MS&E 233
Game Theory, Data Science and AI
Lecture 9

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Assistant Professor

Management Science and Engineering

(by courtesy) Computer Science and Electrical Engineering

Institute for Computational and Mathematical Engineering

Computational Game Theory for Complex Games

1
- Basics of game theory and zero-sum games (T)
- Basics of online learning theory (T)
- Solving zero-sum games via online learning (T)
- *HW1: implement simple algorithms to solve zero-sum games*
- Applications to ML and AI (T+A)
- *HW2: implement boosting as solving a zero-sum game*

2
- Basics of extensive-form games
- Solving extensive-form games via online learning (T)
- *HW3: implement agents to solve very simple variants of poker*

3
- General games, equilibria and online learning (T)
- Online learning in general games
- *HW4: implement no-regret algorithms that converge to correlated equilibria in general games*

Data Science for Auctions and Mechanisms

4
- **Basics and applications of auction theory (T+A)**
- Learning to bid in auctions via online learning (T)
- *HW5: implement bandit algorithms to bid in ad auctions*

5
- Optimal auctions and mechanisms (T)
- Simple vs optimal mechanisms (T)
- *HW6: calculate equilibria in simple auctions, implement simple and optimal auctions, analyze revenue empirically*

6
- Optimizing mechanisms from samples (T)
- Online optimization of auctions and mechanisms (T)
- *HW7: implement procedures to learn approximately optimal auctions from historical samples and in an online manner*

Further Topics

7
- Econometrics in games and auctions (T+A)
- A/B testing in markets (T+A)
- *HW8: implement procedure to estimate values from bids in an auction, empirically analyze inaccuracy of A/B tests in markets*

Guest Lectures

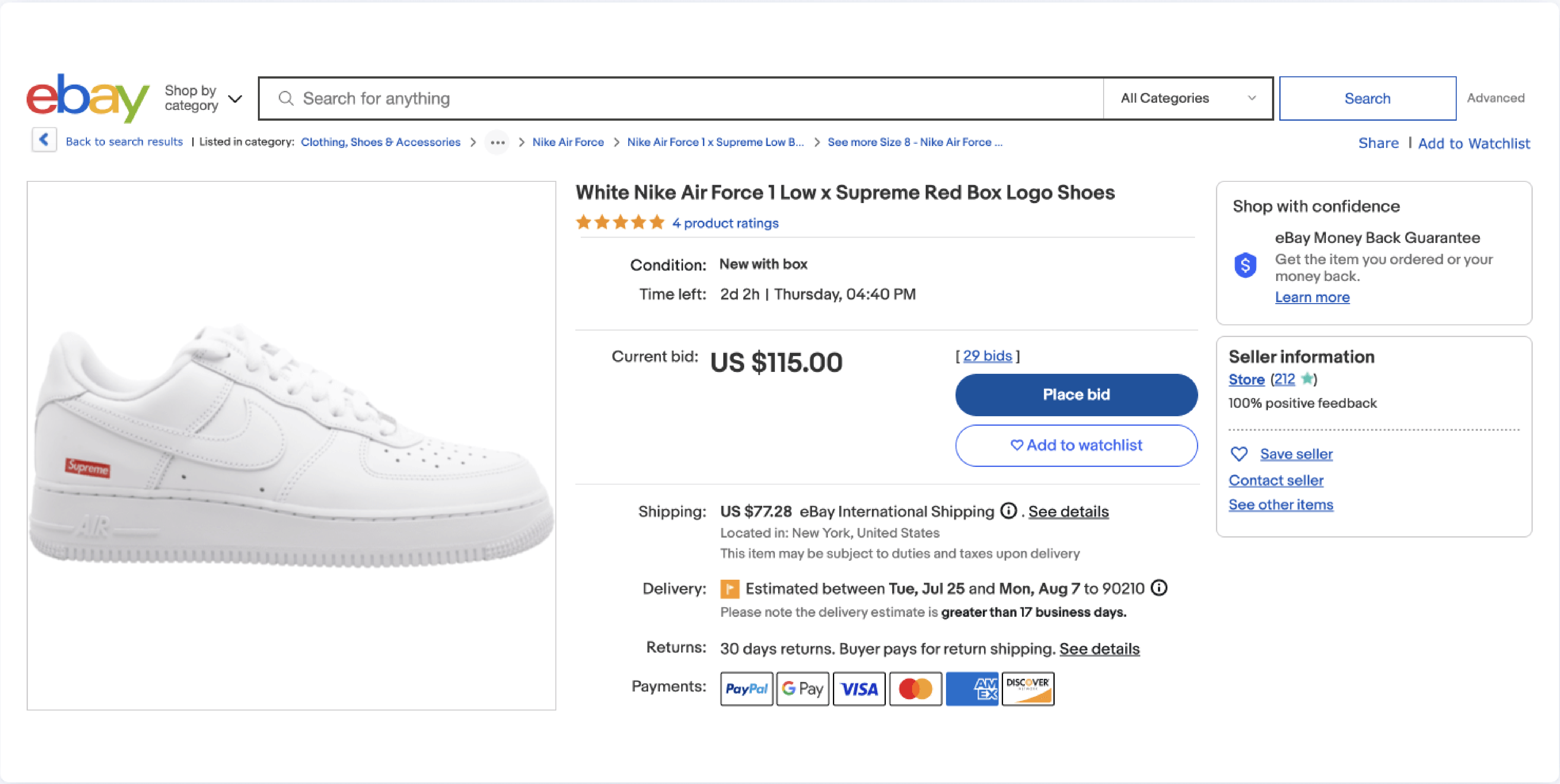
- Mechanism Design for LLMs, Renato Paes Leme, Google Research
- Auto-bidding in Sponsored Search Auctions, Kshipra Bhawalkar, Google Research

Auctions

Sotheby's Sells $7.3 Billion in Art, Fueled by Moneyed Millennials

Auction house says millennials drove art market rebound by bidding up everything from luxury goods to NFTs

Credits: https://www.wsj.com/articles/sothebys-sells-7-3-billion-in-art-fueled-by-moneyed-millennials-11639581146



Credits: https://www.3dsellers.com/blog/best-time-to-end-an-ebay-auction

Auction & dates

Here you can find the important dates of the current program:

Register by April 30, 2024

We bundle all the registered households and present them as a group to electricity providers. We conduct a reverse auction where electricity providers bid against each other to determine who can provide the lowest offer. Register now.

Auction on April 30, 2024

All interested electricity providers requested to participate in the auction, but only those who met our specific quality requirements made the cut. It's our way of ensuring customers get a smooth and reliable switch. We will be sending out an email about the winning provider and our competitive rates by **7 May**.

Sign up today

Already registered?

More Information

How it works

Signing up

Auction & dates

Credits: https://texaspowerswitch.com/texas/en/info/auction

California ISO

Search...

ABOUT US | PARTICIPATE | STAY INFORMED | PLANNING | MARKET & OPERATIONS | RULES | ISO EN ESPAÑOL

- About Us
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- Planning
- Market & Operations
- Market Processes
  - Congestion revenue rights
  - Network and Resource Modeling
  - Outage Management
  - Interchange Scheduling
  - Metering and Telemetry
  - Settlements
  - Transmission Operations
  - Power Contracts Bulletin Board
  - Reports and Bulletins
  - Market Monitoring

Market processes and products

The ISO wholesale energy market is comprised of distinct day-ahead and real-time processes. The energy products and services traded in our market allow us to meet reliability needs and serve load. The market also offers services in which qualified entities can buy and sell congestion revenue rights and engage in convergence bidding activities.

Day-ahead market

The day-ahead market is made up of three market processes that run sequentially. First, the ISO runs a market power mitigation test. Bids that fail the test are revised to predetermined limits. Then the integrated forward market establishes the generation needed to meet forecast demand. And last, the residual unit commitment process designates additional power plants that will be needed for the next day and must be ready to generate electricity. Market prices set are based on bids.

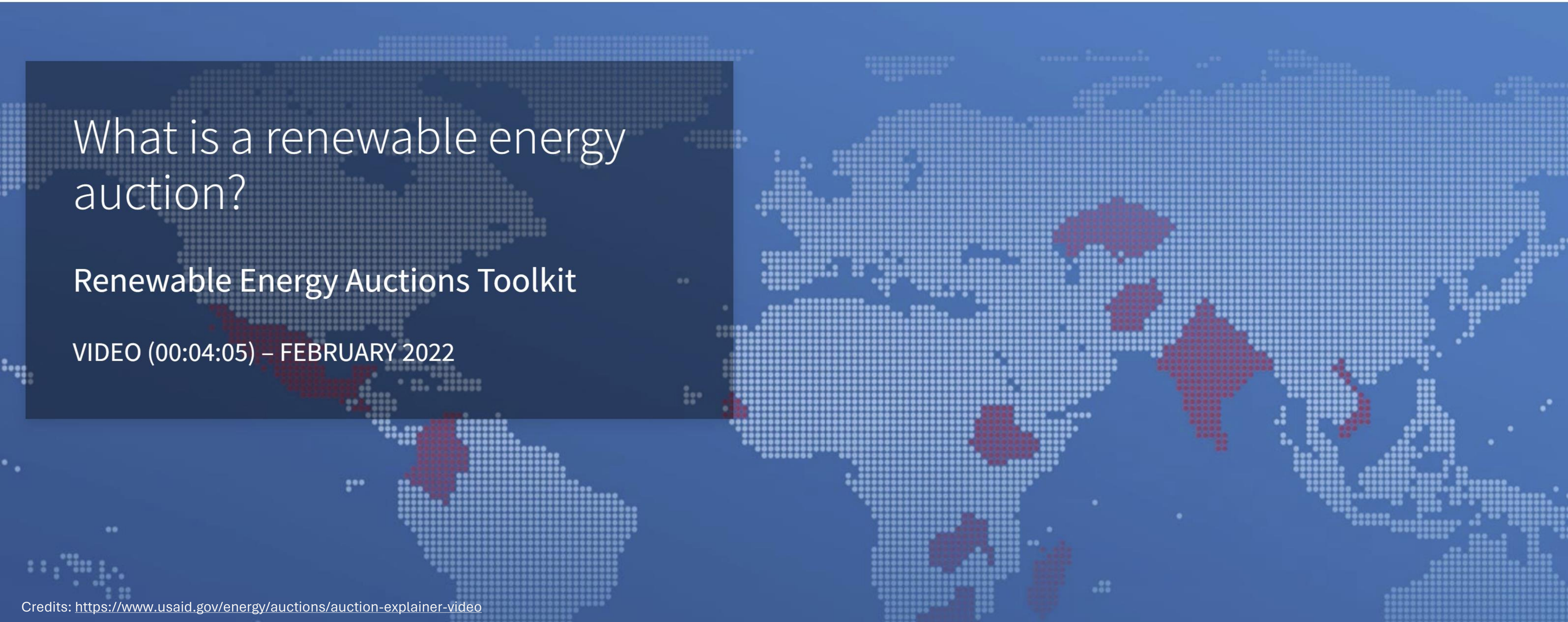
A major component of the market is the full network model, which analyzes the active transmission and generation resources to find the least cost energy to serve demand. The model produces prices that show the cost of producing and delivering energy from individual nodes, or locations on the grid where transmission lines and generation interconnect.

Scheduling coordinators (SCs) are pre-qualified entities authorized to transact in the ISO market. The day-ahead market opens for bids and schedules seven days before and closes the day prior to the trade date. Results are published at 1:00 p.m.

Real-time market

The real-time market is a spot market in which utilities can buy power to meet the last few increments of demand not covered in their day ahead schedules. It is also the market that secures energy reserves, held ready and available for ISO use if needed, and the energy needed to regulate transmission line stability.

Credits: https://www.caiso.com/market/Pages/MarketProcesses.aspx



What is a renewable energy auction?

Renewable Energy Auctions Toolkit

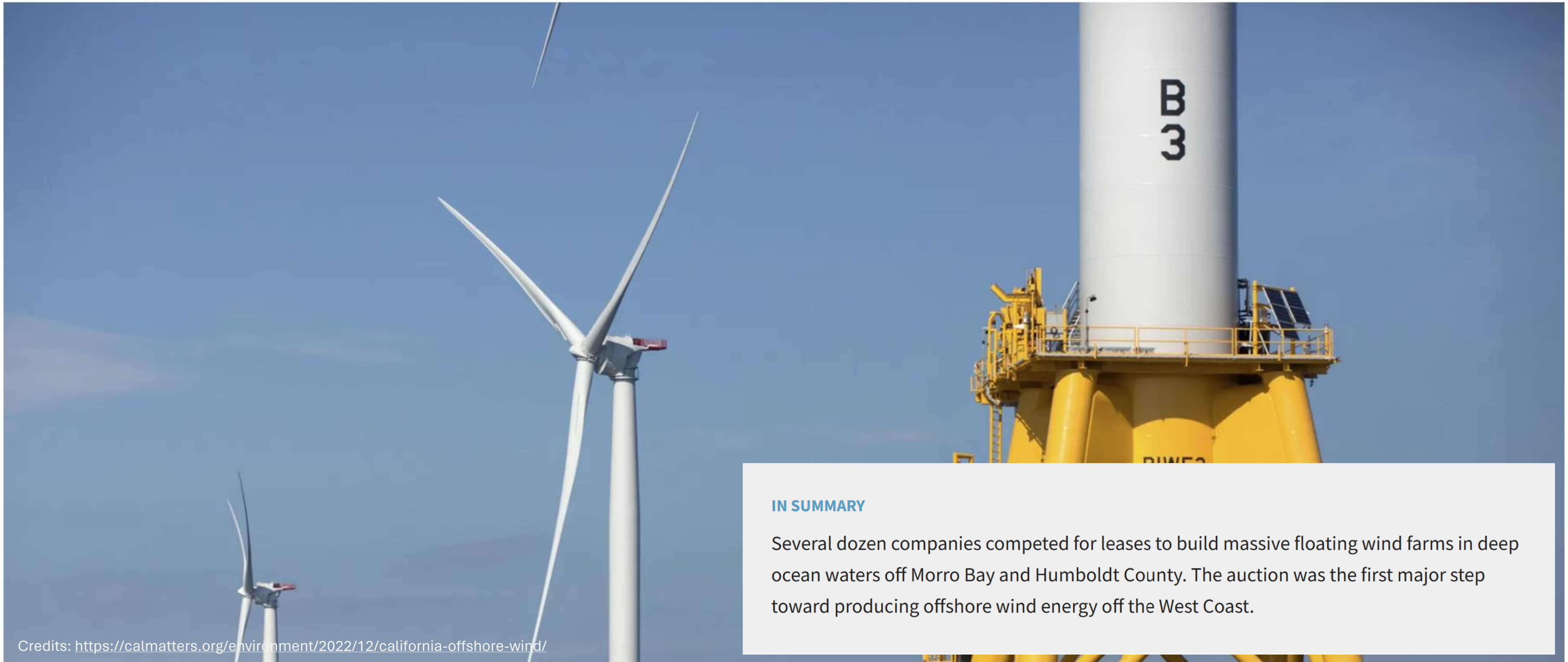
VIDEO (00:04:05) – FEBRUARY 2022

Credits: https://www.usaid.gov/energy/auctions/auction-explainer-video

First-ever California offshore wind auction nets $757 million

BY NADIA LOPEZ

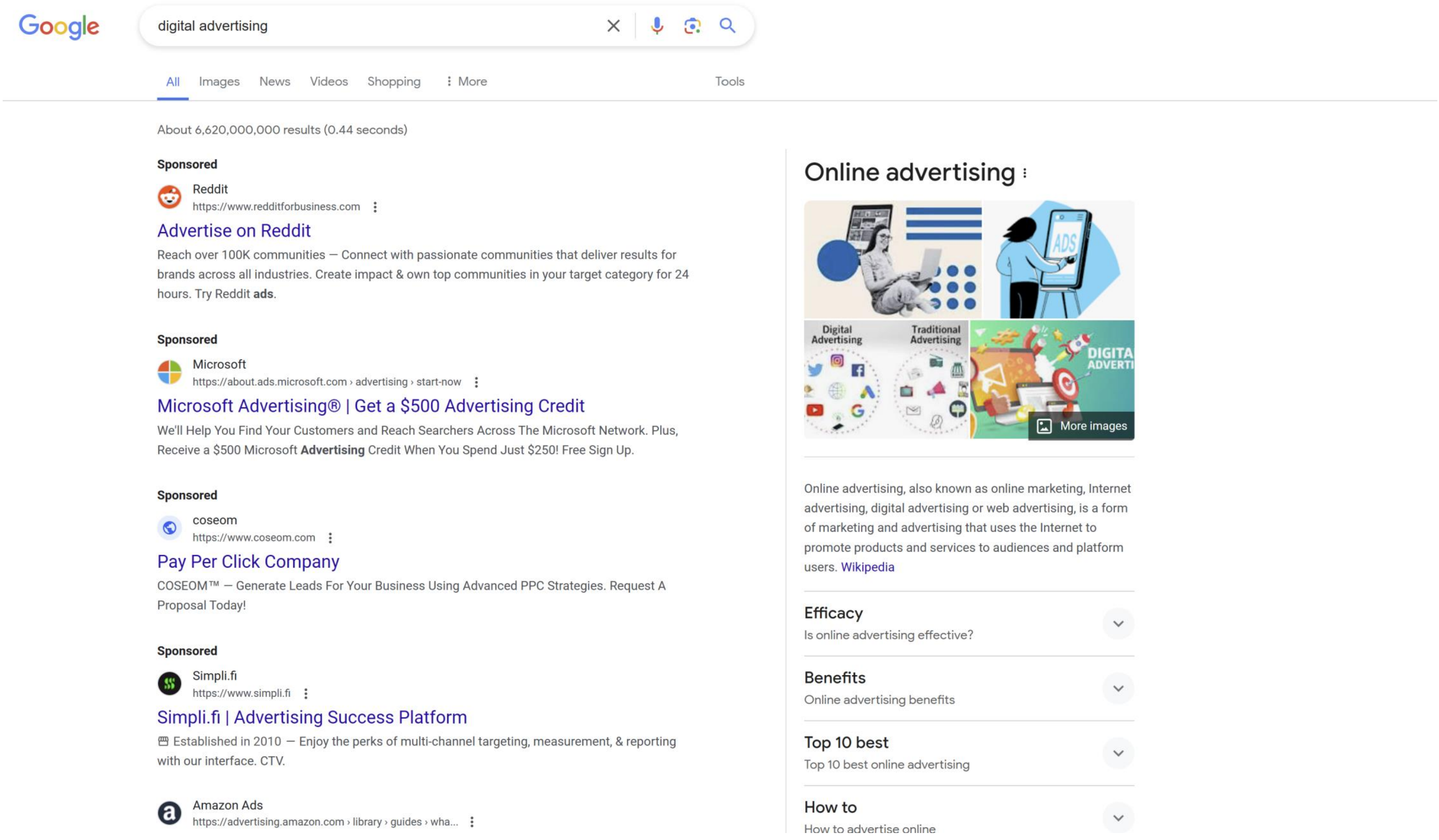
DECEMBER 6, 2022 UPDATED DECEMBER 7, 2022



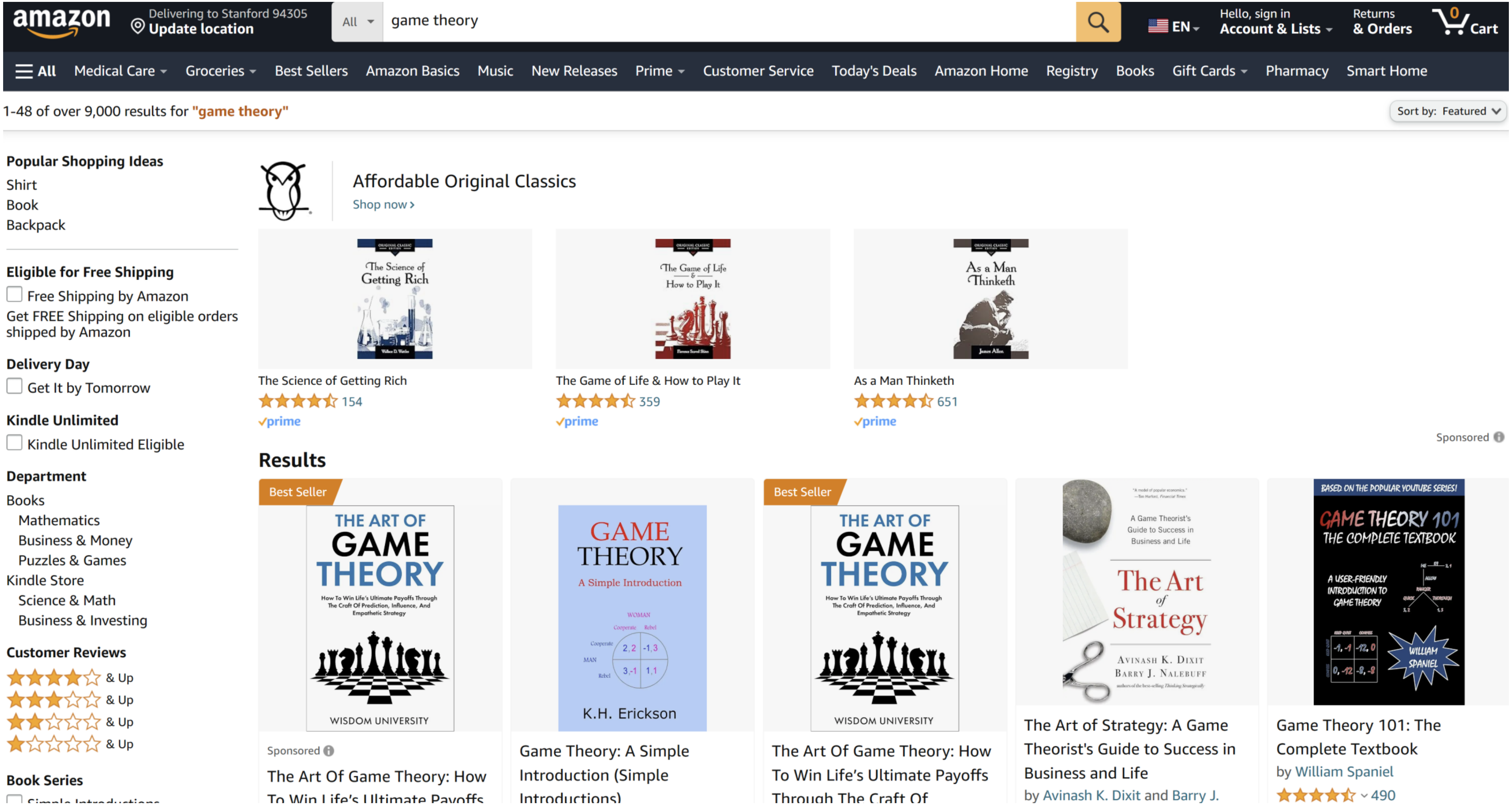
IN SUMMARY

Several dozen companies competed for leases to build massive floating wind farms in deep ocean waters off Morro Bay and Humboldt County. The auction was the first major step toward producing offshore wind energy off the West Coast.

Credits: https://calmatters.org/environment/2022/12/california-offshore-wind/



Credits: https://www.google.com/search?q=digital+advertising



Credits: https://www.amazon.com/s?k=game+theory

Auctions Summary

Auctions

Summary
General Releases
Archived Auction Releases
About Auctions
Broadcast Incentive Auction
Prohibited Communications

Completed Spectrum Auctions

	Auction	Licenses Auctioned	Licenses Won	Net Winning Bids	Rounds
1	**Nationwide Narrowband (PCS)** 7/25/1994 - 7/29/1994	10	10	$617,006,674	47
2	**Interactive Video and Data Services (IVDS)** 7/28/1994 - 7/29/1994 Map: CMA (MSA & RSA) (pdf)	594	594	$213,892,375	Oral Outcry
3	**Regional Narrowband (PCS)**				

Credits: https://www.fcc.gov/auctions-summary

Auction 97: Advanced Wireless Services (AWS-3)

Summary

November 13, 2014 - January 29, 2015

Rounds:	341
Bidding Days:	45
Qualified Bidders:	70
Winning Bidders:	31 Bidders won 1611 Licenses
Licenses Held by FCC:	3
Gross Bids:	$44,899,451,600
Net Bids:	$41,329,673,325

Credits: https://www.fcc.gov/auctions-summary

Federal Communications Commission

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About the FCC | Proceedings & Actions | Licensing & Databases | Reports & Research | News & Events | For Consumers

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Auction 107: 3.7 GHz Service

Go to an auction

Select an Auction

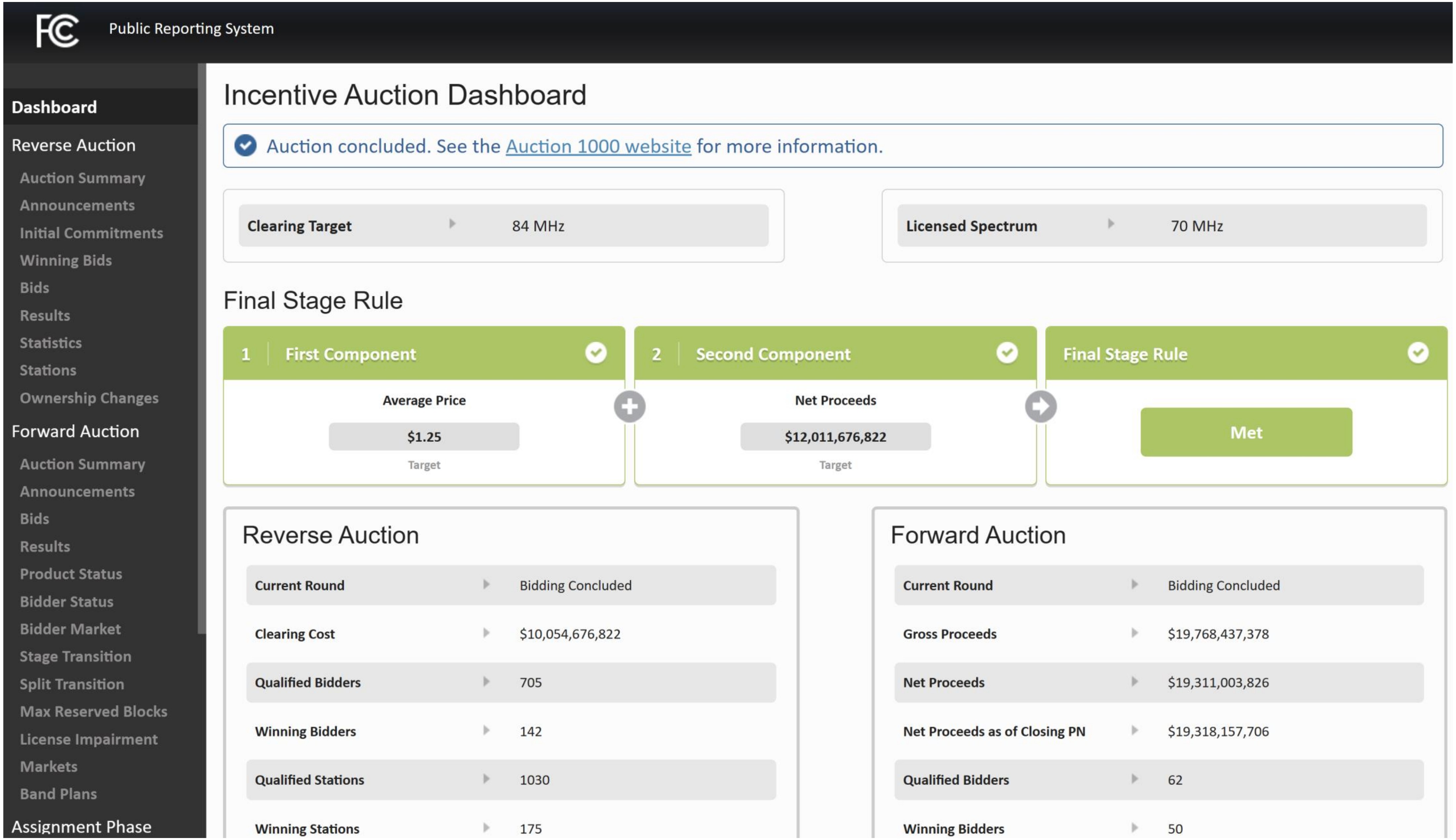
- Summary
- General Releases
- Archived Auction Releases
- About Auctions

Summary | Fact Sheet | Releases | Education | Results | Application Search

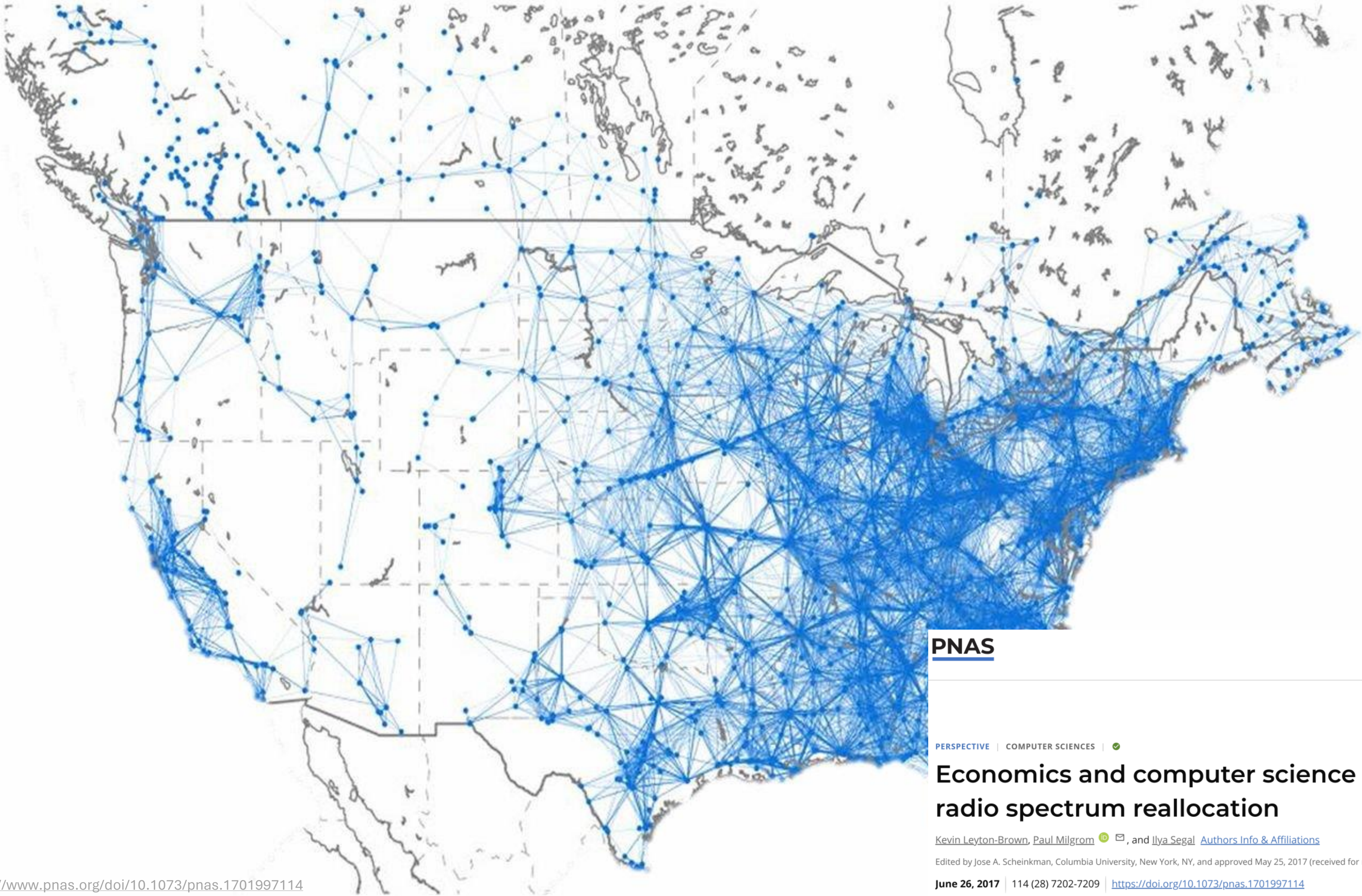
Fact Sheet

Winning Bidders:	21
Qualified Bidders:	57
Licenses Won:	5684
Licenses Held by FCC:	0
Total Licenses:	5684
Net Bids:	$81,114,481,921
Gross Bids:	$81,168,677,645

Credits: https://www.fcc.gov/auctions-summary



Credits: https://auctiondata.fcc.gov/public/projects/1000



PNAS

PERSPECTIVE | COMPUTER SCIENCES

Economics and computer science of a radio spectrum reallocation

Kevin Leyton-Brown, Paul Milgrom, and Ilya Segal Authors Info & Affiliations

Edited by Jose A. Scheinkman, Columbia University, New York, NY, and approved May 25, 2017 (received for review February 4, 2017)

June 26, 2017 | 114 (28) 7202-7209 | https://doi.org/10.1073/pnas.1701997114

Credits: https://www.pnas.org/doi/10.1073/pnas.1701997114

What is EIP-1559?

Ethereum Improvement Proposal (EIP) 1559 is an upgrade that happened on August 5, 2021 to change how Ethereum calculates and processes network transaction fees (called "gas fees"). The upgrade made Ethereum transactions more efficient by using a system of block-based base fees, and sender-specified max fees, rather than bidding on gas prices to more evenly incentivize miners in periods of high or low network congestion. It was packaged with the London hard fork. Four other EIPs will join EIP 1559 in London.

Credits: https://help.coinbase.com/en/coinbase/getting-started/crypto-education/eip-1559

Transaction Fee Mechanism Design for the Ethereum Blockchain: An Economic Analysis of EIP-1559

Tim Roughgarden

EIP-1559 is a proposal to make several tightly coupled additions to Ethereum's transaction fee mechanism, including variable-size blocks and a burned base fee that rises and falls with demand. This report assesses the game-theoretic strengths and weaknesses of the proposal and explores some alternative designs.

Subjects: **Computer Science and Game Theory (cs.GT)**; Distributed, Parallel, and Cluster Computing (cs.DC); Data Structures and Algorithms (cs.DS); Theoretical Economics (econ.TH)

Cite as: arXiv:2012.00854 **[cs.GT]**
(or arXiv:2012.00854v1 **[cs.GT]** for this version)
https://doi.org/10.48550/arXiv.2012.00854

Credits: https://arxiv.org/abs/2012.00854
Further reading: https://arxiv.org/abs/2106.01340v3

Auction Basics

Auction Basics

- n bidders are interested in acquiring an item

- Bidder i has value v_i for the item
- Value is known only to them (private information)
- If bidder wins the item ($x_i = 1$) they gain a value v_i

- If at the end they are asked to pay a price p_i they gain

$$u_i(x_i, p_i; v_i) = v_i \cdot x_i - p_i$$

Sealed-Bid Auctions

- Each bidder privately communicates a bid b_i to the auctioneer

- Auctioneer applies an allocation rule x to bid vector $b = (b_1, \ldots, b_n)$

$$x_i(b) = \text{Probability bidder } i \text{ gets the item}, \qquad \sum_i x_i(b) \leq 1$$

- Auctioneer applies a price rule p to bid vector $b = (b_1, \ldots, b_n)$

$$p_i(b) = \text{Price that bidder } i \text{ is asked to pay}$$

Welfare of the outcome

- The expected welfare of the outcome of an auction is the expected value of the winner!

$$SW(b) = \sum_i x_i(b) \cdot v_i$$

- It depends on the allocation rule of the auction and on how bidders bid as a function of their value (which in turn depends on the auction allocation and payment rules)

Revenue of the outcome

- The expected revenue of the outcome of an auction is the expected total payments made to the auctioneer!

$$Rev(b) = \sum_i p_i(b)$$

Total utility of the outcome

- The expected total utility of the outcome of an auction is the expected total net gains made by the bidders!

$$U(b) = \sum_i v_i \cdot x_i(b) - p_i(b) = SW(b) - Rev(b)$$

How would you maximize welfare?

What if we just elicit bids, give it to the highest bidder, and don't charge anything…

First-Price Auction

- Arguably the simplest auction to describe

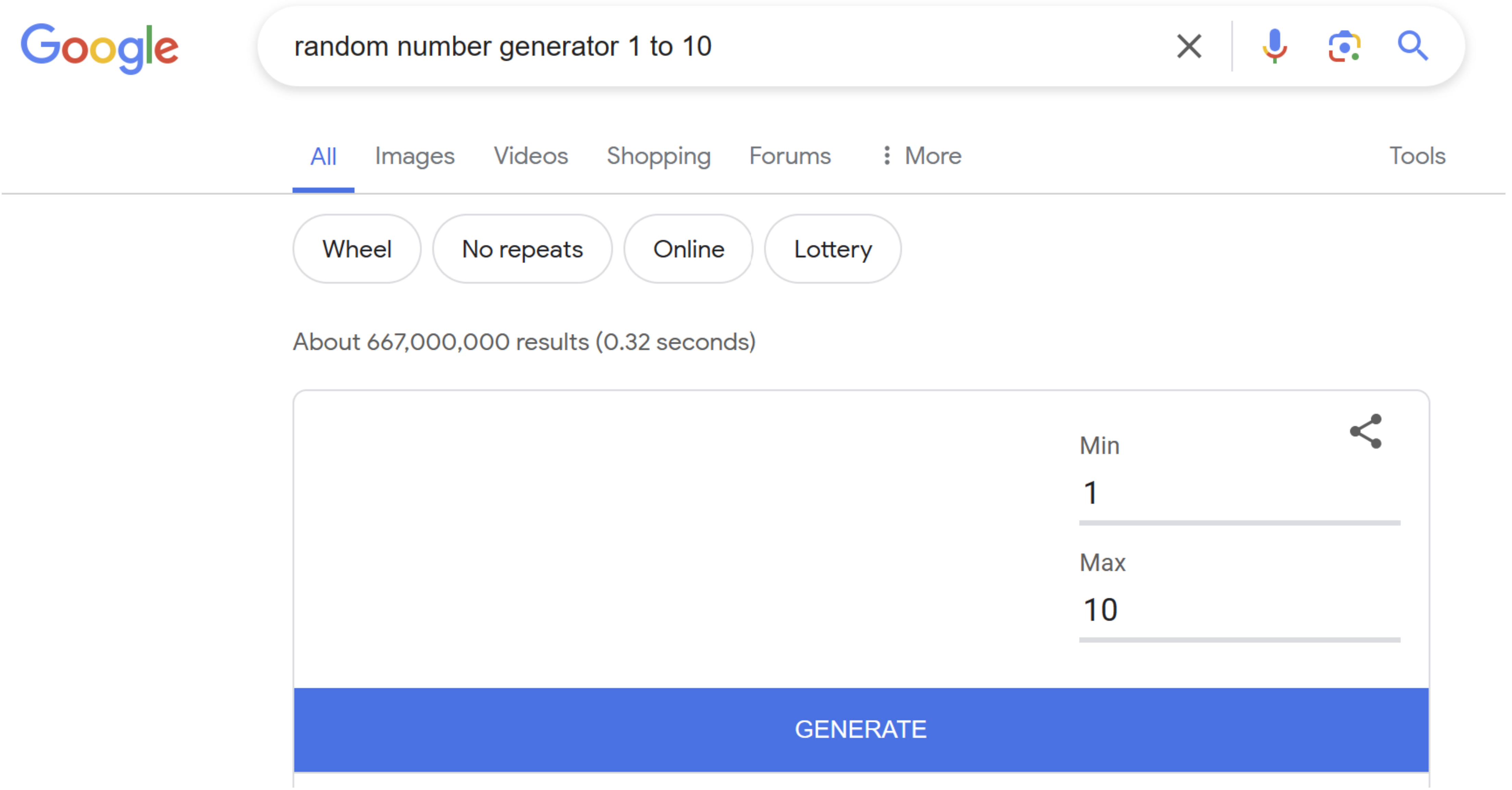
- Each bidder submits a bid b_i
- The highest bidder wins the item (ties broken at random)
- The winner pays their bid

- Utility of a bidder with value v_i under a bid profile b:

$$u_i(b; v_i) = (v_i - b_i) \cdot 1\{b_i \geq \max b_j\}$$

How would you bid!

https://www.google.com/search?q=random+number+generator+1+to+10



You will be participate in a first price auction for an item, competing with 1 other randomly chosen student in class. Submit your bid!

- 0
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10

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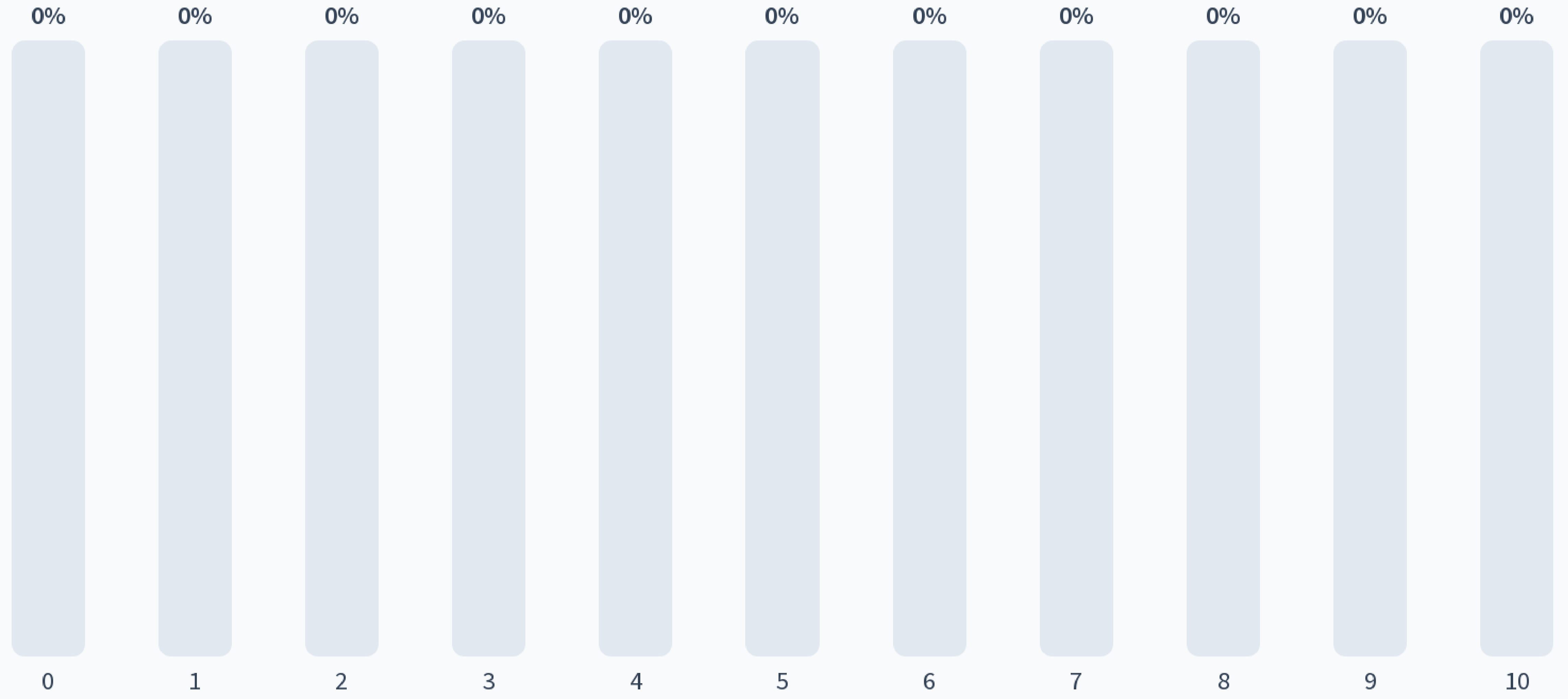

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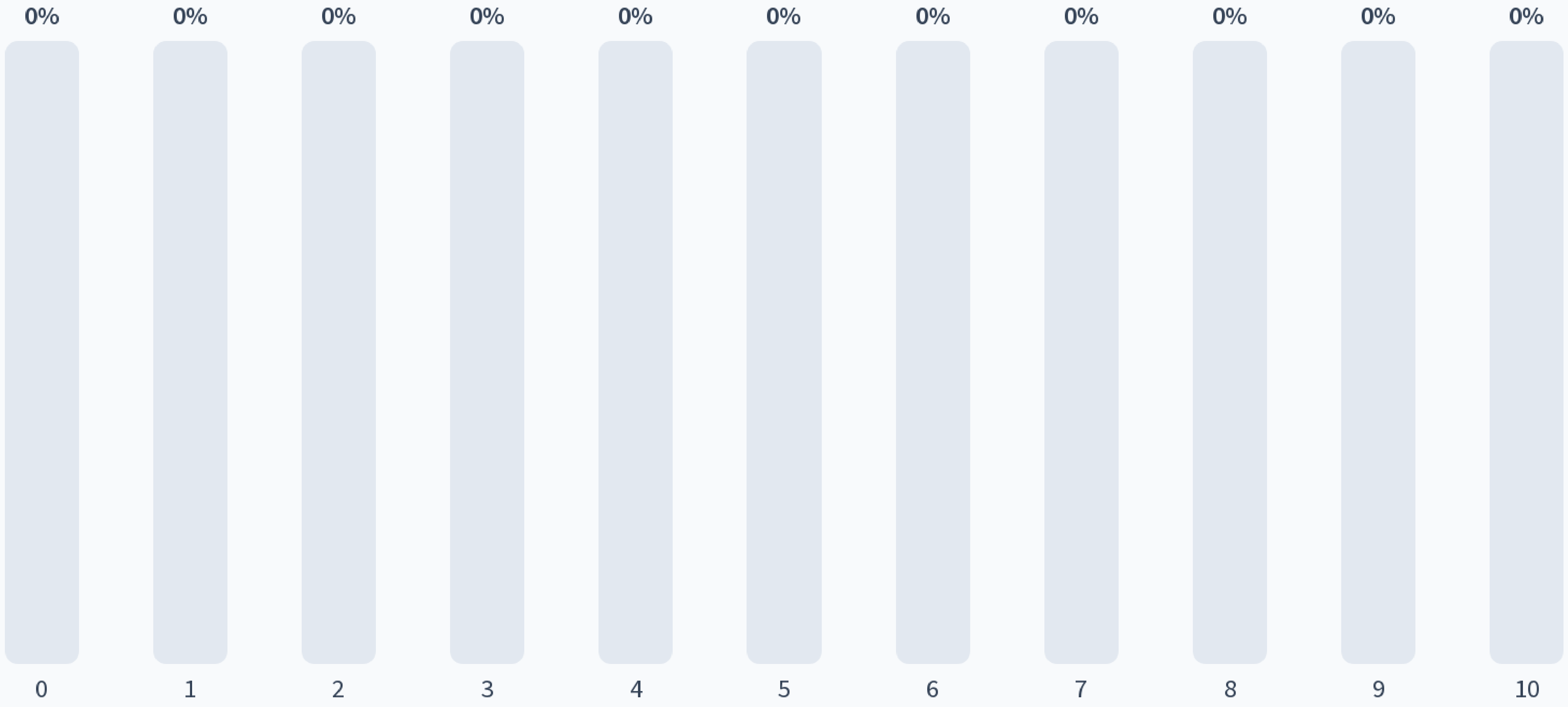

You will be participate in a first price auction for an item, competing with 10 other randomly chosen students in class. Submit your bid!

- 0
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10

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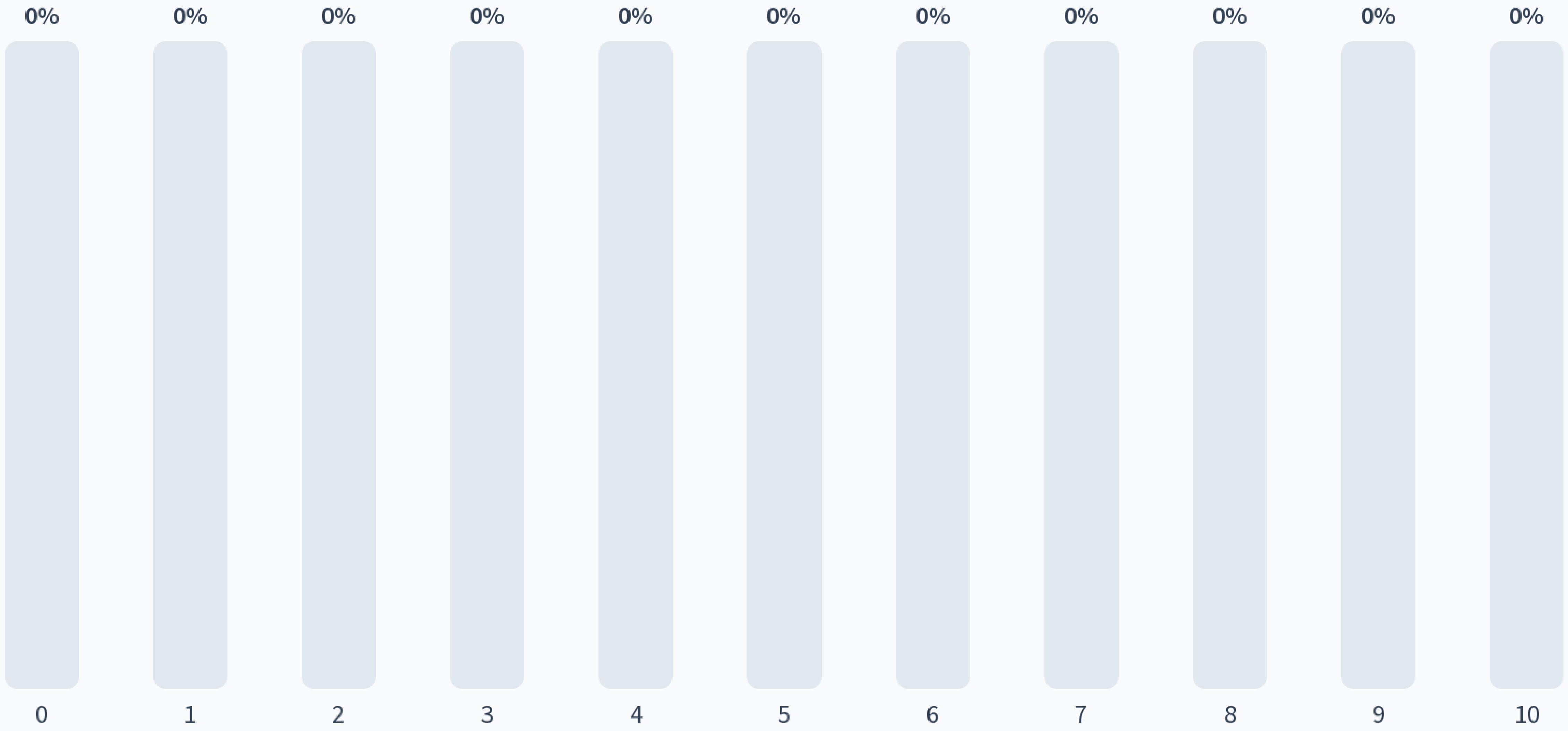

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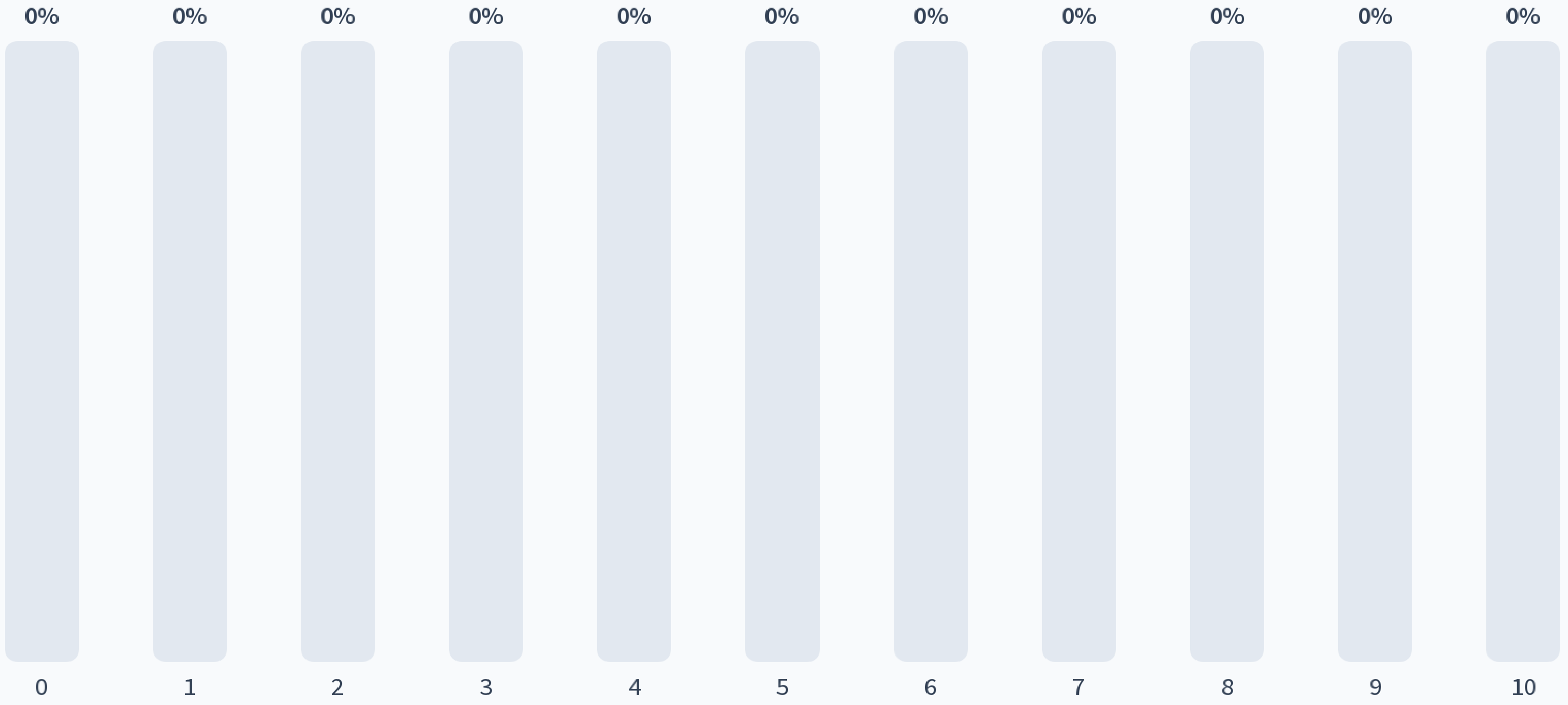

Bayesian Formulation

- Each bidder's value is drawn from some distribution

$$v_i \sim F_i, \qquad v = (v_1, \dots, v_n) \sim F = F_1 \times \cdots \times F_n$$

- Bidders submit a bid as a function of their value

$$s_i(v_i) = \text{Bid of player } i \text{ when their value is } v_i$$

Bayes-Nash Equilibrium. A bidding strategy profile $s = (s_1, \dots, s_n)$ is a Bayes-Nash equilibrium, if players cannot gain by deviating in expectation, assuming others follow their strategies

$$E_{v\sim F}[u_i(s(v); v_i)] \geq E_{v\sim F}\left[u_i\left(b_i', s_{-i}(v_{-i}); v_i\right)\right]$$

Equilibrium of a First Price Auction (FPA)

- Consider a FPA with two bidders
- Each player's value is distributed uniformly in $[0, 1]$
- It suffices to look at symmetric bidding strategies

Theorem. The following is a Bayes-Nash equilibrium

$$s_i = \frac{1}{2} v_i$$

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Theorem. The following is a Bayes-Nash equilibrium

$$s_i = \frac{1}{2} v_i$$

Proof. Suppose bidder 2 follows strategy. Bidder 1 utility from $b_1 \in \left[0, \frac{1}{2}\right]$:

$$(v_1 - b_1) \Pr\left(b_1 > \frac{1}{2} v_2\right) = (v_1 - b_1) \Pr(2b_1 > v_2) = (v_1 - b_1) 2b_1$$

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FOC with respect to b_1: $2v_1 - 4b_1 = 0 \Rightarrow b_1 = v_1/2$!

Equilibrium of a First Price Auction (FPA)

- Consider a FPA with n bidders
- Each player's value is distributed uniformly in $[0, 1]$

Theorem. The following is a Bayes-Nash equilibrium

$$s_i = \left(1 - \frac{1}{n}\right) v_i$$

Equilibrium of a First Price Auction (FPA)

- Consider a FPA with n bidders
- Each player's value is distributed uniformly in $[0, 1]$

Theorem. The following is a Bayes-Nash equilibrium

$$s_i = \left(1 - \frac{1}{n}\right) v_i$$

Proof. Suppose others follow strategy. Bidder 1 utility from $b_1 \in [0, 1/2]$:

$$(v_1 - b_1) \Pr\left(b_1 > \frac{n-1}{n} \max_{j \neq i} v_j\right) = (v_1 - b_1) \prod_{j \neq i} \Pr\left(\frac{n}{n-1} b_1 > v_j\right) = (v_1 - b_1) \left(\frac{n}{n-1} b_1\right)^{n-1}$$

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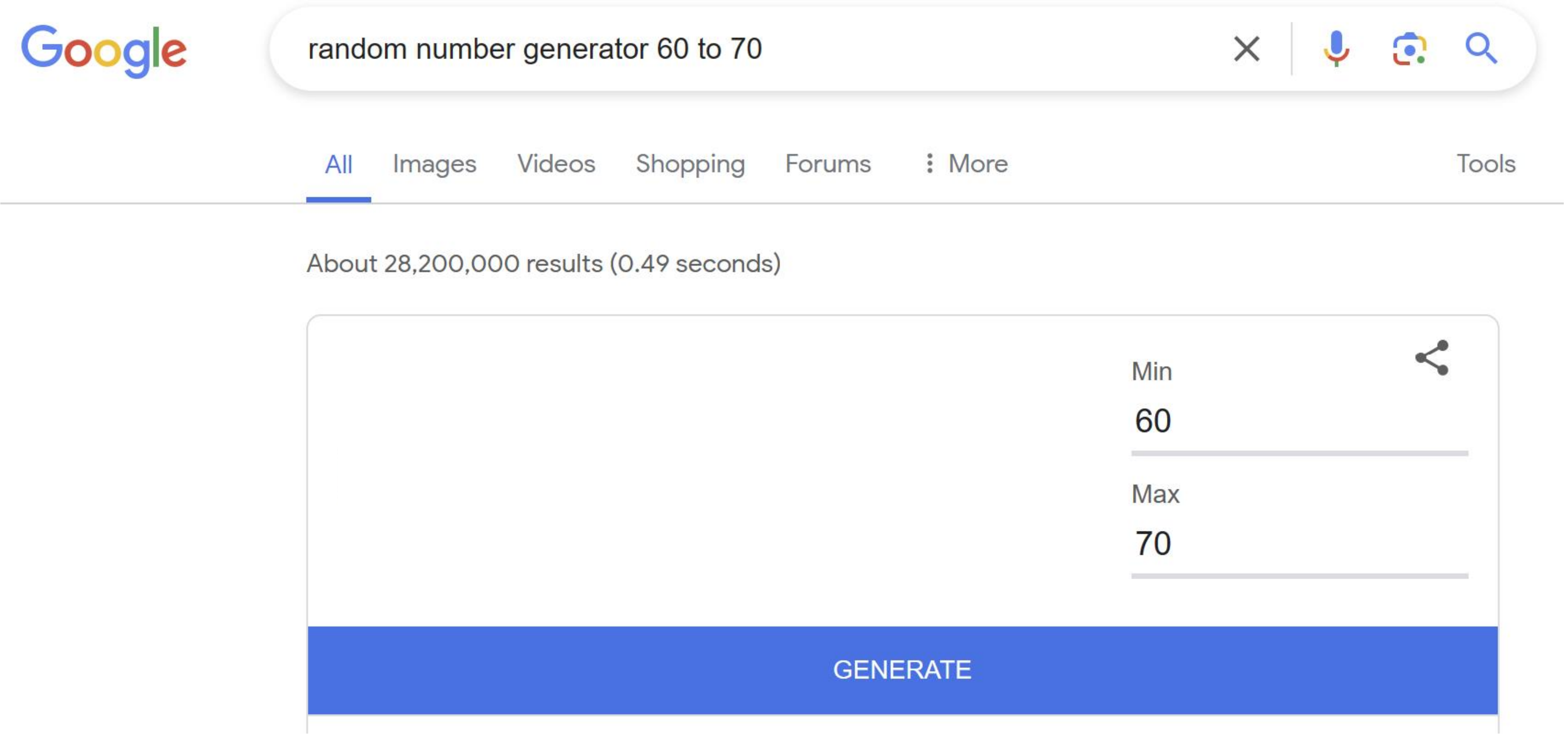
FOC with respect to b_1:

$$(n-1) \frac{n}{n-1} \left(\frac{n}{n-1} b_1\right)^{n-2} (v_1 - b_1) - \left(\frac{n}{n-1} b_1\right)^{n-1} = 0 \Rightarrow n(v_1 - b_1) - \frac{n}{n-1} b_1 = 0$$

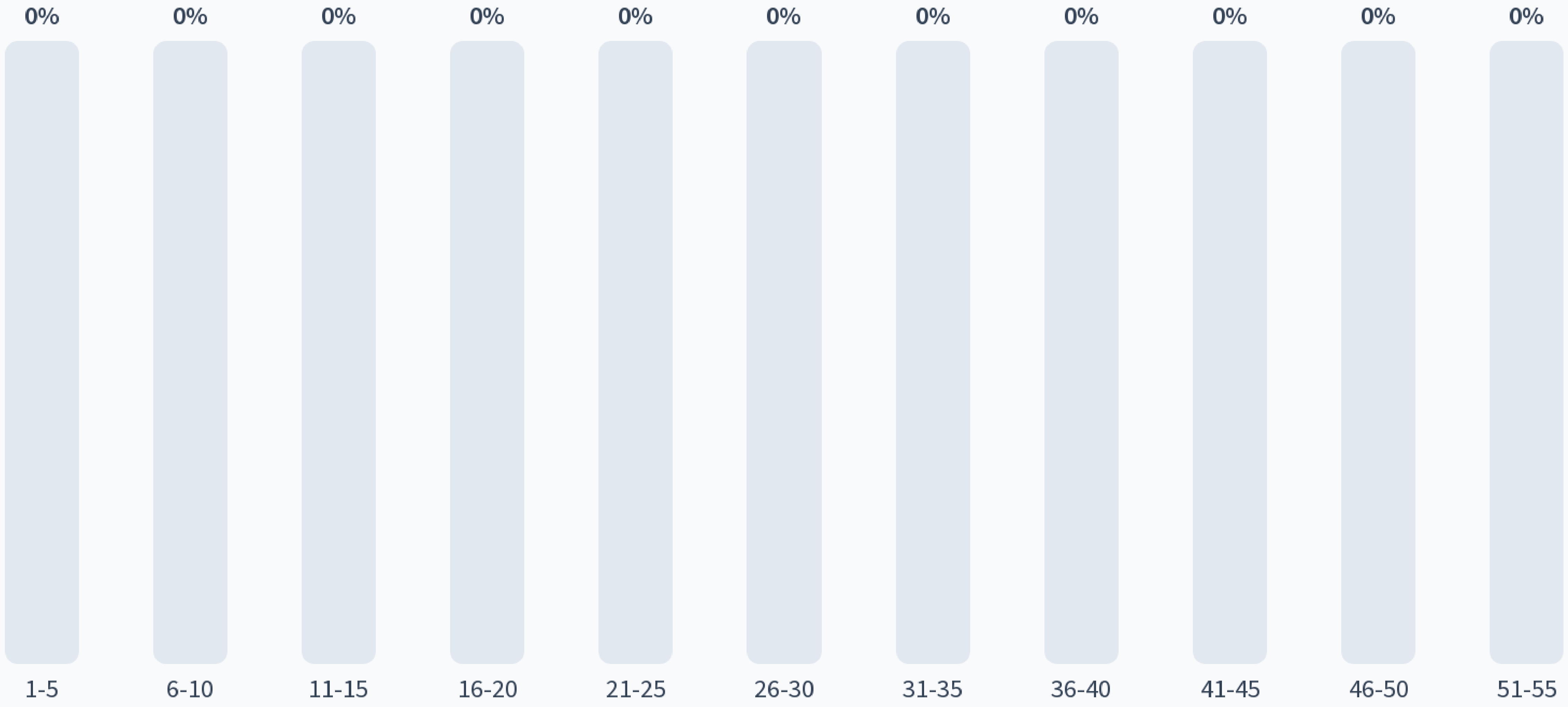
https://www.google.com/search?q=random+number+generator+60+to+70



You will be participate in a first price auction with a competitor whose value is distributed uniformly in [0, 50]. Submit your bid!

- 1-5
- 6-10
- 11-15
- 16-20
- 21-25
- 26-30
- 31-35
- 36-40
- 41-45
- 46-50
- 51-55

You will be participate in a first price auction with a competitor whose value is distributed uniformly in [0, 50]. Submit your bid!



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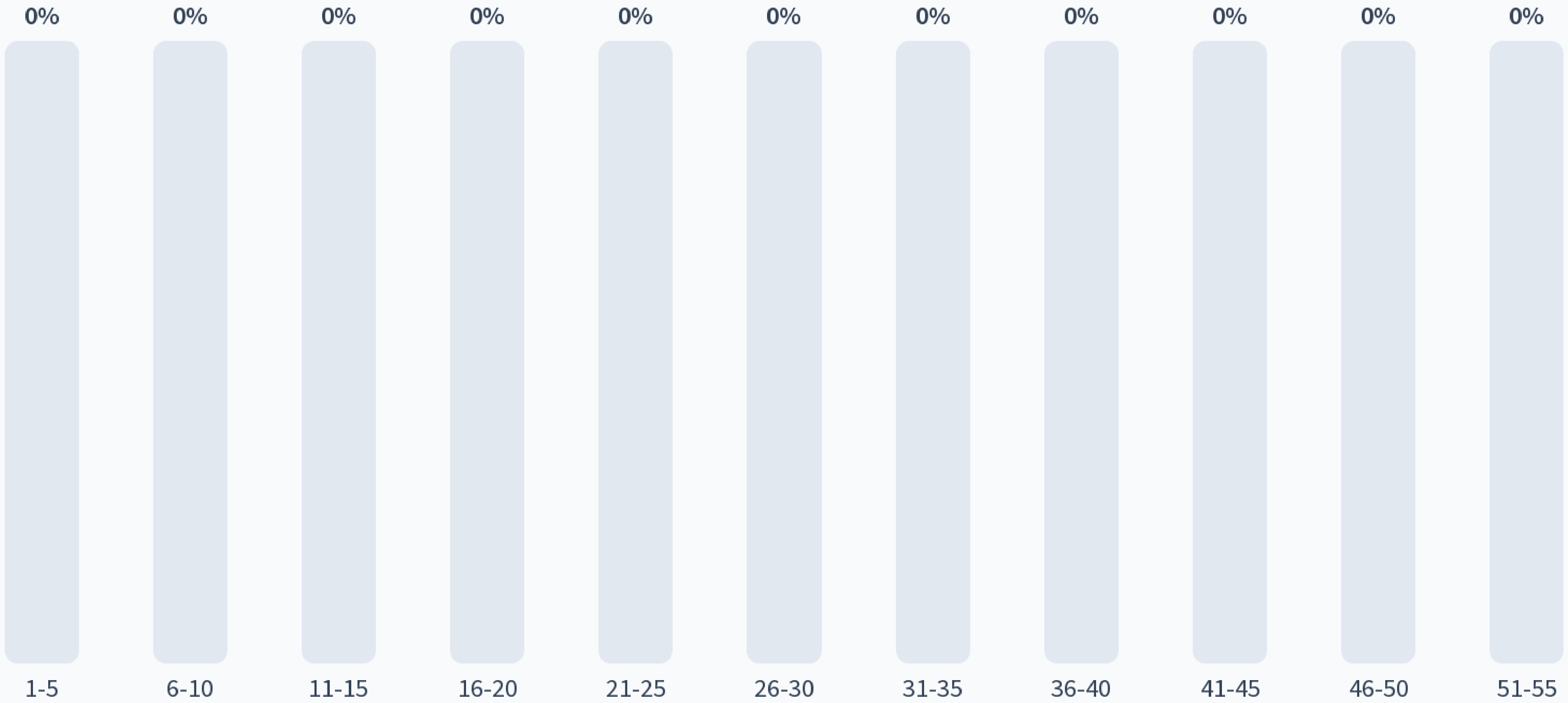

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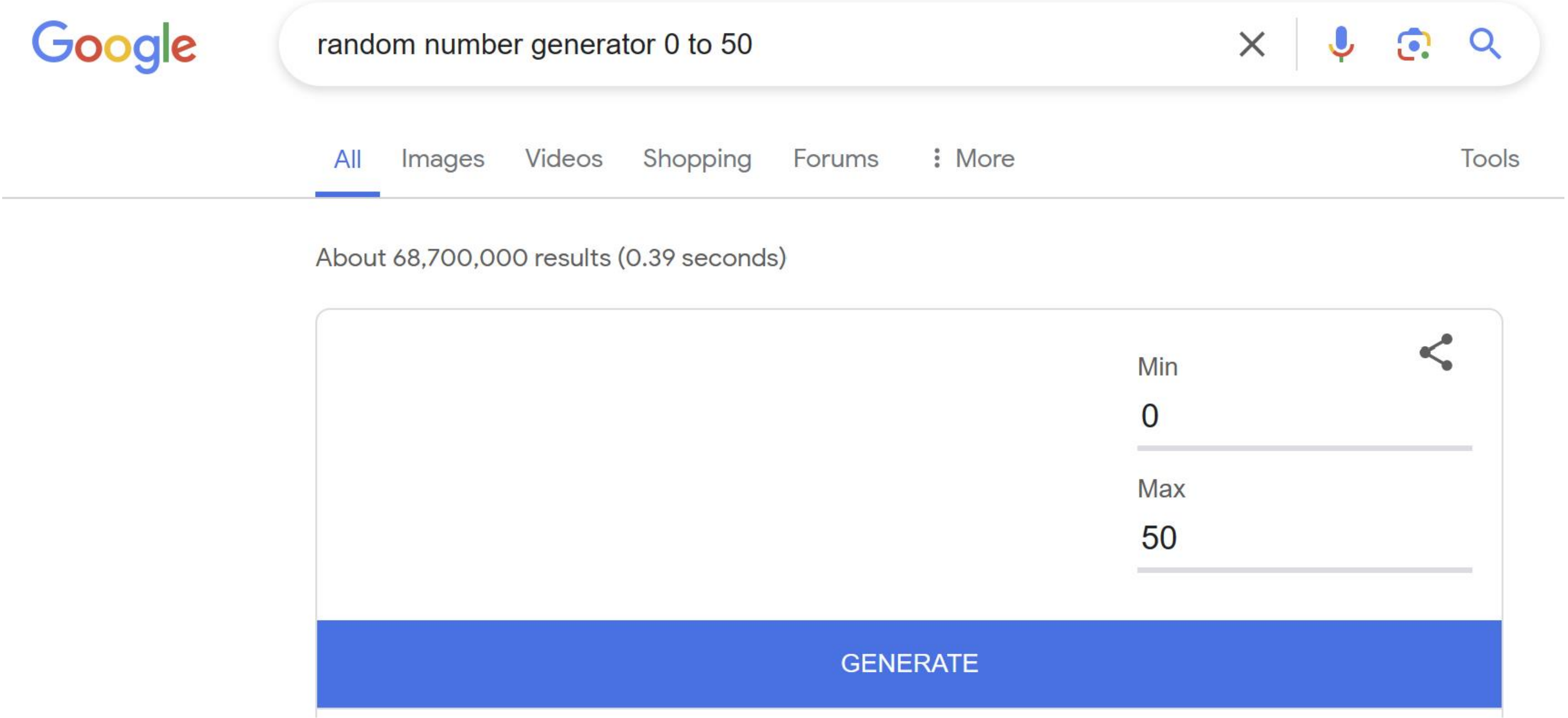

You will be participate in a first price auction with a competitor whose value is distributed uniformly in [60, 70]. Submit your bid!

- 1-5
- 6-10
- 11-15
- 16-20
- 21-25
- 26-30
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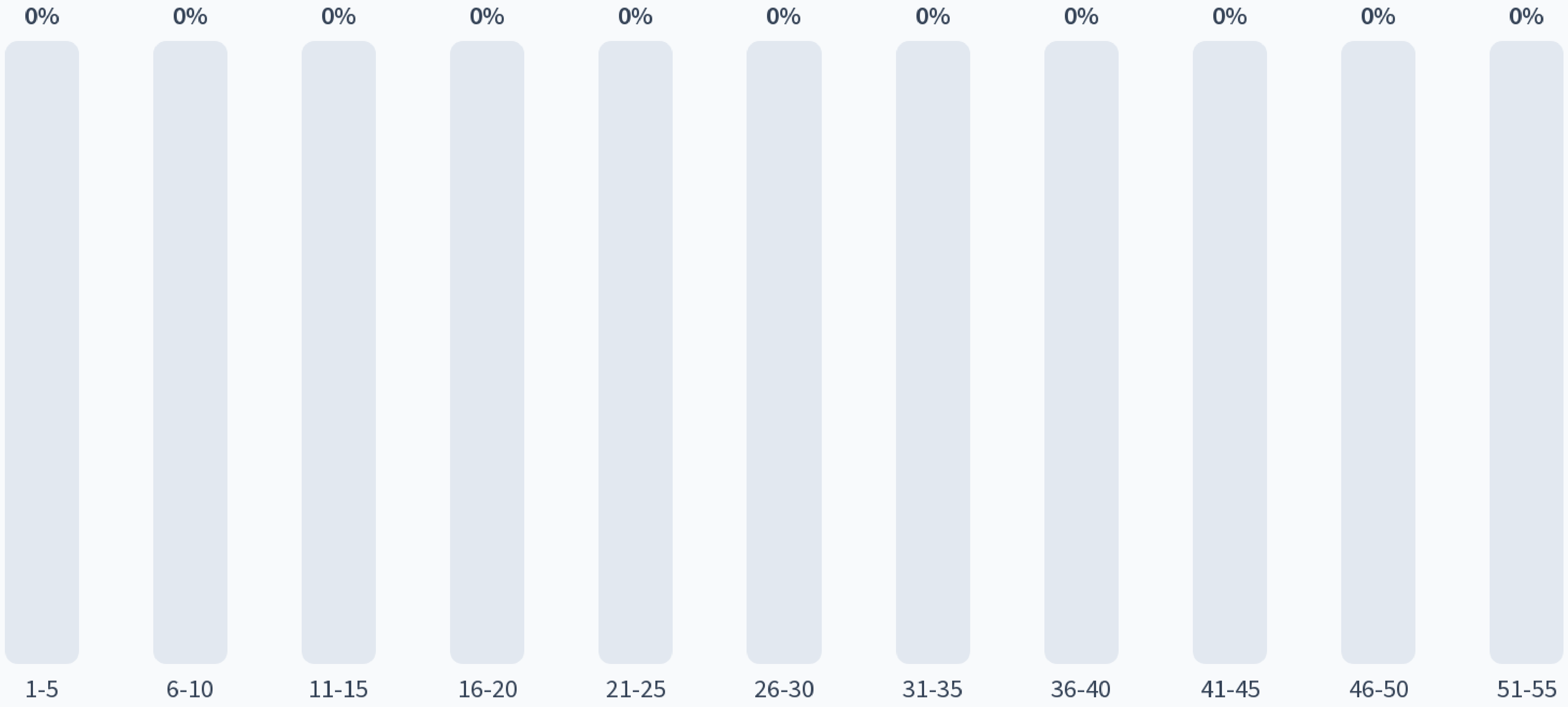

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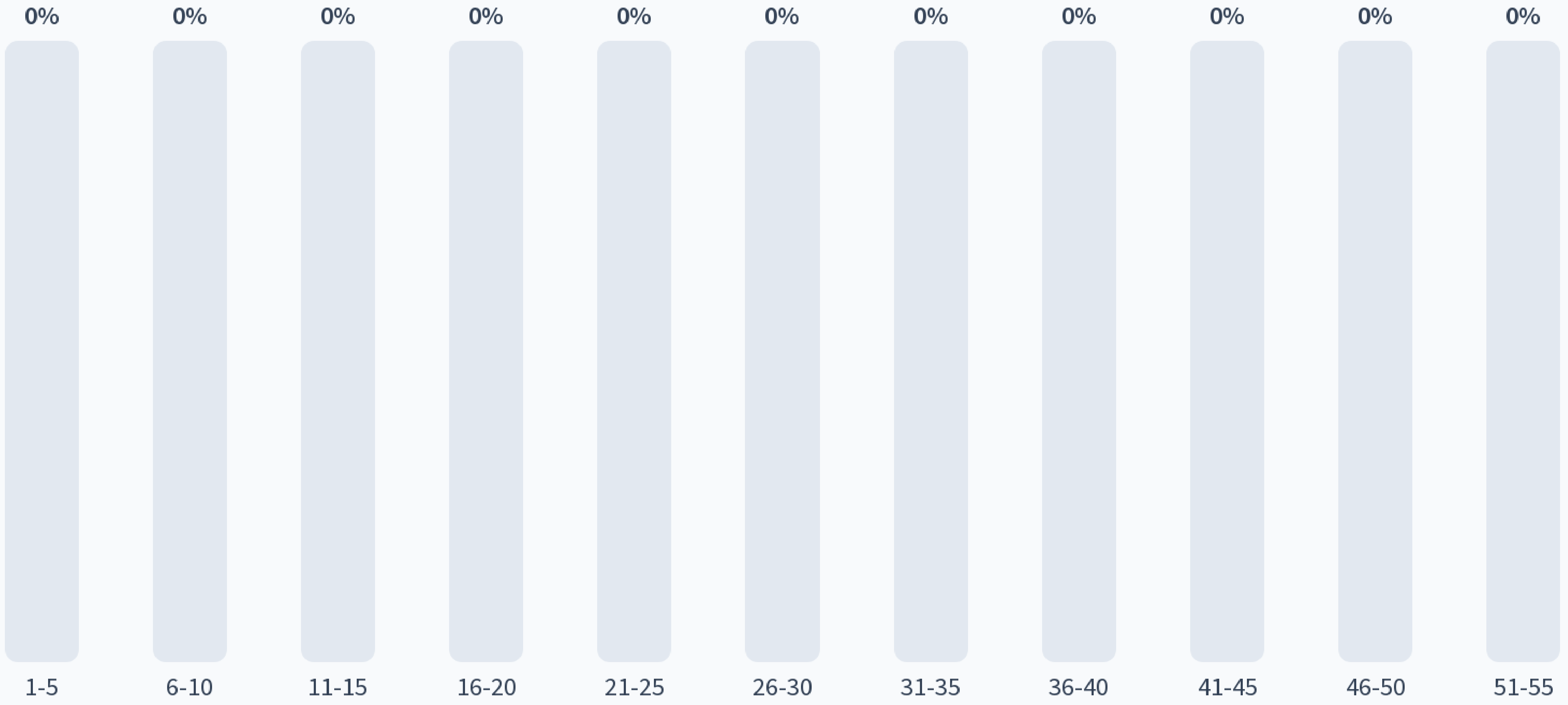

Equilibrium with Asymmetric Bidders

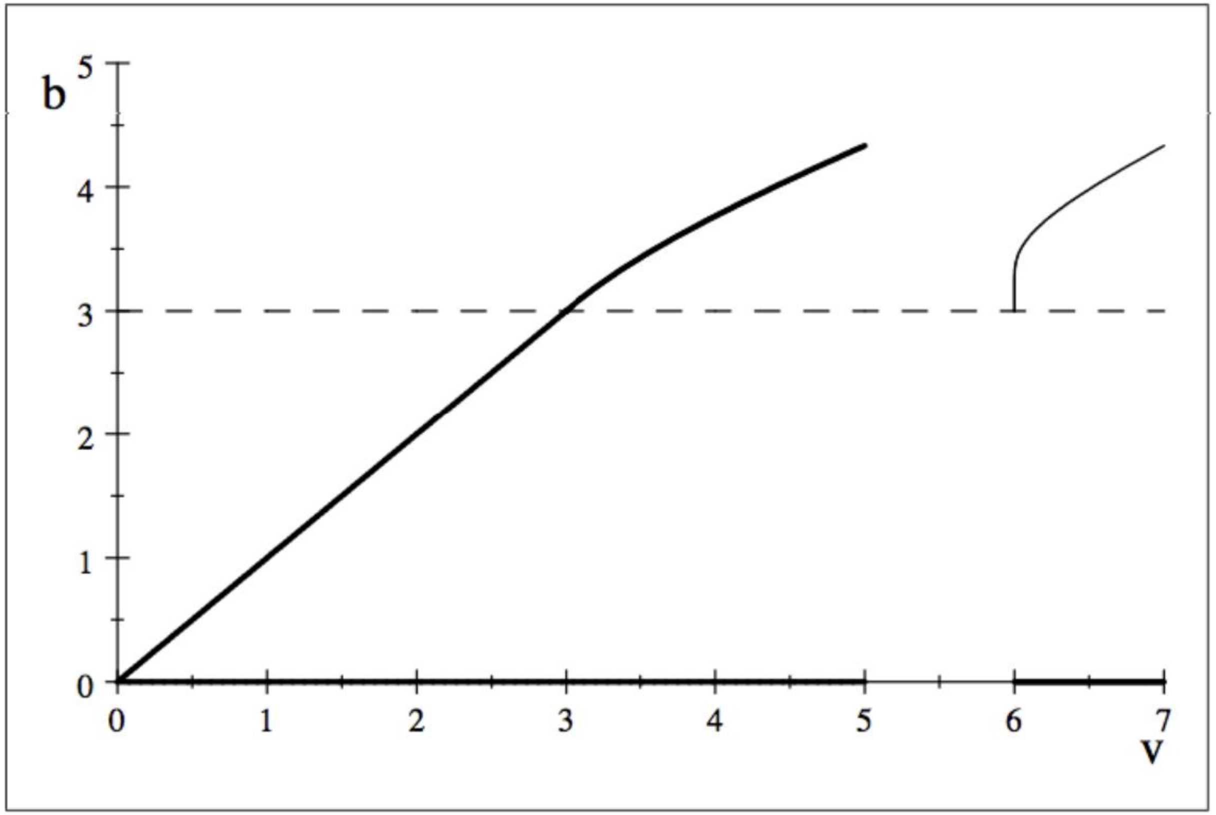

Figure 1: Equilibrium 1. The thicker line is buyer 1's bid function.

Inefficiency of First Price Auction

- At equilibrium, with positive probability, lower value player wins!
- Expected welfare of auction, not equal to expected highest value!
- The auction can be “inefficient”!
- How inefficient?

Efficiency Guarantees

- At any equilibrium, each bidder does not want to deviate to bidding half of their value!
- Either wins with such a bid and gains:

$$U_i' = v_i - \frac{v_i}{2} = \frac{v_i}{2}$$

- Or loses, in which case highest bid is that high:

$$\max_j b_j \geq \frac{v_i}{2}$$

$$\max_j b_j + U_i' \geq \frac{v_i}{2}$$

$$\Downarrow$$

$$U_i' \geq \frac{v_i}{2} - \max_j b_j$$

Efficiency Guarantees

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$$\max_j b_j + U_i' \geq \frac{v_i}{2}$$

⇓

$$U_i' \geq \frac{v_i}{2} - \max_j b_j$$

- Equilibrium utility is at least this much and always non-negative:

$$U_i' \geq \left(\frac{v_i}{2} - \max_j b_j\right) 1(\text{highest value})$$

Efficiency Guarantees

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$$\max_j b_j + U_i' \geq \frac{v_i}{2}$$

$$\Downarrow$$

$$U_i' \geq \frac{v_i}{2} - \max_j b_j$$

- Equilibrium utility is at least this much and always non-negative:

$$E\left[U_i'\right] \geq E\left[\left(\frac{v_i}{2} - \max_j b_j\right) 1(\text{highest value})\right]$$

Efficiency Guarantees

- At any equilibrium, each bidder does not want to deviate to bidding half of their value!
- Either wins with such a bid and gains:

$$U_i' = v_i - \frac{v_i}{2} = \frac{v_i}{2}$$

- Or loses, in which case highest bid is that high:

$$\max_j b_j \geq \frac{v_i}{2}$$

$$\max_j b_j + U_i' \geq \frac{v_i}{2}$$

$$\Downarrow$$

$$U_i' \geq \frac{v_i}{2} - \max_j b_j$$

- Equilibrium utility is at least this much and always non-negative:

$$E[U_i(b)] \geq E[U_i'] \geq E\left[\left(\frac{v_i}{2} - \max_j b_j\right) 1(\text{highest value})\right]$$

Efficiency Guarantees

- At any equilibrium, each bidder does not want to deviate to bidding half of their value!
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⬇

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$$E[U(b)] \geq \frac{1}{2} E\left[\sum_i v_i \, 1(\text{highest value})\right] - E[Rev(b)]$$

Efficiency Guarantees

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$$E[Rev(b) + U(b)] \geq \frac{1}{2} E\left[\sum_i v_i \, 1(\text{highest value})\right]$$

Efficiency Guarantees

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- Either wins with such a bid and gains:

$$U_i' = v_i - \frac{v_i}{2} = \frac{v_i}{2}$$

- Or loses, in which case highest bid is that high:

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$$\max_j b_j + U_i' \geq \frac{v_i}{2}$$

$$\Downarrow$$

$$U_i' \geq \frac{v_i}{2} - \max_j b_j$$

- Equilibrium utility is at least this much and always non-negative:

$$E[SW(b)] \geq \frac{1}{2} E[\text{highest value}]$$

For more details see [1607.07684] The Price of Anarchy in Auctions (arxiv.org)

Sum: First Price

- First Price is arguably the simplest auction rule
- It can be hard to strategize in such an auction
- The auction can lead to inefficient allocations

- Though approximately efficient
- Still used in practice in many settings (e.g. online advertising, government procurement)
- Primarily because it has very transparent rules

Second-Price (Vickrey) Auction

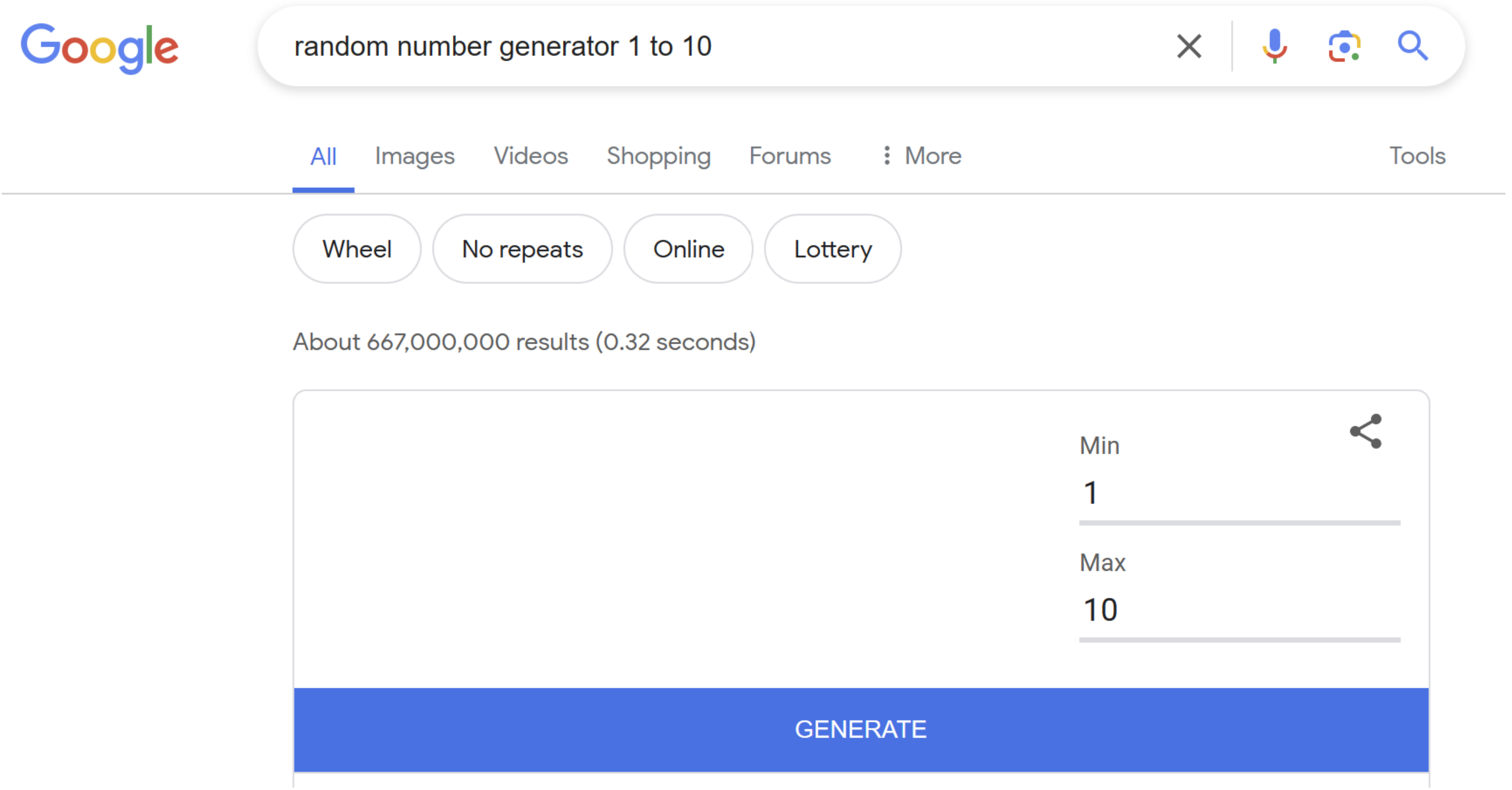
- Each bidder submits a bid b_i
- The highest bidder wins the item (ties broken at random)
- The winner pays the second highest bid

- Utility of a bidder with value v_i under a bid profile b:

$$u_i(b; v_i) = \left(v_i - \boxed{b_{(2)}}\right) \cdot 1\left\{b_i \geq \max b_j\right\}$$

Second highest bid

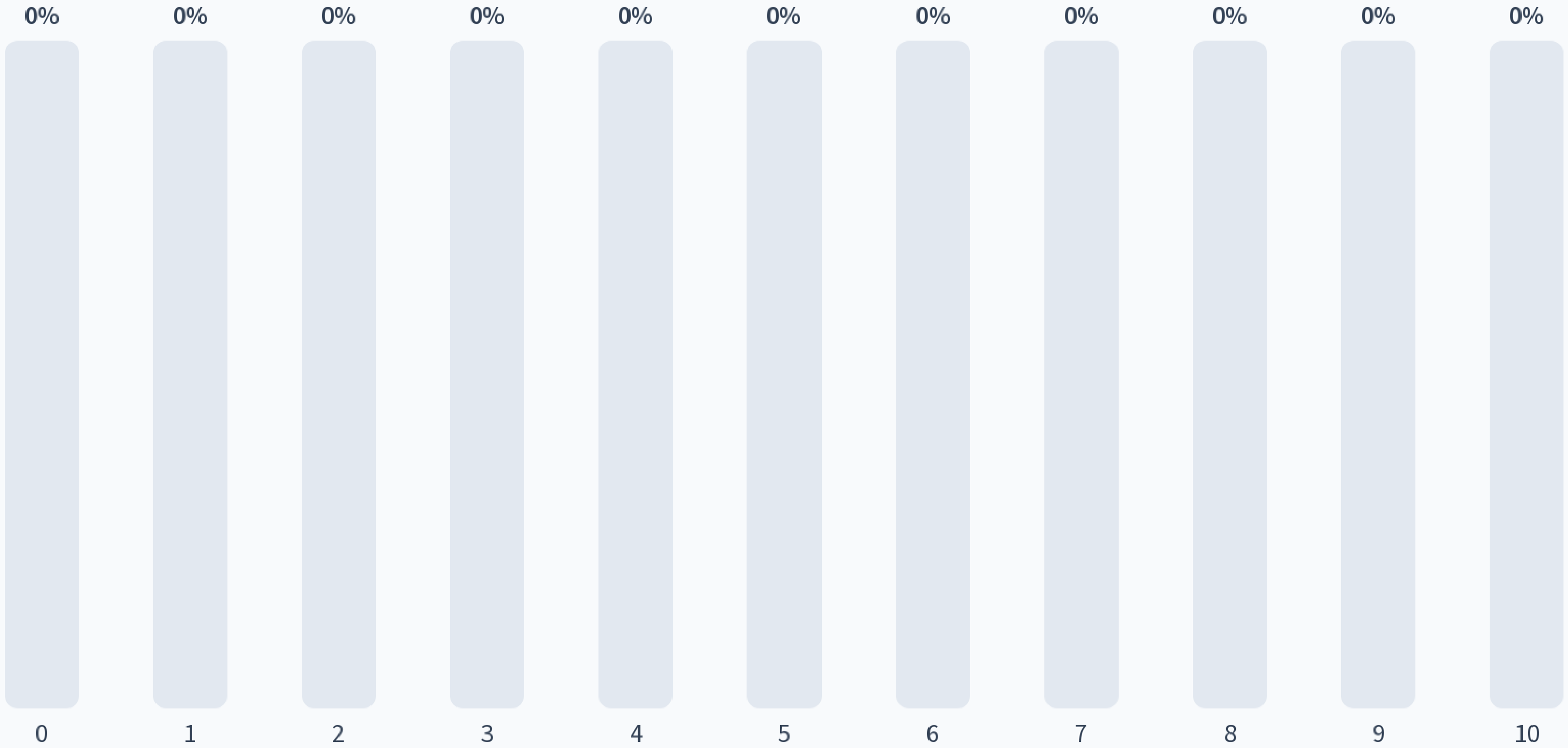
https://www.google.com/search?q=random+number+generator+1+to+10



Submit your bid to the second-price auction!

- 0
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10

Submit your bid to the second-price auction!



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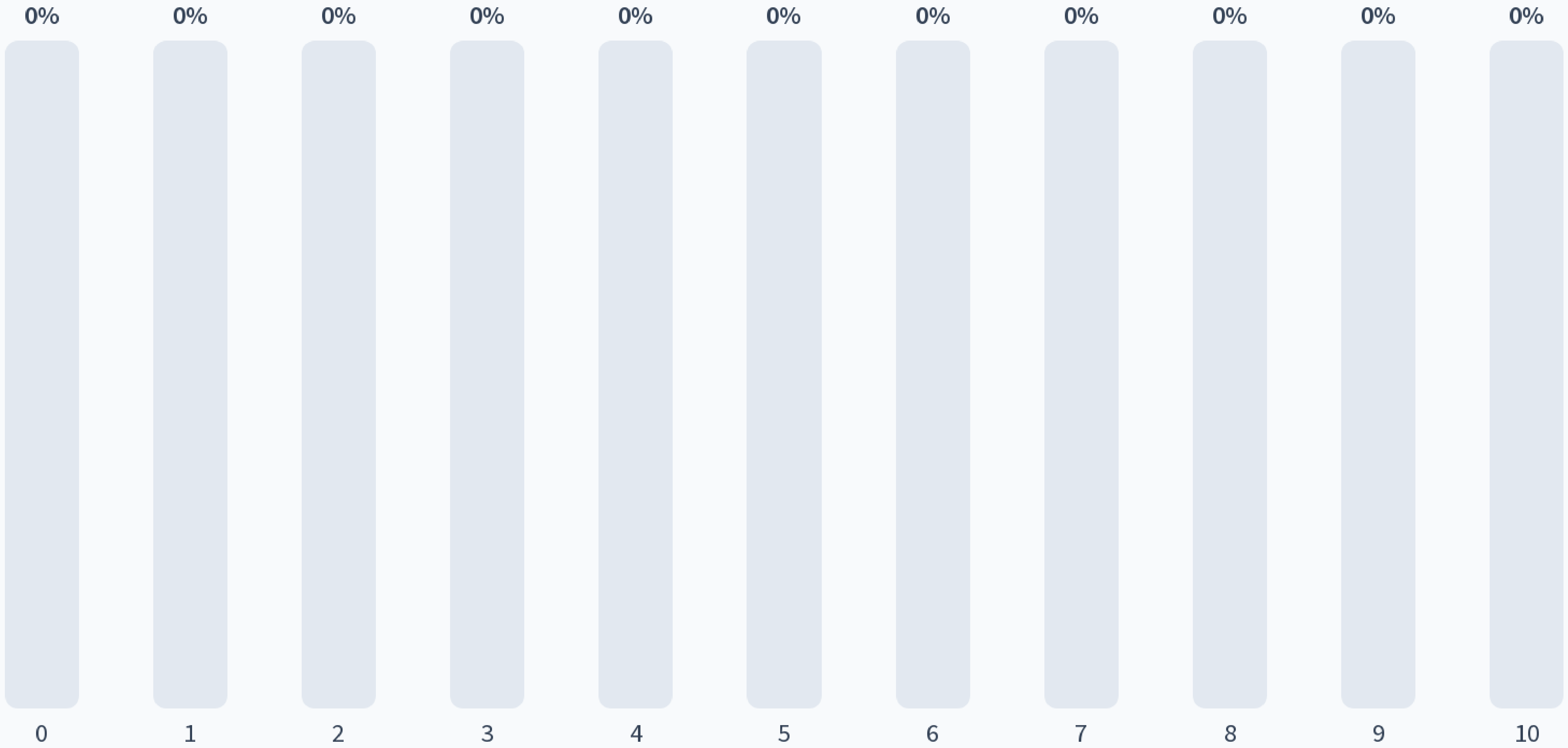

Truthfulness of Second Price Auction

- In a second price auction it is **dominant strategy** to bid your value
- No matter what the value is and no matter how others behave

Suppose I bid my value. Would I want to deviate?

Truthfulness of Second Price Auction

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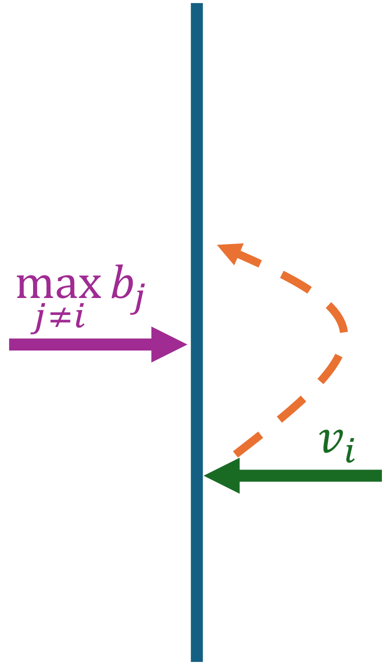
Suppose I bid my value. Would I want to deviate?

- **Case 1.** My value is below highest other bid
- Only way to change anything is bid above
- But then I get negative utility as I pay more than value

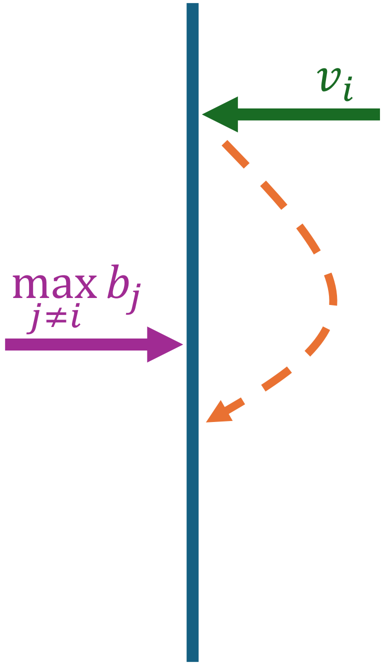


Truthfulness of Second Price Auction

- In a second price auction it is **dominant strategy** to bid your value
- No matter what the value is and no matter how others behave

Suppose I bid my value. Would I want to deviate?

- **Case 2.** My value is above highest other bid
- I get non-negative utility
- Only way to change anything is bid below
- But then I get zero utility as I lose



Sum: Second Price

- Second Price is arguably the simplest truthful auction rule
- It is very easy to strategize in such an auction (be truthful)
- Auction always leads to efficient allocations (highest value wins)
- Auction can be run very quickly (computationally efficient)

- Still not always the auction used in many places
- Primarily because it has not very transparent rules
- Susceptible to collusion and manipulations by the auctioneer

Sponsored Search Auctions

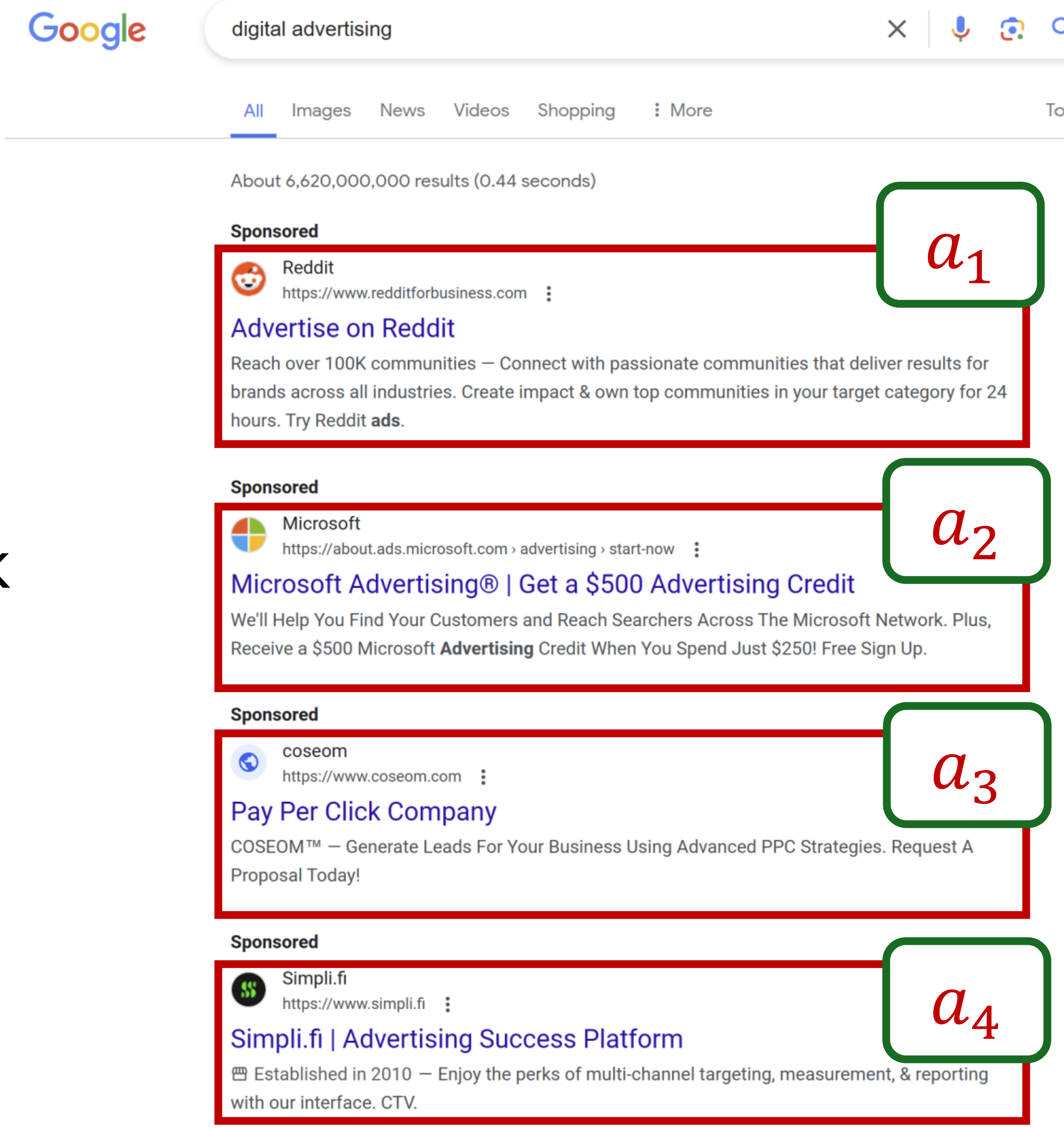
Sponsored Search Auctions

- Now we have many items to sell
- Slots on a web impressions

- Higher slots get more clicks!
- Each slot has some probability of click

$$a_1 > a_2 > \cdots > a_m$$

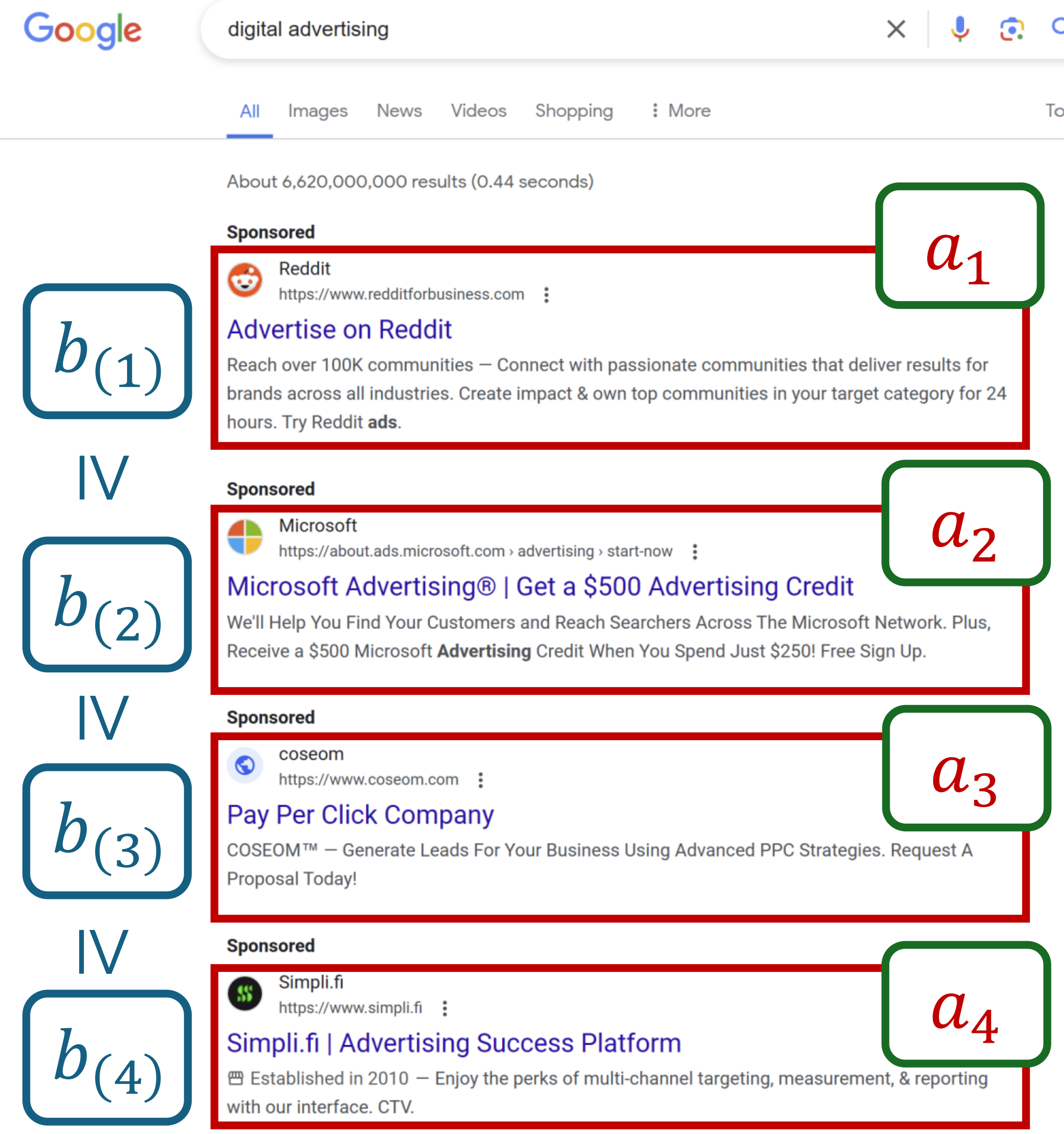
- Bidders have a value-per-click v_i



Generalized First Price (GFP) Auction

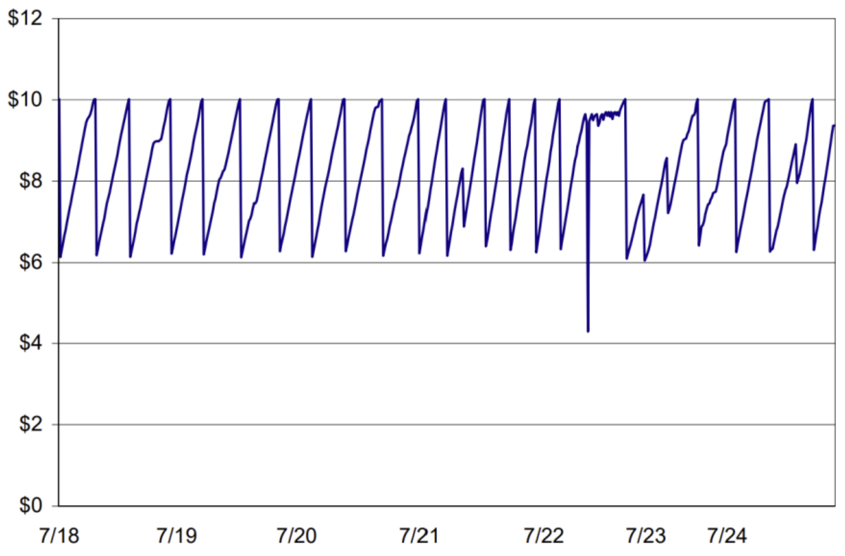
- Bidders submit a bid-per-click b_i
- Slots allocated in decreasing order of bids
- Bidder i is allocated slot $j_i(b)$
- Bidder pays their bid when clicked

$$u_i(b; v_i) = a_{j_i(b)} \cdot (v_i - b_i)$$

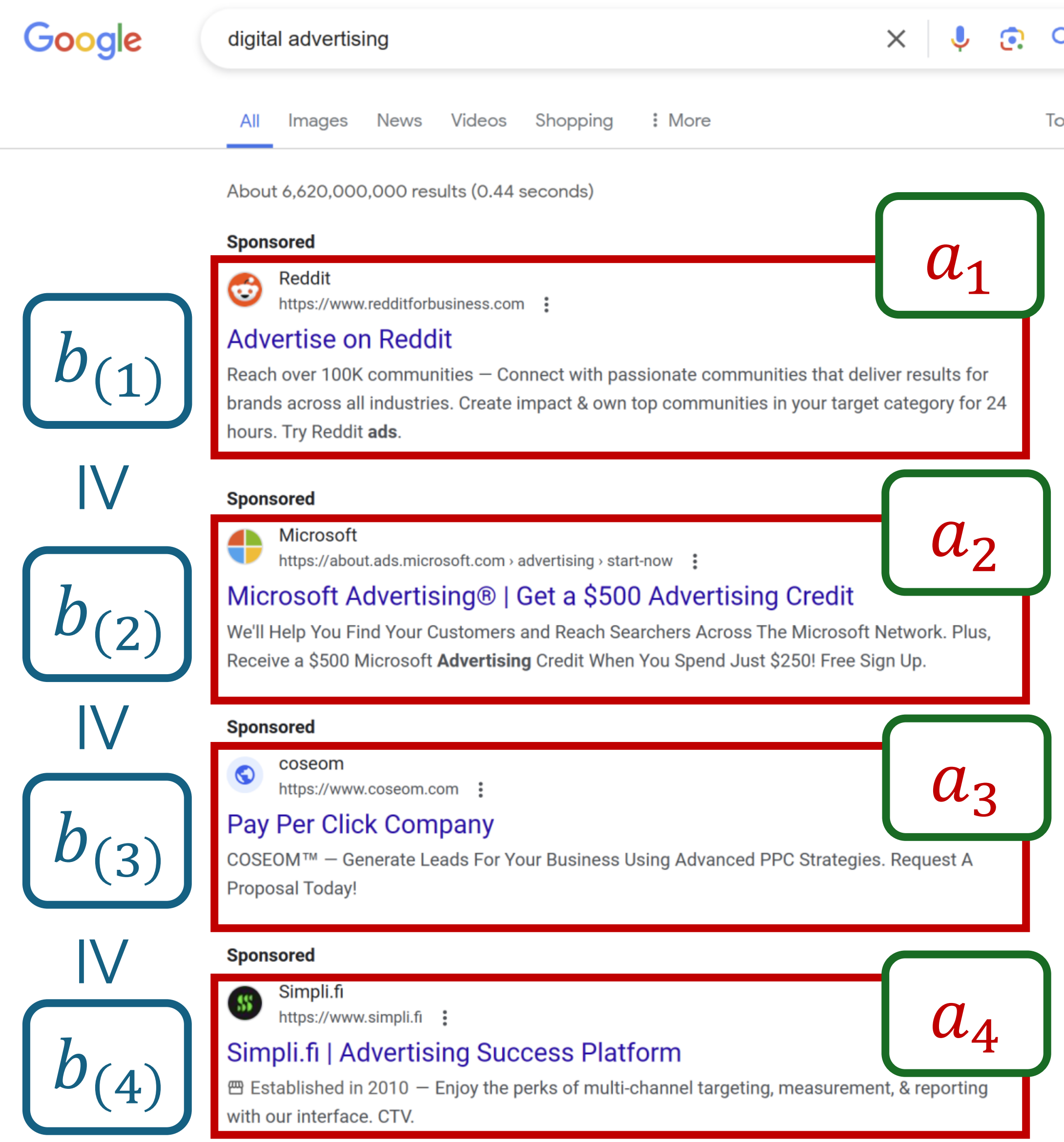


Generalized First Price (GFP) Auction

- The first auction that was used by Overture in late 90s
- Lead to weird bidding patterns



(b) 1 week

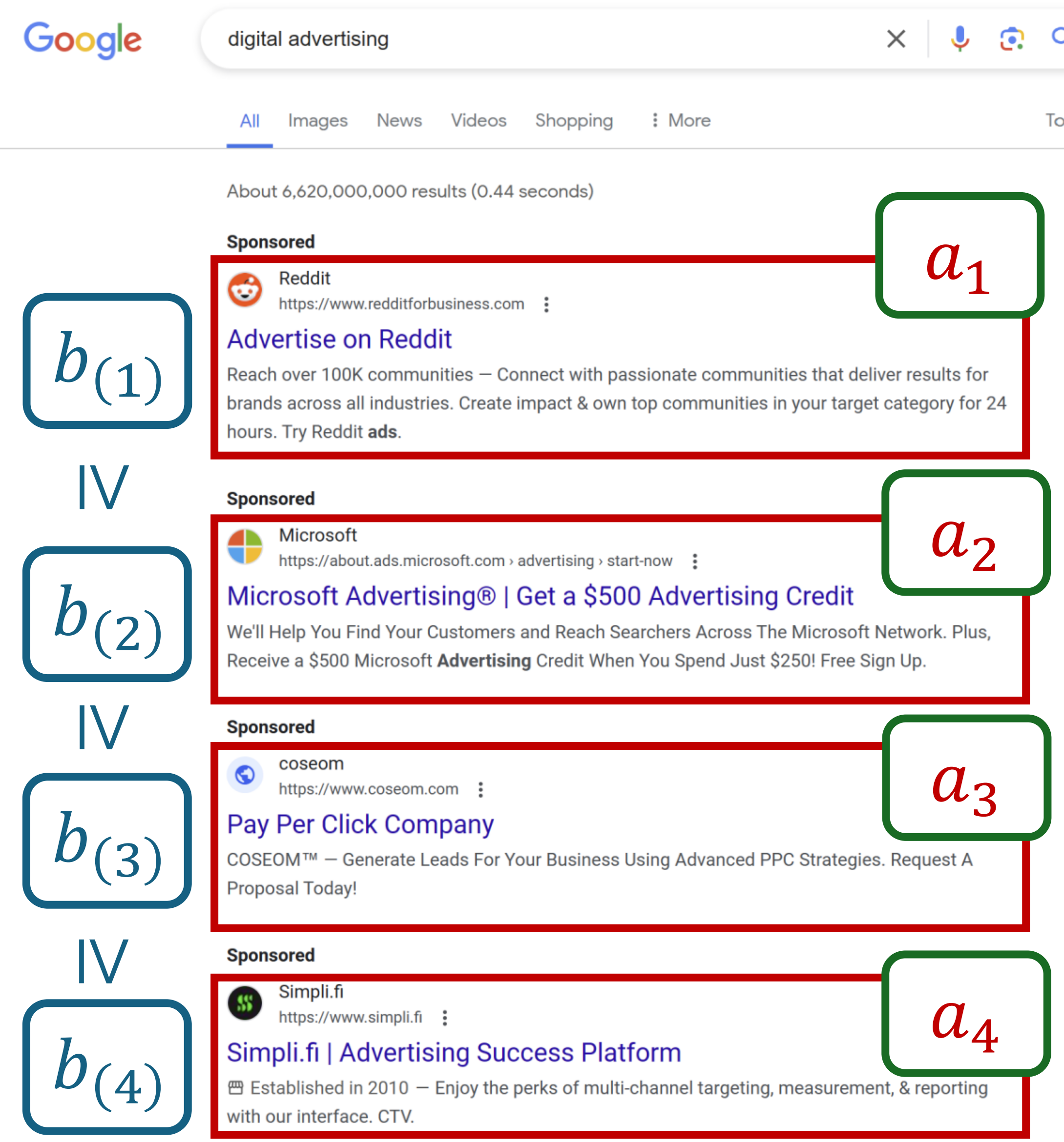


Credits: https://www.benedelman.org/publications/cycling-060703.pdf

Generalized Second Price (GSP) Auction

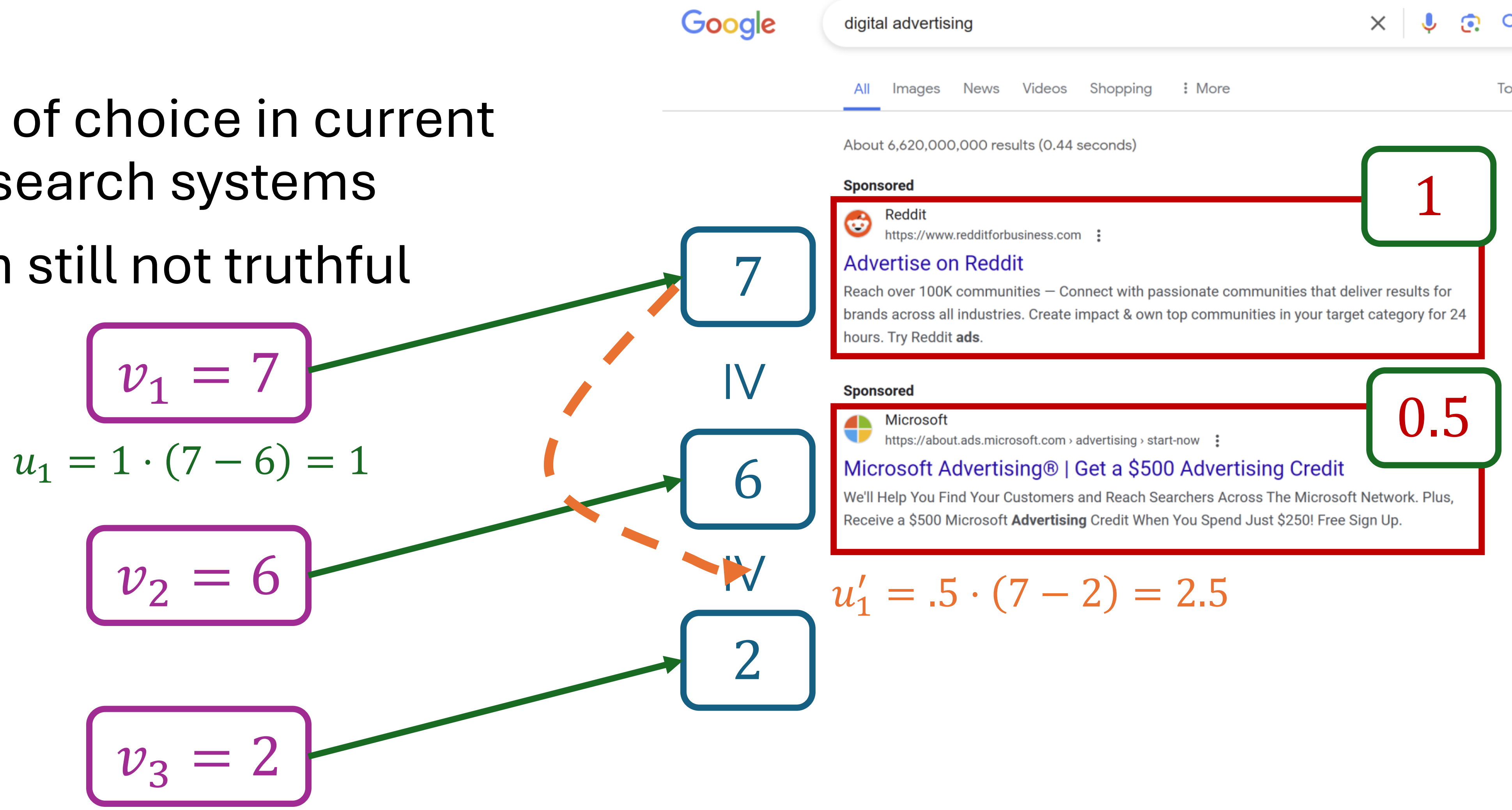
- Bidders submit a bid-per-click b_i
- Slots allocated in decreasing order of bids
- Bidder i is allocated slot $j_i(b)$
- Bidder pays the next highest bid when clicked

$$u_i(b; v_i) = a_{j_i(b)} \cdot \left(v_i - b_{(j_i(b)+1)}\right)$$



Generalized Second Price (GSP) Auction

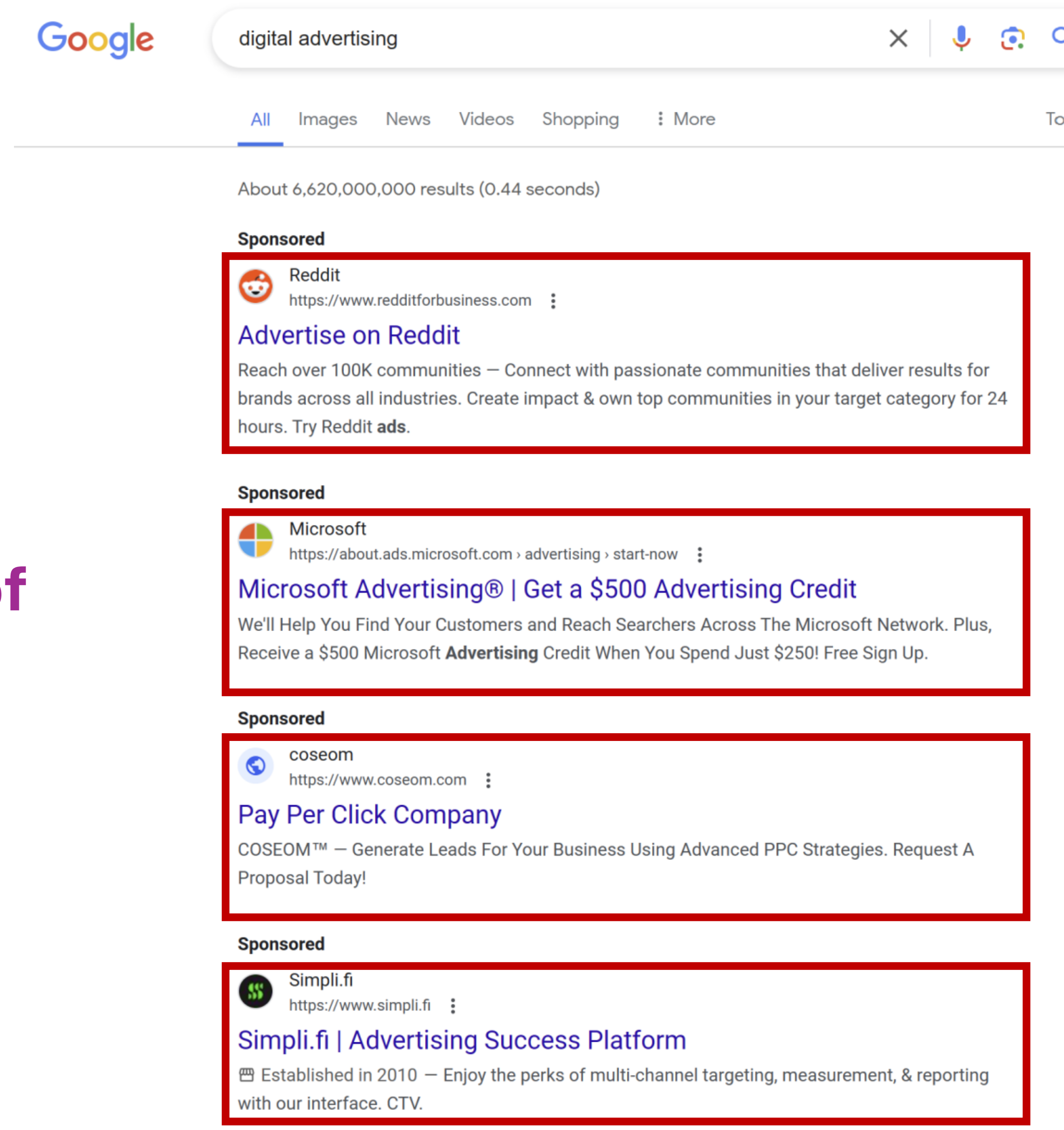
- The auction of choice in current sponsored search systems
- Even though still not truthful

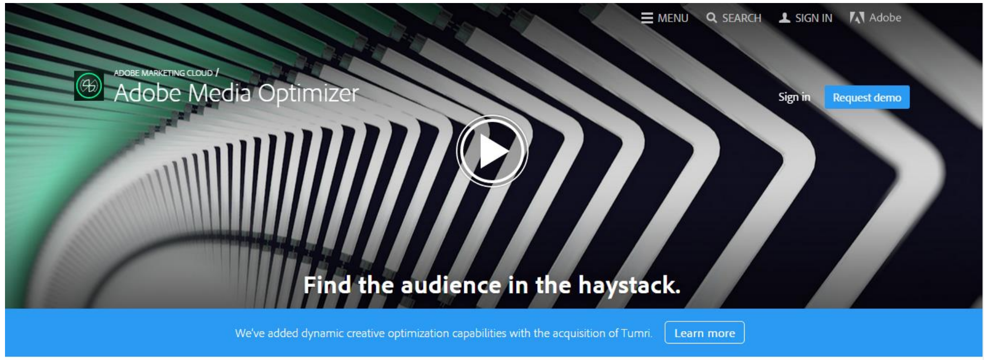


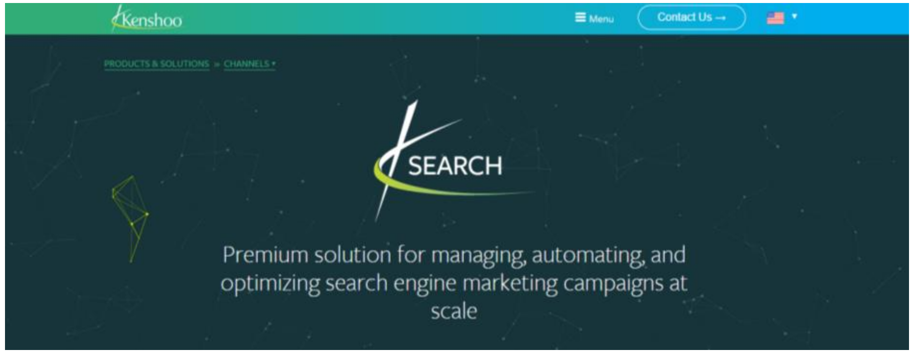
Generalized First Price (GFP) Auction with Many Bells and Whistles

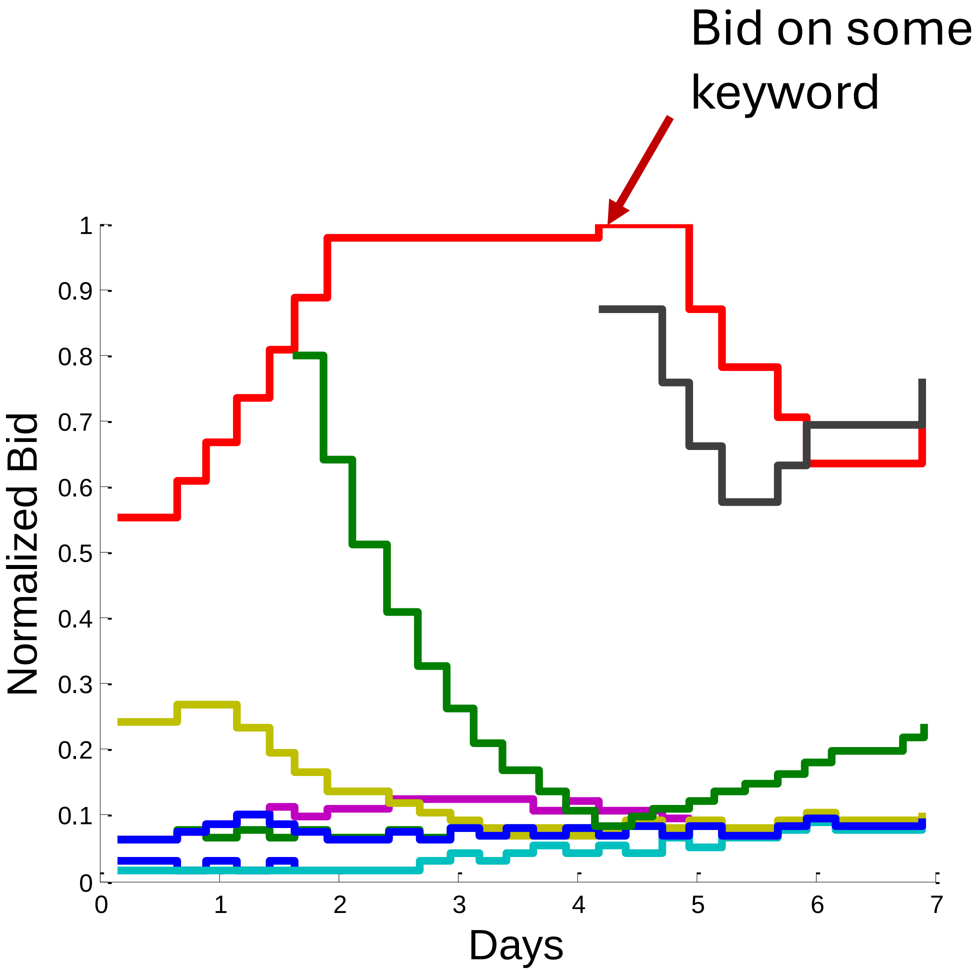
- Bidders submit a bid-per-click b_i
- Each bidder assigned a **quality score** s_i
- Slots allocated in decreasing order of **quality weighted bids** $s_i \cdot b_i$
- Bidder i is allocated slot $j_i(b)$
- Slots have **bidder-specific probability of click** $a_{i,j_i(b)}$
- Each bidder pays, per-click, the **highest bid that still gives them the same slot**

$$p_i(b) = \frac{s_{(j_i(b)+1)} \cdot b_{(j_i(b)+1)}}{s_i}$$



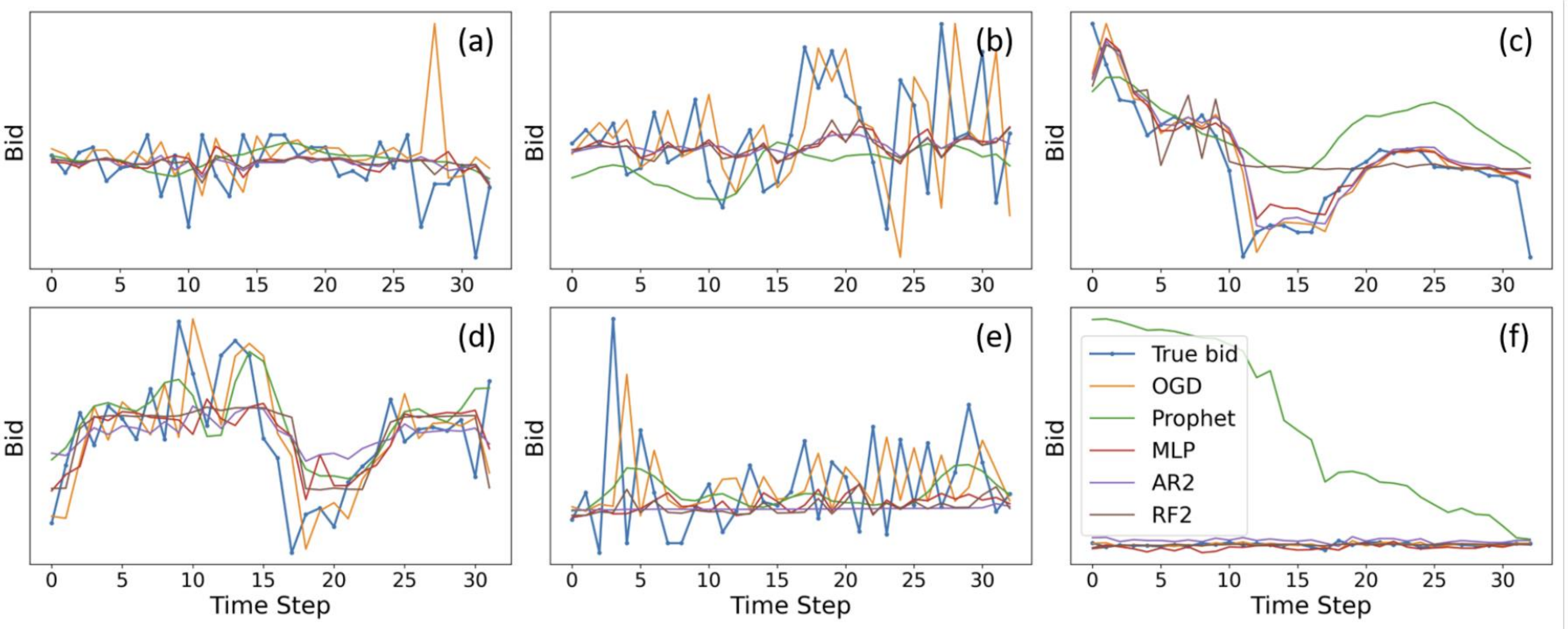






Credits: https://arxiv.org/abs/1505.00720

Simple learning dynamics are good predictors



(b) Stepahead Predictions

Credits: https://arxiv.org/pdf/2007.13193