

Game-theoretic Foundations of Multi-agent Systems

Lecture 1: Introduction

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Outline

1. Course Mechanics
2. Course Outline
3. Overview of Game Theory and Mechanism Design



Course Mechanics

- Course website
 - <https://ece.uwaterloo.ca/~smzahedi/crs/ece750/>
 - All course information, lecture notes, assignments, etc.
- Office hours
 - Book appointment online
 - Or catch me after class, or send me email to setup meeting
- Prerequisites
 - None
 - Helpful knowledge: algorithms, probability, comp. complexity, and optimization
- Anti-requisites
 - ECON.412 (Topics in Game Theory), CO.456 (Introduction to Game Theory), CS.886 (Multi-agent Systems), and MSCI.724 (Game Theory and Recent App)



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Tentative Topics

- **Introduction to multi-agent systems**
 - Overview of game theory and mechanism design, rationality and self-interest, utility theorem, risk attitudes
- **Games in normal form**
 - Pure and mixed-strategy Nash equilibrium, iterative elimination of dominated strategies, price of anarchy, correlated equilibrium, computing solution concepts of normal-form games
- **Games in extensive form**
 - Perfect and imperfect-information games, finite and infinite-horizon games, subgame-perfect equilibrium, backward induction, one-shot deviation principle



Tentative Topics (cont.)

- **Beyond normal and extensive-form games**
 - Repeated games, stochastic games, Bayesian games, congestion games, trigger strategies, folk theorems, Bayes-Nash equilibrium, auctions, optimal auctions, revenue-equivalence theorem, incentive compatibility, VCG mechanisms
- **Learning in multi-agent systems**
 - Multi-agent reinforcement learning, fictitious play, Bayesian learning, regret-minimization learning



Textbook and References

- Y. Shoham and K. Leyton-Brown, **Multi-agent Systems: Algorithmic, Game-theoretic, and Logical Foundations** ([available online](#))
- N. Nisan, et al. **Algorithmic Game Theory** ([available online](#))
- T. Roughgarden, **Twenty Lectures on Algorithmic Game Theory** ([notes available online](#))
- D. Fudenberg and D. Levine, **The Theory of Learning in Games**
- D. Fudenberg and J. Tirole, **Game Theory**
- M. J. Osborne and A. Rubinstein, **A Course in Game Theory** ([available online](#))



Course Requirements

- Graduate offering (available to undergraduate students)
 - Quizzes 5%
 - Assignments 20%
 - Research project 45%
 - Final exam 30%

- Undergraduate offering (available to MEng students)
 - Quizzes 5%
 - Assignments 20%
 - Survey project 25%
 - Final exam 50%



Survey Project (Undergraduate Offering)

- Goal is to review existing literature in sub-area of multi-agent systems and possibly explore open research questions in that sub-area
- Two milestones: proposal (15%) and final written report (85%)



Research Project (Graduate Offering)

- Goal is to try to do something novel, rather than merely surveying existing work
- Only real constraint is that it has to have something to do with covered material
- Projects may be theoretical, experimental (based on simulations), experimental (based on real-world data), a useful software artifact, or any combination thereof
- Three milestones: proposal (15%), oral progress report (optional), and final written report (85%)
- Creativity is encouraged



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Questions and Problems in (Computational) Game Theory

- How should we represent games (i.e., strategic settings)?
 - Game-theoretic representations not always concise enough
- What does it mean to solve a game?
 - Solution concepts from game theory, e.g., Nash equilibrium
- How computationally hard is it to solve a games?
 - Can we solve them approximately?
- Is there a role for (machine) learning in games?
- What types of modeling problems do we face when addressing real-world games?



Optimization vs. Game Theory

- **Optimization theory:** Optimize a single objective over a decision variable

$$\begin{aligned} \min . \quad & u(x) \\ \text{s.t.} \quad & x \in X \end{aligned}$$

- **Game theory:** Study of multi-agent decision problems
 - Model **cooperation** and **competition** between **intelligent** and **rational** decision makers
 - n agents, each chooses some $x_i \in X_i$, and has a utility function

$$u_i(x_i, x_{-i}), \quad x_{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$$

- What are the possible outcomes?
- Steady-state, stable operating point, characteristics?
- How do you get there (learning dynamics, computation of equilibrium)?



Guess $2/3$ of the Average ($2/3$ -Beauty Contest Game)

- Everyone writes down an integer between 1 and 100
- Person closest to $2/3$ of the average wins
- Example:
 - A says 50 - B says 10 - C says 90
 - $2/3$ of average(50, 10, 90) = $2/3 \times 50 = 33.33$
 - A is the closest ($|50 - 33.33| = 16.67$), so A wins



Mechanism Design

- **Inverse Game Theory**: Design of game forms to implement desirable outcomes
 - E.g. to incentivize independent agents to reveal their types truthfully
- In Economics, MD is all about designing the right incentives
 - Mechanisms map **signals** from independent agents into allocations and payments
 - Optimal Mechanisms (Myerson): Design a mechanism that maximizes profits
 - Efficient Mechanisms (Vickrey-Clarke-Groves (VCG) Mechanisms): Design a mechanism to maximize a **social** or system-wide objective
- In CS/Engineering, focus is more on the design of efficient decentralized protocols that take into account incentives
 - Mechanisms in networks, distributed and online mechanisms, mechanisms that operate with limited information



Example: Single-item Auction

- **Sealed-bid auction**: every bidder submits bid in sealed envelope
- **First-price** sealed-bid auction: highest bid wins, pays amount of own bid
- **Second-price** sealed-bid auction: highest bid wins, pays amount of 2nd-highest bid



Which Auction Generates More Revenue?

- Each bid depends on
 - bidder's true valuation for the item (utility = valuation - payment)
 - bidder's beliefs over what others will bid
 - and ... auction mechanism used
- In first-price auction, it does not make sense to bid your true valuation
 - Even if you win, your utility will be 0
- In second-price auction, it always makes sense to bid your true valuation (we will see this later)



What is Mechanism Design Again?

- Designing mechanism = designing a game
- We can use game theory to predict what will happen under mechanism
 - If agents act strategically
- When is a mechanism “good”?
 - Should it result in outcomes that are “good” for **reported** or for **true** preferences?
 - Should agents ever end up lying about their preferences (in game-theoretic solution)?
 - Should it always generate the best allocation?
 - Should agents ever burn money?(!?)
- Can we solve for the optimal mechanism?



Acknowledgment

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