

MS&E 233

Game Theory, Data Science and AI

Lecture 9

Vasilis Syrgkanis

Assistant Professor

Management Science and Engineering

(by courtesy) Computer Science and Electrical Engineering

Institute for Computational and Mathematical Engineering

Computational Game Theory for Complex Games

- 1
 - Basics of game theory and zero-sum games (T)
 - Basics of online learning theory (T)
 - Solving zero-sum games via online learning (T)
 - *HW1: implement simple algorithms to solve zero-sum games*
 - Applications to ML and AI (T+A)
 - *HW2: implement boosting as solving a zero-sum game*

- 2
 - Basics of extensive-form games
 - Solving extensive-form games via online learning (T)
 - *HW3: implement agents to solve very simple variants of poker*

- 3
 - General games, equilibria and online learning (T)
 - Online learning in general games
 - *HW4: implement no-regret algorithms that converge to correlated equilibria in general games*

Data Science for Auctions and Mechanisms

- 4
 - **Basics and applications of auction theory (T+A)**
 - **Learning to bid in auctions via online learning (T)**
 - *HW5: implement bandit algorithms to bid in ad auctions*

- 5
 - **Optimal auctions and mechanisms (T)**
 - **Simple vs optimal mechanisms (T)**
 - *HW6: calculate equilibria in simple auctions, implement simple and optimal auctions, analyze revenue empirically*

- 6
 - **Optimizing mechanisms from samples (T)**
 - **Online optimization of auctions and mechanisms (T)**
 - *HW7: implement procedures to learn approximately optimal auctions from historical samples and in an online manner*

Further Topics

- 7
 - **Econometrics in games and auctions (T+A)**
 - **A/B testing in markets (T+A)**
 - *HW8: implement procedure to estimate values from bids in an auction, empirically analyze inaccuracy of A/B tests in markets*

Guest Lectures

- Mechanism Design for LLMs, Renato Paes Leme, Google Research
- Auto-bidding in Sponsored Search Auctions, Kshipra Bhawalkar, Google Research

Auctions



Sotheby's EST. 1744

Sotheby's Sells \$7.3 Billion in Art, Fueled by Moneyed Millennials

Auction house says millennials drove art market rebound by bidding up everything from luxury goods to NFTs



Shop by category

Search for anything

All Categories

Search

Advanced

Back to search results | Listed in category: Clothing, Shoes & Accessories > Nike Air Force > Nike Air Force 1 x Supreme Low B... > See more Size 8 - Nike Air Force ...

Share | Add to Watchlist



White Nike Air Force 1 Low x Supreme Red Box Logo Shoes

★★★★★ 4 product ratings

Condition: **New with box**

Time left: 2d 2h | Thursday, 04:40 PM

Current bid: **US \$115.00**

[29 bids]

Place bid

Add to watchlist

Shipping: **US \$77.28** eBay International Shipping ⓘ . [See details](#)

Located in: New York, United States

This item may be subject to duties and taxes upon delivery

Delivery: **Estimated between Tue, Jul 25 and Mon, Aug 7 to 90210** ⓘ

Please note the delivery estimate is **greater than 17 business days**.

Returns: 30 days returns. Buyer pays for return shipping. [See details](#)

Payments:

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Get the item you ordered or your money back.

[Learn more](#)

Seller information

[Store](#) (212 ★)

100% positive feedback

[Save seller](#)

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[See other items](#)



Auction & dates

Here you can find the important dates of the current program:

Register by April 30, 2024

We bundle all the registered households and present them as a group to electricity providers. We conduct a reverse auction where electricity providers bid against each other to determine who can provide the lowest offer. [Register now.](#)

Auction on April 30, 2024

All interested electricity providers requested to participate in the auction, but only those who met our specific quality requirements made the cut. It's our way of ensuring customers get a smooth and reliable switch. We will be sending out an email about the winning provider and our competitive rates by **7 May**.

[Sign up today](#) >

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[Who can participate?](#)

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Market processes and products

The ISO wholesale energy market is comprised of distinct day-ahead and real-time processes. The energy products and services traded in our market allow us to meet reliability needs and serve load. The market also offers services in which qualified entities can buy and sell congestion revenue rights and engage in convergence bidding activities.

Day-ahead market

The day-ahead market is made up of three market processes that run sequentially. First, the ISO runs a market power mitigation test. Bids that fail the test are revised to predetermined limits. Then the integrated forward market establishes the generation needed to meet forecast demand. And last, the residual unit commitment process designates additional power plants that will be needed for the next day and must be ready to generate electricity. Market prices set are based on bids.

A major component of the market is the full network model, which analyzes the active transmission and generation resources to find the least cost energy to serve demand. The model produces prices that show the cost of producing and delivering energy from individual nodes, or locations on the grid where transmission lines and generation interconnect.

Scheduling coordinators (SCs) are pre-qualified entities authorized to transact in the ISO market. The day-ahead market opens for bids and schedules seven days before and closes the day prior to the trade date. Results are published at 1:00 p.m.

Real-time market

The real-time market is a spot market in which utilities can buy power to meet the last few increments of demand not covered in their day ahead schedules. It is also the market that secures energy reserves, held ready and available for ISO use if needed, and the energy needed to regulate transmission line stability.



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What is a renewable energy auction?

Renewable Energy Auctions Toolkit

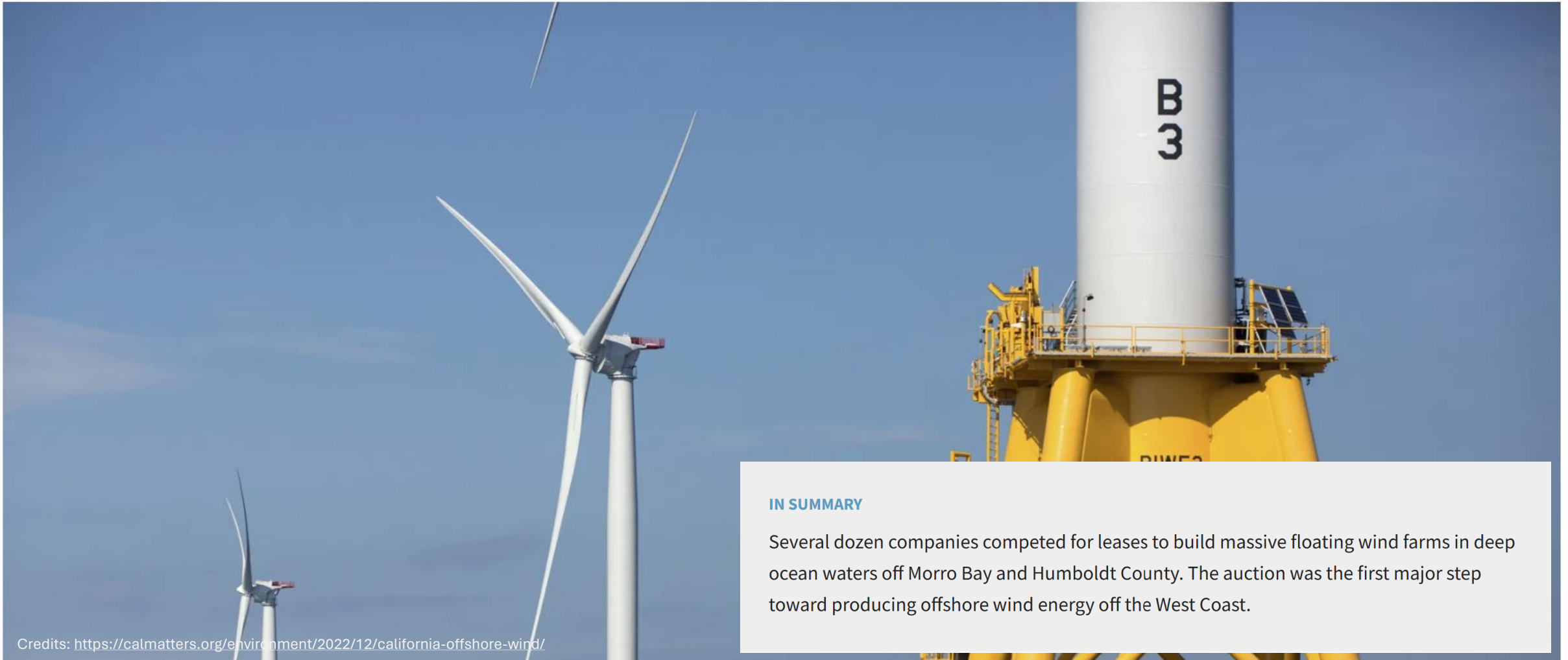
VIDEO (00:04:05) – FEBRUARY 2022

First-ever California offshore wind auction nets \$757 million



BY NADIA LOPEZ

DECEMBER 6, 2022 UPDATED DECEMBER 7, 2022



IN SUMMARY

Several dozen companies competed for leases to build massive floating wind farms in deep ocean waters off Morro Bay and Humboldt County. The auction was the first major step toward producing offshore wind energy off the West Coast.

About 6,620,000,000 results (0.44 seconds)

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https://www.redditforbusiness.com

Advertise on Reddit

Reach over 100K communities — Connect with passionate communities that deliver results for brands across all industries. Create impact & own top communities in your target category for 24 hours. Try Reddit ads.

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https://www.simpli.fi

Simpli.fi | Advertising Success Platform

Established in 2010 — Enjoy the perks of multi-channel targeting, measurement, & reporting with our interface. CTV.

Amazon Ads
https://advertising.amazon.com > library > guides > wha...

Online advertising :



Online advertising, also known as online marketing, Internet advertising, digital advertising or web advertising, is a form of marketing and advertising that uses the Internet to promote products and services to audiences and platform users. Wikipedia

Efficacy

Is online advertising effective?

Benefits

Online advertising benefits

Top 10 best

Top 10 best online advertising

How to

How to advertise online

1-48 of over 9,000 results for "game theory"

Sort by: Featured

Popular Shopping Ideas

- Shirt
- Book
- Backpack



Affordable Original Classics

Shop now >



The Science of Getting Rich

★★★★☆ 154

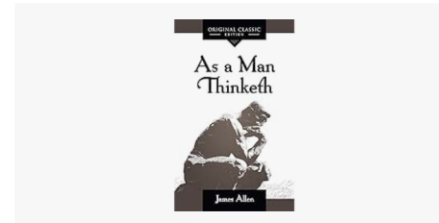
prime



The Game of Life & How to Play It

★★★★☆ 359

prime



As a Man Thinketh

★★★★☆ 651

prime

Eligible for Free Shipping

- Free Shipping by Amazon
- Get FREE Shipping on eligible orders shipped by Amazon

Delivery Day

- Get It by Tomorrow

Kindle Unlimited

- Kindle Unlimited Eligible

Department

- Books
 - Mathematics
 - Business & Money
 - Puzzles & Games
- Kindle Store
 - Science & Math
 - Business & Investing

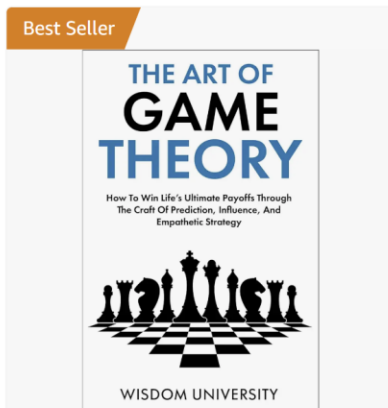
Customer Reviews

- ★★★★☆ & Up
- ★★★★☆ & Up
- ★★★★☆ & Up
- ★★★★☆ & Up

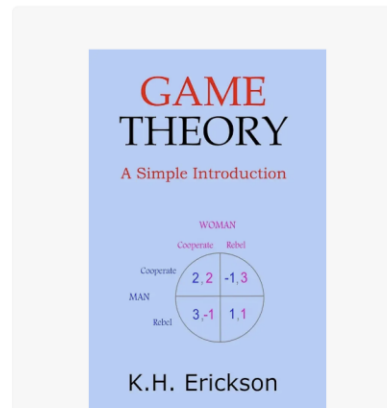
Book Series

- Simple Introductions

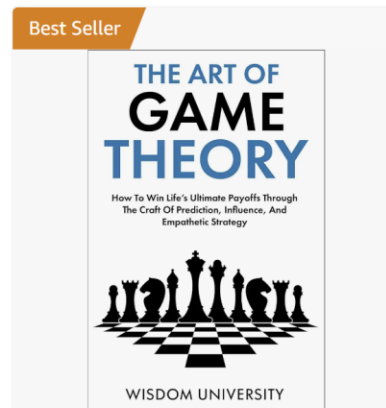
Results



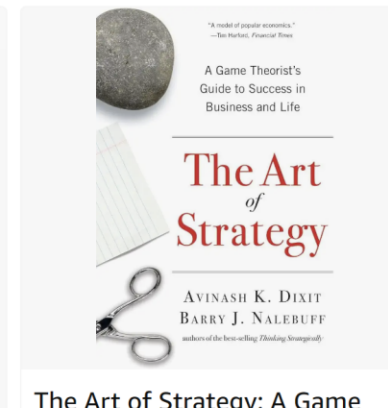
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The Art Of Game Theory: How To Win Life's Ultimate Payoffs



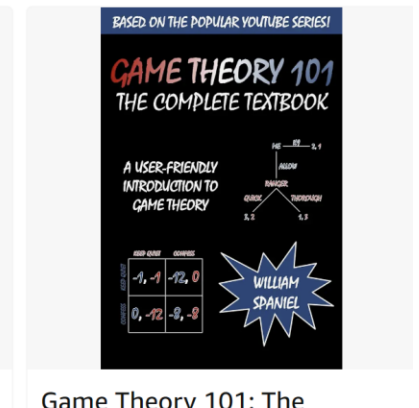
Game Theory: A Simple Introduction (Simple Introductions)



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The Art Of Game Theory: How To Win Life's Ultimate Payoffs Through The Craft Of



The Art of Strategy: A Game Theorist's Guide to Success in Business and Life by Avinash K. Dixit and Barry J.



Game Theory 101: The Complete Textbook by William Spaniel
★★★★☆ ~ 490

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Auctions Summary

Auctions

Summary

General Releases

Archived Auction Releases

About Auctions

Broadcast Incentive Auction

Prohibited Communications

Completed Spectrum Auctions

Auction	Licenses Auctioned	Licenses Won	Net Winning Bids	Rounds
1 Nationwide Narrowband (PCS) 7/25/1994 - 7/29/1994	10	10	\$617,006,674	47
2 Interactive Video and Data Services (IVDS) 7/28/1994 - 7/29/1994 Map: CMA (MSA & RSA) (pdf)	594	594	\$213,892,375	Oral Outcry
3 Regional Narrowband (PCS)			\$200,700,707	



Home / Economics and Analytics / Auctions

Auction 97: Advanced Wireless Services (AWS-3)

Go to an auction

Select an Auction

Summary

Fact Sheet

Releases

Education

Results

Application Search

Summary

General Releases

Archived Auction Releases

About Auctions

Broadcast Incentive Auction

Summary

November 13, 2014 - January 29, 2015

Rounds:	341
Bidding Days:	45
Qualified Bidders:	70
Winning Bidders:	31 Bidders won 1611 Licenses
Licenses Held by FCC:	3
Gross Bids:	\$44,899,451,600
Net Bids:	\$41,329,673,325

Home / Economics and Analytics / Auctions

Auction 107: 3.7 GHz Service

Go to an auction

Select an Auction 

Summary

Fact Sheet

Releases

Education

Results

Application Search

Fact Sheet

Winning Bidders:	21
Qualified Bidders:	57
Licenses Won:	5684
Licenses Held by FCC:	0
Total Licenses:	5684
Net Bids:	\$81,114,481,921
Gross Bids:	\$81,168,677,645

Summary

General Releases

Archived Auction Releases

About Auctions



Dashboard

Reverse Auction

- Auction Summary
- Announcements
- Initial Commitments
- Winning Bids
- Bids
- Results
- Statistics
- Stations
- Ownership Changes

Forward Auction

- Auction Summary
- Announcements
- Bids
- Results
- Product Status
- Bidder Status
- Bidder Market
- Stage Transition
- Split Transition
- Max Reserved Blocks
- License Impairment
- Markets
- Band Plans
- Assignment Phase

Incentive Auction Dashboard

✓ Auction concluded. See the [Auction 1000 website](#) for more information.

Clearing Target ▶ 84 MHz

Licensed Spectrum ▶ 70 MHz

Final Stage Rule

1 First Component ✓	2 Second Component ✓	Final Stage Rule ✓
Average Price	Net Proceeds	Met
\$1.25 <small>Target</small>	\$12,011,676,822 <small>Target</small>	

Reverse Auction

Current Round ▶ Bidding Concluded

Clearing Cost ▶ \$10,054,676,822

Qualified Bidders ▶ 705

Winning Bidders ▶ 142

Qualified Stations ▶ 1030

Winning Stations ▶ 175

Forward Auction

Current Round ▶ Bidding Concluded

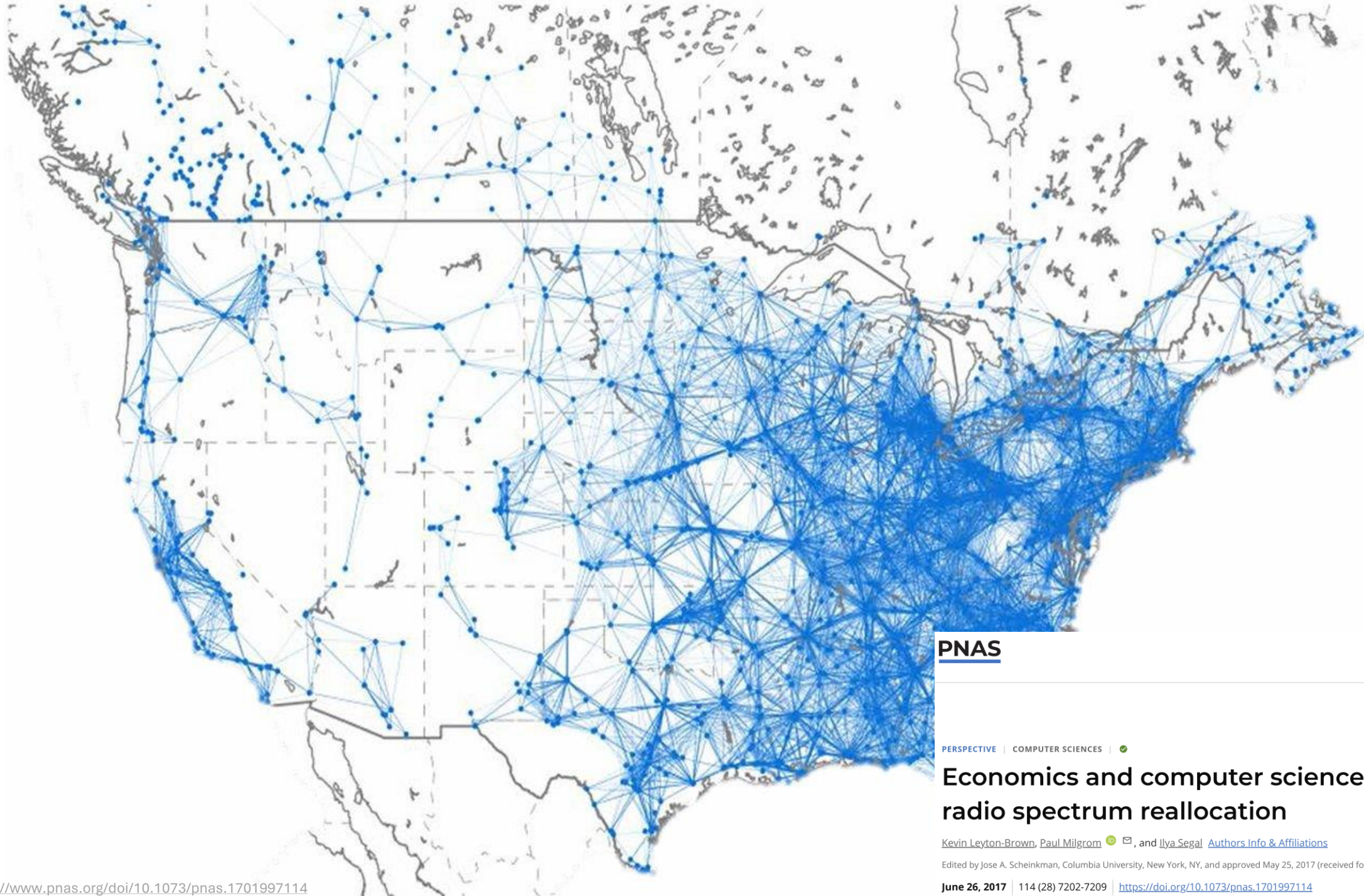
Gross Proceeds ▶ \$19,768,437,378

Net Proceeds ▶ \$19,311,003,826

Net Proceeds as of Closing PN ▶ \$19,318,157,706

Qualified Bidders ▶ 62

Winning Bidders ▶ 50



PNAS



PERSPECTIVE | COMPUTER SCIENCES |



Economics and computer science of a radio spectrum reallocation

Kevin Leyton-Brown, Paul Milgrom , and Ilya Segal [Authors Info & Affiliations](#)

Edited by Jose A. Scheinkman, Columbia University, New York, NY, and approved May 25, 2017 (received for review February 4, 2017)

June 26, 2017 | 114 (28) 7202-7209 | <https://doi.org/10.1073/pnas.1701997114>



Due to increased support volume, wait times are longer than usual.

What is EIP-1559?

Ethereum Improvement Proposal (EIP) 1559 is an [upgrade](#) that happened on August 5, 2021 to change how Ethereum calculates and processes network transaction fees (called "gas fees"). The upgrade made Ethereum transactions more efficient by using a system of block-based base fees, and sender-specified max fees, rather than bidding on gas prices to more evenly incentivize miners in periods of high or low network congestion. It was packaged with the London hard fork. Four other EIPs will join EIP 1559 in London.

Transaction Fee Mechanism Design for the Ethereum Blockchain: An Economic Analysis of EIP-1559

Tim Roughgarden

EIP-1559 is a proposal to make several tightly coupled additions to Ethereum's transaction fee mechanism, including variable-size blocks and a burned base fee that rises and falls with demand. This report assesses the game-theoretic strengths and weaknesses of the proposal and explores some alternative designs.

Subjects: **Computer Science and Game Theory (cs.GT)**; Distributed, Parallel, and Cluster Computing (cs.DC); Data Structures and Algorithms (cs.DS); Theoretical Economics (econ.TH)

Cite as: [arXiv:2012.00854](https://arxiv.org/abs/2012.00854) [cs.GT]

(or [arXiv:2012.00854v1](https://arxiv.org/abs/2012.00854v1) [cs.GT] for this version)

<https://doi.org/10.48550/arXiv.2012.00854> 

Auction Basics

Auction Basics

- n bidders are interested in acquiring an item
- Bidder i has value v_i for the item
- Value is known only to them (private information)
- If bidder wins the item ($x_i = 1$) they gain a value v_i
- If at the end they are asked to pay a price p_i they gain

$$u_i(x_i, p_i; v_i) = v_i \cdot x_i - p_i$$

Sealed-Bid Auctions

- Each bidder privately communicates a bid b_i to the auctioneer
- Auctioneer applies an allocation rule x to bid vector $b = (b_1, \dots, b_n)$

$$x_i(b) = \text{Probability bidder } i \text{ gets the item,} \quad \sum_i x_i(b) \leq 1$$

- Auctioneer applies a price rule p to bid vector $b = (b_1, \dots, b_n)$

$$p_i(b) = \text{Price that bidder } i \text{ is asked to pay}$$

Welfare of the outcome

- The expected welfare of the outcome of an auction is the expected value of the winner!

$$SW(b) = \sum_i x_i(b) \cdot v_i$$

- It depends on the allocation rule of the auction and on how bidders bid as a function of their value (which in turn depends on the auction allocation and payment rules)

Revenue of the outcome

- The expected revenue of the outcome of an auction is the expected total payments made to the auctioneer!

$$Rev(b) = \sum_i p_i(b)$$

Total utility of the outcome

- The expected total utility of the outcome of an auction is the expected total net gains made by the bidders!

$$U(b) = \sum_i v_i \cdot x_i(b) - p_i(b) = SW(b) - Rev(b)$$

How would you maximize welfare?

What if we just elicit bids, give it to the highest bidder, and don't charge anything...

First-Price Auction

- Arguably the simplest auction to describe
- Each bidder submits a bid b_i
- The highest bidder wins the item (ties broken at random)
- The winner pays their bid
- Utility of a bidder with value v_i under a bid profile b :
$$u_i(b; v_i) = (v_i - b_i) \cdot 1\{b_i \geq \max b_j\}$$

How would you bid!

<https://www.google.com/search?q=random+number+generator+1+to+10>



random number generator 1 to 10



All

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Wheel

No repeats

Online

Lottery

About 667,000,000 results (0.32 seconds)

Min

1

Max

10



GENERATE

You will be participate in a first price auction for an item, competing with 1 other randomly chosen student in class. Submit your bid!

0

1

2

3

4

5

6

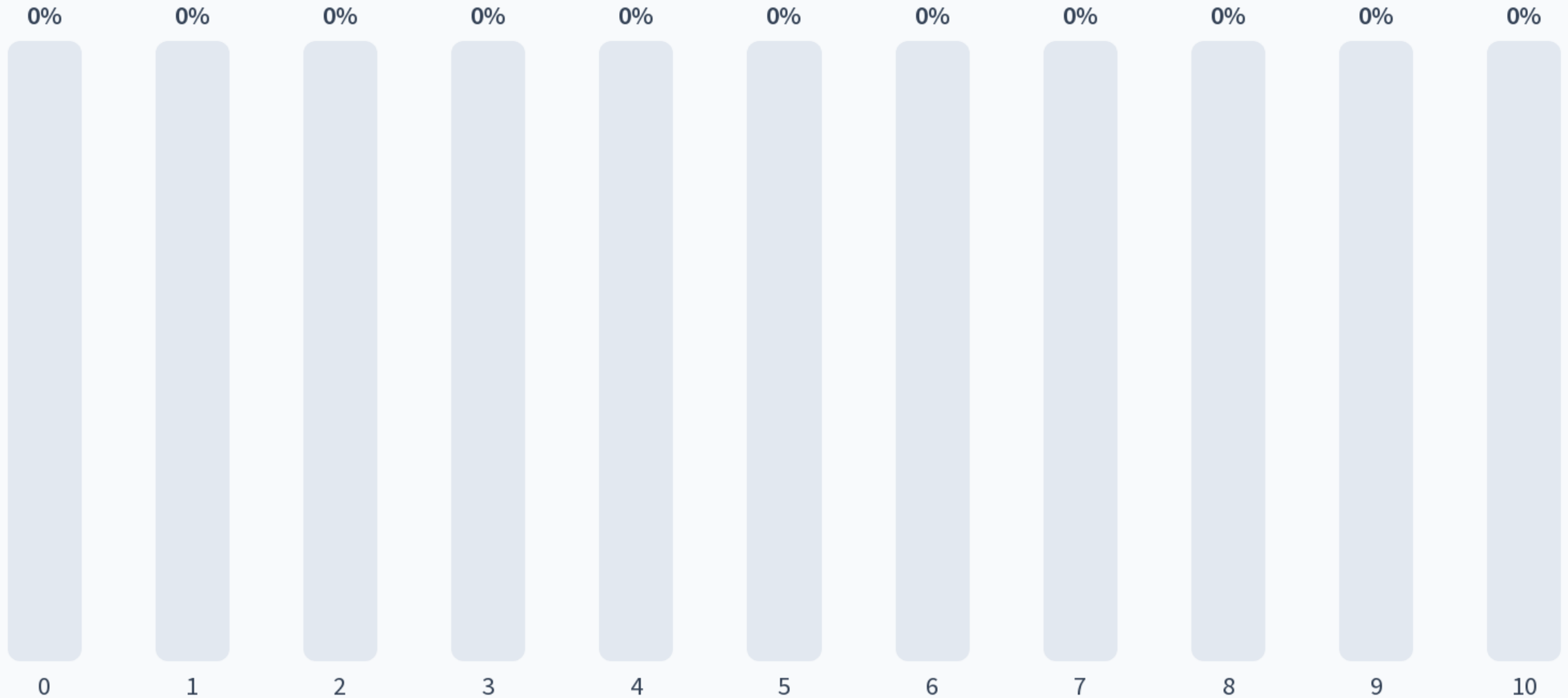
7

8

9

10

You will be participate in a first price auction for an item, competing with 1 other randomly chosen student in class. Submit your bid!



You will be participate in a first price auction for an item, competing with 1 other randomly chosen student in class. Submit your bid!



You will be participate in a first price auction for an item, competing with 10 other randomly chosen students in class. Submit your bid!

0

1

2

3

4

5

6

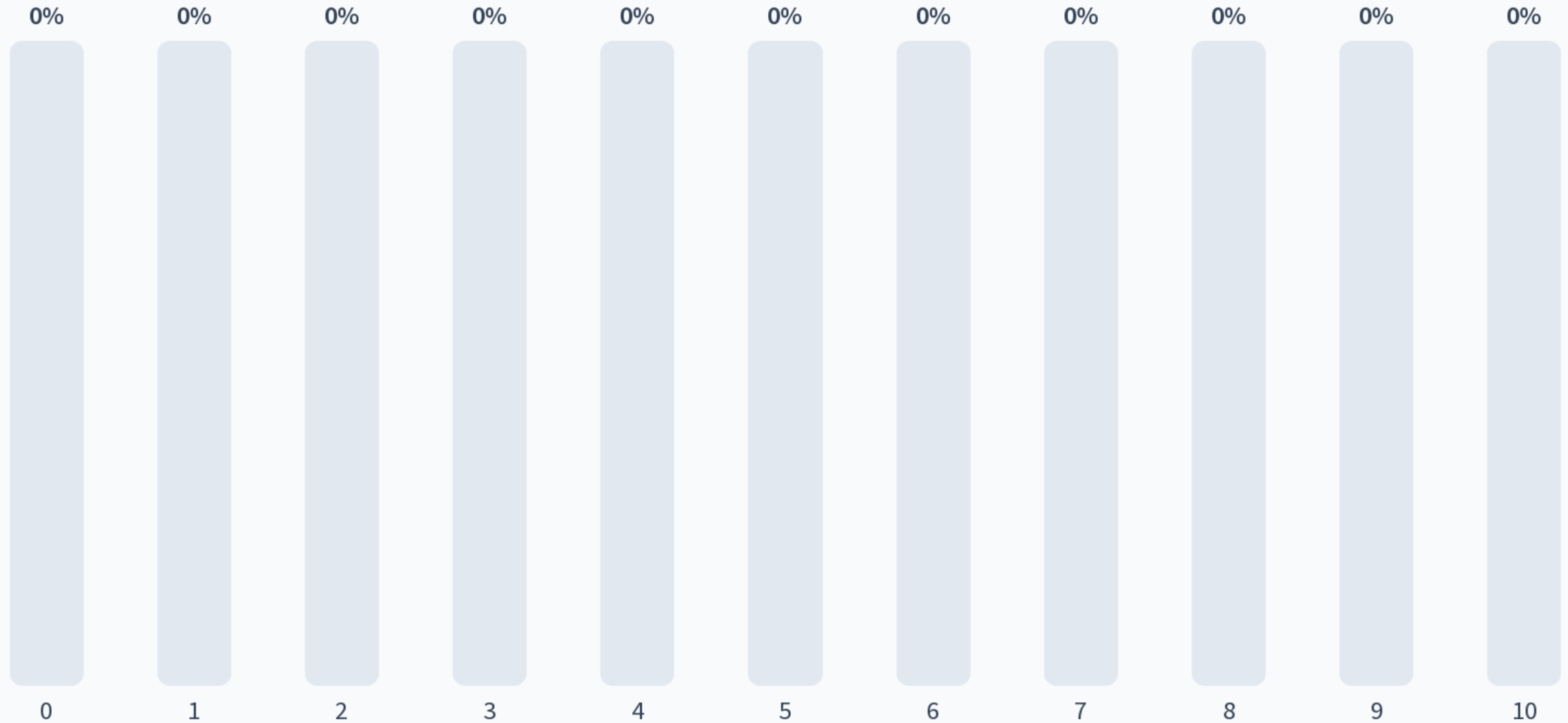
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10

You will be participate in a first price auction for an item, competing with 10 other randomly chosen students in class. Submit your bid!



You will be participate in a first price auction for an item, competing with 10 other randomly chosen students in class. Submit your bid!



Bayesian Formulation

- Each bidder's value is drawn from some distribution

$$v_i \sim F_i, \quad v = (v_1, \dots, v_n) \sim F = F_1 \times \dots \times F_n$$

- Bidders submit a bid as a function of their value

$$s_i(v_i) = \text{Bid of player } i \text{ when their value is } v_i$$

Bayes-Nash Equilibrium. A bidding strategy profile $s = (s_1, \dots, s_n)$ is a Bayes-Nash equilibrium, if players cannot gain by deviating in expectation, assuming others follow their strategies

$$E_{v \sim F} [u_i(s(v); v_i)] \geq E_{v \sim F} [u_i(b'_i, s_{-i}(v_{-i}); v_i)]$$

Equilibrium of a First Price Auction (FPA)

- Consider a FPA with two bidders
- Each player's value is distributed uniformly in $[0, 1]$
- It suffices to look at symmetric bidding strategies

Theorem. The following is a Bayes-Nash equilibrium

$$s_i = \frac{1}{2} v_i$$

Equilibrium of a First Price Auction (FPA)

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Theorem. The following is a Bayes-Nash equilibrium

$$s_i = \frac{1}{2} v_i$$

Proof. Suppose bidder 2 follows strategy. Bidder 1 utility from $b_1 \in \left[0, \frac{1}{2}\right]$:

$$(v_1 - b_1) \Pr\left(b_1 > \frac{1}{2} v_2\right) = (v_1 - b_1) \Pr(2b_1 > v_2) = (v_1 - b_1) 2b_1$$

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FOC with respect to b_1 : $2v_1 - 4b_1 = 0 \Rightarrow b_1 = v_1/2!$

Equilibrium of a First Price Auction (FPA)

- Consider a FPA with n bidders
- Each player's value is distributed uniformly in $[0, 1]$

Theorem. The following is a Bayes-Nash equilibrium

$$s_i = \left(1 - \frac{1}{n}\right) v_i$$

Equilibrium of a First Price Auction (FPA)

- Consider a FPA with n bidders
- Each player's value is distributed uniformly in $[0, 1]$

Theorem. The following is a Bayes-Nash equilibrium

$$s_i = \left(1 - \frac{1}{n}\right) v_i$$

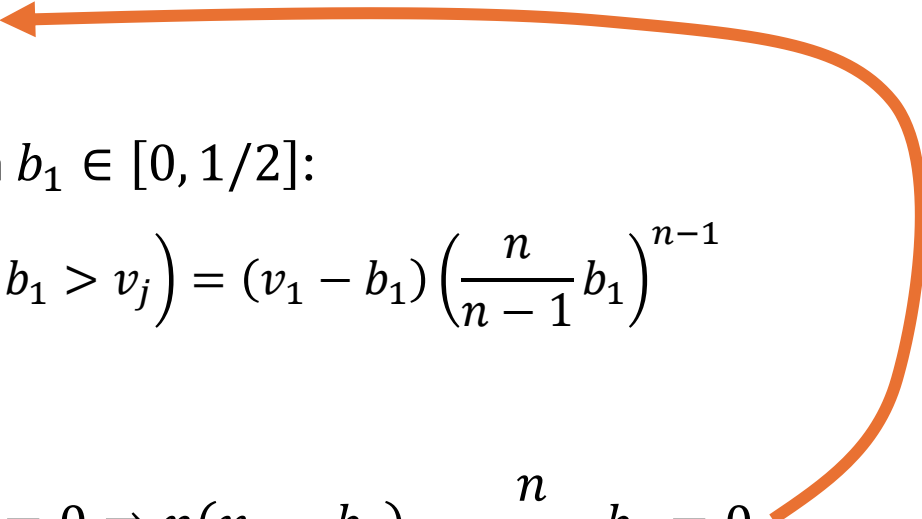
Proof. Suppose others follow strategy. Bidder 1 utility from $b_1 \in [0, 1/2]$:

$$(v_1 - b_1) \Pr\left(b_1 > \frac{n-1}{n} \max_{j \neq i} v_j\right) = (v_1 - b_1) \prod_{j \neq i} \Pr\left(\frac{n}{n-1} b_1 > v_j\right) = (v_1 - b_1) \left(\frac{n}{n-1} b_1\right)^{n-1}$$

Equilibrium of a First Price Auction (FPA)

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FOC with respect to b_1 :

$$(n-1) \frac{n}{n-1} \left(\frac{n}{n-1} b_1\right)^{n-2} (v_1 - b_1) - \left(\frac{n}{n-1} b_1\right)^{n-1} = 0 \Rightarrow n(v_1 - b_1) - \frac{n}{n-1} b_1 = 0$$

<https://www.google.com/search?q=random+number+generator+60+to+70>



random number generator 60 to 70



All

Images

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Tools

About 28,200,000 results (0.49 seconds)

Min

60

Max

70



GENERATE

You will be participate in a first price auction with a competitor whose value is distributed uniformly in $[0, 50]$. Submit your bid!

1-5

6-10

11-15

16-20

21-25

26-30

31-35

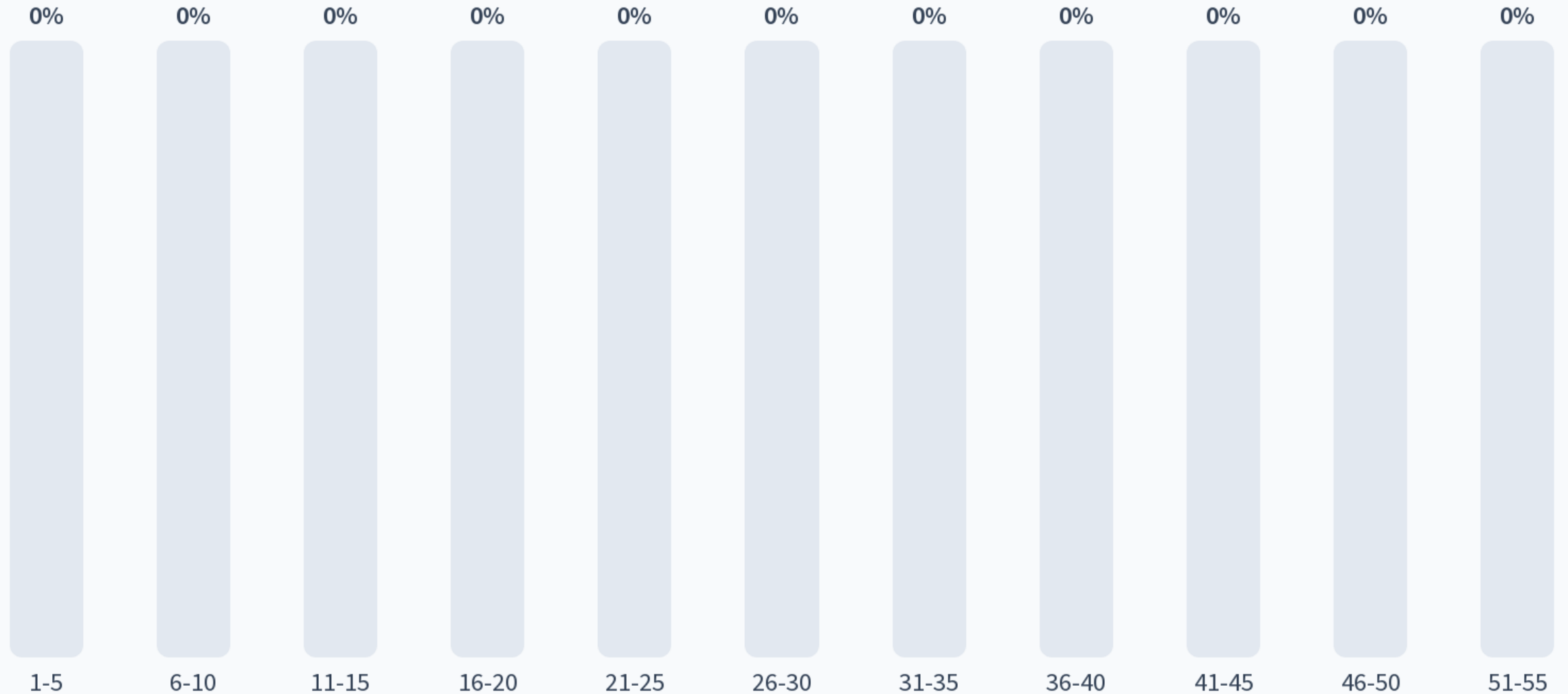
36-40

41-45

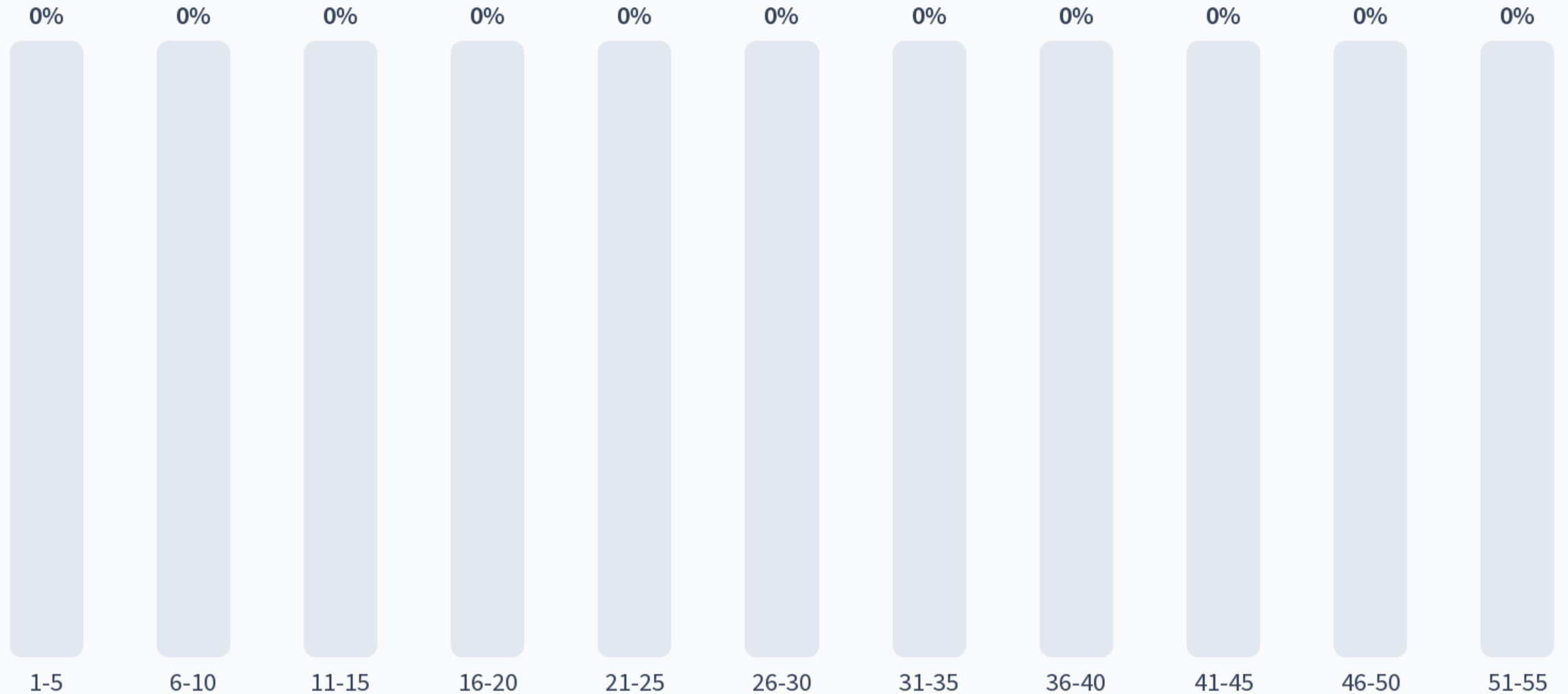
46-50

51-55

You will be participate in a first price auction with a competitor whose value is distributed uniformly in $[0, 50]$. Submit your bid!



You will be participate in a first price auction with a competitor whose value is distributed uniformly in $[0, 50]$. Submit your bid!



<https://www.google.com/search?q=random+number+generator+0+to+50>



random number generator 0 to 50



All

Images

Videos


Shopping

Forums

More

Tools

About 68,700,000 results (0.39 seconds)

Min 

0

Max

50

GENERATE

You will be participate in a first price auction with a competitor whose value is distributed uniformly in $[60, 70]$. Submit your bid!

1-5

6-10

11-15

16-20

21-25

26-30

31-35

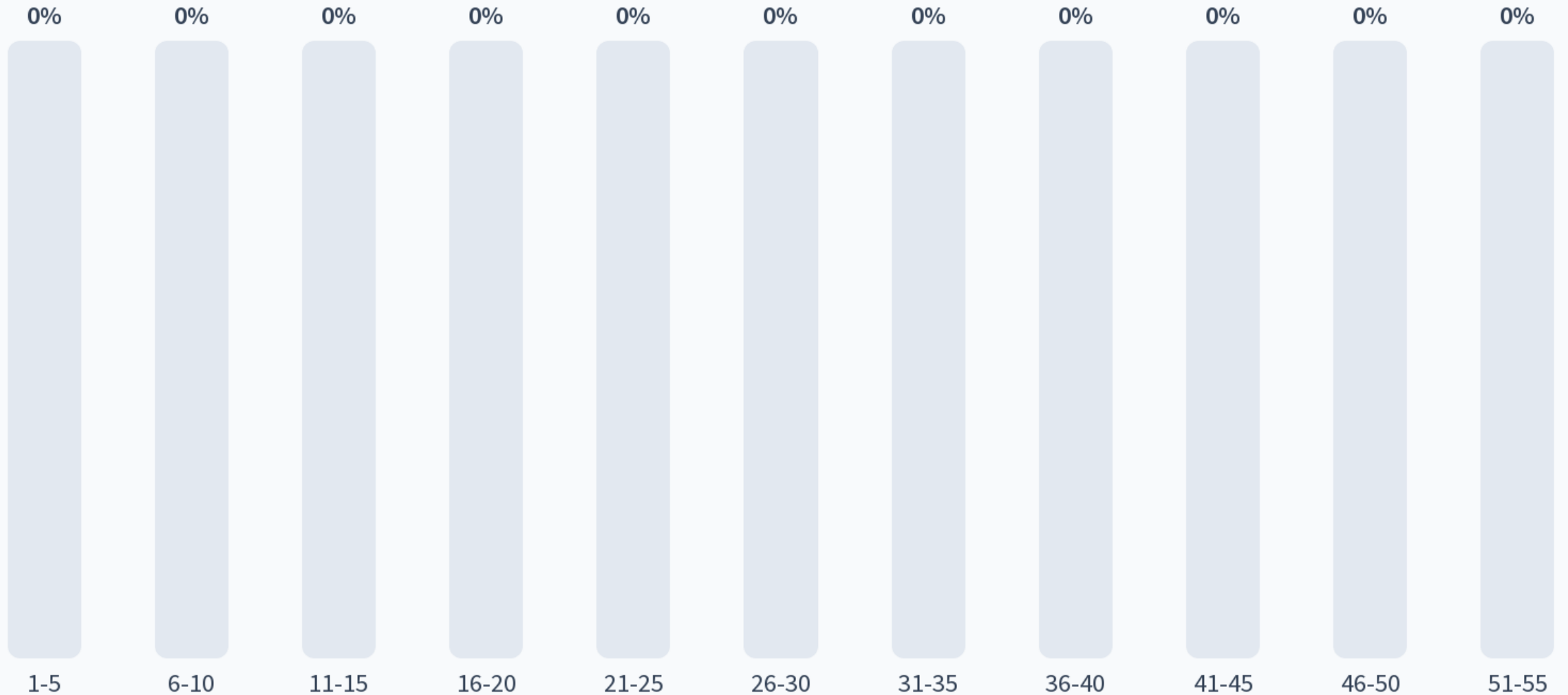
36-40

41-45

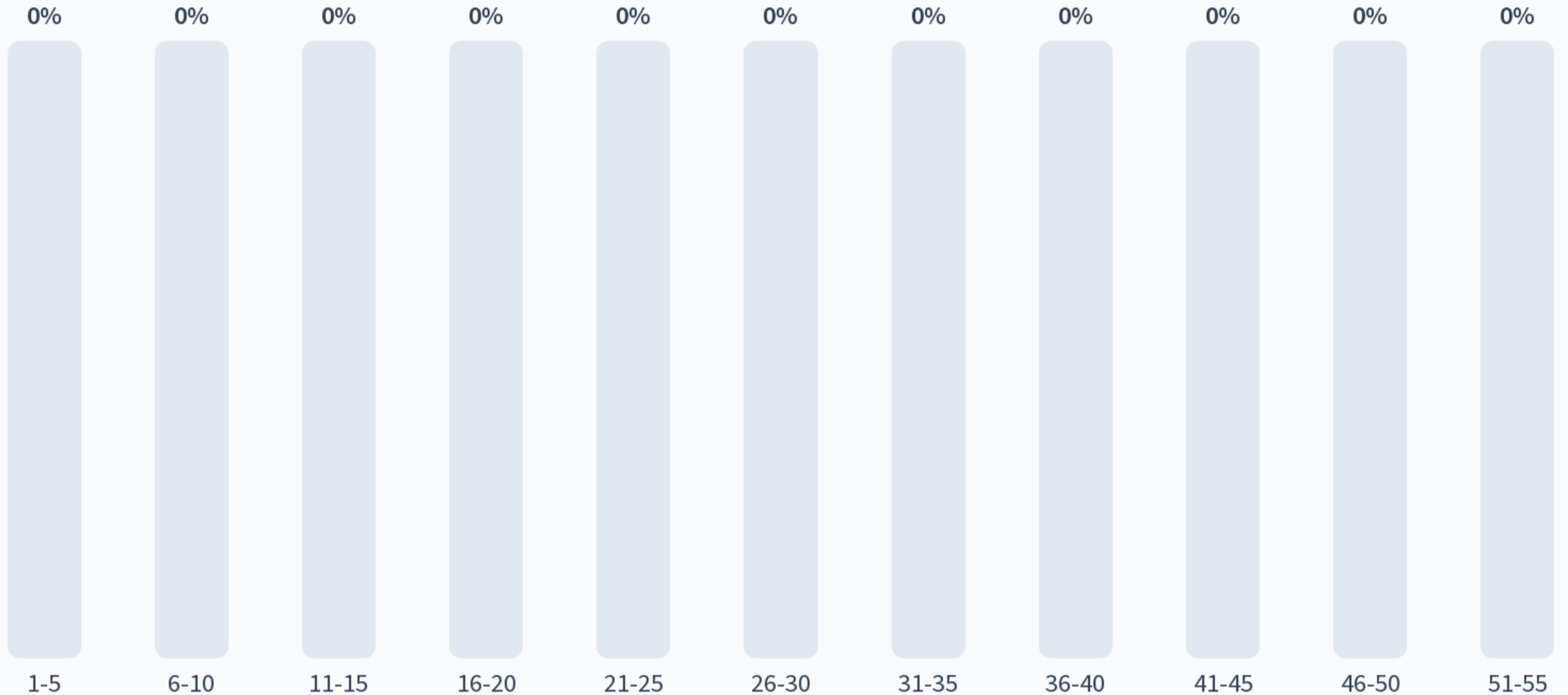
46-50

51-55

You will be participate in a first price auction with a competitor whose value is distributed uniformly in $[60, 70]$. Submit your bid!



You will be participate in a first price auction with a competitor whose value is distributed uniformly in $[60, 70]$. Submit your bid!



Equilibrium with Asymmetric Bidders

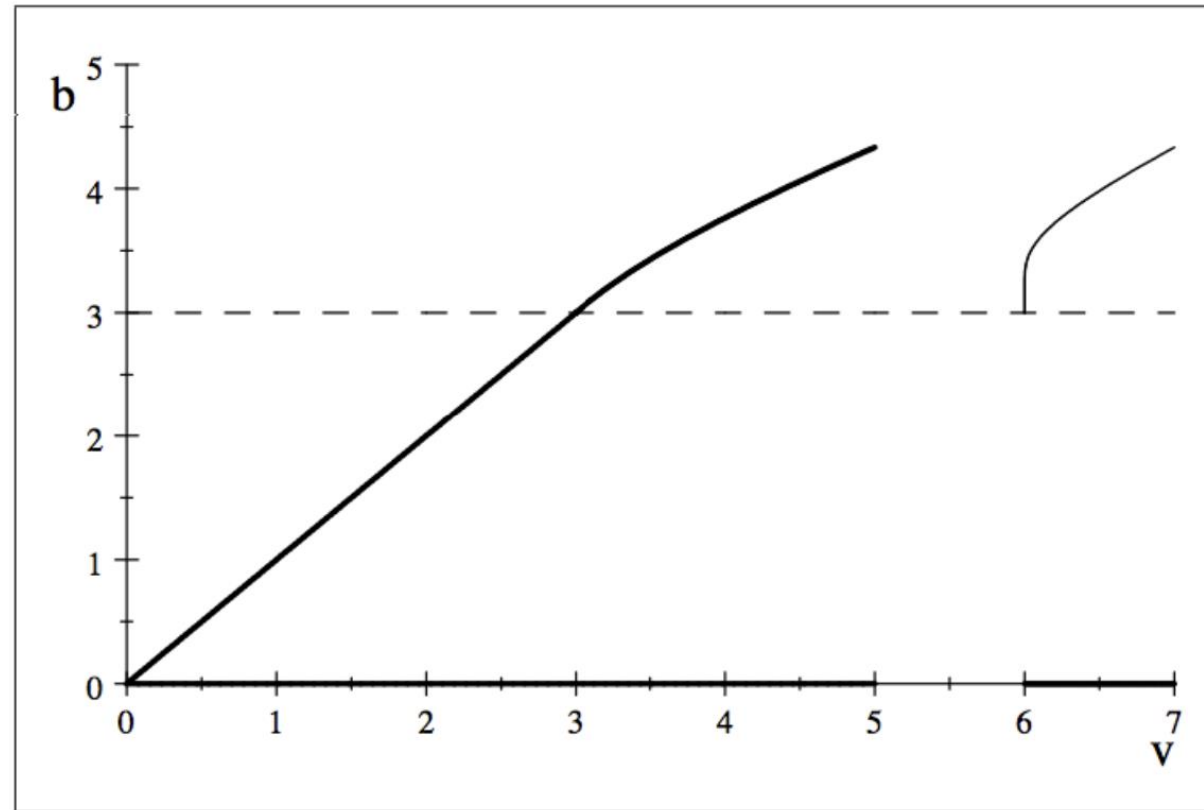


Figure 1: Equilibrium 1. The thicker line is buyer 1's bid function.

Inefficiency of First Price Auction

- At equilibrium, with positive probability, lower value player wins!
- Expected welfare of auction, not equal to expected highest value!
- The auction can be “inefficient”!
- How inefficient?

Efficiency Guarantees

- At any equilibrium, each bidder does not want to deviate to bidding half of their value!

- Either wins with such a bid and gains:

$$U'_i = v_i - \frac{v_i}{2} = \frac{v_i}{2}$$

- Or loses, in which case highest bid is that high:

$$\max_j b_j \geq \frac{v_i}{2}$$

$$\left. \begin{array}{l} \max_j b_j + U'_i \geq \frac{v_i}{2} \\ U'_i \geq \frac{v_i}{2} - \max_j b_j \end{array} \right\}$$

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$$\max_j b_j + U'_i \geq \frac{v_i}{2}$$



$$U'_i \geq \frac{v_i}{2} - \max_j b_j$$

- Equilibrium utility is at least this much and always non-negative:

$$U'_i \geq \left(\frac{v_i}{2} - \max_j b_j \right) \mathbb{1}(\text{highest value})$$

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$$E[U'_i] \geq E \left[\left(\frac{v_i}{2} - \max_j b_j \right) 1(\text{highest value}) \right]$$

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$$E[U_i(b)] \geq E[U'_i] \geq E \left[\left(\frac{v_i}{2} - \max_j b_j \right) 1(\text{highest value}) \right]$$

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$$E[U(b)] \geq E \left[\sum_i \frac{v_i}{2} 1(\text{highest value}) - \max_j b_j \right]$$

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$$U'_i \geq \frac{v_i}{2} - \max_j b_j$$

- Equilibrium utility is at least this much and always non-negative:

$$E[U(b)] \geq \frac{1}{2} E \left[\sum_i v_i \mathbf{1}(\text{highest value}) \right] - E[\text{Rev}(b)]$$

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$$U'_i \geq \frac{v_i}{2} - \max_j b_j$$

- Equilibrium utility is at least this much and always non-negative:

$$E[Rev(b) + U(b)] \geq \frac{1}{2} E \left[\sum_i v_i 1(\text{highest value}) \right]$$

Efficiency Guarantees

- At any equilibrium, each bidder does not want to deviate to bidding half of their value!

- Either wins with such a bid and gains:

$$U'_i = v_i - \frac{v_i}{2} = \frac{v_i}{2}$$

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$$\max_j b_j + U'_i \geq \frac{v_i}{2}$$



$$U'_i \geq \frac{v_i}{2} - \max_j b_j$$

- Equilibrium utility is at least this much and always non-negative:

$$E[SW(b)] \geq \frac{1}{2} E[\text{highest value}]$$

Sum: First Price

- First Price is arguably the simplest auction rule
- It can be hard to strategize in such an auction
- The auction can lead to inefficient allocations

- Though approximately efficient
- Still used in practice in many settings (e.g. online advertising, government procurement)
- Primarily because it has very transparent rules

Second-Price (Vickrey) Auction

- Each bidder submits a bid b_i
- The highest bidder wins the item (ties broken at random)
- The winner pays the second highest bid
- Utility of a bidder with value v_i under a bid profile b :

$$u_i(b; v_i) = (v_i - b_{(2)}) \cdot 1\{b_i \geq \max b_j\}$$

Second highest bid

<https://www.google.com/search?q=random+number+generator+1+to+10>



random number generator 1 to 10



All

Images

Videos

Shopping

Forums

More

Tools

Wheel

No repeats

Online

Lottery

About 667,000,000 results (0.32 seconds)

Min

1

Max

10



GENERATE

Submit your bid to the second-price auction!

0

1

2

3

4

5

6

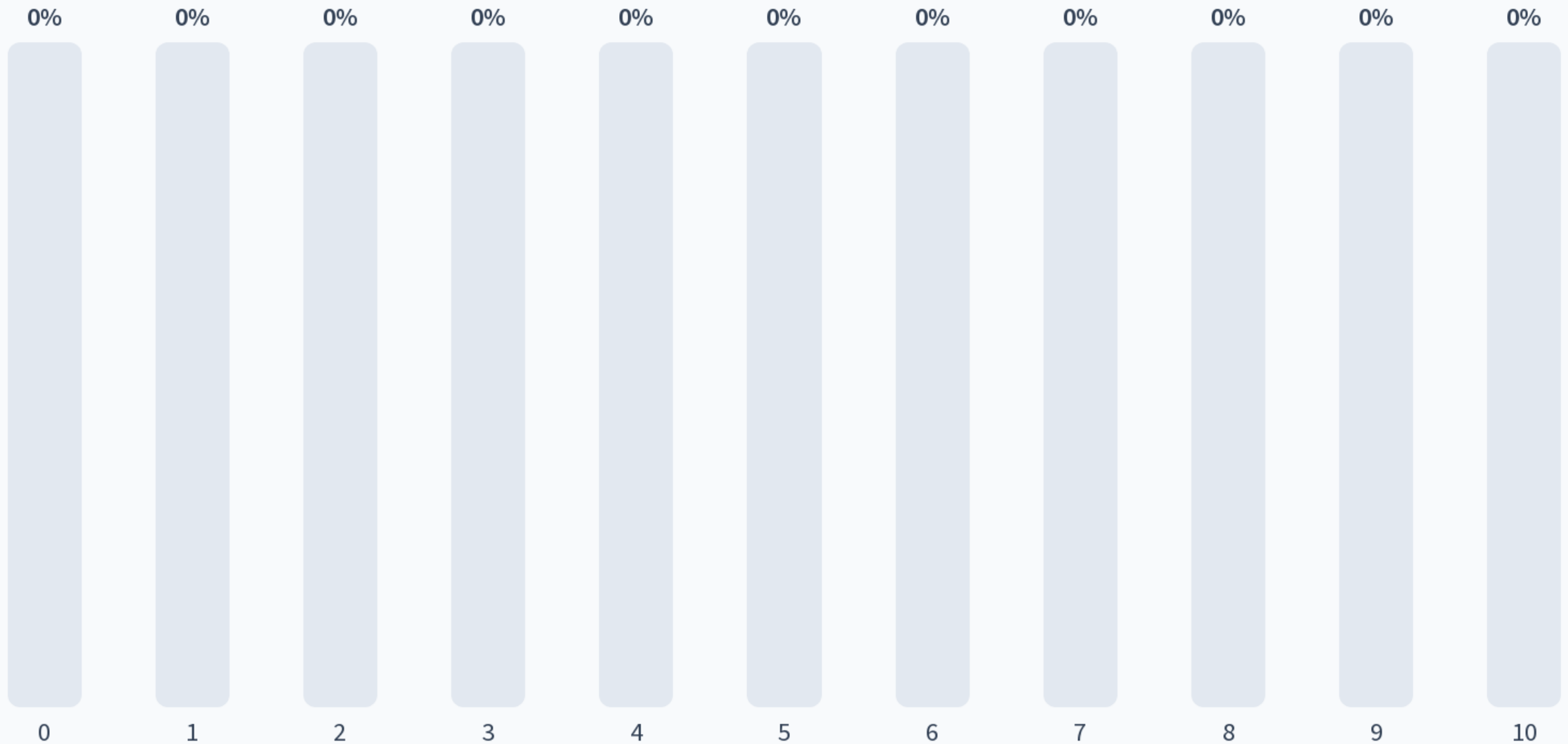
7

8

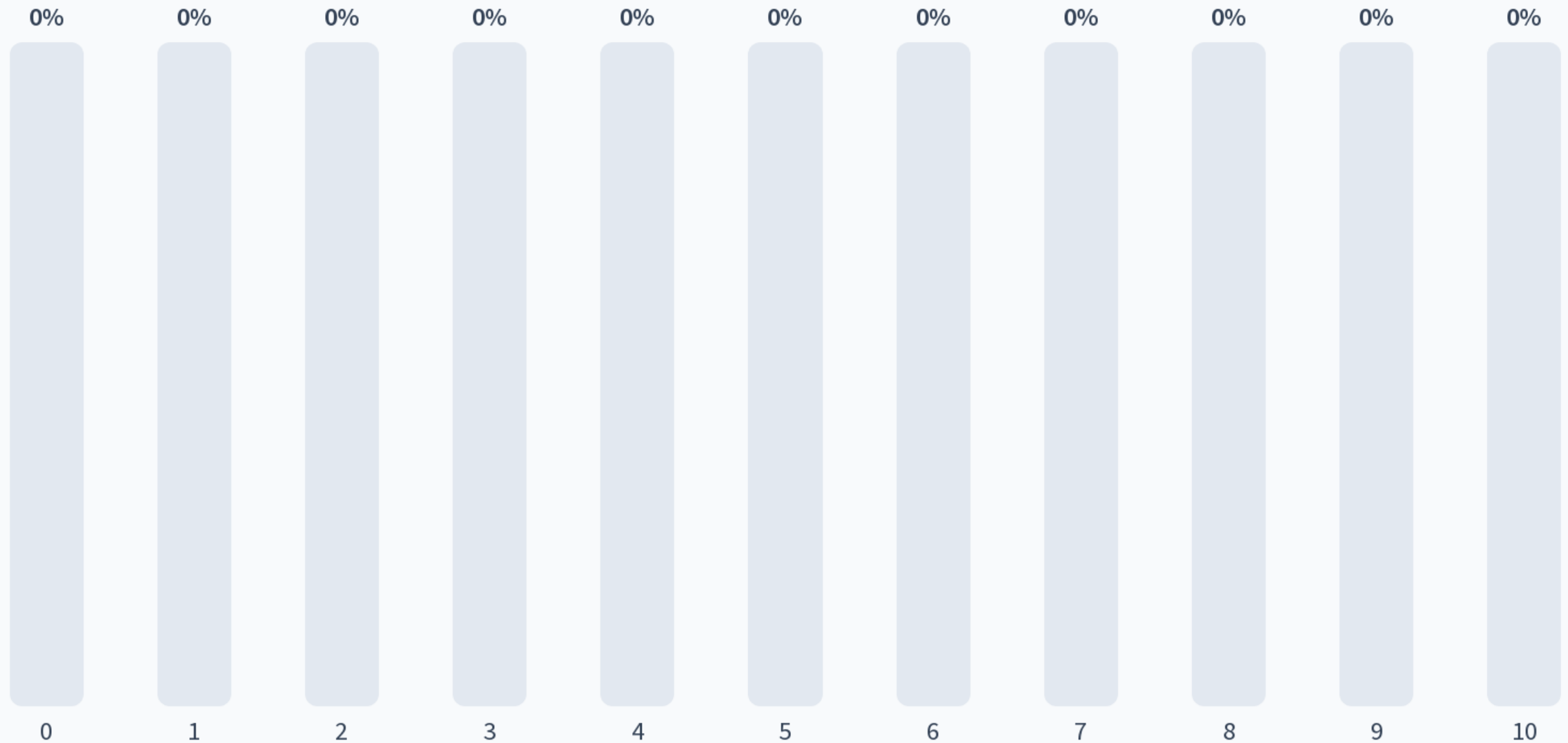
9

10

Submit your bid to the second-price auction!



Submit your bid to the second-price auction!



Truthfulness of Second Price Auction

- In a second price auction it is **dominant strategy** to bid your value
- No matter what the value is and no matter how others behave

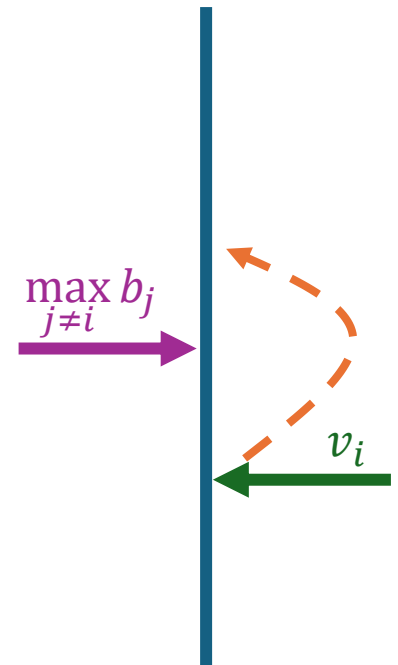
Suppose I bid my value. Would I want to deviate?

Truthfulness of Second Price Auction

- In a second price auction it is **dominant strategy** to bid your value
- No matter what the value is and no matter how others behave

Suppose I bid my value. Would I want to deviate?

- **Case 1.** My value is below highest other bid
- Only way to change anything is bid above
- But then I get negative utility as I pay more than value

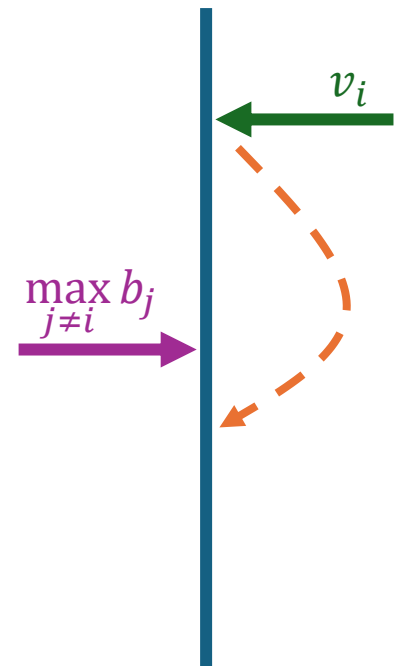


Truthfulness of Second Price Auction

- In a second price auction it is **dominant strategy** to bid your value
- No matter what the value is and no matter how others behave

Suppose I bid my value. Would I want to deviate?

- **Case 2.** My value is above highest other bid
- I get non-negative utility
- Only way to change anything is bid below
- But then I get zero utility as I lose



Sum: Second Price

- Second Price is arguably the simplest truthful auction rule
- It is very easy to strategize in such an auction (be truthful)
- Auction always leads to efficient allocations (highest value wins)
- Auction can be run very quickly (computationally efficient)

- Still not always the auction used in many places
- Primarily because it has not very transparent rules
- Susceptible to collusion and manipulations by the auctioneer

Sponsored Search Auctions

Sponsored Search Auctions

- Now we have many items to sell
- Slots on a web impressions
- Higher slots get more clicks!
- Each slot has some probability of click

$$a_1 > a_2 > \dots > a_m$$

- Bidders have a value-per-click v_i

Google digital advertising

All Images News Videos Shopping More

About 6,620,000,000 results (0.44 seconds)

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https://www.redditforbusiness.com

Advertise on Reddit

Reach over 100K communities — Connect with passionate communities that deliver results for brands across all industries. Create impact & own top communities in your target category for 24 hours. Try Reddit ads.

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Established in 2010 — Enjoy the perks of multi-channel targeting, measurement, & reporting with our interface. CTV.

a_1

a_2

a_3

a_4

Generalized First Price (GFP) Auction

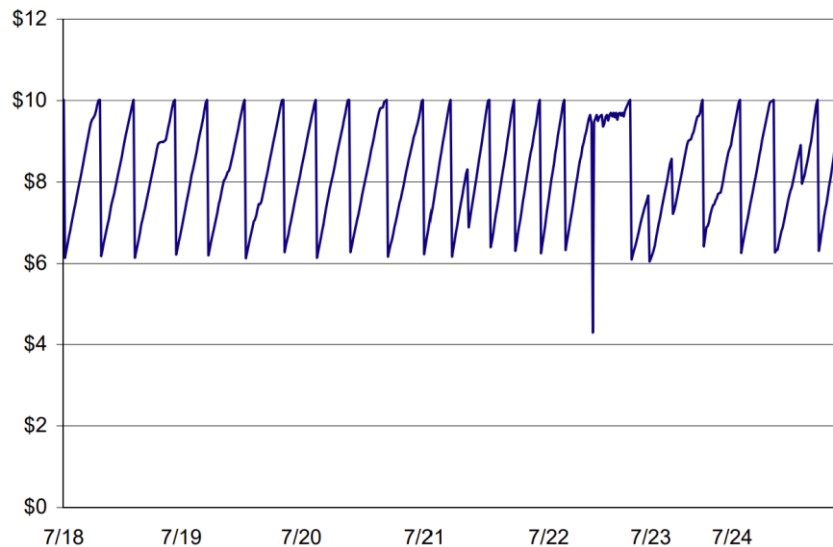
- Bidders submit a bid-per-click b_i
- Slots allocated in decreasing order of bids
- Bidder i is allocated slot $j_i(b)$
- Bidder pays their bid when clicked

$$u_i(b; v_i) = a_{j_i(b)} \cdot (v_i - b_i)$$

The image shows a Google search results page for the query "digital advertising". The page displays four sponsored advertisements. Each advertisement is annotated with a red box and a red label a_i on the right side, representing the value of the slot. To the left of the advertisements, there are four blue boxes containing the bids $b_{(1)}$, $b_{(2)}$, $b_{(3)}$, and $b_{(4)}$, arranged vertically. Roman numerals "IV" are placed between the bid boxes, indicating the order of the bids. The advertisements are: 1. Reddit (advertising on Reddit), 2. Microsoft (Microsoft Advertising), 3. coseom (Pay Per Click Company), and 4. Simpli.fi (Advertising Success Platform).

Generalized First Price (GFP) Auction

- The first auction that was used by Overture in late 90s
- Lead to weird bidding patterns



(b) 1 week

The screenshot shows a Google search for "digital advertising" with four sponsored ads. Annotations are placed as follows:

- $b_{(1)}$ (blue box) is next to the first ad (Reddit).
- a_1 (red box) is next to the first ad.
- $b_{(2)}$ (blue box) is next to the second ad (Microsoft).
- a_2 (red box) is next to the second ad.
- $b_{(3)}$ (blue box) is next to the third ad (COSEOM).
- a_3 (red box) is next to the third ad.
- $b_{(4)}$ (blue box) is next to the fourth ad (Simpli.fi).
- a_4 (red box) is next to the fourth ad.

Red lines connect each a_i box to the corresponding ad. Roman numerals IV are placed between the b boxes.

Generalized Second Price (GSP) Auction

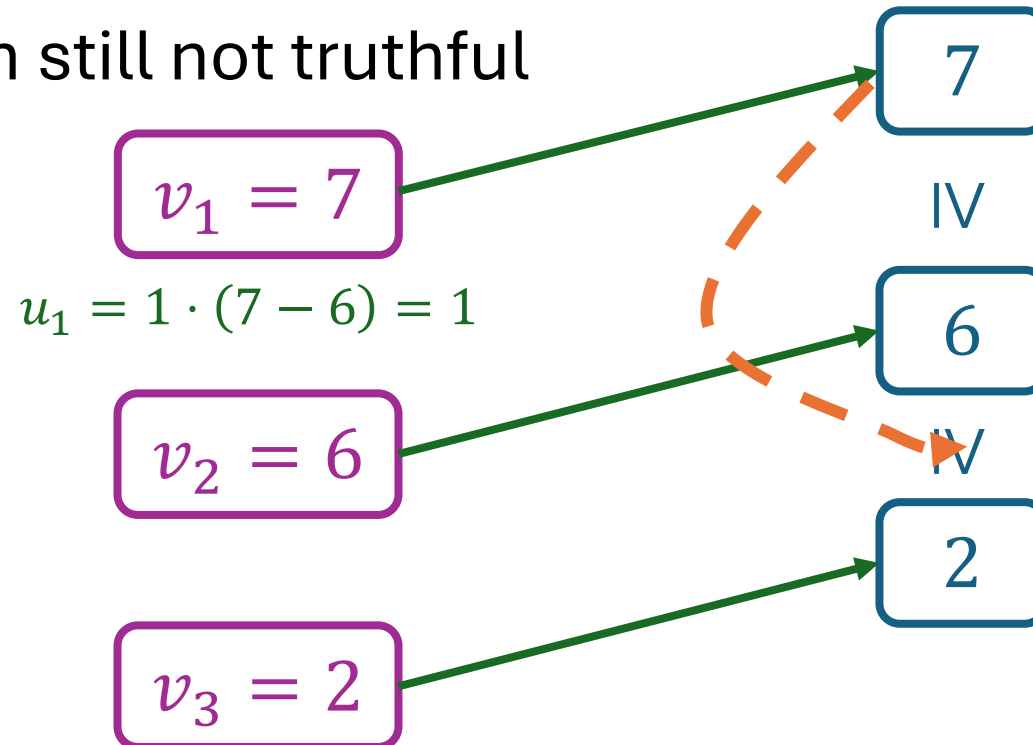
- Bidders submit a bid-per-click b_i
- Slots allocated in decreasing order of bids
- Bidder i is allocated slot $j_i(b)$
- Bidder pays the next highest bid when clicked

$$u_i(b; v_i) = a_{j_i(b)} \cdot (v_i - b_{(j_i(b)+1)})$$

The image shows a Google search results page for the query "digital advertising". The search results are annotated to illustrate a Generalized Second Price (GSP) auction. On the left side, four blue boxes labeled $b_{(1)}$, $b_{(2)}$, $b_{(3)}$, and $b_{(4)}$ are connected by vertical lines to the corresponding sponsored ads. The ads are: 1. Reddit (URL: https://www.redditforbusiness.com), 2. Microsoft Advertising (URL: https://about.ads.microsoft.com), 3. COSEOM (URL: https://www.coseom.com), and 4. Simpli.fi (URL: https://www.simpli.fi). On the right side, four green boxes labeled a_1 , a_2 , a_3 , and a_4 are connected by horizontal lines to the corresponding sponsored ads. The ads are sorted in descending order of their bid values b_i . The text "IV" is placed between the bid boxes, likely indicating a specific bid value or a step in the auction process. The search results page shows "About 6,620,000,000 results (0.44 seconds)".

Generalized Second Price (GSP) Auction

- The auction of choice in current sponsored search systems
- Even though still not truthful



Google digital advertising

All Images News Videos Shopping More

About 6,620,000,000 results (0.44 seconds)

Sponsored 1

Reddit
https://www.redditforbusiness.com

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Microsoft Advertising® | Get a \$500 Advertising Credit

We'll Help You Find Your Customers and Reach Searchers Across The Microsoft Network. Plus, Receive a \$500 Microsoft Advertising Credit When You Spend Just \$250! Free Sign Up.

$u'_1 = .5 \cdot (7 - 2) = 2.5$

Generalized First Price (GFP) Auction with Many Bells and Whistles

- Bidders submit a bid-per-click b_i
- Each bidder assigned a **quality score** s_i
- Slots allocated in decreasing order of **quality weighted bids** $s_i \cdot b_i$
- Bidder i is allocated slot $j_i(b)$
- Slots have **bidder-specific probability of click** $a_{i,j_i(b)}$
- Each bidder pays, per-click, the **highest bid that still gives them the same slot**

$$p_i(b) = \frac{s_{(j_i(b)+1)} \cdot b_{(j_i(b)+1)}}{s_i}$$

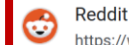
Google

digital advertising

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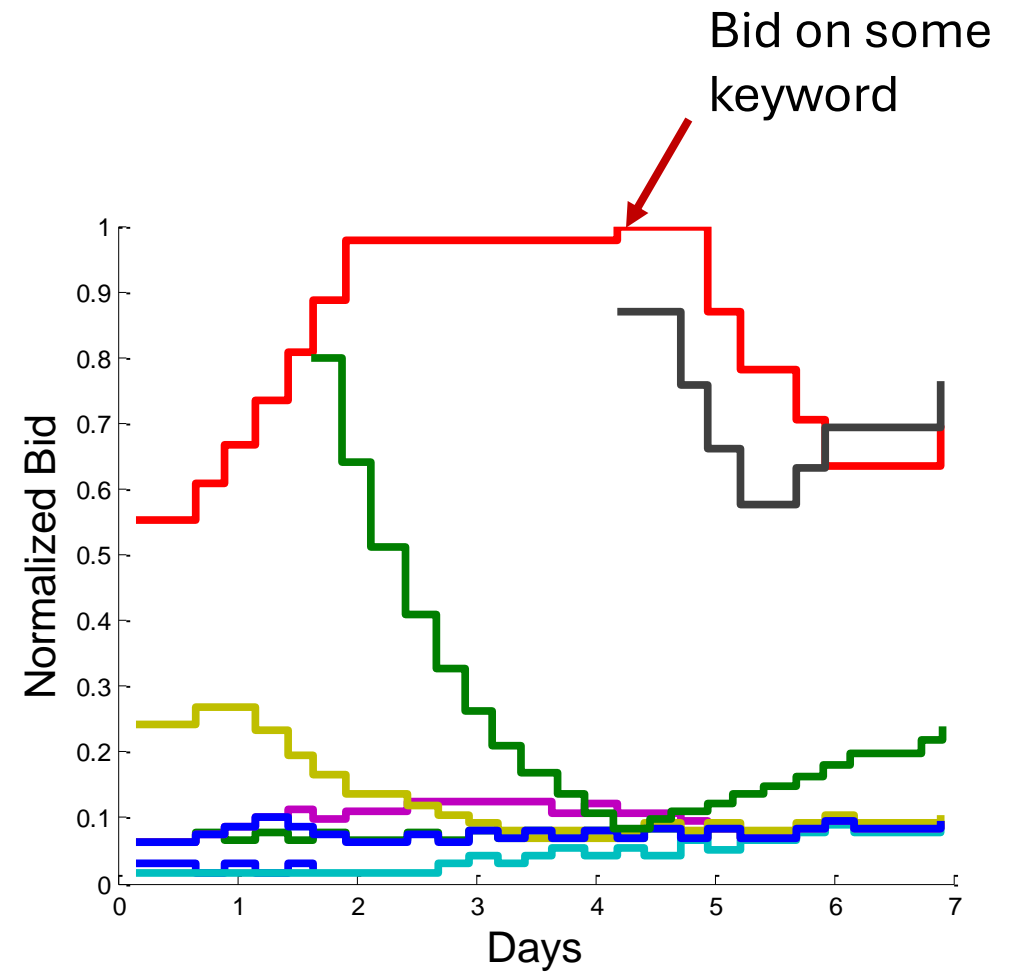
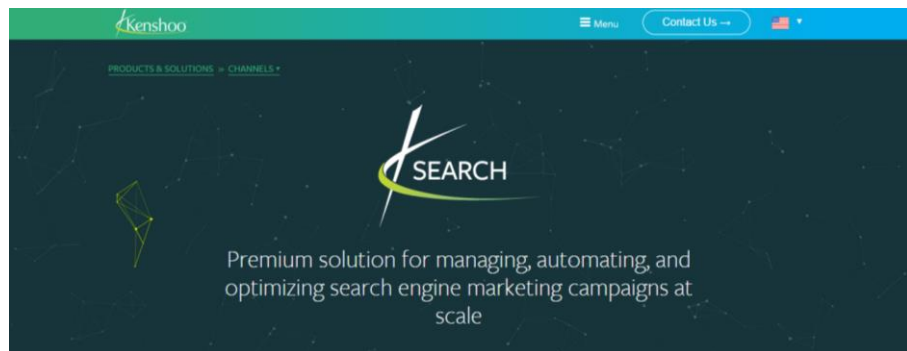
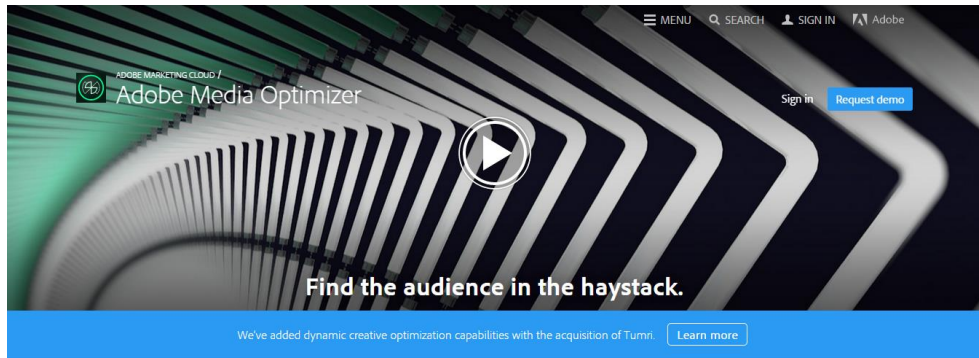


Simpli.fi

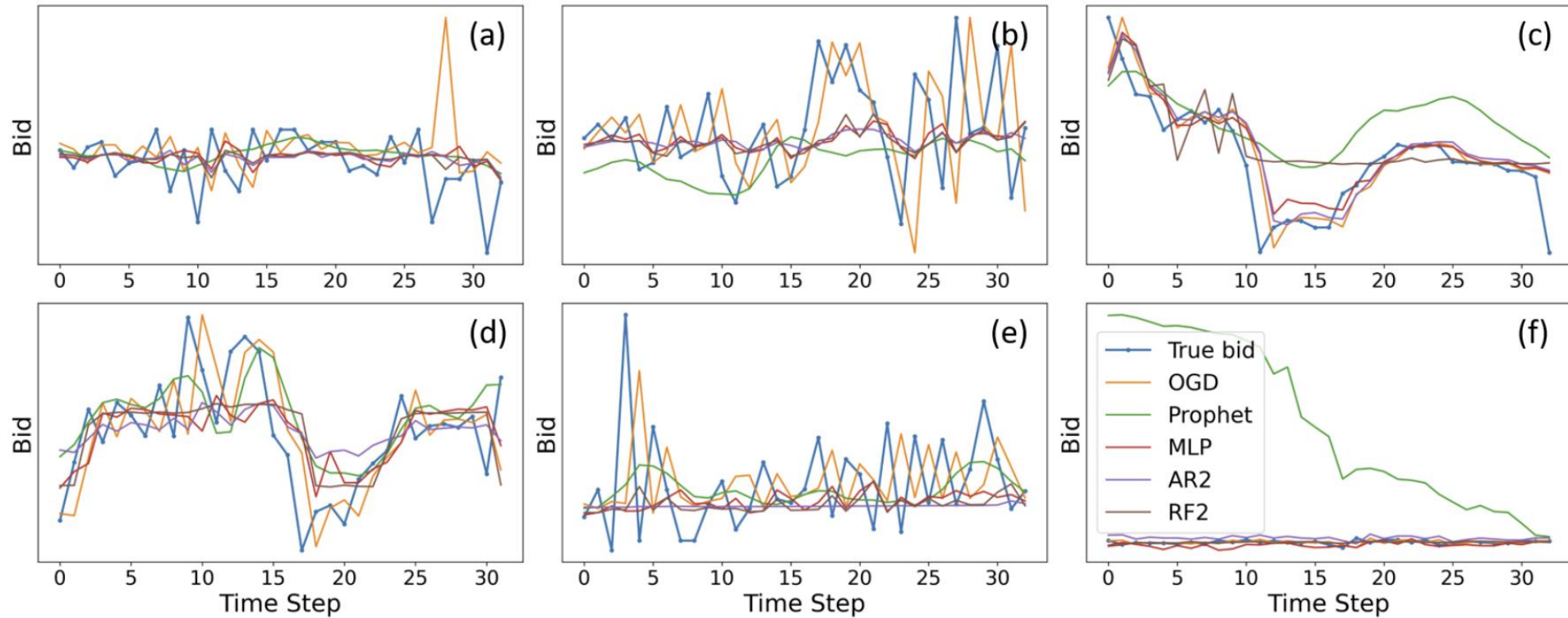
<https://www.simpli.fi>

[Simpli.fi | Advertising Success Platform](#)

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Simple learning dynamics are good predictors



(b) Stepahead Predictions