)	9.34e−02 [3.13e−05] - (7.94e−02%)
3K	9.33e−02 [1.05e−05]	9.38e−02 [1.29e−04] - (5.25e−01%)	9.37e−02 [1.54e−04] - (4.78e−01%)	9.34e−02 [3.62e−05] - (9.78e−02%)
4K	9.34e−02 [7.62e−06]	9.39e−02 [1.27e−04] - (5.59e−01%)	9.39e−02 [2.81e−04] - (6.02e−01%)	9.35e−02 [2.77e−05] - (9.99e−02%)
5K	9.32e−02 [1.15e−05]	9.37e−02 [1.19e−04] - (5.59e−01%)	9.38e−02 [2.44e−04] - (5.86e−01%)	9.33e−02 [3.49e−05] - (1.22e−01%)
6K	9.33e−02 [9.07e−06]	9.39e−02 [1.12e−04] - (6.72e−01%)	9.39e−02 [2.44e−04] - (6.83e−01%)	9.34e−02 [3.18e−05] - (1.07e−01%)
7K	9.31e−02 [7.75e−06]	9.37e−02 [1.21e−04] - (6.64e−01%)	9.39e−02 [3.38e−04] - (8.07e−01%)	9.32e−02 [4.18e−05] - (1.14e−01%)
8K	9.33e−02 [9.84e−06]	9.39e−02 [1.03e−04] - (7.29e−01%)	9.40e−02 [2.82e−04] - (8.21e−01%)	9.34e−02 [3.83e−05] - (1.40e−01%)
9K	9.31e−02 [9.30e−06]	9.38e−02 [1.43e−04] - (7.30e−01%)	9.38e−02 [3.16e−04] - (7.78e−01%)	9.32e−02 [3.86e−05] - (1.23e−01%)
10K	9.31e−02 [1.48e−05]	9.39e−02 [1.53e−04] - (7.88e−01%)	9.39e−02 [2.85e−04] - (8.47e−01%)	9.33e−02 [4.01e−05] - (1.41e−01%)

(e) NR2

n	LS	LS-N	LS-R	LS-RN
1K	1.31e+01 [4.55e−03]	1.30e+01 [4.09e−02] - (8.17e−01%)	1.31e+01 [1.04e−02] - (3.86e−01%)	1.31e+01 [6.49e−03] - (1.53e−01%)
2K	1.33e+01 [3.11e−03]	1.32e+01 [3.47e−02] - (9.86e−01%)	1.32e+01 [1.95e−02] - (8.10e−01%)	1.33e+01 [1.04e−02] - (2.70e−01%)
3K	1.34e+01 [4.20e−03]	1.33e+01 [4.00e−02] - (9.35e−01%)	1.33e+01 [1.97e−02] - (1.02e+00%)	1.34e+01 [8.66e−03] - (2.50e−01%)
4K	1.34e+01 [4.68e−03]	1.32e+01 [6.23e−02] - (1.20e+00%)	1.32e+01 [2.40e−02] - (1.05e+00%)	1.33e+01 [1.19e−02] - (3.10e−01%)
5K	1.34e+01 [4.00e−03]	1.33e+01 [3.24e−02] - (9.95e−01%)	1.33e+01 [1.94e−02] - (1.04e+00%)	1.34e+01 [9.85e−03] - (2.93e−01%)
6K	1.34e+01 [3.80e−03]	1.32e+01 [5.07e−02] - (1.47e+00%)	1.32e+01 [2.11e−02] - (1.37e+00%)	1.34e+01 [1.45e−02] - (3.90e−01%)
7K	1.35e+01 [4.48e−03]	1.33e+01 [3.24e−02] - (1.14e+00%)	1.33e+01 [2.41e−02] - (1.27e+00%)	1.34e+01 [1.44e−02] - (3.39e−01%)
8K	1.34e+01 [4.01e−03]	1.32e+01 [4.05e−02] - (1.36e+00%)	1.32e+01 [2.37e−02] - (1.40e+00%)	1.34e+01 [1.25e−02] - (3.61e−01%)
9K	1.35e+01 [4.13e−03]	1.33e+01 [4.59e−02] - (1.39e+00%)	1.33e+01 [2.84e−02] - (1.56e+00%)	1.34e+01 [1.50e−02] - (4.14e−01%)
10K	1.35e+01 [4.34e−03]	1.33e+01 [4.29e−02] - (1.21e+00%)	1.33e+01 [2.00e−02] - (1.47e+00%)	1.34e+01 [1.18e−02] - (3.45e−01%)

(f) ϵ

n	LS	LS-N	LS-R	LS-RN
1K	1.19e−01 [1.55e−02]	1.22e−01 [1.07e−02] ≈ (2.52e+00%)	1.21e−01 [1.31e−02] ≈ (1.33e+00%)	1.21e−01 [1.25e−02] ≈ (1.48e+00%)
2K	1.11e−01 [2.30e−02]	1.14e−01 [2.34e−02] ≈ (2.77e+00%)	1.14e−01 [2.35e−02] ≈ (2.70e+00%)	1.14e−01 [2.35e−02] ≈ (2.70e+00%)
3K	1.14e−01 [1.70e−02]	1.21e−01 [1.56e−02] ≈ (5.51e+00%)	1.16e−01 [1.88e−02] ≈ (1.12e+00%)	1.16e−01 [1.88e−02] ≈ (1.12e+00%)
4K	1.16e−01 [1.27e−02]	1.15e−01 [1.12e−02] ≈ (−9.55e−01%)	1.16e−01 [1.27e−02] ≈ (0.00e+00%)	1.16e−01 [1.27e−02] ≈ (0.00e+00%)
5K	1.16e−01 [1.66e−02]	1.18e−01 [1.73e−02] ≈ (1.94e+00%)	1.14e−01 [1.55e−02] ≈ (−1.94e+00%)	1.14e−01 [1.55e−02] ≈ (−1.94e+00%)
6K	1.22e−01 [1.93e−02]	1.26e−01 [1.91e−02] ≈ (3.48e+00%)	1.21e−01 [1.97e−02] ≈ (−8.70e−01%)	1.21e−01 [1.97e−02] ≈ (−8.70e−01%)
7K	1.17e−01 [1.54e−02]	1.21e−01 [1.67e−02] ≈ (3.42e+00%)	1.16e−01 [1.74e−02] ≈ (−4.32e−01%)	1.16e−01 [1.78e−02] ≈ (−8.55e−01%)
8K	1.15e−01 [1.58e−02]	1.19e−01 [1.37e−02] ≈ (3.49e+00%)	1.11e−01 [1.46e−02] ≈ (−3.31e+00%)	1.11e−01 [1.46e−02] ≈ (−3.31e+00%)
9K	1.23e−01 [1.57e−02]	1.27e−01 [1.41e−02] ≈ (2.92e+00%)	1.20e−01 [1.72e−02] ≈ (−2.60e+00%)	1.21e−01 [1.54e−02] ≈ (−1.46e+00%)
10K	1.18e−01 [1.51e−02]	1.24e−01 [1.59e−02] ≈ (5.07e+00%)	1.20e−01 [1.51e−02] ≈ (1.45e+00%)	1.20e−01 [1.35e−02] ≈ (1.45e+00%)

(g) s-energy

n	LS	LS-N	LS-R	LS-RN
1K	7.22e+04 [3.19e+02]	8.57e+04 [6.80e+03] - (1.87e+01%)	8.56e+04 [1.85e+03] - (1.85e+01%)	7.34e+04 [6.04e+02] - (1.62e+00%)
2K	7.21e+04 [3.87e+02]	8.30e+04 [3.91e+03] - (1.51e+01%)	8.75e+04 [1.59e+03] - (2.13e+01%)	7.35e+04 [5.38e+02] - (1.99e+00%)
3K	6.92e+04 [3.17e+02]	8.43e+04 [8.33e+03] - (2.19e+01%)	8.66e+04 [2.35e+03] - (2.52e+01%)	7.04e+04 [5.58e+02] - (1.72e+00%)
4K	6.24e+04 [2.09e+02]	7.64e+04 [8.20e+03] - (2.24e+01%)	8.27e+04 [2.18e+03] - (3.26e+01%)	6.37e+04 [6.97e+02] - (2.09e+00%)
5K	6.66e+04 [2.43e+02]	7.87e+04 [6.46e+03] - (1.82e+01%)	8.60e+04 [2.38e+03] - (2.92e+01%)	6.80e+04 [5.48e+02] - (2.13e+00%)
6K	6.27e+04 [1.62e+02]	7.60e+04 [1.22e+04] - (2.11e+01%)	8.37e+04 [2.19e+03] - (3.34e+01%)	6.39e+04 [6.43e+02] - (1.92e+00%)
7K	6.53e+04 [2.30e+02]	7.70e+04 [4.72e+03] - (1.78e+01%)	8.55e+04 [1.79e+03] - (3.08e+01%)	6.65e+04 [4.39e+02] - (1.79e+00%)
8K	6.04e+04 [1.85e+02]	7.19e+04 [4.51e+03] - (1.91e+01%)	8.30e+04 [2.11e+03] - (3.74e+01%)	6.19e+04 [5.69e+02] - (2.51e+00%)
9K	6.36e+04 [2.13e+02]	7.62e+04 [6.15e+03] - (1.98e+01%)	8.55e+04 [1.81e+03] - (3.44e+01%)	6.49e+04 [5.24e+02] - (2.11e+00%)
10K	6.57e+04 [2.83e+02]	8.04e+04 [6.32e+03] - (2.23e+01%)	8.76e+04 [1.82e+03] - (3.33e+01%)	6.73e+04 [6.53e+02] - (2.45e+00%)

Table S.6: Comparison of LS-N, LS-R, and LS-RN on the ISSP instances with the discontinuous PF and HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy. The symbols + , - , and $\approx$ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to LS-RN.

(a) HV

n	LS-N	LS-R
1K	-	-
2K	-	-
3K	-	-
4K	-	-
5K	-	-
6K	-	-
7K	-	-
8K	-	-
9K	-	-
10K	-	-

(b) IGD

n	LS-N	LS-R
1K	-	-
2K	-	-
3K	-	-
4K	-	-
5K	-	-
6K	-	-
7K	-	-
8K	-	-
9K	-	-
10K	-	-

(c) IGD$^+$

n	LS-N	LS-R
1K	-	-
2K	-	-
3K	-	-
4K	-	-
5K	-	-
6K	-	-
7K	-	-
8K	-	-
9K	-	-
10K	-	-

(d) R2

n	LS-N	LS-R
1K	-	-
2K	-	-
3K	-	-
4K	-	-
5K	-	-
6K	-	-
7K	-	-
8K	-	-
9K	-	-
10K	-	-

(e) NR2

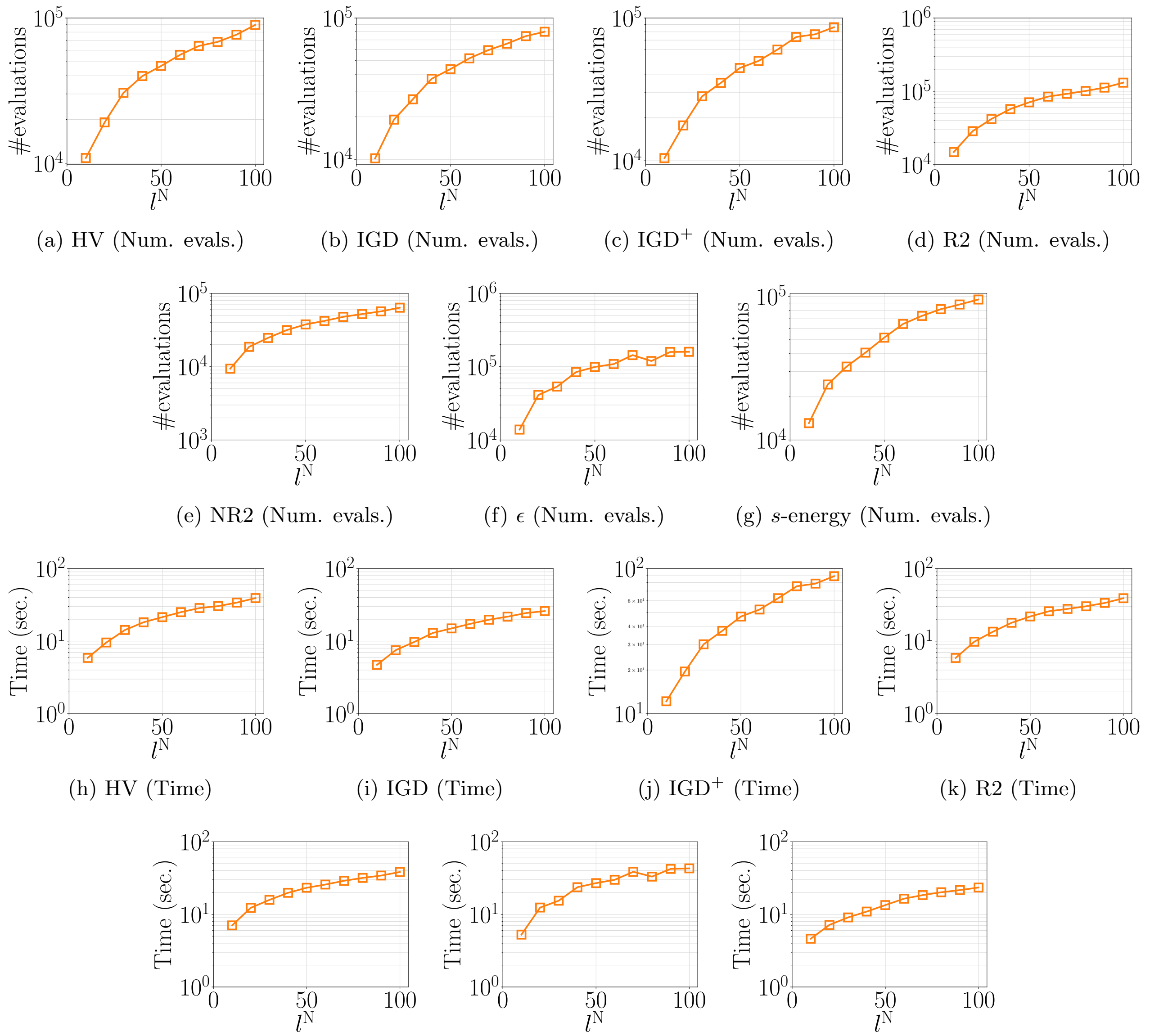
n	LS-N	LS-R
1K	-	-
2K	-	-
3K	-	-
4K	-	-
5K	-	-
6K	-	-
7K	-	-
8K	-	-
9K	-	-
10K	-	-

(f) ϵ

n	LS-N	LS-R
1K	$\approx$	$\approx$
2K	$\approx$	$\approx$
3K	$\approx$	$\approx$
4K	$\approx$	$\approx$
5K	$\approx$	$\approx$
6K	$\approx$	$\approx$
7K	$\approx$	$\approx$
8K	-	$\approx$
9K	$\approx$	$\approx$
10K	$\approx$	$\approx$

(g) s–energy

n	LS-N	LS-R
1K	-	-
2K	-	-
3K	-	-
4K	-	-
5K	-	-
6K	-	-
7K	-	-
8K	-	-
9K	-	-
10K	-	-



(a) HV (Num. evals.) (b) IGD (Num. evals.) (c) IGD$^+$ (Num. evals.) (d) R2 (Num. evals.)

(e) NR2 (Num. evals.) (f) ϵ (Num. evals.) (g) s-energy (Num. evals.)

(h) HV (Time) (i) IGD (Time) (j) IGD$^+$ (Time) (k) R2 (Time)

(l) NR2 (Time) (m) ϵ (Time) (n) s-energy (Time)

Fig. S.14: Results of LS-N with $l^{\mathrm{N}} \in \{10, 20, \ldots, 100\}$ on the ISSP with linear PF, $d = 4$, $n = 5\,000$, and $k = 100$ using HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy.

Table S.7: Comparison of LS and LS-N with $l^{\mathrm{N}} \in \{10, 20, \dots, 100\}$ on the ISSP with the linear PF, $d = 4$, $n = 5\,000$, and $k = 100$ using HV, IGD, IGD^{+}, R2, NR2, ϵ, and s-energy. The mean, [standard deviation], and (relative error based on LS) of the quality indicator values of the subsets found by each LS method are shown. The symbols +, -, and $\approx$ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to conventional LS.

(a) HV

LS	LS-N ($l^{\mathrm{N}} = 10$)	LS-N ($l^{\mathrm{N}} = 20$)	LS-N ($l^{\mathrm{N}} = 30$)	LS-N ($l^{\mathrm{N}} = 40$)	LS-N ($l^{\mathrm{N}} = 50$)	LS-N ($l^{\mathrm{N}} = 60$)	LS-N ($l^{\mathrm{N}} = 70$)	LS-N ($l^{\mathrm{N}} = 80$)	LS-N ($l^{\mathrm{N}} = 90$)	LS-N ($l^{\mathrm{N}} = 100$)
1.37e+00 [6.64e−05]	1.37e+00 [8.41e−05]	1.37e+00 [8.72e−05]	1.37e+00 [7.93e−05]	1.37e+00 [7.13e−05]	1.37e+00 [6.65e−05]	1.37e+00 [6.66e−05]	1.37e+00 [8.33e−05]	1.37e+00 [6.92e−05]	1.37e+00 [7.29e−05]	1.37e+00 [7.58e−05]
	- (1.09e−02%)	- (7.30e−03%)	- (6.06e−03%)	- (7.72e−03%)	- (7.78e−03%)	- (6.28e−03%)	- (6.07e−03%)	- (6.14e−03%)	- (5.83e−03%)	- (5.49e−03%)

(b) IGD

LS	LS-N ($l^{\mathrm{N}} = 10$)	LS-N ($l^{\mathrm{N}} = 20$)	LS-N ($l^{\mathrm{N}} = 30$)	LS-N ($l^{\mathrm{N}} = 40$)	LS-N ($l^{\mathrm{N}} = 50$)	LS-N ($l^{\mathrm{N}} = 60$)	LS-N ($l^{\mathrm{N}} = 70$)	LS-N ($l^{\mathrm{N}} = 80$)	LS-N ($l^{\mathrm{N}} = 90$)	LS-N ($l^{\mathrm{N}} = 100$)
7.94e−02 [1.16e−04]	7.99e−02 [2.15e−04]	7.97e−02 [1.53e−04]	7.97e−02 [1.60e−04]	7.96e−02 [1.71e−04]	7.96e−02 [2.02e−04]	7.96e−02 [1.90e−04]	7.96e−02 [1.72e−04]	7.96e−02 [1.69e−04]	7.96e−02 [1.79e−04]	7.96e−02 [1.79e−04]
	- (6.99e−01%)	- (3.82e−01%)	- (3.83e−01%)	- (3.66e−01%)	- (3.56e−01%)	- (3.26e−01%)	- (3.09e−01%)	- (3.04e−01%)	- (3.16e−01%)	- (3.36e−01%)

(c) IGD^{+}

LS	LS-N ($l^{\mathrm{N}} = 10$)	LS-N ($l^{\mathrm{N}} = 20$)	LS-N ($l^{\mathrm{N}} = 30$)	LS-N ($l^{\mathrm{N}} = 40$)	LS-N ($l^{\mathrm{N}} = 50$)	LS-N ($l^{\mathrm{N}} = 60$)	LS-N ($l^{\mathrm{N}} = 70$)	LS-N ($l^{\mathrm{N}} = 80$)	LS-N ($l^{\mathrm{N}} = 90$)	LS-N ($l^{\mathrm{N}} = 100$)
5.28e−02 [6.97e−05]	5.33e−02 [1.49e−04]	5.31e−02 [1.49e−04]	5.31e−02 [1.38e−04]	5.31e−02 [1.33e−04]	5.30e−02 [1.23e−04]	5.30e−02 [1.26e−04]	5.30e−02 [1.23e−04]	5.30e−02 [1.08e−04]	5.30e−02 [8.88e−05]	5.30e−02 [1.10e−04]
	- (9.96e−01%)	- (6.71e−01%)	- (5.63e−01%)	- (5.51e−01%)	- (4.90e−01%)	- (4.79e−01%)	- (4.83e−01%)	- (4.29e−01%)	- (4.61e−01%)	- (4.53e−01%)

(d) R2

LS	LS-N ($l^{\mathrm{N}} = 10$)	LS-N ($l^{\mathrm{N}} = 20$)	LS-N ($l^{\mathrm{N}} = 30$)	LS-N ($l^{\mathrm{N}} = 40$)	LS-N ($l^{\mathrm{N}} = 50$)	LS-N ($l^{\mathrm{N}} = 60$)	LS-N ($l^{\mathrm{N}} = 70$)	LS-N ($l^{\mathrm{N}} = 80$)	LS-N ($l^{\mathrm{N}} = 90$)	LS-N ($l^{\mathrm{N}} = 100$)
2.13e−02 [1.27e−05]	2.15e−02 [4.68e−05]	2.15e−02 [3.98e−05]	2.14e−02 [3.27e−05]	2.14e−02 [3.13e−05]	2.14e−02 [3.11e−05]	2.14e−02 [2.83e−05]	2.14e−02 [2.74e−05]	2.14e−02 [2.39e−05]	2.14e−02 [2.03e−05]	2.13e−02 [1.64e−05]
	- (1.20e+00%)	- (8.62e−01%)	- (6.42e−01%)	- (5.38e−01%)	- (4.68e−01%)	- (4.10e−01%)	- (4.08e−01%)	- (3.32e−01%)	- (2.74e−01%)	- (2.47e−01%)

(e) NR2

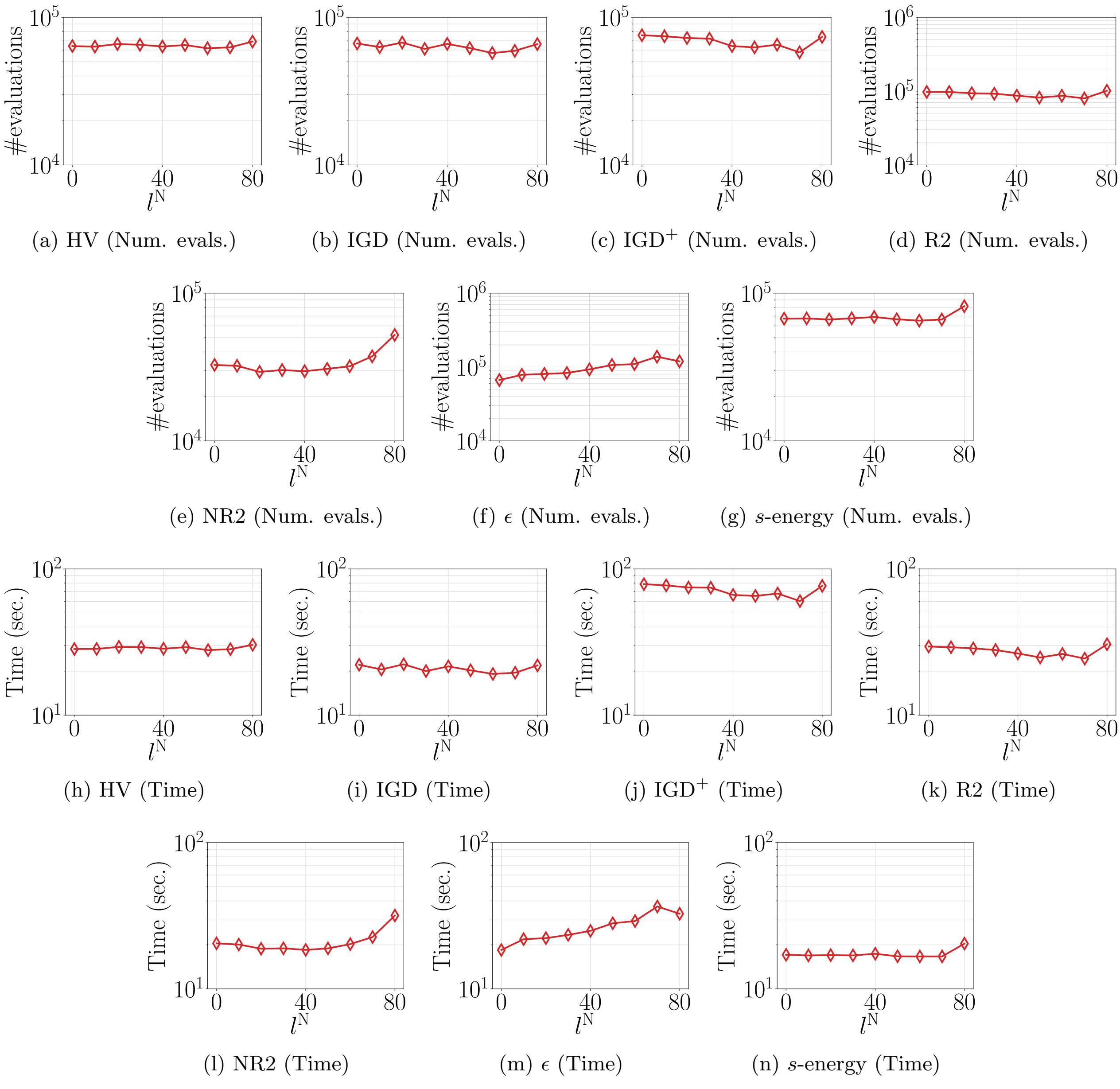
LS	LS-N ($l^{\mathrm{N}} = 10$)	LS-N ($l^{\mathrm{N}} = 20$)	LS-N ($l^{\mathrm{N}} = 30$)	LS-N ($l^{\mathrm{N}} = 40$)	LS-N ($l^{\mathrm{N}} = 50$)	LS-N ($l^{\mathrm{N}} = 60$)	LS-N ($l^{\mathrm{N}} = 70$)	LS-N ($l^{\mathrm{N}} = 80$)	LS-N ($l^{\mathrm{N}} = 90$)	LS-N ($l^{\mathrm{N}} = 100$)
2.90e+01 [7.22e−15]	2.89e+01 [3.54e−02]	2.90e+01 [1.87e−02]	2.90e+01 [1.68e−02]	2.90e+01 [1.78e−02]	2.90e+01 [1.62e−02]	2.90e+01 [1.53e−02]	2.90e+01 [1.39e−02]	2.90e+01 [1.28e−02]	2.90e+01 [1.08e−02]	2.90e+01 [9.16e−03]
	- (3.56e−01%)	- (1.51e−01%)	- (1.04e−01%)	- (9.00e−02%)	- (8.97e−02%)	- (6.31e−02%)	- (5.73e−02%)	- (5.43e−02%)	- (4.10e−02%)	- (3.47e−02%)

(f) ϵ

LS	LS-N ($l^{\mathrm{N}} = 10$)	LS-N ($l^{\mathrm{N}} = 20$)	LS-N ($l^{\mathrm{N}} = 30$)	LS-N ($l^{\mathrm{N}} = 40$)	LS-N ($l^{\mathrm{N}} = 50$)	LS-N ($l^{\mathrm{N}} = 60$)	LS-N ($l^{\mathrm{N}} = 70$)	LS-N ($l^{\mathrm{N}} = 80$)	LS-N ($l^{\mathrm{N}} = 90$)	LS-N ($l^{\mathrm{N}} = 100$)
7.28e−02 [9.71e−03]	9.89e−02 [9.32e−03]	9.00e−02 [1.40e−02]	8.88e−02 [1.43e−02]	8.50e−02 [1.57e−02]	8.27e−02 [1.54e−02]	8.47e−02 [1.61e−02]	7.95e−02 [1.42e−02]	8.78e−02 [1.59e−02]	8.24e−02 [1.54e−02]	8.34e−02 [1.59e−02]
	- (3.59e+01%)	- (2.37e+01%)	- (2.20e+01%)	- (1.68e+01%)	- (1.36e+01%)	- (1.64e+01%)	- (9.30e+00%)	- (2.06e+01%)	- (1.32e+01%)	- (1.47e+01%)

(g) s-energy

LS	LS-N ($l^{\mathrm{N}} = 10$)	LS-N ($l^{\mathrm{N}} = 20$)	LS-N ($l^{\mathrm{N}} = 30$)	LS-N ($l^{\mathrm{N}} = 40$)	LS-N ($l^{\mathrm{N}} = 50$)	LS-N ($l^{\mathrm{N}} = 60$)	LS-N ($l^{\mathrm{N}} = 70$)	LS-N ($l^{\mathrm{N}} = 80$)	LS-N ($l^{\mathrm{N}} = 90$)	LS-N ($l^{\mathrm{N}} = 100$)
1.33e+05 [2.96e+02]	1.34e+05 [4.68e+02]	1.34e+05 [4.70e+02]	1.34e+05 [5.43e+02]	1.34e+05 [5.56e+02]	1.34e+05 [4.92e+02]	1.34e+05 [5.35e+02]	1.34e+05 [5.94e+02]	1.34e+05 [4.64e+02]	1.33e+05 [3.59e+02]	1.34e+05 [5.45e+02]
	- (9.39e−01%)	- (6.51e−01%)	- (6.27e−01%)	- (5.66e−01%)	- (5.38e−01%)	- (4.35e−01%)	- (5.13e−01%)	- (4.18e−01%)	- (3.82e−01%)	- (4.64e−01%)



(a) HV (Num. evals.) (b) IGD (Num. evals.) (c) IGD$^+$ (Num. evals.) (d) R2 (Num. evals.)

(e) NR2 (Num. evals.) (f) ϵ (Num. evals.) (g) s-energy (Num. evals.)

(h) HV (Time) (i) IGD (Time) (j) IGD$^+$ (Time) (k) R2 (Time)

(l) NR2 (Time) (m) ϵ (Time) (n) s-energy (Time)

Fig. S.15: Results of LS-RN with $l^{\mathrm{N}} \in \{0, 10, \dots, 80\}$ on the ISSP with the linear PF, $d = 4$, $n = 5\,000$, $k = 100$, using HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy, where $l^{\mathrm{R}} = 80 - l^{\mathrm{N}}$ in LS-RN.

Table S.8: Comparison of LS and LS-RN with $l^{\mathrm{N}} \in \{0, 10, \ldots, 80\}$ on the ISSP with the linear PF, $d = 4$, $n = 5\,000$, $k = 100$, using HV, IGD, IGD$^+$, R2, NR2, ϵ, and s-energy, where $l^{\mathrm{R}} = 80 - l^{\mathrm{N}}$ in LS-RN. The mean, [standard deviation], and (relative error based on LS) of the quality indicator values of the subsets found by each LS method are shown. The symbols + , - , and ≈ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to conventional LS.

(a) HV

LS	LS-RN ($l^{\mathrm{N}} = 0$)	LS-RN ($l^{\mathrm{N}} = 10$)	LS-RN ($l^{\mathrm{N}} = 20$)	LS-RN ($l^{\mathrm{N}} = 30$)	LS-RN ($l^{\mathrm{N}} = 40$)	LS-RN ($l^{\mathrm{N}} = 50$)	LS-RN ($l^{\mathrm{N}} = 60$)	LS-RN ($l^{\mathrm{N}} = 70$)	LS-RN ($l^{\mathrm{N}} = 80$)
1.37e+00 [6.64e−05]	1.37e+00 [1.62e−04] - (1.14e−01%)	1.37e+00 [9.19e−05] - (9.63e−03%)	1.37e+00 [8.11e−05] - (7.61e−03%)	1.37e+00 [7.43e−05] - (6.63e−03%)	1.37e+00 [6.43e−05] - (6.30e−03%)	1.37e+00 [6.65e−05] - (7.15e−03%)	1.37e+00 [9.58e−05] - (5.24e−03%)	1.37e+00 [7.88e−05] - (6.02e−03%)	1.37e+00 [6.92e−05] - (6.14e−03%)

(b) IGD

LS	LS-RN ($l^{\mathrm{N}} = 0$)	LS-RN ($l^{\mathrm{N}} = 10$)	LS-RN ($l^{\mathrm{N}} = 20$)	LS-RN ($l^{\mathrm{N}} = 30$)	LS-RN ($l^{\mathrm{N}} = 40$)	LS-RN ($l^{\mathrm{N}} = 50$)	LS-RN ($l^{\mathrm{N}} = 60$)	LS-RN ($l^{\mathrm{N}} = 70$)	LS-RN ($l^{\mathrm{N}} = 80$)
7.94e−02 [1.16e−04]	8.24e−02 [2.68e−04] - (3.82e+00%)	7.97e−02 [1.34e−04] - (4.34e−01%)	7.96e−02 [1.55e−04] - (2.64e−01%)	7.95e−02 [1.37e−04] - (2.06e−01%)	7.95e−02 [1.55e−04] - (2.08e−01%)	7.95e−02 [1.49e−04] - (2.32e−01%)	7.95e−02 [1.50e−04] - (2.23e−01%)	7.95e−02 [1.13e−04] - (1.93e−01%)	7.96e−02 [1.69e−04] - (3.04e−01%)

(c) IGD$^+$

LS	LS-RN ($l^{\mathrm{N}} = 0$)	LS-RN ($l^{\mathrm{N}} = 10$)	LS-RN ($l^{\mathrm{N}} = 20$)	LS-RN ($l^{\mathrm{N}} = 30$)	LS-RN ($l^{\mathrm{N}} = 40$)	LS-RN ($l^{\mathrm{N}} = 50$)	LS-RN ($l^{\mathrm{N}} = 60$)	LS-RN ($l^{\mathrm{N}} = 70$)	LS-RN ($l^{\mathrm{N}} = 80$)
5.28e−02 [6.97e−05]	5.48e−02 [2.07e−04] - (3.92e+00%)	5.31e−02 [1.08e−04] - (5.16e−01%)	5.30e−02 [9.26e−05] - (4.08e−01%)	5.29e−02 [9.71e−05] - (3.19e−01%)	5.29e−02 [1.03e−04] - (2.97e−01%)	5.29e−02 [1.00e−04] - (2.54e−01%)	5.29e−02 [8.47e−05] - (2.17e−01%)	5.29e−02 [9.66e−05] - (3.11e−01%)	5.30e−02 [1.08e−04] - (4.29e−01%)

(d) R2

LS	LS-RN ($l^{\mathrm{N}} = 0$)	LS-RN ($l^{\mathrm{N}} = 10$)	LS-RN ($l^{\mathrm{N}} = 20$)	LS-RN ($l^{\mathrm{N}} = 30$)	LS-RN ($l^{\mathrm{N}} = 40$)	LS-RN ($l^{\mathrm{N}} = 50$)	LS-RN ($l^{\mathrm{N}} = 60$)	LS-RN ($l^{\mathrm{N}} = 70$)	LS-RN ($l^{\mathrm{N}} = 80$)
2.13e−02 [1.27e−05]	2.15e−02 [1.00e−04] - (1.19e+00%)	2.14e−02 [1.80e−05] - (4.34e−01%)	2.14e−02 [2.19e−05] - (3.50e−01%)	2.14e−02 [2.13e−05] - (3.04e−01%)	2.14e−02 [2.49e−05] - (2.79e−01%)	2.14e−02 [2.01e−05] - (3.00e−01%)	2.14e−02 [2.87e−05] - (3.12e−01%)	2.14e−02 [2.89e−05] - (4.20e−01%)	2.14e−02 [2.39e−05] - (3.32e−01%)

(e) NR2

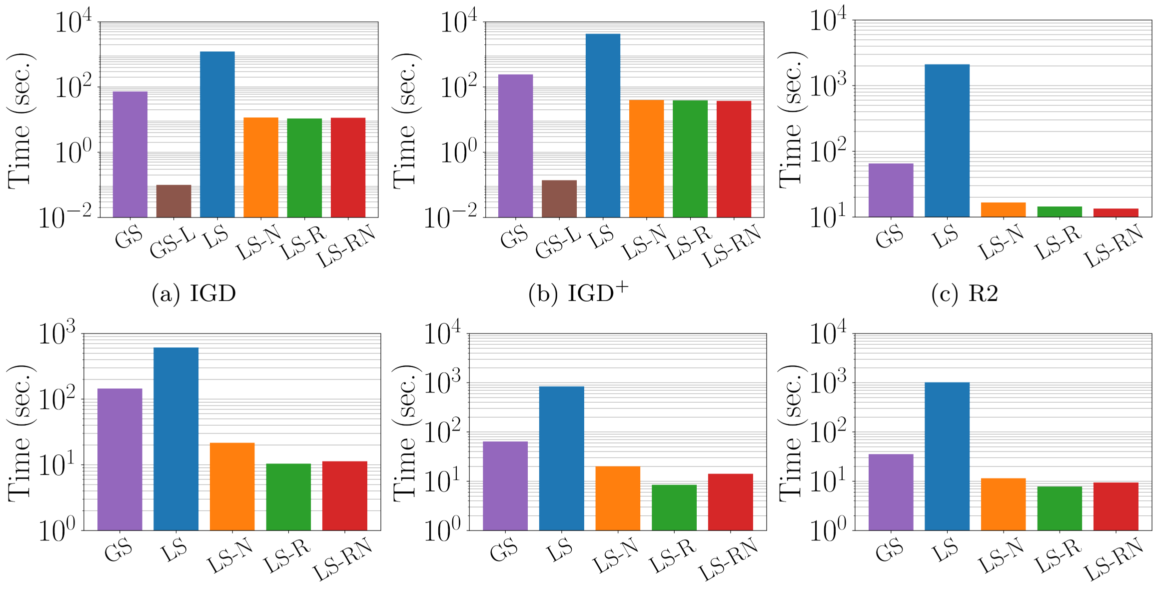
LS	LS-RN ($l^{\mathrm{N}} = 0$)	LS-RN ($l^{\mathrm{N}} = 10$)	LS-RN ($l^{\mathrm{N}} = 20$)	LS-RN ($l^{\mathrm{N}} = 30$)	LS-RN ($l^{\mathrm{N}} = 40$)	LS-RN ($l^{\mathrm{N}} = 50$)	LS-RN ($l^{\mathrm{N}} = 60$)	LS-RN ($l^{\mathrm{N}} = 70$)	LS-RN ($l^{\mathrm{N}} = 80$)
2.90e+01 [7.22e−15]	2.90e+01 [1.82e−02] - (1.25e−01%)	2.90e+01 [5.69e−03] - (1.03e−02%)	2.90e+01 [7.50e−04] ≈ (6.68e−04%)	2.90e+01 [2.77e−03] ≈ (2.35e−03%)	2.90e+01 [3.36e−03] - (3.13e−03%)	2.90e+01 [4.41e−03] - (5.87e−03%)	2.90e+01 [4.27e−03] - (7.04e−03%)	2.90e+01 [7.07e−03] - (1.09e−02%)	2.90e+01 [1.28e−02] - (5.43e−02%)

(f) ϵ

LS	LS-RN ($l^{\mathrm{N}} = 0$)	LS-RN ($l^{\mathrm{N}} = 10$)	LS-RN ($l^{\mathrm{N}} = 20$)	LS-RN ($l^{\mathrm{N}} = 30$)	LS-RN ($l^{\mathrm{N}} = 40$)	LS-RN ($l^{\mathrm{N}} = 50$)	LS-RN ($l^{\mathrm{N}} = 60$)	LS-RN ($l^{\mathrm{N}} = 70$)	LS-RN ($l^{\mathrm{N}} = 80$)
7.28e−02 [9.71e−03]	7.76e−02 [8.46e−03] - (6.61e+00%)	7.25e−02 [5.93e−03] + (−4.00e−01%)	7.13e−02 [1.65e−03] + (−1.99e+00%)	7.24e−02 [5.88e−03] + (−5.34e−01%)	7.07e−02 [1.79e−03] ≈ (−2.82e+00%)	7.10e−02 [5.96e−03] ≈ (−2.39e+00%)	7.26e−02 [8.15e−03] + (−1.98e−01%)	7.36e−02 [9.94e−03] - (1.20e+00%)	8.78e−02 [1.59e−02] - (2.06e+01%)

(g) s-energy

LS	LS-RN ($l^{\mathrm{N}} = 0$)	LS-RN ($l^{\mathrm{N}} = 10$)	LS-RN ($l^{\mathrm{N}} = 20$)	LS-RN ($l^{\mathrm{N}} = 30$)	LS-RN ($l^{\mathrm{N}} = 40$)	LS-RN ($l^{\mathrm{N}} = 50$)	LS-RN ($l^{\mathrm{N}} = 60$)	LS-RN ($l^{\mathrm{N}} = 70$)	LS-RN ($l^{\mathrm{N}} = 80$)
1.33e+05 [2.96e+02]	1.53e+05 [2.63e+03] - (1.49e+01%)	1.34e+05 [9.35e+02] - (6.74e−01%)	1.34e+05 [5.13e+02] - (4.42e−01%)	1.33e+05 [4.34e+02] - (2.68e−01%)	1.34e+05 [4.46e+02] - (4.32e−01%)	1.33e+05 [4.19e+02] - (3.47e−01%)	1.33e+05 [4.25e+02] - (2.98e−01%)	1.33e+05 [3.60e+02] - (2.93e−01%)	1.34e+05 [4.64e+02] - (4.18e−01%)



(a) IGD (b) IGD$^+$ (c) R2 (d) NR2 (e) ϵ (f) s-energy

Fig. S.16: Wall-clock time of the run of the four LS methods and the GS methods on the ISSP instances with the linear PF using IGD, IGD$^+$, R2, NR2, ϵ, and s-energy.

Table S.9: Comparison of the four LS methods and the GS methods on the ISSP instances with the linear PF using IGD, IGD$^+$, R2, NR2, ϵ, and s–energy. The mean, standard deviation, and relative error based on GS of the quality indicator values of the subsets found by each method are shown.

(a) IGD

	mean	std.	rel. error
GS	8.135e−02	NA	
GS-L	8.135e−02	NA	
LS	7.936e−02	1.267e−04	−2.445e+00%
LS-N	7.966e−02	2.012e−04	−2.072e+00%
LS-R	8.287e−02	2.903e−04	1.872e+00%
LS-RN	7.955e−02	1.316e−04	−2.211e+00%

(b) IGD$^+$

	mean	std.	rel. error
GS	5.421e−02	NA	
GS-L	5.421e−02	NA	
LS	5.280e−02	8.287e−05	−2.604e+00%
LS-N	5.308e−02	1.222e−04	−2.093e+00%
LS-R	5.523e−02	2.488e−04	1.882e+00%
LS-RN	5.298e−02	9.853e−05	−2.261e+00%

(c) R2

	mean	std.	rel. error
GS	2.150e−02	NA	
LS	2.131e−02	1.154e−05	−9.134e−01%
LS-N	2.142e−02	2.670e−05	−3.785e−01%
LS-R	2.192e−02	3.857e−04	1.926e+00%
LS-RN	2.142e−02	3.769e−05	−4.018e−01%

(d) NR2

	mean	std.	rel. error
GS	2.900e+01	NA	
LS	2.900e+01	7.223e−15	−3.676e−14%
LS-N	2.897e+01	1.180e−02	8.267e−02%
LS-R	2.892e+01	2.132e−02	2.790e−01%
LS-RN	2.900e+01	3.384e−03	5.458e−03%

(e) ϵ

	mean	std.	rel. error
GS	1.379e−01	NA	
LS	7.718e−02	1.430e−02	−4.404e+01%
LS-N	8.795e−02	1.513e−02	−3.623e+01%
LS-R	8.427e−02	1.300e−02	−3.890e+01%
LS-RN	7.677e−02	1.149e−02	−4.434e+01%

(f) s-energy

	mean	std.	rel. error
GS	1.384e+05	NA	
LS	1.330e+05	3.399e+02	−3.910e+00%
LS-N	1.336e+05	5.121e+02	−3.431e+00%
LS-R	1.578e+05	2.866e+03	1.400e+01%
LS-RN	1.338e+05	5.987e+02	−3.308e+00%

Table S.10: Comparison of the four LS methods and the GS methods on the ISSP instances with the discontinuous PF and IGD, IGD$^+$, R2, NR2, ϵ, and s–energy. The mean, standard deviation, and relative error based on GS of the quality indicator values of the subsets found by each method are shown.

(a) IGD

	mean	std.	rel. error
GS	7.588e−02	NA	
GS-L	7.588e−02	NA	
LS	7.354e−02	9.300e−05	−3.091e+00%
LS-N	7.527e−02	1.034e−03	−8.135e−01%
LS-R	7.738e−02	3.634e−04	1.971e+00%
LS-RN	7.373e−02	1.309e−04	−2.833e+00%

(b) IGD$^+$

	mean	std.	rel. error
GS	3.650e−02	NA	
GS-L	3.654e−02	NA	
LS	3.363e−02	7.604e−05	−7.873e+00%
LS-N	3.532e−02	3.556e−04	−3.244e+00%
LS-R	3.683e−02	3.677e−04	8.951e−01%
LS-RN	3.434e−02	1.878e−04	−5.925e+00%

(c) R2

	mean	std.	rel. error
GS	9.347e−02	NA	
LS	9.323e−02	1.081e−05	−2.551e−01%
LS-N	9.374e−02	1.128e−04	2.900e−01%
LS-R	9.377e−02	2.206e−04	3.225e−01%
LS-RN	9.332e−02	3.126e−05	−1.592e−01%

(d) NR2

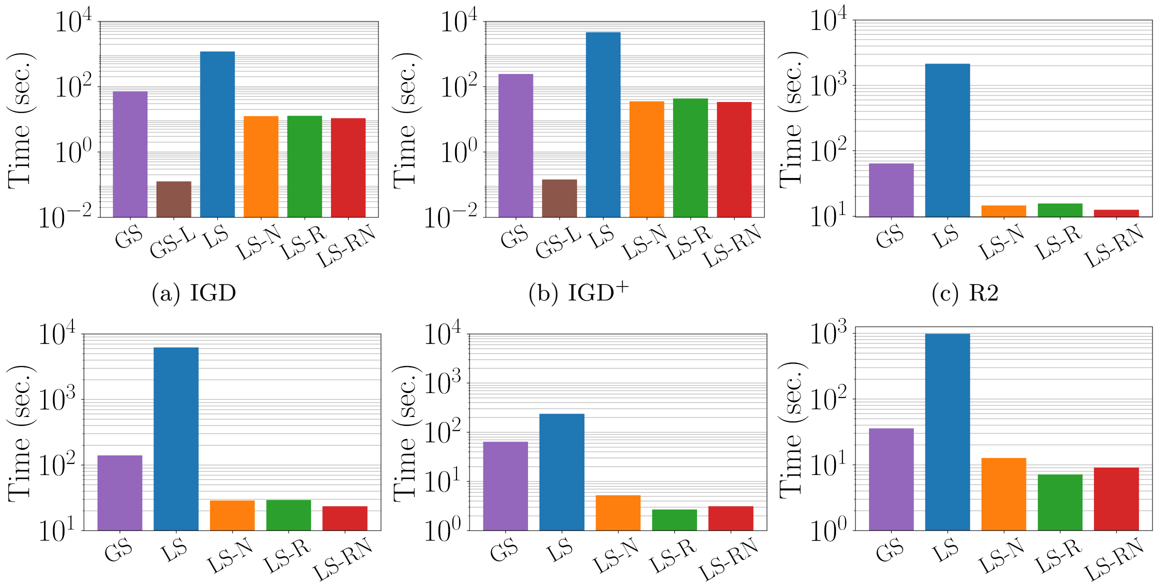
	mean	std.	rel. error
GS	1.336e+01	NA	
LS	1.342e+01	4.254e−03	−4.877e−01%
LS-N	1.329e+01	3.979e−02	5.094e−01%
LS-R	1.328e+01	2.138e−02	6.318e−01%
LS-RN	1.339e+01	6.612e−03	−2.160e−01%

(e) ϵ

	mean	std.	rel. error
GS	2.636e−01	NA	
LS	1.085e−01	1.462e−02	−5.883e+01%
LS-N	1.123e−01	1.352e−02	−5.739e+01%
LS-R	1.087e−01	1.414e−02	−5.877e+01%
LS-RN	1.088e−01	1.150e−02	−5.872e+01%

(f) s-energy

	mean	std.	rel. error
GS	6.896e+04	NA	
LS	6.656e+04	2.426e+02	−3.482e+00%
LS-N	7.870e+04	6.456e+03	1.413e+01%
LS-R	8.603e+04	2.384e+03	2.475e+01%
LS-RN	6.798e+04	5.476e+02	−1.431e+00%



(a) IGD (b) IGD$^+$ (c) R2

(d) NR2 (e) ϵ (f) s-energy

Fig. S.17: Wall-clock time of the run of the four LS methods and the GS methods on the ISSP instances with the discontinuous PF using IGD, IGD$^+$, R2, NR2, ϵ, and s-energy.

Table S.11: Comparison of the four LS methods on the ISSPs using HV, IGD, IGD+, and R2, and the approximated PFs of the 14 real-world (RE) problems. The mean, [standard deviation], and (relative error based on LS) of the quality indicator values of the subsets found by each LS method are shown. The symbols + , - , and ≈ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to conventional LS.

(a) HV

Problem	LS	LS-N	LS-R	LS-RN
RE2-4-1	8.85e−01 [9.85e−06]	8.84e−01 [1.07e−04] - (2.77e−02%)	8.84e−01 [4.37e−05] - (2.79e−02%)	8.85e−01 [1.14e−05] - (5.91e−04%)
RE2-3-3	7.59e−01 [9.73e−06]	7.59e−01 [1.43e−04] - (3.48e−02%)	7.59e−01 [6.80e−05] - (3.52e−02%)	7.59e−01 [1.27e−05] - (1.08e−03%)
RE2-4-3	1.16e+00 [8.57e−07]	1.16e+00 [2.66e−05] - (6.74e−03%)	1.16e+00 [2.39e−06] - (1.71e−03%)	1.16e+00 [1.03e−06] - (4.88e−05%)
RE2-2-4	1.17e+00 [3.10e−06]	1.17e+00 [9.55e−05] - (3.90e−02%)	1.17e+00 [6.95e−06] - (3.32e−03%)	1.17e+00 [3.23e−06] + (−2.72e−04%)
RE2-3-5	1.08e+00 [2.26e−16]	7.74e−01 [2.28e−01] - (2.84e+01%)	1.08e+00 [3.41e−07] ≈ (5.67e−06%)	1.08e+00 [1.46e−03] - (7.35e−02%)
RE3-3-1	1.33e+00 [9.03e−16]	1.33e+00 [3.01e−08] - (8.24e−06%)	1.33e+00 [9.34e−08] - (8.72e−06%)	1.33e+00 [9.03e−16] ≈ (0.00e+00%)
RE3-4-2	1.33e+00 [5.14e−08]	1.33e+00 [1.35e−06] - (5.59e−04%)	1.33e+00 [2.42e−07] - (4.80e−05%)	1.33e+00 [1.75e−07] - (1.53e−05%)
RE3-4-3	1.31e+00 [2.26e−16]	1.31e+00 [3.37e−05] - (3.04e−02%)	1.31e+00 [4.65e−05] - (1.20e−03%)	1.31e+00 [5.39e−05] - (3.00e−03%)
RE3-5-4	1.05e+00 [1.33e−05]	1.04e+00 [4.01e−04] - (1.04e−01%)	1.04e+00 [7.96e−05] - (4.40e−02%)	1.05e+00 [3.52e−05] - (9.78e−03%)
RE3-7-5	1.31e+00 [1.36e−06]	1.31e+00 [3.00e−05] - (8.42e−03%)	1.31e+00 [1.03e−05] - (3.07e−03%)	1.31e+00 [2.94e−06] - (5.66e−04%)
RE3-4-7	8.89e−01 [4.80e−05]	8.89e−01 [2.48e−04] - (8.57e−02%)	8.88e−01 [2.01e−04] - (1.45e−01%)	8.89e−01 [8.56e−05] - (2.45e−02%)
RE4-7-1	8.65e−01 [8.19e−05]	8.58e−01 [9.50e−04] - (8.04e−01%)	8.63e−01 [4.33e−04] - (3.16e−01%)	8.65e−01 [2.17e−04] - (6.62e−02%)
RE4-6-2	8.49e−01 [4.13e−05]	8.46e−01 [9.85e−04] - (3.95e−01%)	8.47e−01 [2.58e−04] - (2.05e−01%)	8.49e−01 [1.16e−04] - (4.26e−02%)
RE6-3-1	1.50e+00 [5.16e−05]	1.50e+00 [4.00e−04] - (9.82e−02%)	1.50e+00 [1.04e−04] - (6.47e−02%)	1.50e+00 [9.45e−05] - (2.01e−02%)

(b) IGD

Problem	LS	LS-N	LS-R	LS-RN
RE2-4-1	3.53e−03 [1.03e−05]	3.79e−03 [9.94e−05] - (7.38e+00%)	3.76e−03 [2.32e−05] - (6.49e+00%)	3.55e−03 [1.75e−05] - (4.96e−01%)
RE2-3-3	3.56e−03 [1.51e−05]	3.84e−03 [1.33e−04] - (7.84e+00%)	3.78e−03 [2.36e−05] - (6.18e+00%)	3.57e−03 [1.53e−05] - (4.15e−01%)
RE2-4-3	3.31e−03 [1.24e−05]	4.15e−03 [6.70e−04] - (2.56e+01%)	3.55e−03 [2.99e−05] - (7.39e+00%)	3.34e−03 [1.94e−05] - (8.14e−01%)
RE2-2-4	4.37e−03 [1.33e−05]	4.73e−03 [1.34e−04] - (8.15e+00%)	4.65e−03 [4.00e−05] - (6.29e+00%)	4.39e−03 [2.24e−05] - (4.85e−01%)
RE2-3-5	4.88e−04 [1.52e−06]	3.84e−03 [1.79e−03] - (6.86e+02%)	5.30e−04 [1.90e−05] - (8.56e+00%)	5.28e−04 [4.85e−05] - (8.12e+00%)
RE3-3-1	2.54e−02 [6.07e−05]	2.68e−02 [4.05e−04] - (5.53e+00%)	2.71e−02 [1.82e−04] - (6.54e+00%)	2.55e−02 [7.64e−05] - (5.75e−01%)
RE3-4-2	5.51e−03 [2.78e−05]	6.23e−03 [2.78e−04] - (1.30e+01%)	5.90e−03 [4.80e−05] - (6.95e+00%)	5.55e−03 [2.92e−05] - (5.85e−01%)
RE3-4-3	1.30e−02 [2.96e−05]	1.76e−02 [1.86e−03] - (3.53e+01%)	1.38e−02 [1.03e−04] - (6.07e+00%)	1.31e−02 [4.55e−05] - (7.28e−01%)
RE3-5-4	1.90e−02 [3.71e−05]	1.97e−02 [3.69e−04] - (3.81e+00%)	2.02e−02 [1.26e−04] - (6.08e+00%)	1.91e−02 [4.65e−05] - (4.15e−01%)
RE3-7-5	8.60e−03 [2.35e−05]	9.51e−03 [3.90e−04] - (1.06e+01%)	9.23e−03 [6.92e−05] - (7.25e+00%)	8.66e−03 [2.88e−05] - (6.97e−01%)
RE3-4-7	3.29e−02 [7.28e−05]	3.37e−02 [3.31e−04] - (2.21e+00%)	3.47e−02 [1.98e−04] - (5.32e+00%)	3.31e−02 [7.99e−05] - (4.22e−01%)
RE4-7-1	7.03e−02 [1.09e−04]	7.12e−02 [3.38e−04] - (1.32e+00%)	7.41e−02 [3.81e−04] - (5.44e+00%)	7.06e−02 [1.50e−04] - (3.92e−01%)
RE4-6-2	3.23e−02 [6.54e−05]	3.32e−02 [4.26e−04] - (2.62e+00%)	3.41e−02 [1.78e−04] - (5.58e+00%)	3.24e−02 [6.97e−05] - (3.97e−01%)
RE6-3-1	6.98e−02 [1.17e−04]	7.08e−02 [3.84e−04] - (1.47e+00%)	7.37e−02 [3.23e−04] - (5.61e+00%)	7.01e−02 [1.51e−04] - (3.56e−01%)

(c) IGD$^+$

Problem	LS	LS-N	LS-R	LS-RN
RE2-4-1	2.10e−03 [8.50e−06]	2.28e−03 [6.81e−05] - (8.38e+00%)	2.27e−03 [1.88e−05] - (7.75e+00%)	2.12e−03 [1.41e−05] - (6.85e−01%)
RE2-3-3	2.07e−03 [6.26e−06]	2.24e−03 [5.88e−05] - (8.04e+00%)	2.22e−03 [1.56e−05] - (7.35e+00%)	2.08e−03 [1.34e−05] - (5.24e−01%)
RE2-4-3	2.68e−04 [1.02e−06]	3.34e−04 [2.27e−05] - (2.46e+01%)	2.94e−04 [2.84e−06] - (9.66e+00%)	2.71e−04 [1.58e−06] - (1.23e+00%)
RE2-2-4	4.11e−04 [1.69e−06]	6.85e−04 [5.90e−05] - (6.68e+01%)	4.39e−04 [3.53e−06] - (6.93e+00%)	4.11e−04 [1.83e−06] ≈ (5.27e−02%)
RE2-3-5	1.12e−09 [4.41e−12]	2.25e−03 [1.67e−03] - (2.01e+08%)	1.12e−06 [6.25e−06] - (1.00e+05%)	1.22e−05 [1.92e−05] - (1.09e+06%)
RE3-3-1	7.93e−08 [2.77e−10]	1.76e−07 [1.58e−08] - (1.21e+02%)	9.84e−08 [1.81e−08] - (2.40e+01%)	8.72e−08 [4.30e−09] - (9.96e+00%)
RE3-4-2	5.57e−06 [1.88e−08]	9.86e−06 [9.40e−07] - (7.72e+01%)	6.15e−06 [1.41e−07] - (1.06e+01%)	5.71e−06 [6.33e−08] - (2.60e+00%)
RE3-4-3	4.51e−09 [1.40e−11]	1.23e−05 [3.09e−06] - (2.72e+05%)	1.82e−07 [3.93e−07] - (3.93e+03%)	1.15e−06 [1.32e−06] - (2.53e+04%)
RE3-5-4	5.92e−03 [1.25e−05]	6.19e−03 [9.91e−05] - (4.68e+00%)	6.31e−03 [4.64e−05] - (6.58e+00%)	5.96e−03 [2.12e−05] - (7.04e−01%)
RE3-7-5	3.05e−04 [7.39e−07]	3.50e−04 [1.37e−05] - (1.47e+01%)	3.32e−04 [4.62e−06] - (8.70e+00%)	3.12e−04 [1.84e−06] - (2.23e+00%)
RE3-4-7	1.10e−02 [1.98e−05]	1.13e−02 [1.68e−04] - (3.01e+00%)	1.18e−02 [1.19e−04] - (7.21e+00%)	1.11e−02 [4.59e−05] - (9.36e−01%)
RE4-7-1	2.24e−02 [3.99e−05]	2.30e−02 [1.89e−04] - (2.64e+00%)	2.37e−02 [1.62e−04] - (5.90e+00%)	2.27e−02 [7.42e−05] - (1.29e+00%)
RE4-6-2	1.15e−02 [2.88e−05]	1.22e−02 [2.16e−04] - (6.13e+00%)	1.22e−02 [1.25e−04] - (6.78e+00%)	1.16e−02 [4.74e−05] - (1.39e+00%)
RE6-3-1	8.56e−03 [2.55e−05]	9.18e−03 [1.65e−04] - (7.22e+00%)	9.18e−03 [6.20e−05] - (7.20e+00%)	8.69e−03 [3.40e−05] - (1.49e+00%)

(d) R2

Problem	LS	LS-N	LS-R	LS-RN
RE2-4-1	1.29e−01 [2.67e−06]	1.29e−01 [4.41e−05] - (1.26e−01%)	1.29e−01 [7.97e−06] - (4.80e−02%)	1.29e−01 [3.33e−06] - (1.91e−03%)
RE2-3-3	1.57e−01 [3.53e−06]	1.57e−01 [3.82e−05] - (7.66e−02%)	1.57e−01 [7.54e−06] - (3.80e−02%)	1.57e−01 [4.91e−06] - (4.56e−03%)
RE2-4-3	3.13e−02 [2.78e−07]	3.13e−02 [1.46e−05] - (1.48e−01%)	3.13e−02 [1.41e−06] - (2.56e−02%)	3.13e−02 [4.74e−07] - (3.25e−03%)
RE2-2-4	3.86e−02 [1.07e−06]	3.91e−02 [9.32e−05] - (1.28e+00%)	3.86e−02 [2.92e−06] - (4.46e−02%)	3.86e−02 [1.34e−06] - (7.99e−03%)
RE2-3-5	7.34e−02 [1.41e−17]	1.95e−01 [1.05e−01] - (1.66e+02%)	7.35e−02 [4.63e−04] ≈ (1.13e−01%)	7.39e−02 [5.16e−04] - (6.05e−01%)
RE3-3-1	9.41e−05 [5.51e−20]	9.42e−05 [2.86e−08] - (8.91e−02%)	1.42e−04 [1.85e−04] - (5.07e+01%)	9.41e−05 [4.15e−09] - (3.96e−03%)
RE3-4-2	1.18e−03 [6.61e−19]	1.20e−03 [5.54e−06] - (1.58e+00%)	1.19e−03 [1.07e−05] - (5.27e−01%)	1.18e−03 [4.52e−07] - (2.17e−02%)
RE3-4-3	9.25e−03 [1.76e−18]	9.29e−03 [4.51e−05] - (5.10e−01%)	9.25e−03 [5.56e−06] - (4.92e−02%)	9.26e−03 [2.28e−05] - (1.03e−01%)
RE3-5-4	7.14e−02 [1.67e−06]	7.17e−02 [8.61e−05] - (4.09e−01%)	7.15e−02 [1.43e−05] - (9.97e−02%)	7.15e−02 [8.00e−06] - (3.13e−02%)
RE3-7-5	1.35e−02 [2.04e−07]	1.36e−02 [1.70e−05] - (6.93e−01%)	1.35e−02 [1.69e−06] - (3.69e−02%)	1.35e−02 [9.55e−07] - (1.67e−02%)
RE3-4-7	1.00e−01 [5.31e−06]	1.00e−01 [5.47e−05] - (1.86e−01%)	1.00e−01 [3.00e−05] - (1.82e−01%)	1.00e−01 [1.26e−05] - (3.34e−02%)
RE4-7-1	7.80e−02 [5.03e−06]	7.84e−02 [9.64e−05] - (5.16e−01%)	7.83e−02 [4.93e−05] - (3.95e−01%)	7.81e−02 [3.08e−05] - (1.21e−01%)
RE4-6-2	8.38e−02 [2.06e−06]	8.41e−02 [8.75e−05] - (3.53e−01%)	8.40e−02 [2.43e−05] - (1.75e−01%)	8.39e−02 [1.28e−05] - (5.06e−02%)
RE6-3-1	2.71e−02 [7.86e−07]	2.74e−02 [4.05e−05] - (9.32e−01%)	2.72e−02 [1.52e−05] - (2.36e−01%)	2.72e−02 [9.50e−06] - (1.15e−01%)

Table S.11: Comparison of the four LS methods on the ISSPs using NR2, ϵ, and s-energy and the approximated PFs of the 14 real-world (RE) problems. The mean, [standard deviation], and (relative error based on LS) of the quality indicator values of the subsets found by each LS method are shown. The symbols +, -, and ≈ indicate the result of the Wilcoxon rank-sum test with $\alpha < 0.05$ compared to conventional LS.

(e) NR2

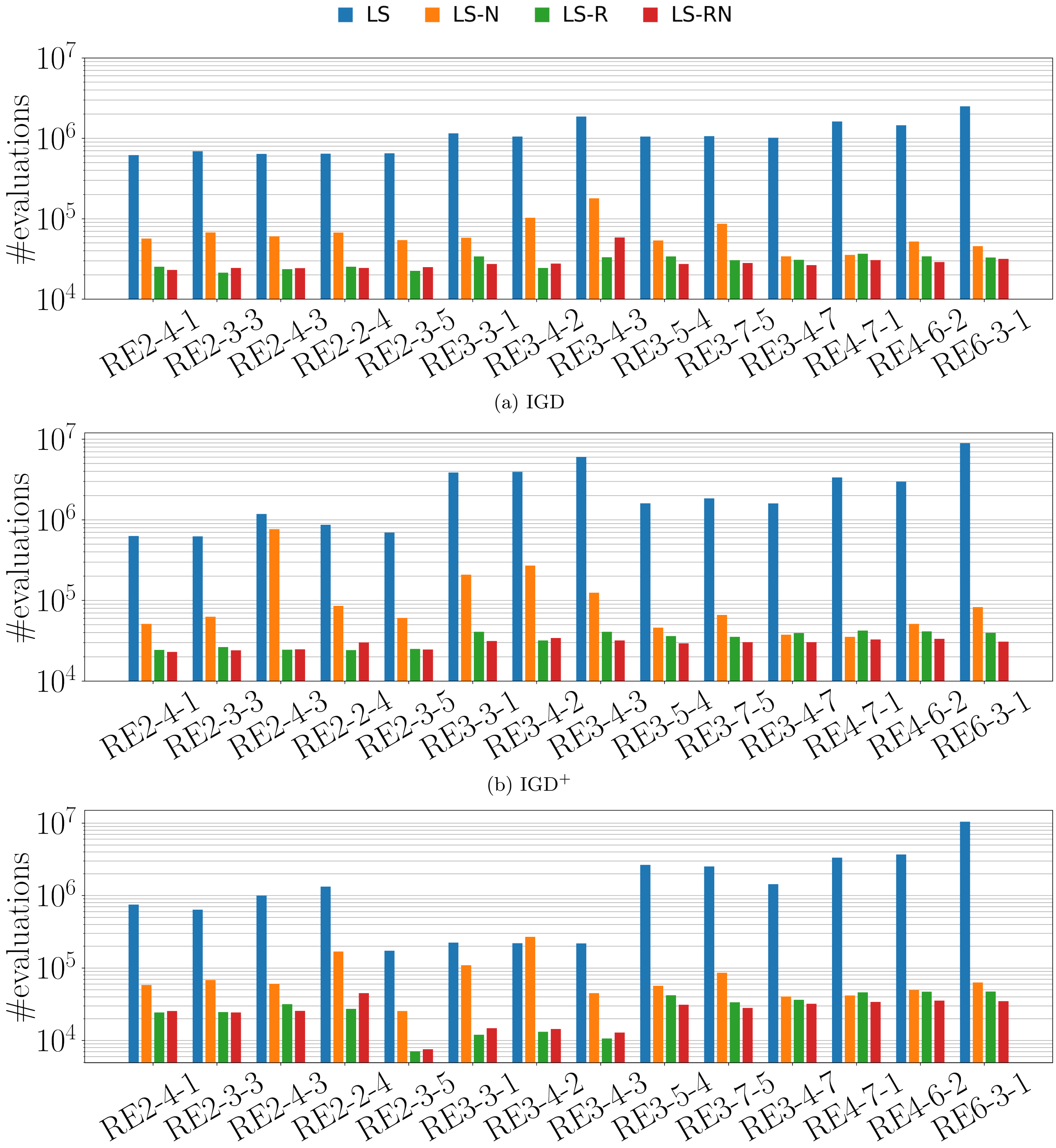
Problem	LS	LS-N	LS-R	LS-RN
RE2-4-1	1.77e+00 [3.16e−05]	1.77e+00 [2.40e−04] - (4.03e−02%)	1.77e+00 [1.04e−04] - (4.81e−02%)	1.77e+00 [4.02e−05] - (3.05e−03%)
RE2-3-3	1.52e+00 [2.89e−05]	1.52e+00 [3.06e−04] - (5.57e−02%)	1.52e+00 [1.06e−04] - (5.75e−02%)	1.52e+00 [4.93e−05] - (6.50e−03%)
RE2-4-3	2.33e+00 [2.78e−06]	2.33e+00 [6.37e−05] - (8.12e−03%)	2.33e+00 [1.84e−05] - (4.83e−03%)	2.33e+00 [5.04e−06] - (4.94e−04%)
RE2-2-4	2.34e+00 [9.42e−06]	2.34e+00 [2.98e−04] - (5.26e−02%)	2.34e+00 [4.13e−05] - (9.38e−03%)	2.34e+00 [1.25e−05] - (9.59e−04%)
RE2-3-5	2.16e+00 [4.51e−16]	1.55e+00 [4.56e−01] - (2.84e+01%)	2.16e+00 [4.51e−16] ≈ (0.00e+00%)	2.16e+00 [2.76e−03] - (5.79e−02%)
RE3-3-1	7.66e+00 [3.61e−15]	7.66e+00 [7.37e−07] - (6.36e−06%)	7.66e+00 [1.44e−07] ≈ (3.37e−07%)	7.66e+00 [3.41e−08] - (1.68e−07%)
RE3-4-2	7.66e+00 [1.80e−08]	7.66e+00 [4.02e−05] - (1.46e−03%)	7.66e+00 [6.79e−07] - (4.22e−05%)	7.66e+00 [2.09e−06] - (1.61e−05%)
RE3-4-3	7.55e+00 [0.00e+00]	7.55e+00 [3.58e−03] - (5.21e−02%)	7.55e+00 [6.84e−04] - (6.92e−03%)	7.55e+00 [5.40e−04] - (8.83e−03%)
RE3-5-4	6.05e+00 [6.00e−05]	6.04e+00 [2.89e−03] - (1.71e−01%)	6.04e+00 [9.76e−04] - (7.91e−02%)	6.05e+00 [3.93e−04] - (1.97e−02%)
RE3-7-5	7.51e+00 [9.45e−06]	7.51e+00 [3.97e−04] - (2.05e−02%)	7.51e+00 [5.65e−05] - (3.81e−03%)	7.51e+00 [2.29e−05] - (9.84e−04%)
RE3-4-7	5.18e+00 [5.07e−04]	5.17e+00 [3.43e−03] - (1.55e−01%)	5.16e+00 [1.60e−03] - (2.28e−01%)	5.17e+00 [8.88e−04] - (5.77e−02%)
RE4-7-1	1.84e+01 [2.84e−03]	1.82e+01 [5.43e−02] - (1.20e+00%)	1.83e+01 [1.72e−02] - (5.08e−01%)	1.84e+01 [8.54e−03] - (1.76e−01%)
RE4-6-2	1.79e+01 [7.93e−04]	1.78e+01 [2.37e−02] - (5.54e−01%)	1.78e+01 [7.97e−03] - (2.72e−01%)	1.79e+01 [3.66e−03] - (8.01e−02%)
RE6-3-1	4.75e+02 [2.95e−03]	4.74e+02 [1.89e−01] - (1.35e−01%)	4.74e+02 [8.01e−02] - (5.83e−02%)	4.75e+02 [4.28e−02] - (3.26e−02%)

(f) ϵ

Problem	LS	LS-N	LS-R	LS-RN
RE2-4-1	5.57e−03 [2.06e−04]	9.56e−03 [1.97e−03] - (7.16e+01%)	6.14e−03 [1.89e−04] - (1.03e+01%)	5.93e−03 [2.80e−04] - (6.56e+00%)
RE2-3-3	5.57e−03 [2.49e−04]	9.43e−03 [2.05e−03] - (6.92e+01%)	6.07e−03 [1.69e−04] - (8.89e+00%)	5.96e−03 [2.90e−04] - (6.99e+00%)
RE2-4-3	8.98e−04 [3.84e−05]	2.32e−03 [7.73e−04] - (1.58e+02%)	1.01e−03 [3.20e−05] - (1.20e+01%)	9.99e−04 [6.50e−05] - (1.13e+01%)
RE2-2-4	1.67e−03 [3.95e−05]	7.63e−03 [2.16e−03] - (3.57e+02%)	2.04e−03 [1.74e−04] - (2.23e+01%)	2.25e−03 [3.18e−04] - (3.49e+01%)
RE2-3-5	3.27e−09 [1.48e−10]	3.57e−01 [2.43e−01] - (1.09e+10%)	2.44e−03 [9.61e−03] - (7.46e+07%)	3.71e−02 [1.85e−02] - (1.13e+09%)
RE3-3-1	7.12e−07 [2.31e−08]	1.20e−04 [3.98e−05] - (1.67e+04%)	6.65e−04 [1.78e−03] - (9.33e+04%)	8.47e−05 [3.99e−05] - (1.18e+04%)
RE3-4-2	3.99e−05 [1.95e−06]	1.14e−03 [4.88e−04] - (2.77e+03%)	4.81e−04 [4.04e−04] - (1.11e+03%)	4.16e−04 [1.50e−04] - (9.43e+02%)
RE3-4-3	9.03e−08 [3.70e−09]	5.88e−03 [2.21e−03] - (6.51e+06%)	2.07e−03 [2.17e−03] - (2.30e+06%)	2.71e−03 [1.37e−03] - (3.00e+06%)
RE3-5-4	1.46e−02 [5.00e−04]	2.18e−02 [3.76e−03] - (4.89e+01%)	1.63e−02 [7.62e−04] - (1.11e+01%)	1.57e−02 [6.28e−04] - (7.38e+00%)
RE3-7-5	1.01e−03 [2.95e−05]	3.13e−03 [9.44e−04] - (2.09e+02%)	1.38e−03 [2.03e−04] - (3.56e+01%)	1.49e−03 [2.12e−04] - (4.72e+01%)
RE3-4-7	2.70e−02 [7.23e−04]	3.66e−02 [4.36e−03] - (3.54e+01%)	2.97e−02 [8.95e−04] - (9.87e+00%)	2.93e−02 [1.31e−03] - (8.58e+00%)
RE4-7-1	5.11e−02 [2.91e−03]	7.31e−02 [1.21e−02] - (4.29e+01%)	5.60e−02 [1.81e−03] - (9.52e+00%)	5.59e−02 [3.13e−03] - (9.30e+00%)
RE4-6-2	2.57e−02 [7.23e−04]	4.05e−02 [4.97e−03] - (5.79e+01%)	2.84e−02 [7.93e−04] - (1.08e+01%)	2.76e−02 [1.28e−03] - (7.71e+00%)
RE6-3-1	2.13e−02 [5.39e−04]	3.84e−02 [3.49e−03] - (8.03e+01%)	2.78e−02 [1.76e−03] - (3.02e+01%)	2.66e−02 [2.00e−03] - (2.49e+01%)

(g) s-energy

Problem	LS	LS-N	LS-R	LS-RN
RE2-4-1	5.57e+04 [3.09e+01]	5.67e+04 [5.13e+02] - (1.83e+00%)	5.68e+04 [1.74e+02] - (1.92e+00%)	5.57e+04 [2.31e+01] ≈ (1.51e−02%)
RE2-3-3	5.57e+04 [3.85e+01]	5.72e+04 [5.34e+02] - (2.65e+00%)	5.68e+04 [1.32e+02] - (1.95e+00%)	5.57e+04 [4.66e+01] ≈ (2.84e−02%)
RE2-4-3	4.61e+04 [3.53e+01]	5.26e+04 [1.32e+03] - (1.42e+01%)	4.72e+04 [1.76e+02] - (2.44e+00%)	4.63e+04 [1.93e+02] - (4.56e−01%)
RE2-2-4	4.55e+04 [1.99e+01]	4.65e+04 [4.36e+02] - (2.28e+00%)	4.64e+04 [1.33e+02] - (1.89e+00%)	4.55e+04 [1.98e+01] - (2.21e−02%)
RE2-3-5	3.06e+05 [2.06e+02]	3.93e+05 [1.44e+04] - (2.83e+01%)	3.13e+05 [1.63e+03] - (2.31e+00%)	3.08e+05 [3.37e+03] ≈ (6.73e−01%)
RE3-3-1	1.04e+05 [2.13e+02]	1.09e+05 [1.45e+03] - (4.37e+00%)	1.11e+05 [8.86e+02] - (6.86e+00%)	1.04e+05 [2.89e+02] - (2.31e−01%)
RE3-4-2	5.36e+05 [2.23e+03]	7.73e+05 [1.22e+05] - (4.42e+01%)	5.94e+05 [6.76e+03] - (1.07e+01%)	5.37e+05 [2.98e+03] ≈ (1.38e−01%)
RE3-4-3	1.99e+05 [3.58e+02]	2.21e+07 [2.43e+07] - (1.10e+04%)	2.13e+05 [2.65e+03] - (7.16e+00%)	2.02e+05 [2.84e+03] - (1.63e+00%)
RE3-5-4	1.82e+05 [4.91e+02]	2.15e+05 [9.66e+03] - (1.79e+01%)	2.00e+05 [2.01e+03] - (9.76e+00%)	1.83e+05 [5.62e+02] - (3.86e−01%)
RE3-7-5	3.47e+05 [1.44e+03]	4.08e+05 [2.04e+04] - (1.74e+01%)	3.85e+05 [4.12e+03] - (1.08e+01%)	3.48e+05 [1.60e+03] - (3.32e−01%)
RE3-4-7	8.90e+04 [1.81e+02]	9.02e+04 [6.37e+02] - (1.29e+00%)	9.51e+04 [8.49e+02] - (6.75e+00%)	8.92e+04 [2.19e+02] - (2.31e−01%)
RE4-7-1	1.13e+05 [5.54e+02]	1.15e+05 [6.69e+02] - (1.47e+00%)	1.28e+05 [1.68e+03] - (1.33e+01%)	1.15e+05 [7.74e+02] - (1.52e+00%)
RE4-6-2	5.29e+05 [3.68e+03]	5.62e+05 [1.28e+04] - (6.25e+00%)	6.25e+05 [1.49e+04] - (1.82e+01%)	5.39e+05 [5.79e+03] - (1.99e+00%)
RE6-3-1	1.09e+06 [1.71e+04]	1.22e+06 [4.31e+04] - (1.22e+01%)	1.68e+06 [7.38e+04] - (5.42e+01%)	1.16e+06 [3.42e+04] - (6.30e+00%)

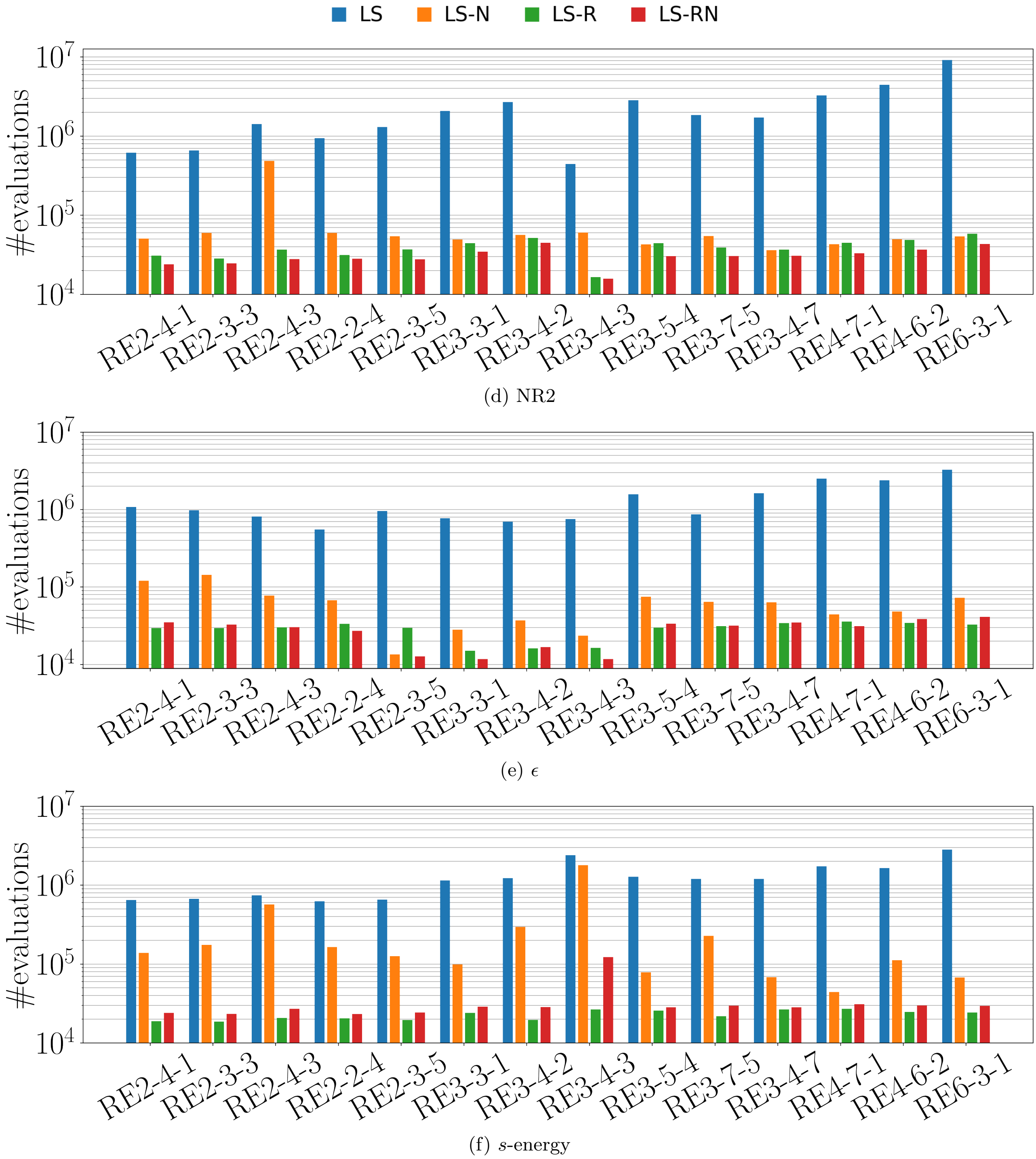


(a) IGD

(b) IGD$^+$

(c) R2

Fig. S.18: Number of subset evaluations by $\mathcal{I}$ for the four LS methods on the ISSPs using IGD, IGD$^+$, and R2, and the approximated PFs of the 14 real-world (RE) problems.



(d) NR2

(e) ϵ

(f) s-energy

Fig. S.18: Number of subset evaluations by $\mathcal{I}$ for the four LS methods on the ISSPs using NR2, ϵ, and s-energy, and the approximated PFs of the 14 real-world (RE) problems.