Game-theoretic Foundations of Multi-agent Systems

Lecture 1: Introduction

Seyed Majid Zahedi

UNIVERSITY OF WATERLOO

Outline

Course Mechanics

- Course website
  - https://ece.uwaterloo.ca/~smzahedi/crs/ece750/
  - All course information, lecture notes, assignments, etc.

Course Mechanics

- Course website
  - https://ece.uwaterloo.ca/~smzahedi/crs/ece750/
  - All course information, lecture notes, assignments, etc.

- Office hours
  - Book appointment online
  - Or catch me after class, or send me email to setup meeting

Course Mechanics

- Course website
  - https://ece.uwaterloo.ca/~smzahedi/crs/ece750/
  - All course information, lecture notes, assignments, etc.

- Office hours
  - Book appointment online
  - Or catch me after class, or send me email to setup meeting
- Prerequisites
  - None
  - Helpful knowledge: algorithms, probability, comp. complexity, and optimization

Course Mechanics

- Course website
  - https://ece.uwaterloo.ca/~smzahedi/crs/ece750/
  - All course information, lecture notes, assignments, etc.

- Office hours
  - Book appointment online
  - Or catch me after class, or send me email to setup meeting
- Prerequisites
  - None
  - Helpful knowledge: algorithms, probability, comp. complexity, and optimization
- Anti-requisites
  - ECON.412 (Topics in Game Theory), CO.456 (Introduction to Game Theory), CS.886 (Multi-agent Systems), and MSCI.724 (Game Theory and Recent App)

Outline

Tentative Topics

- Introduction to multi-agent systems
  - Overview of game theory and mechanism design, rationality and self-interest, utility theorem, risk attitudes

Tentative Topics

- Introduction to multi-agent systems
  - Overview of game theory and mechanism design, rationality and self-interest, utility theorem, risk attitudes
- Games in normal form
  - Pure and mixed-strategy Nash equilibrium, iterative elimination of dominated strategies, price of anarchy, correlated equilibrium, computing solution concepts of normal-form games

Tentative Topics

- Introduction to multi-agent systems
  - Overview of game theory and mechanism design, rationality and self-interest, utility theorem, risk attitudes
- Games in normal form
  - Pure and mixed-strategy Nash equilibrium, iterative elimination of dominated strategies, price of anarchy, correlated equilibrium, computing solution concepts of normal-form games
- Games in extensive form
  - Perfect and imperfect-information games, finite and infinite-horizon games, subgame-perfect equilibrium, backward induction, one-shot deviation principle

Tentative Topics (cont.)

- Beyond normal and extensive-form games
  - Repeated games, stochastic games, Bayesian games, congestion games, trigger strategies, folk theorems, Bayes-Nash equilibrium, auctions, optimal auctions, revenue-equivalence theorem, incentive compatibility, VCG mechanisms

Tentative Topics (cont.)

- Beyond normal and extensive-form games
  - Repeated games, stochastic games, Bayesian games, congestion games, trigger strategies, folk theorems, Bayes-Nash equilibrium, auctions, optimal auctions, revenue-equivalence theorem, incentive compatibility, VCG mechanisms
- Learning in multi-agent systems
  - Multi-agent reinforcement learning, fictitious play, Bayesian learning, regret-minimization learning

Textbook and References

- Y. Shoham and K. Leyton-Brown, **Multi-agent Systems: Algorithmic, Game-theoretic, and Logical Foundations** (available online)
- N. Nisan, et al. **Algorithmic Game Theory** (available online)
- T. Roughgarden, **Twenty Lectures on Algorithmic Game Theory** (notes available online)
- D. Fudenberg and D. Levine, **The Theory of Learning in Games**
- D. Fudenberg and J. Tirole, **Game Theory**
- M. J. Osborne and A. Rubinstein, **A Course in Game Theory** (available online)

Course Requirements

- Graduate offering (available to undergraduate students)

Course Requirements

- Graduate offering (available to undergraduate students)
  - Quizzes 5%

Course Requirements

- Graduate offering (available to undergraduate students)
  - Quizzes 5%
  - Assignments 20%

Course Requirements

- Graduate offering (available to undergraduate students)
  - Quizzes 5%
  - Assignments 20%
  - Research project 45%

Course Requirements

- Graduate offering (available to undergraduate students)
  - Quizzes 5%
  - Assignments 20%
  - Research project 45%
  - Final exam 30%

Course Requirements

- Graduate offering (available to undergraduate students)
  - Quizzes 5%
  - Assignments 20%
  - Research project 45%
  - Final exam 30%

- Undergraduate offering (available to MEng students)

Course Requirements

- Graduate offering (available to undergraduate students)
  - Quizzes 5%
  - Assignments 20%
  - Research project 45%
  - Final exam 30%

- Undergraduate offering (available to MEng students)
  - Quizzes 5%

Course Requirements

- Graduate offering (available to undergraduate students)
  - Quizzes 5%
  - Assignments 20%
  - Research project 45%
  - Final exam 30%

- Undergraduate offering (available to MEng students)
  - Quizzes 5%
  - Assignments 20%

Course Requirements

- Graduate offering (available to undergraduate students)
  - Quizzes 5%
  - Assignments 20%
  - Research project 45%
  - Final exam 30%

- Undergraduate offering (available to MEng students)
  - Quizzes 5%
  - Assignments 20%
  - Survey project 25%

Course Requirements

- Graduate offering (available to undergraduate students)
  - Quizzes 5%
  - Assignments 20%
  - Research project 45%
  - Final exam 30%

- Undergraduate offering (available to MEng students)
  - Quizzes 5%
  - Assignments 20%
  - Survey project 25%
  - Final exam 50%

Survey Project (Undergraduate Offering)

- Goal is to review existing literature in sub-area of multi-agent systems and possibly explore open research questions in that sub-area
- Two milestones: proposal (15%) and final written report (85%)

Research Project (Graduate Offering)

- Goal is to try to do something novel, rather than merely surveying existing work
- Only real constraint is that it has to have something to do with covered material
- Projects may be theoretical, experimental (based on simulations), experimental (based on real-world data), a useful software artifact, or any combination thereof
- Three milestones: proposal (15%), oral progress report (optional), and final written report (85%)
- Creativity is encouraged

Outline

Questions and Problems in (Computational) Game Theory

- How should we represent games (i.e., strategic settings)?
  - Game-theoretic representations not always concise enough

Questions and Problems in (Computational) Game Theory

- How should we represent games (i.e., strategic settings)?
  - Game-theoretic representations not always concise enough
- What does it mean to solve a game?
  - Solution concepts from game theory, e.g., Nash equilibrium

Questions and Problems in (Computational) Game Theory

- How should we represent games (i.e., strategic settings)?
  - Game-theoretic representations not always concise enough
- What does it mean to solve a game?
  - Solution concepts from game theory, e.g., Nash equilibrium
- How computationally hard is it to solve a games?
  - Can we solve them approximately?

Questions and Problems in (Computational) Game Theory

- How should we represent games (i.e., strategic settings)?
  - Game-theoretic representations not always concise enough
- What does it mean to solve a game?
  - Solution concepts from game theory, e.g., Nash equilibrium
- How computationally hard is it to solve a games?
  - Can we solve them approximately?
- Is there a role for (machine) learning in games?

Questions and Problems in (Computational) Game Theory

- How should we represent games (i.e., strategic settings)?
  - Game-theoretic representations not always concise enough
- What does it mean to solve a game?
  - Solution concepts from game theory, e.g., Nash equilibrium
- How computationally hard is it to solve a games?
  - Can we solve them approximately?
- Is there a role for (machine) learning in games?
- What types of modeling problems do we face when addressing real-world games?

Optimization vs. Game Theory

- Optimization theory: Optimize a single objective over a decision variable

$$\begin{aligned} \min. \quad & u(x) \\ \text{s.t.} \quad & x \in X \end{aligned}$$

Optimization vs. Game Theory

- Optimization theory: Optimize a single objective over a decision variable

$$\begin{aligned} \min. \quad & u(x) \\ \text{s.t.} \quad & x \in X \end{aligned}$$

- Game theory: Study of multi-agent decision problems

Optimization vs. Game Theory

- Optimization theory: Optimize a single objective over a decision variable

$$\begin{aligned} \min. \quad & u(x) \\ \text{s.t.} \quad & x \in X \end{aligned}$$

- Game theory: Study of multi-agent decision problems
  - Model cooperation and competition between intelligent and rational decision makers

Optimization vs. Game Theory

- Optimization theory: Optimize a single objective over a decision variable

$$\begin{aligned} \min. \quad & u(x) \\ \text{s.t.} \quad & x \in X \end{aligned}$$

- Game theory: Study of multi-agent decision problems
  - Model cooperation and competition between intelligent and rational decision makers
  - n agents, each chooses some $x_i \in X_i$, and has a utility function

$$u_i(x_i, x_{-i}), \quad x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$$

Optimization vs. Game Theory

- Optimization theory: Optimize a single objective over a decision variable

$$\begin{aligned} \min. \quad & u(x) \\ \text{s.t.} \quad & x \in X \end{aligned}$$

- Game theory: Study of multi-agent decision problems
  - Model cooperation and competition between intelligent and rational decision makers
  - n agents, each chooses some $x_i \in X_i$, and has a utility function

$$u_i(x_i, x_{-i}), \quad x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$$

  - What are the possible outcomes?

Optimization vs. Game Theory

- Optimization theory: Optimize a single objective over a decision variable

$$\begin{aligned} \min. \quad & u(x) \\ \text{s.t.} \quad & x \in X \end{aligned}$$

- Game theory: Study of multi-agent decision problems
  - Model cooperation and competition between intelligent and rational decision makers
  - n agents, each chooses some $x_i \in X_i$, and has a utility function

$$u_i(x_i, x_{-i}), \quad x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$$

  - What are the possible outcomes?
  - Steady-state, stable operating point, characteristics?

Optimization vs. Game Theory

- Optimization theory: Optimize a single objective over a decision variable

$$\begin{aligned} \min. \quad & u(x) \\ \text{s.t.} \quad & x \in X \end{aligned}$$

- Game theory: Study of multi-agent decision problems
  - Model cooperation and competition between intelligent and rational decision makers
  - n agents, each chooses some $x_i \in X_i$, and has a utility function

$$u_i(x_i, x_{-i}), \quad x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$$

  - What are the possible outcomes?
  - Steady-state, stable operating point, characteristics?
  - How do you get there (learning dynamics, computation of equilibrium)?

Guess 2/3 of the Average (2/3-Beauty Contest Game)

- Everyone writes down an integer between 1 and 100
- Person closest to 2/3 of the average wins

Guess 2/3 of the Average (2/3-Beauty Contest Game)

- Everyone writes down an integer between 1 and 100
- Person closest to 2/3 of the average wins
- Example:
  - A says 50 - B says 10 - C says 90
  - 2/3 of average(50, 10, 90) $= 2/3 \times 50 = 33.33$
  - A is the closest ($|50 - 33.33| = 16.67$), so A wins

Mechanism Design

- Inverse Game Theory: Design of game forms to implement desirable outcomes
  - E.g. to incentivize independent agents to reveal their types truthfully

Mechanism Design

- Inverse Game Theory: Design of game forms to implement desirable outcomes
  - E.g. to incentivize independent agents to reveal their types truthfully
- In Economics, MD is all about designing the right incentives
  - Mechanisms map signals from independent agents into allocations and payments
  - Optimal Mechanisms (Myerson): Design a mechanism that maximizes profits
  - Efficient Mechanisms (Vickrey-Clarke-Groves (VCG) Mechanisms): Design a mechanism to maximize a social or system-wide objective

Mechanism Design

- Inverse Game Theory: Design of game forms to implement desirable outcomes
  - E.g. to incentivize independent agents to reveal their types truthfully
- In Economics, MD is all about designing the right incentives
  - Mechanisms map signals from independent agents into allocations and payments
  - Optimal Mechanisms (Myerson): Design a mechanism that maximizes profits
  - Efficient Mechanisms (Vickrey-Clarke-Groves (VCG) Mechanisms): Design a mechanism to maximize a social or system-wide objective
- In CS/Engineering, focus is more on the design of efficient decentralized protocols that take into account incentives
  - Mechanisms in networks, distributed and online mechanisms, mechanisms that operate with limited information

Example: Single-item Auction

- Sealed-bid auction: every bidder submits bid in sealed envelope
- First-price sealed-bid auction: highest bid wins, pays amount of own bid
- Second-price sealed-bid auction: highest bid wins, pays amount of 2nd-highest bid

Which Auction Generates More Revenue?

- Each bid depends on
  - bidder's true valuation for the item (utility = valuation - payment)
  - bidder's beliefs over what others will bid
  - and ... auction mechanism used
- In first-price auction, it does not make sense to bid your true valuation
  - Even if you win, your utility will be 0
- In second-price auction, it always makes sense to bid your true valuation (we will see this later)

What is Mechanism Design Again?

- Designing mechanism = designing a game

What is Mechanism Design Again?

- Designing mechanism = designing a game
- We can use game theory to predict what will happen under mechanism

What is Mechanism Design Again?

- Designing mechanism = designing a game
- We can use game theory to predict what will happen under mechanism
  - If agents act strategically

What is Mechanism Design Again?

- Designing mechanism = designing a game
- We can use game theory to predict what will happen under mechanism
  - If agents act strategically
- When is a mechanism "good"?

What is Mechanism Design Again?

- Designing mechanism = designing a game
- We can use game theory to predict what will happen under mechanism
  - If agents act strategically
- When is a mechanism "good"?
  - Should it result in outcomes that are "good" for reported or for true preferences?

What is Mechanism Design Again?

- Designing mechanism = designing a game
- We can use game theory to predict what will happen under mechanism
  - If agents act strategically
- When is a mechanism "good"?
  - Should it result in outcomes that are "good" for reported or for true preferences?
  - Should agents ever end up lying about their preferences (in game-theoretic solution)?

What is Mechanism Design Again?

- Designing mechanism = designing a game
- We can use game theory to predict what will happen under mechanism
  - If agents act strategically
- When is a mechanism "good"?
  - Should it result in outcomes that are "good" for reported or for true preferences?
  - Should agents ever end up lying about their preferences (in game-theoretic solution)?
  - Should it always generate the best allocation?

What is Mechanism Design Again?

- Designing mechanism = designing a game
- We can use game theory to predict what will happen under mechanism
  - If agents act strategically
- When is a mechanism "good"?
  - Should it result in outcomes that are "good" for reported or for true preferences?
  - Should agents ever end up lying about their preferences (in game-theoretic solution)?
  - Should it always generate the best allocation?
  - Should agents ever burn money?(!?)

What is Mechanism Design Again?

- Designing mechanism = designing a game
- We can use game theory to predict what will happen under mechanism
  - If agents act strategically
- When is a mechanism "good"?
  - Should it result in outcomes that are "good" for reported or for true preferences?
  - Should agents ever end up lying about their preferences (in game-theoretic solution)?
  - Should it always generate the best allocation?
  - Should agents ever burn money?(!?)
- Can we solve for the optimal mechanism?

Acknowledgment

- This lecture is a slightly modified version of ones prepared by
  - Asu Ozdaglar [MIT 6.254]
  - Vincent Conitzer [Duke CPS 590.4]