

# Lecture 04

## Morphology and Segmentation

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**1. Introduction**

2. Mathematical Morphology

3. Image Segmentation

4. Analyze segmented image

Previous lecture:

**convolution:**  $f(x, y), g(x, y), \underline{\mathbf{w}}: \mathbb{N} \rightarrow \mathbb{R}$

where  $w = \underline{\text{filter kernel}}$

→ (mostly) linear operators

Today:

**morphology:**  $f(x, y), g(x, y), \underline{\mathbf{b}}: \mathbb{N} \rightarrow \{0, 1\}$

where  $b = \underline{\text{structuring element}}$

→ non-linear operators

→ concerned with connectivity and shape (close to [set theory](#))

**segmentation:**

→ labeling image pixels to partition an image into regions

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1. Introduction

2. Mathematical Morphology

1. Basic concepts
2. Primitive Morphological Operations
3. Composite Morphological Operations

3. Image Segmentation

4. Analyze segmented image

- Initially proposed for binary images (*Matheron and Serra, 1964*)  
→ later extended to gray-scale images, and later color images
  
- Binary images produced by simple thresholding are imperfect due to image noise, etc.  
⇒ morphological image processing attempts to remove these imperfections
  
- Main applications:
  - Image pre-processing (noise filtering, shape simplification)
  - Enhancing object structure (skeletonizing, convex hull, ...)
  - Segmentation
  - Quantitative description of objects (area, perimeter, ...)

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Morphological filtering mechanics are similar to spatial filtering using convolutions:

1) a kernel called a **structuring element** is used to determine filtering operation:

- the size is determined by the matrix dimensions
- the shape is determined by the pattern of 1 and 0 in the matrix
- the origin is usually the matrix center, although it can also off-centered or even outside it

NB: like convolution kernels, it is common to have structuring elements of odd dimensions with the center as the origin.

NB: the shape, size, and orientation of the structuring element depends on application

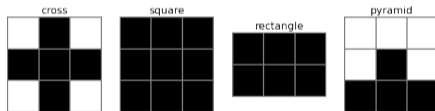
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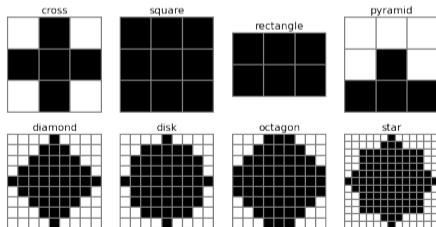
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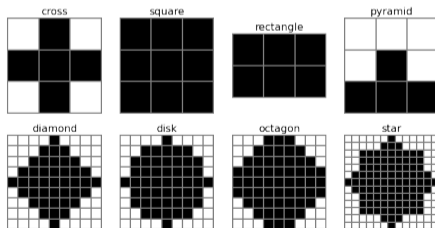
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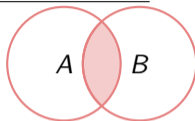
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2) the image is first **padded**, and the structuring element then **slides** across it

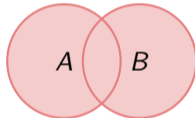
## Morphological filters are essentially set operations

Intersection (AND)



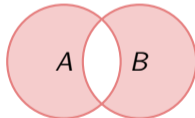
$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Union (OR)



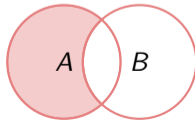
$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Symmetric difference (XOR)



$$\overline{A \cap B}$$

Difference

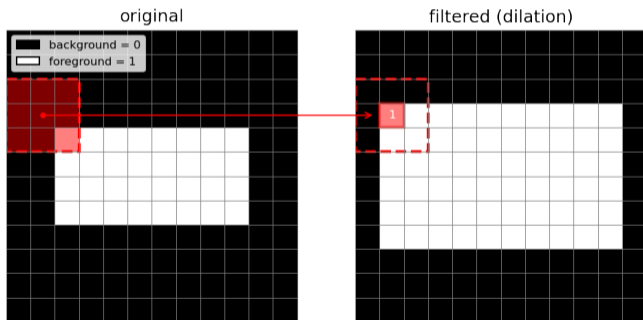


$$A - B$$

- Primary Morphological Operations are: dilation and erosion
- Concatenation of dilation and erosion result in higher level operations
  - ⇒ Composite Morphological Operations: closing and opening

1. **Dilation**: the dilation of a set  $F$  with a structuring element  $b$  is defined as:

$$G = F \oplus b = \{x : (\hat{b})_x \cap F \neq \emptyset\}$$

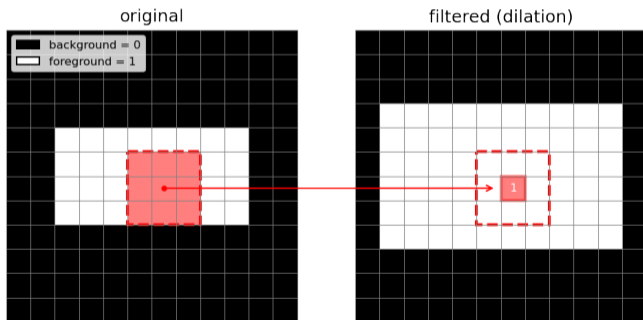


if  $\geq 1$  pixel within the mask = "1", the result is "1", otherwise "0"



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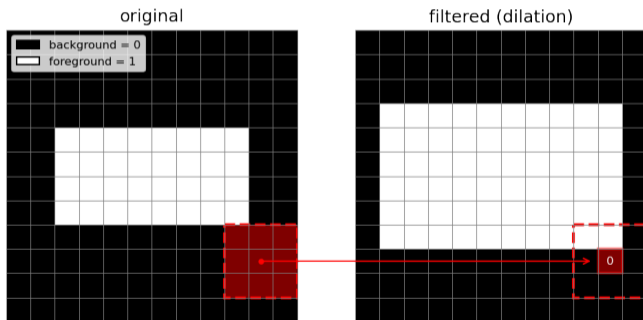
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## 2.2. Primitive Morphological Operations

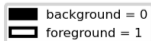
1. **Dilation**: the dilation of a set  $F$  with a structuring element  $b$  is defined as:

- ⇒ Foreground objects get larger
- ⇒ Background objects get smaller
- ⇒ Small gaps are closed

original



dilation (b=3x3)



## 2.2. Primitive Morphological Operations

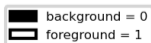
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dilation (b=7x7)



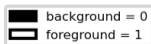
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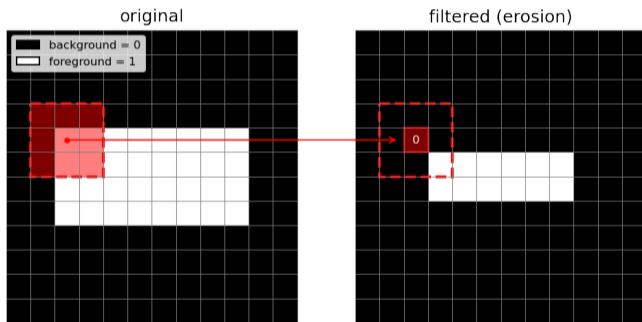


dilation (b=11x11)



2. **Erosion**: the erosion of a set  $F$  with a structuring element  $b$  is defined as:

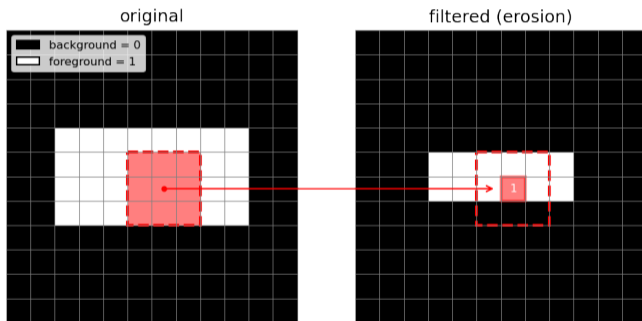
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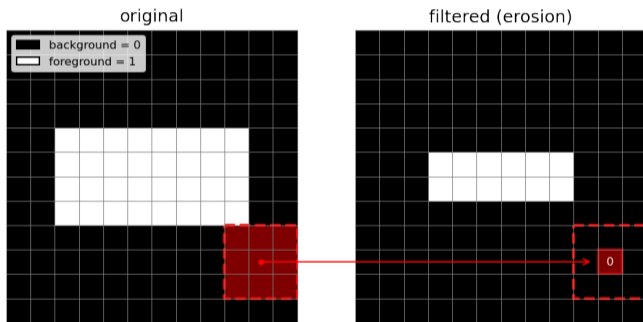
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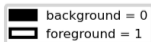
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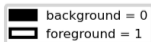
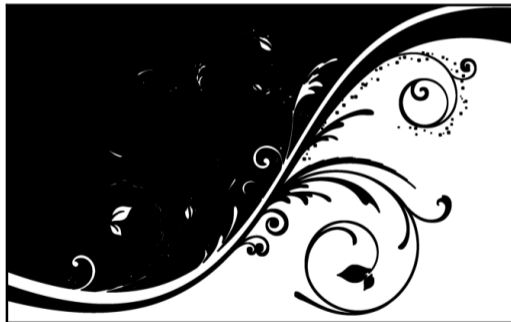
original

erosion ( $b=3 \times 3$ )

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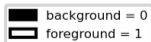
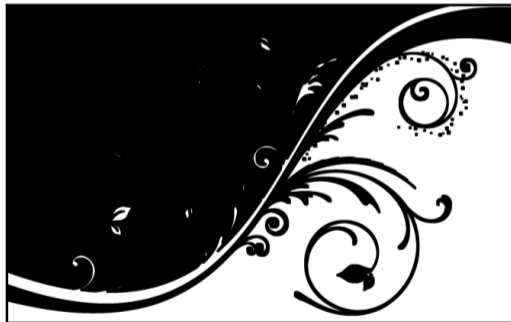
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erosion (b=11x11)



Concatenation of **dilation** and **erosion** result in higher level operations: **closing**, **opening**

1. **Opening**:

Problem: erosion causes deletion of small objects, BUT other objects shrink

Solution: after *erosion*, apply *dilation* with the same structuring element  $\Rightarrow$  **opening**

$$G = F \circ b = (F \ominus b) \oplus b$$

Usage example: removing small isolated “bright spots” (EX: volcanic  $SO_2$  detection from Sentinel-5P as foreground ( $mask=1$ ))

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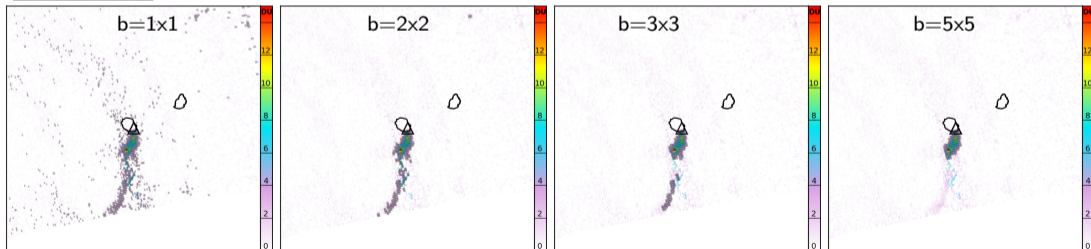
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Concatenation of **dilation** and **erosion** result in higher level operations: **closing**, **opening**

## 2. **Closing**:

Problem: dilation closes small holes and fractions, BUT objects get enlarged

Solution: after *dilation*, apply *erosion* with the same structuring element  $\Rightarrow$  **closing**

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Usage example: removing small isolated "dark spots" (binary mask value = 0)



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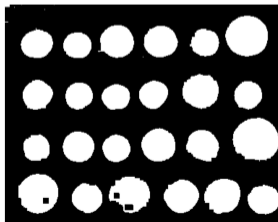
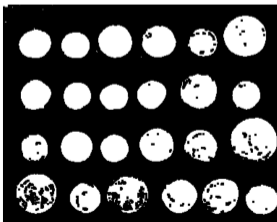
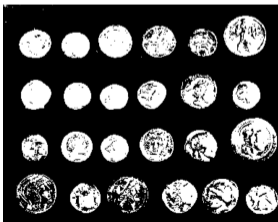
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original

binarized

closing b=3x3

closing b=7x7



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2. Mathematical Morphology

**3. Image Segmentation**

1. histogram-based segmentation

2. edge-based segmentation

3. region-based segmentation

4. Analyze segmented image

**Image segmentation** = labeling image pixels to partition an image into regions

- Histogram-based segmentation

⇒ based on thresholding of pixel values

ex: manual thresholding

ex: automatic thresholding (e.g., Otsu)

ex: k-means clustering

- Edge-based segmentation

⇒ based on local contrast → uses gradients rather than the grey values

- Region-based segmentation

⇒ based on image gradients and region properties

ex: Watershed transform

ex: Random Walker

ex: Flood Fill

- Many other!

ex: Graph-cuts

ex: Active Contours, Region Growing, Weighted Pyramid Linking, Mean-Shift, etc.

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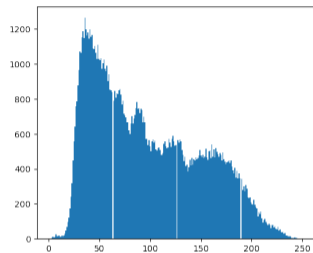
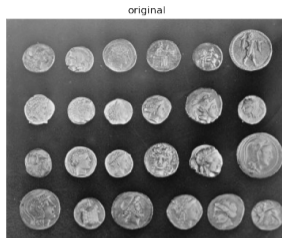
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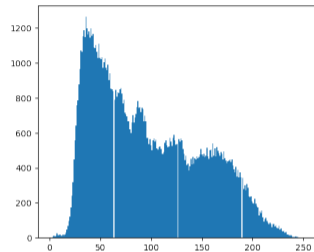
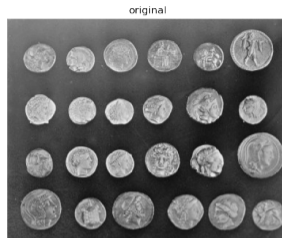
- global thresholding
  - manual
  - automatic (e.g. [Otsu's method](#))  
(threshold calculated to separate pixels into two classes, minimizing intra-class intensity variance)



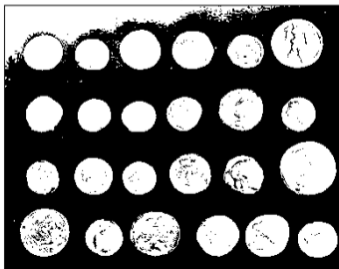
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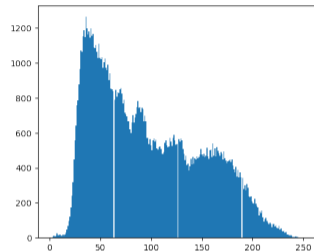
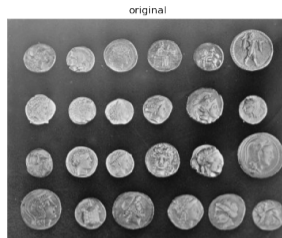
automatic threshold (Otsu thresh=107)



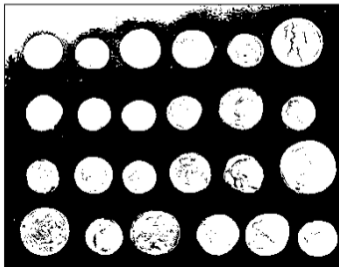
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⇒ based on thresholding pixel values

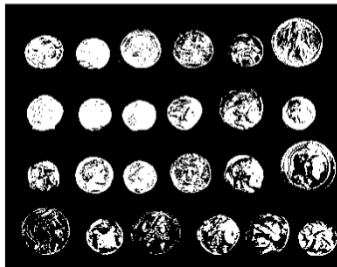
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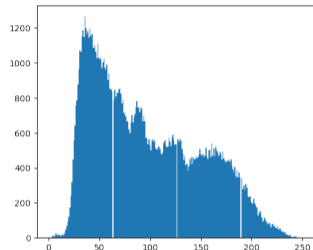
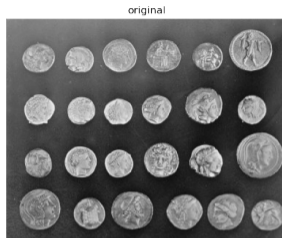
manual threshold (thresh=150)



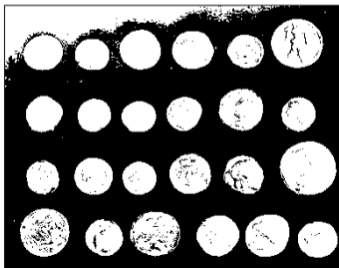
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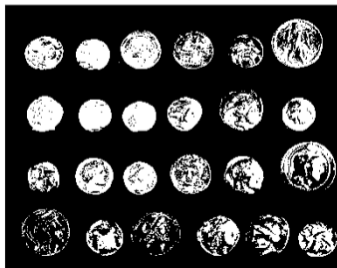
- global thresholding
  - manual
  - automatic (e.g. [Otsu's method](#))  
(threshold calculated to separate pixels into two classes, minimizing intra-class intensity variance)
- local thresholding (adaptive)  
(thresholds calculated based on pixel local neighborhood)



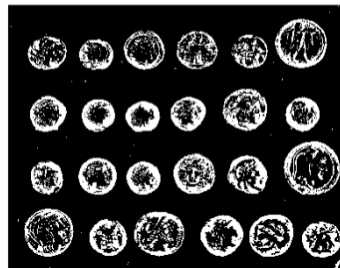
automatic threshold (Otsu thresh=107)



manual threshold (thresh=150)



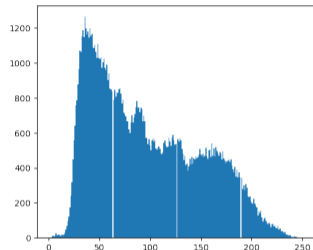
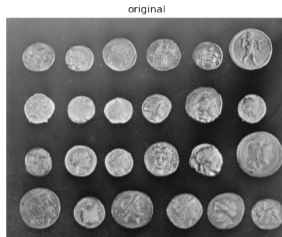
local thresholds (blocksize=41, offset=-20)



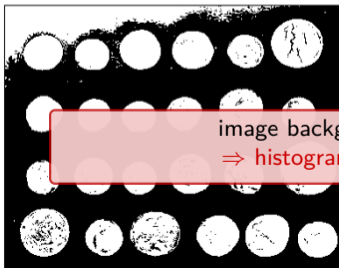
## Histogram-based segmentation

⇒ based on thresholding pixel values

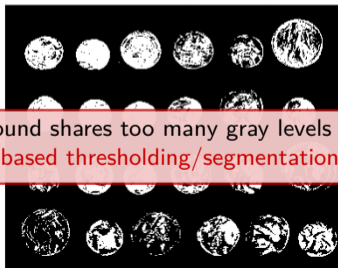
- global thresholding
  - manual
  - automatic (e.g. [Otsu's method](#))  
(threshold calculated to separate pixels into two classes, minimizing intra-class intensity variance)
- local thresholding (adaptive)  
(thresholds calculated based on pixel local neighborhood)



automatic threshold (Otsu thresh=107)



manual threshold (thresh=150)



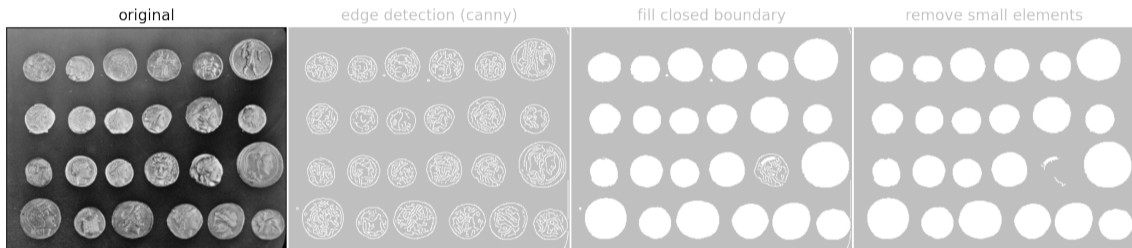
local thresholds (blocksize=41, offset=-20)



image background shares too many gray levels with the coins  
 ⇒ histogram-based thresholding/segmentation is insufficient

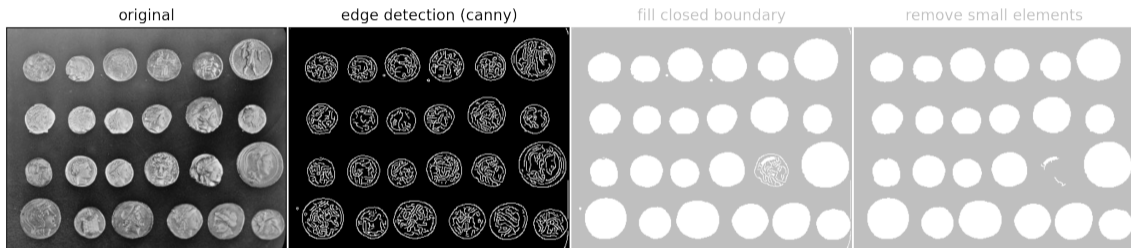
## Edge-based segmentation

⇒ based on image gradients



## Edge-based segmentation

⇒ based on image gradients

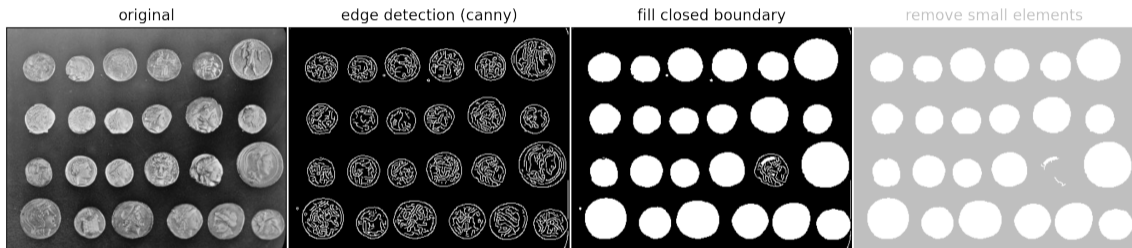


1. apply **Canny** edge detection algorithm (involves gradient detection using e.g. Sobel operator)



## Edge-based segmentation

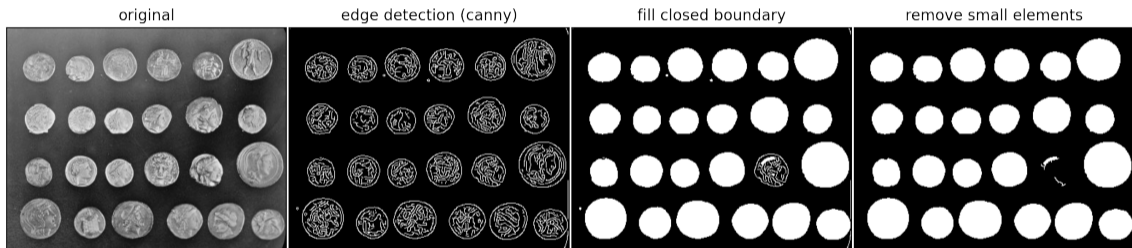
⇒ based on image gradients



1. apply **Canny** edge detection algorithm (involves gradient detection using e.g. Sobel operator)
2. apply mathematical morphology to fill inner part of the coins

## Edge-based segmentation

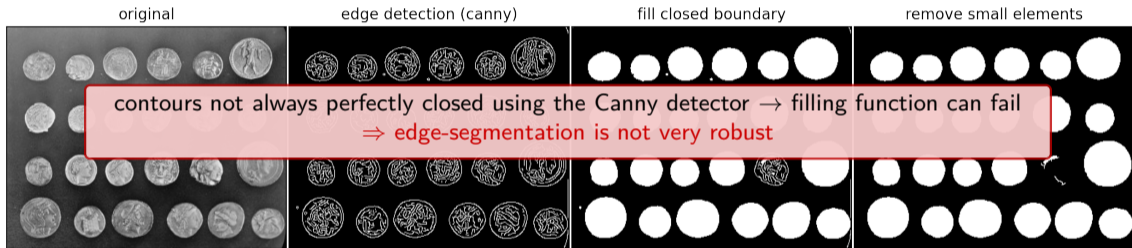
⇒ based on image gradients



1. apply **Canny** edge detection algorithm (involves gradient detection using e.g. Sobel operator)
2. apply mathematical morphology to fill inner part of the coins
3. remove objects smaller than a threshold

## Edge-based segmentation

⇒ based on image gradients



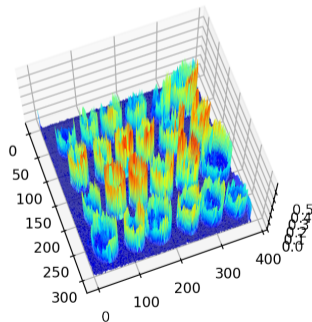
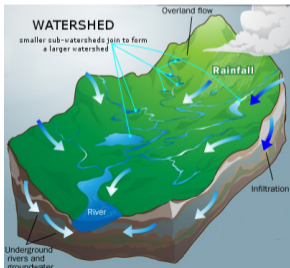
1. apply **Canny** edge detection algorithm (involves gradient detection using e.g. Sobel operator)
2. apply mathematical morphology to fill inner part of the coins
3. remove objects smaller than a threshold

### Region-based segmentation: *watershed transform*

⇒ region-growing approach that fills “basins” in the image

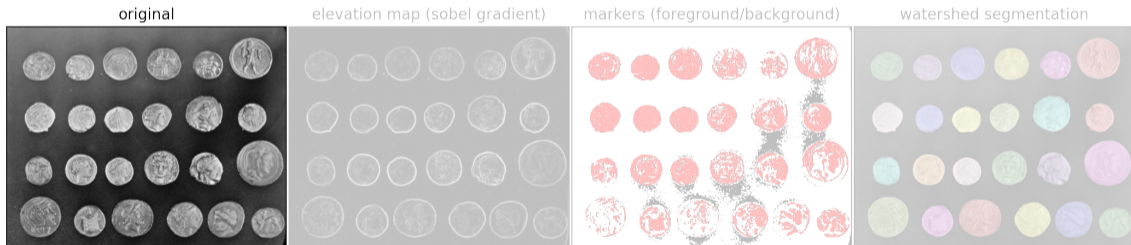
## Region-based segmentation: *watershed transform*

- ⇒ region-growing approach that fills “basins” in the image
- ⇒ the name “watershed” comes from an analogy with hydrology:
  - the *watershed transform* “floods” a “topographic” representation of the image
  - flooding starts from “markers”, in order to determine the catchment basins of these markers



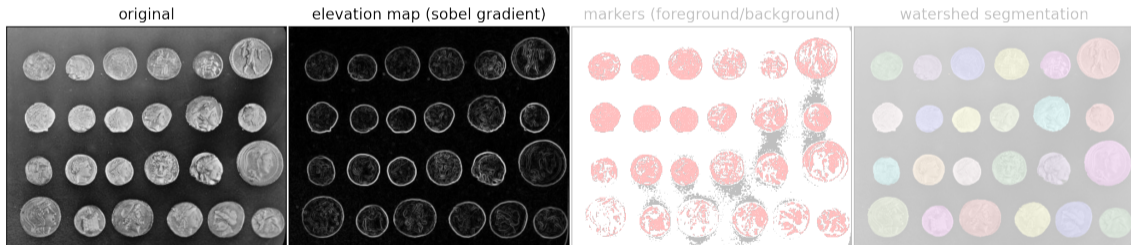
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## Region-based segmentation: *watershed transform*

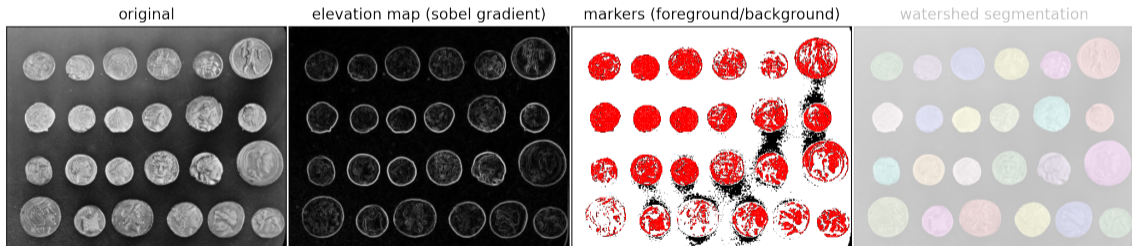
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1. build “elevation map” from image gradient amplitude (using the Sobel operator)

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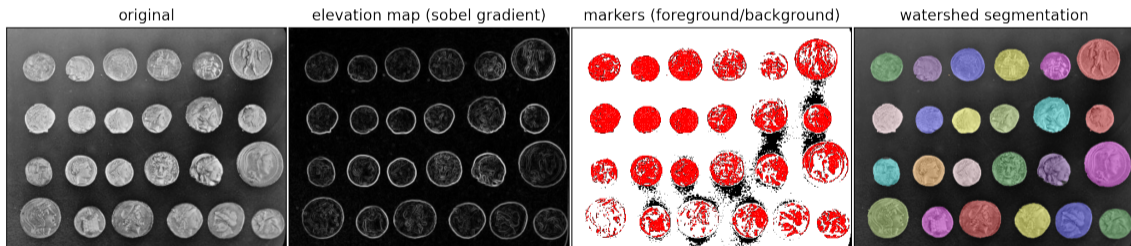


1. build “elevation map” from image gradient amplitude (using the Sobel operator)
2. define markers for background / foreground (here based on the extreme parts of the histogram)



## Region-based segmentation: *watershed transform*

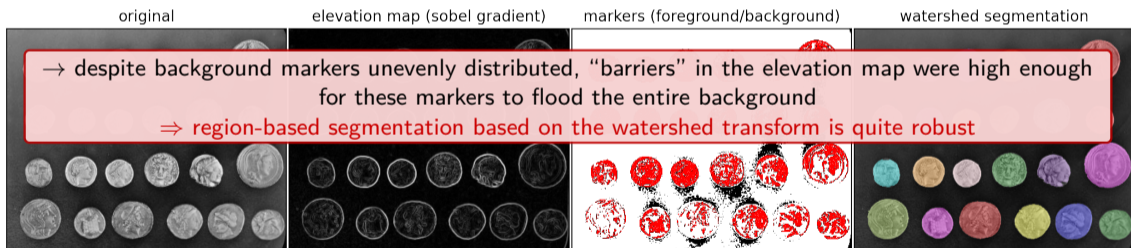
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1. build “elevation map” from image gradient amplitude (using the Sobel operator)
2. define markers for background / foreground (here based on the extreme parts of the histogram)
3. apply **watershed transform** (and colorize segmented elements)

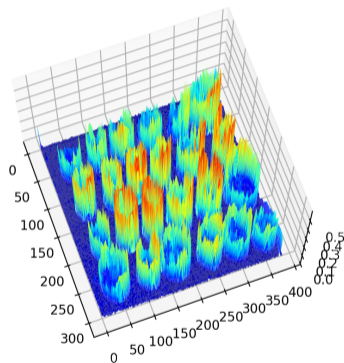
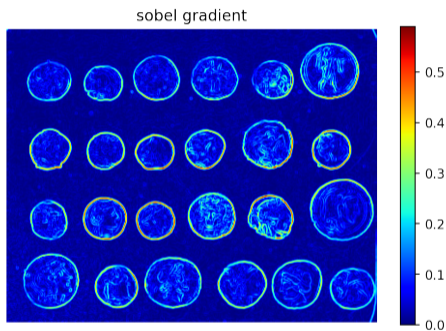
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## Region-based segmentation: *watershed transform*



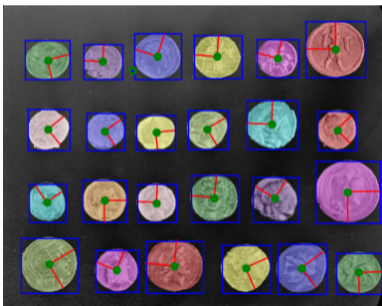
1. Start with lowest "altitude" (Gradient amplitude)
2. Increase the "water level" each time by 1
3. Merge all connected pixel with same/less level

1. Introduction
2. Mathematical Morphology
3. Image Segmentation
4. Analyze segmented image

The segmented elements can be analysed individually to:

→ provide statistics on their shape, distribution, orientation, etc.

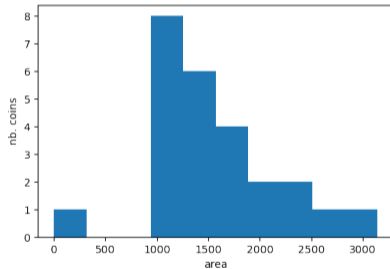
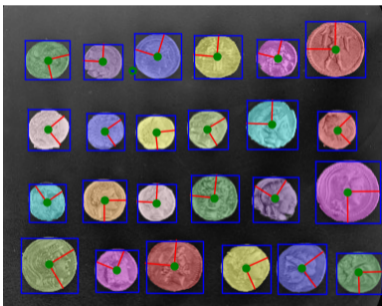
(e.g. fields in a satellite image, crystal/bubble shape distribution in a rock sample, etc.)



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(e.g. fields in a satellite image, crystal/bubble shape distribution in a rock sample, etc.)



## Exercises:

1. Exercise 1:
  - ⇒ histogram-based segmentation of Popocatépetl
2. Exercise 2:
  - ⇒ analyze a thermal infrared image of a lava lake
  - segment the crustal plates from the incandescent cracks and analyze