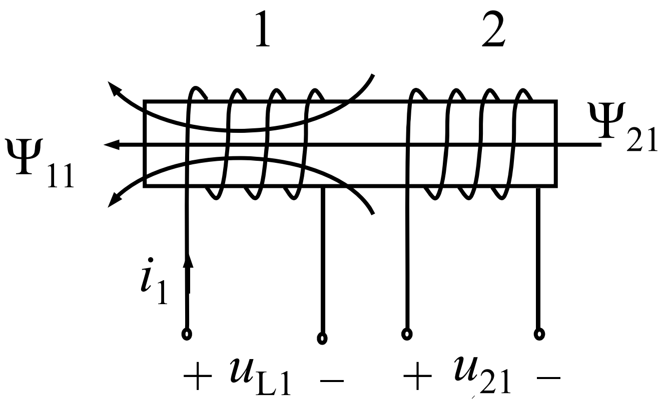
第七章 含有互感的电路

§7-1 互感与互感电路

一、互感与互感电路



Ψ_{11} ——自感磁链

自感系数：$L_1 = \dfrac{\Psi_{11}}{i_1}$

Ψ_{21}——互感磁链

互感系数：$M_{21} = \dfrac{\Psi_{21}}{i_1}$ 简称互感。

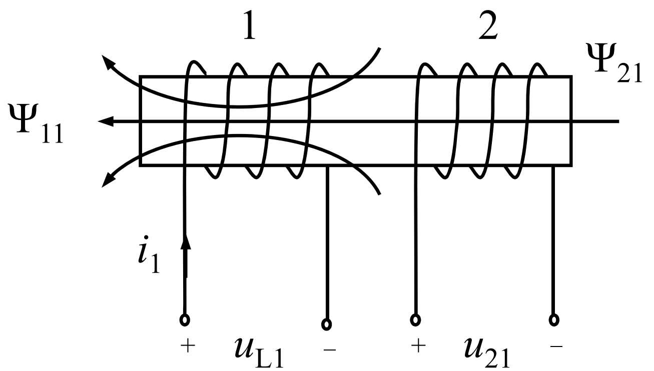
同样有

$$L_2 = \frac{\Psi_{22}}{i_2} \qquad M_{12} = \frac{\Psi_{12}}{i_2}$$

$M_{12} = M_{21} = M$ 单位：亨。

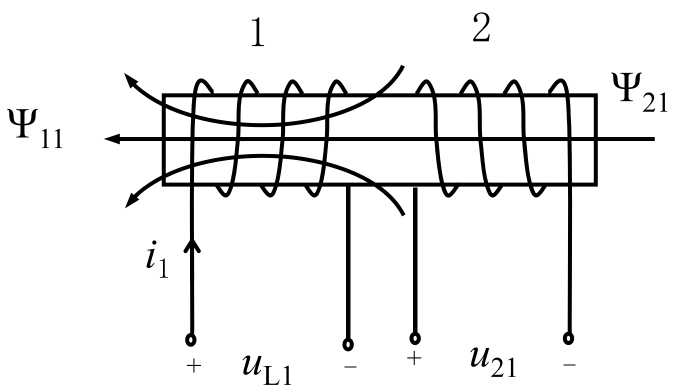
自感电压：　　（关联）

$$u_{L1}=\frac{d\psi_{11}}{dt}=L_1\frac{di_1}{dt}$$



互感电压u_{21}：压降方向与 Ψ_{21}成右螺旋

$$u_{21}=\frac{d\psi_{21}}{dt}=M\frac{di_1}{dt}$$

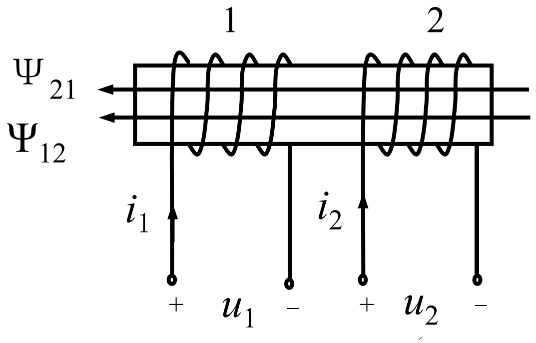


反之

$$u_{21}=\frac{d\psi_{21}}{dt}=-M\frac{di_1}{dt}$$

$$u_{12}=\pm M\frac{di_2}{dt}$$

两线圈均有电流，且取关联参考方向：

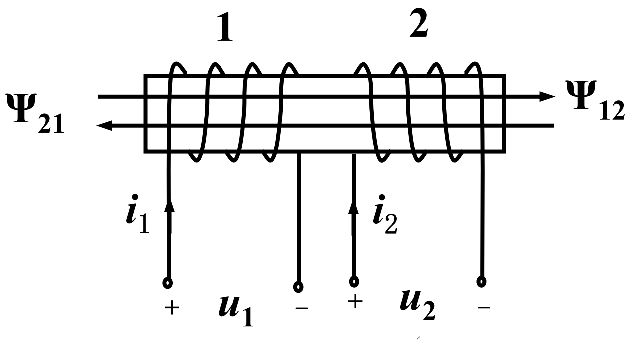


$$u_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$u_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

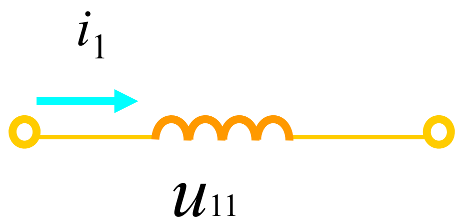
$$u_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$



二、同名端

对自感电压，当u, i 取关联参考方向，u、i与Φ符合右螺旋定则，其表达式为

$$u_{11}=\frac{\mathrm{d}\Psi_{11}}{\mathrm{d}t}=N_1\frac{\mathrm{d}\Phi_{11}}{\mathrm{d}t}=L_1\frac{\mathrm{d}i_1}{\mathrm{d}t}$$



上式 说明，对于自感电压由于电压电流为同一线圈上的，只要参考方向确定了，其数学描述便可容易地写出，可不用考虑线圈绕向。

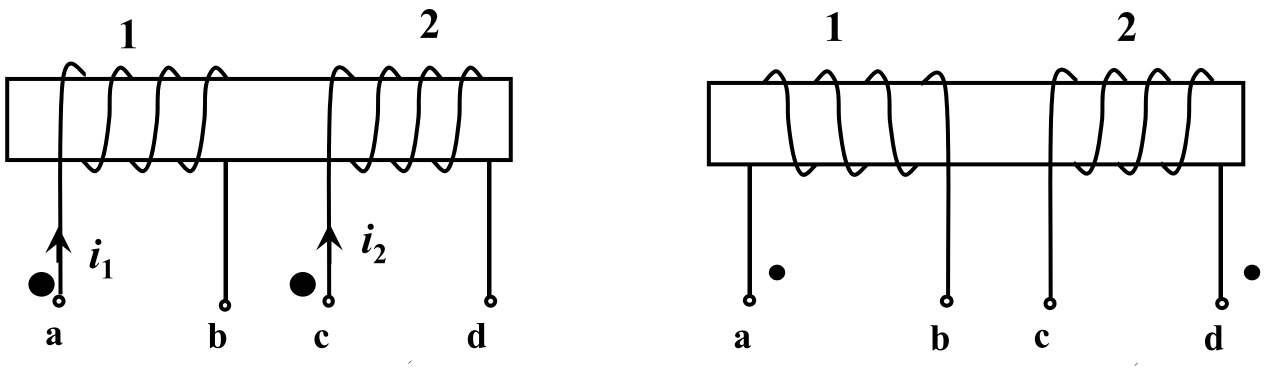
对互感电压，因产生该电压的的电流在另一线圈上，因此，要确定其符号，就必须知道两个线圈的绕向。这在电路分析中显得很不方便。为解决这个问题引入同名端的概念。

方便作图，用“•”或“*”表示两线圈绕向及其相对位置的关系。

同名端标记方法：

（1）若知线圈的结构（绕向）

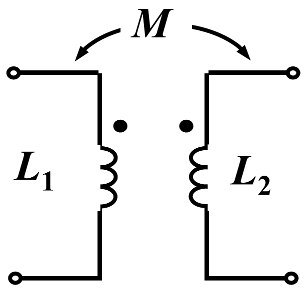
当两线圈的电流均由同名端流入时，两电流所产生的磁通应相互增强。

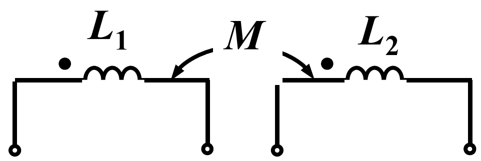


a与c为同名端，

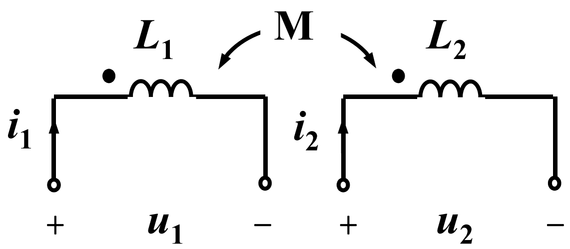
a与d为异名端。

画法：





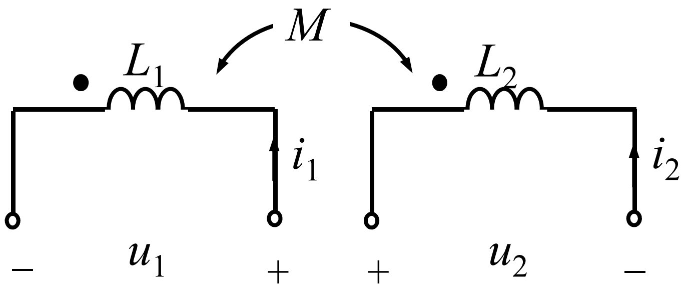
端口电压：



$$u_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

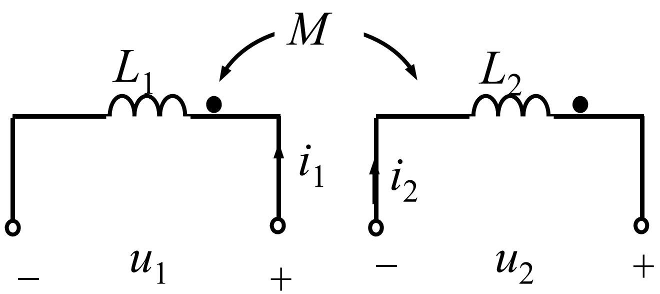
$$u_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

当电流从a端流入时，那么在另一线圈的同名端处互感电压取“+”。



$$u_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u_2 = -L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

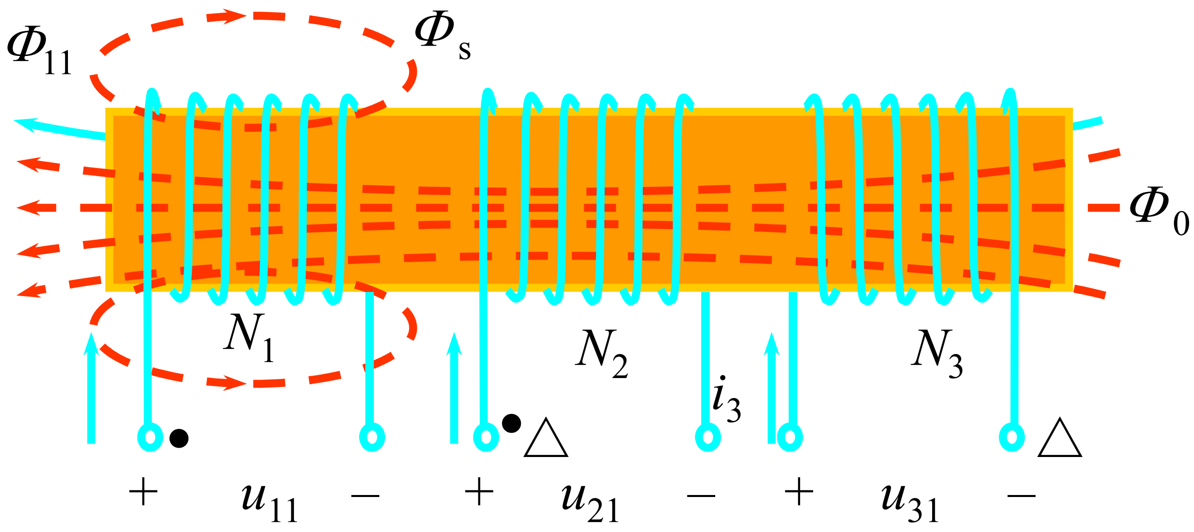


$$u_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$u_2 = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

同名端

当两个电流分别从两个线圈的对应端子同时流入或流出，若所产生的磁通相互加强时，则这两个对应端子称为两互感线圈的同名端。



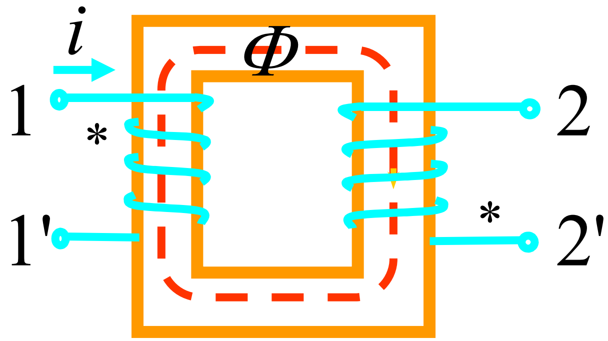
$$u_{21} = M_{21}\frac{\mathrm{d}i_1}{\mathrm{d}t} \qquad u_{31} = -M_{31}\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

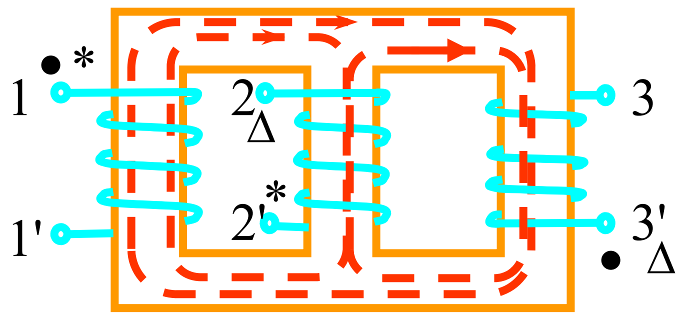
注意：线圈的同名端必须两两确定。

确定同名端的方法：

（1） 当两个线圈中电流同时由同名端流入(或流出)时，两个电流产生的磁场相互增强。

例





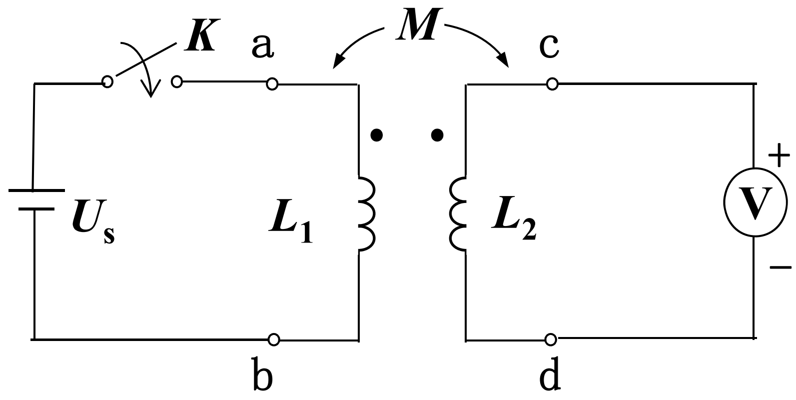
（2） 当随时间增大的时变电流从一线圈的一端流入时，将会引起另一线圈相应同名端的电位升高。

（2）若不知线圈内部结构：实验法判别同名端

当K闭合时，如电压表正偏，则a与c为同名端；反之a与d为同名端。

闭合时 $\dfrac{di_1}{dt} > 0.$

$$u_{cd} = M\frac{di_1}{dt}$$



（a、c端为同名端时）

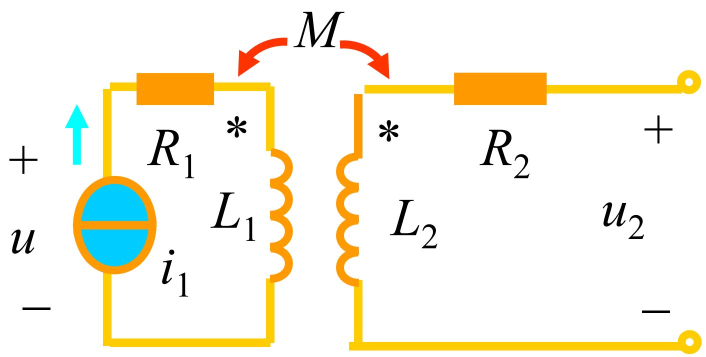
耦合线圈通正弦交流电，则

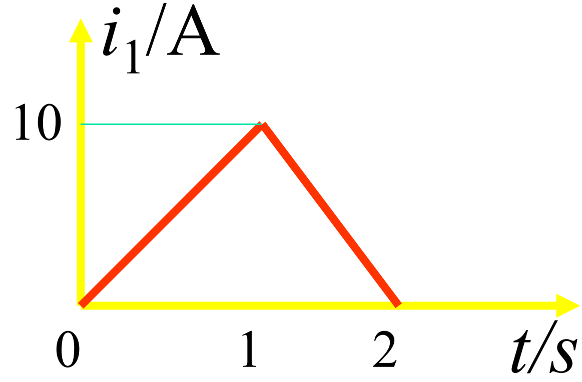
$$\dot{U}_1 = \pm j\omega L_1\dot{I}_1 \pm j\omega M\dot{I}_2$$

$$\dot{U}_2 = \pm j\omega L_2\dot{I}_2 \pm j\omega M\dot{I}_1$$

例

已知 $R_1=10\Omega, L_1=5\mathrm{H}, L_2=2\mathrm{H}, M=1\mathrm{H}$,求$u(t)$和$u_2(t)$





解

$$u_2(t)=M\frac{\mathrm{d}i_1}{\mathrm{d}t}=\begin{cases}10V & 0\le t\le 1s\\ -10V & 1\le t\le 2s\\ 0 & 2\le t\end{cases}$$

$$u(t)=R_1i_1+L\frac{\mathrm{d}i_1}{\mathrm{d}t}=\begin{cases}100\ t+50V & 0\le t\le 1s\\ -100\ t+150V & 1\le t\le 2s\\ 0 & 2\le t\end{cases}$$

第七章 含有互感的电路

§7-2 含有互感的电路的分析计算

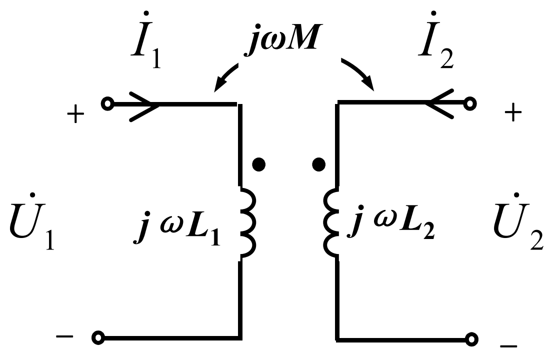
仍用相量法分析

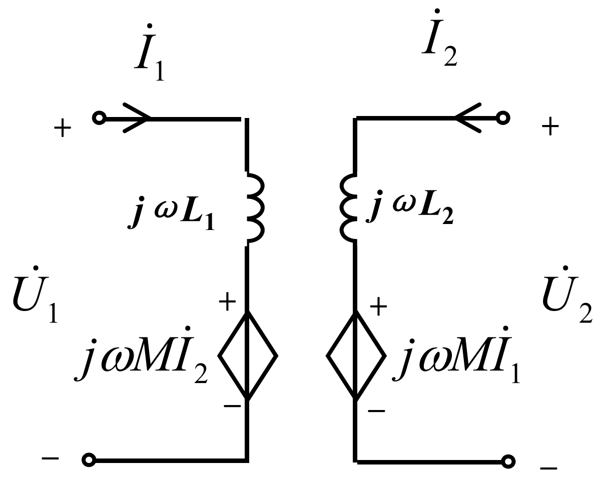
处理互感的方法：

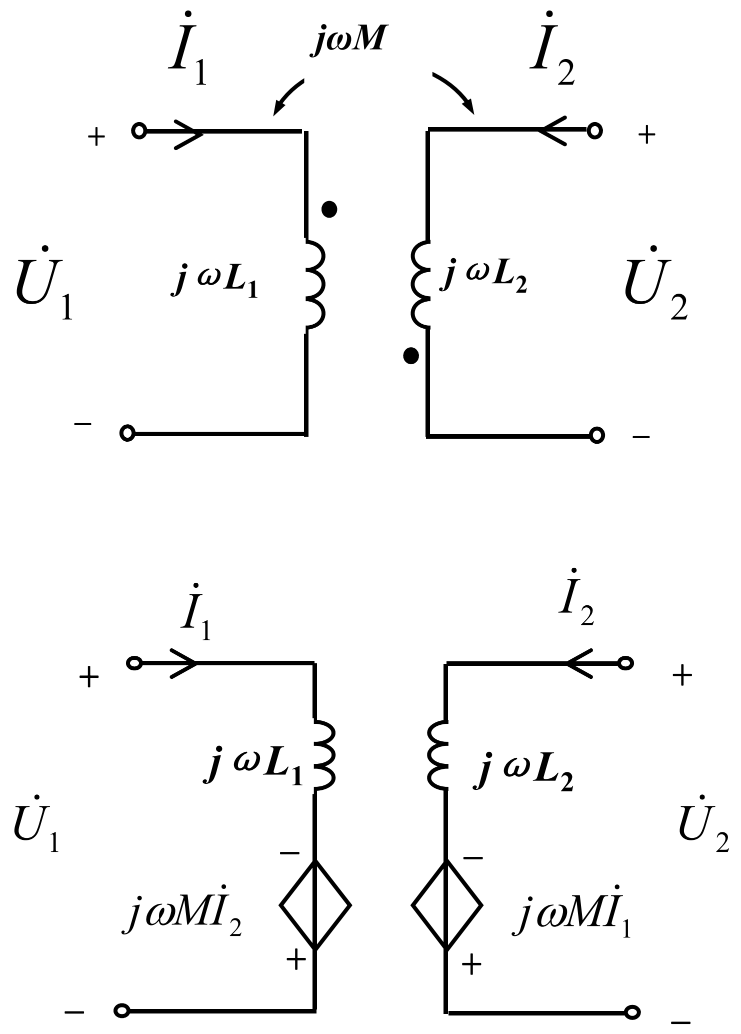
①用受控源表示互感电压

②去耦法（互感消去法）

一、用受控源表示互感电压



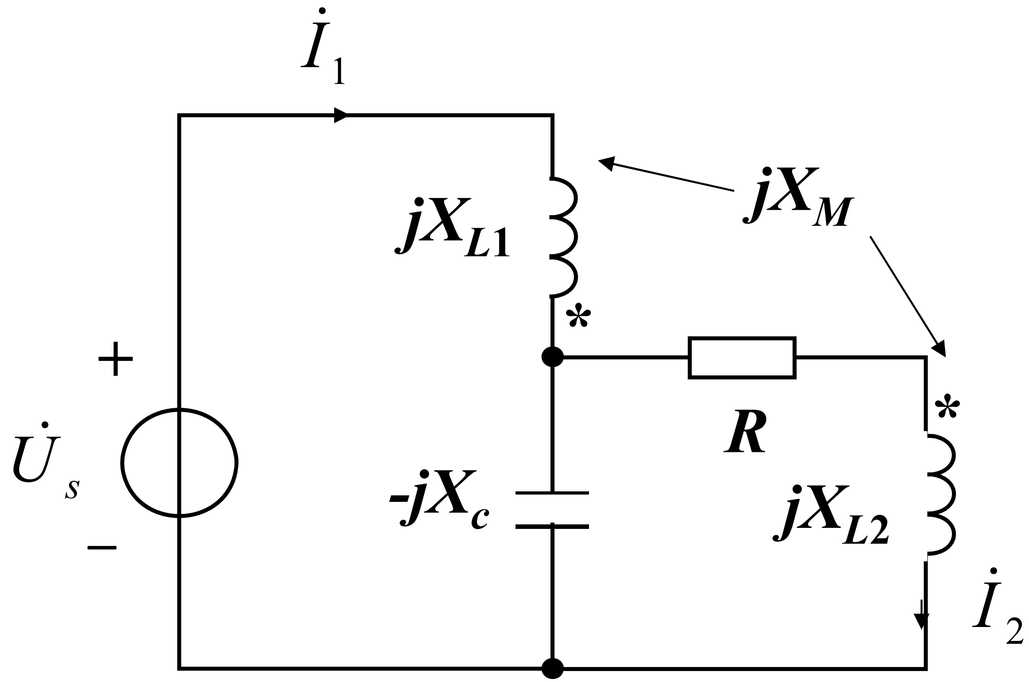


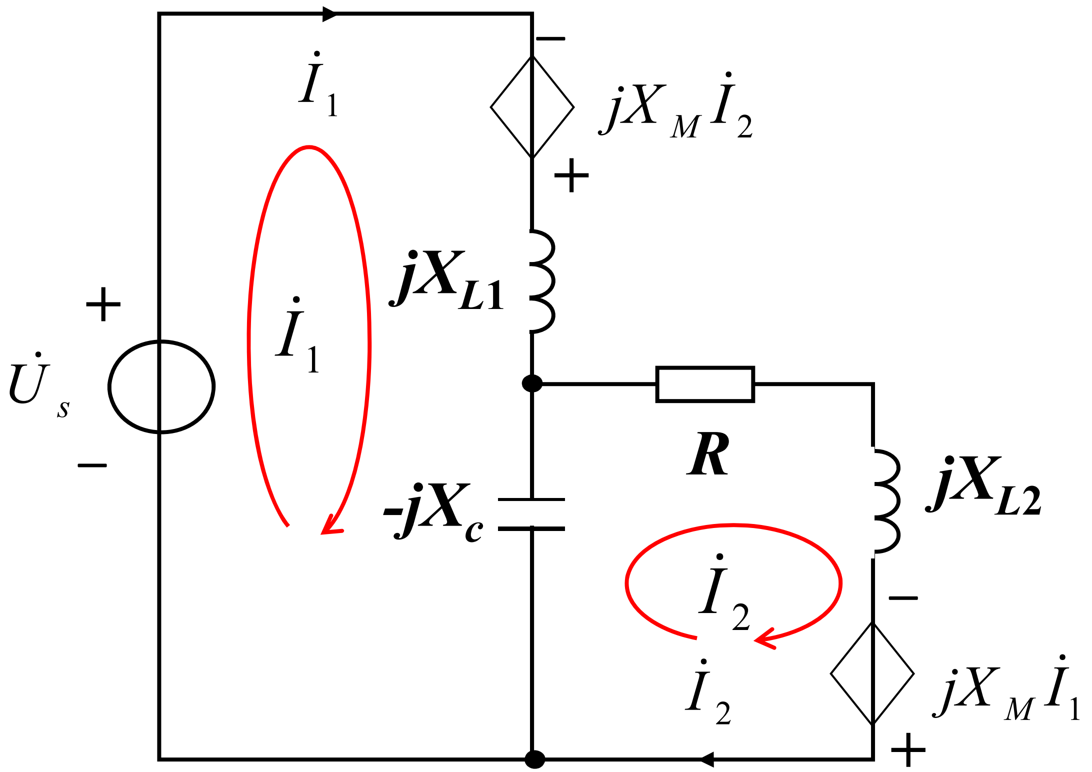


$\dot{I}_1$
$j\omega M$
$\dot{I}_2$
$\dot{U}_1$
$j\omega L_1$
$j\omega L_2$
$\dot{U}_2$
$\dot{I}_1$
$\dot{I}_2$
$j\omega L_1$
$j\omega L_2$
$\dot{U}_1$
$\dot{U}_2$
$j\omega M\dot{I}_2$
$j\omega M\dot{I}_1$

例1 图示电路。已知 $\dot{U}_s = 50\angle 0° V$，$X_{L1} = 10\Omega$，

$X_{L2} = 20\Omega$，$X_M = X_C = R = 5\Omega$。求 $\dot{I}_2$





$$(j10-j5)\dot{I}_1+[-j5-(-j5)]\dot{I}_2=50$$

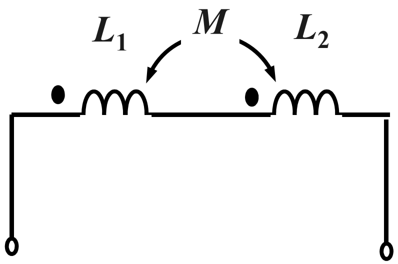
$$[-j5-(-j5)]\dot{I}_1+(5+j20-j5)\dot{I}_2=0$$

解得 $\dot{I}_1=10\angle-90°A$ $\dot{I}_2=0$

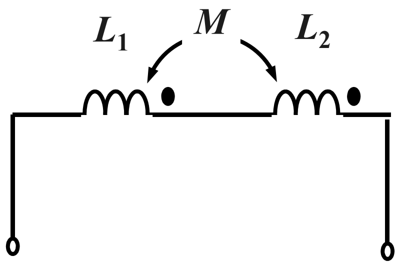
二、去耦法（互感消去法）

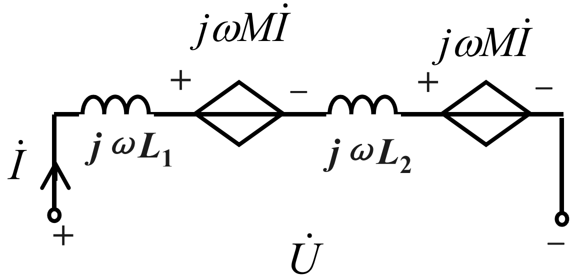
1. 两线圈的串联

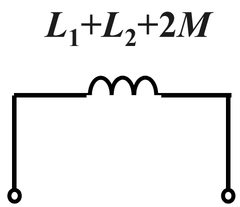
a. 顺接



或





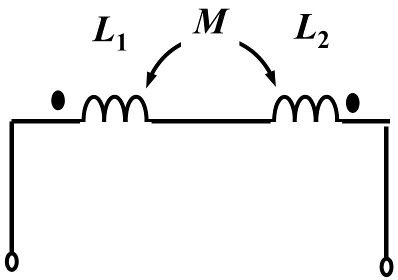


$$\dot{U}=(j\omega L_1+j\omega L_2)\dot{I}+j\omega M\dot{I}+j\omega M\dot{I}$$

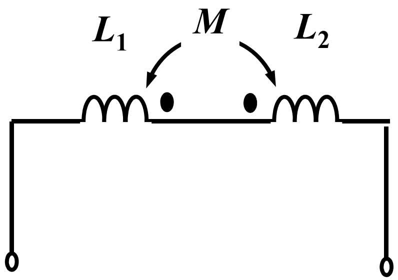
$$=j\omega(L_1+L_2+2M)\dot{I}=j\omega L\dot{I}$$

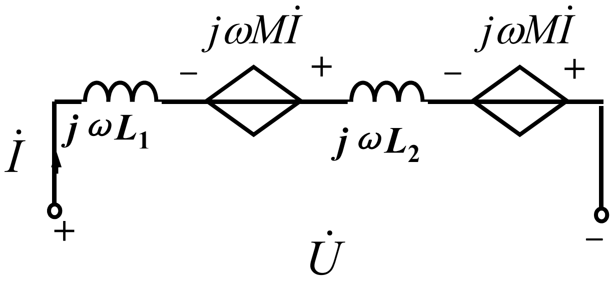
等效电感　$L=L_1+L_2+2M$

b. 反接



或

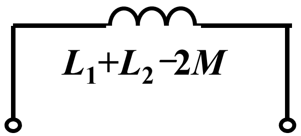




$$\dot{U} = j\omega L_1 \dot{I} - j\omega M\dot{I} + j\omega L_2 \dot{I} - j\omega M\dot{I}$$

$$= j\omega(L_1 + L_2 - 2M)\dot{I} = j\omega L\dot{I}$$

$$\therefore L = L_1 + L_2 - 2M$$



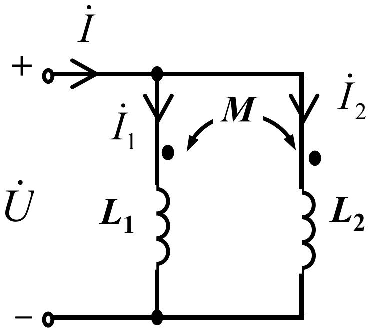
串联时： $L = L_1 + L_2 \pm 2M$ （顺接取“+”，反接取“−”）

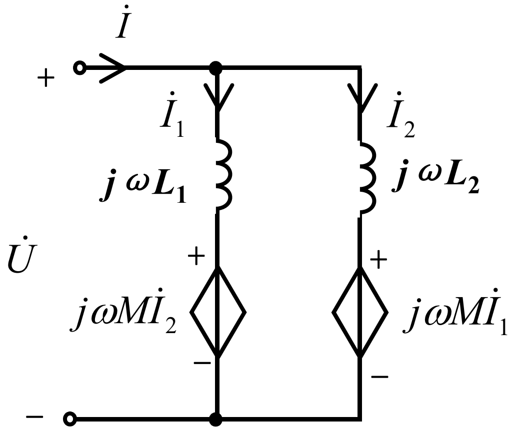
$$\because \begin{aligned} L' &= L_1 + L_2 + 2M \\ L'' &= L_1 + L_2 - 2M \end{aligned} \qquad \therefore \underline{\underline{M = \frac{L' - L''}{4}}}$$

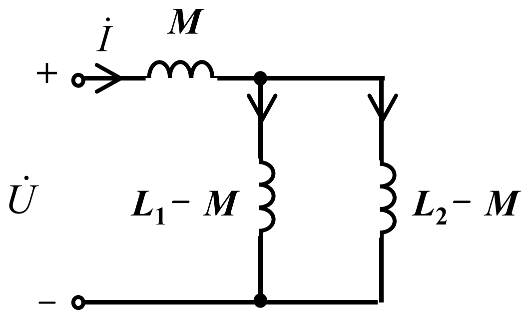
$$L_1 + L_2 - 2M \geq 0 \qquad \mathrm{M} \leq \frac{1}{2}(L_1 + L_2)$$

2. 两线圈的并联

a. 同名端相联：又称同向并联







$$\begin{cases} j\omega L_1\dot I_1 + j\omega M\dot I_2 = \dot U \\ j\omega L_2\dot I_2 + j\omega M\dot I_1 = \dot U \\ \dot I_1 + \dot I_2 = \dot I \end{cases}$$

$\xrightarrow{消\dot I_2}$ $\quad$ $\xrightarrow{消\dot I_1}$

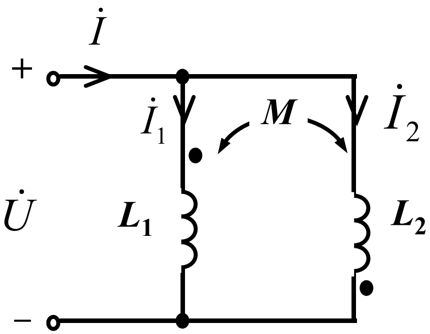
$$\begin{cases} j\omega M\dot I + j\omega(L_1-M)\dot I_1 = \dot U \\ j\omega M\dot I + j\omega(L_2-M)\dot I_2 = \dot U \end{cases}$$

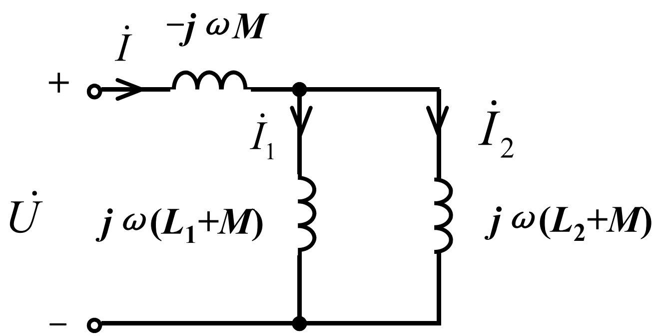
$$Z = \frac{\dot U}{\dot I} = j\omega M + \frac{j^2\omega^2(L_1-M)(L_2-M)}{j\omega(L_1-M)+j\omega(L_2-M)}$$

$$= j\omega\frac{L_1L_2-M^2}{L_1+L_2-2M} = j\omega L$$

$$\therefore \quad L = \frac{L_1L_2-M^2}{L_1+L_2-2M}$$

b. 异名端相联





$$\begin{cases} j\omega L_1\dot{I}_1 - j\omega M\dot{I}_2 = \dot{U} \\ j\omega L_2\dot{I}_2 - j\omega M\dot{I}_1 = \dot{U} \end{cases} \begin{matrix} \xrightarrow{\text{消}\dot{I}_2} \\ \xrightarrow{\text{消}\dot{I}_1} \end{matrix} \begin{cases} -j\omega M\dot{I} + j\omega(L_1+M)\dot{I}_1 = \dot{U} \\ -j\omega M\dot{I} + j\omega(L_2+M)\dot{I}_2 = \dot{U} \end{cases}$$

$$Z = \frac{\dot{U}}{\dot{I}} = -j\omega M + \frac{(j\omega)^2(L_1+M)(L_2+M)}{j\omega(L_1+L_2+2M)}$$

$$= j\omega\frac{L_1L_2 - M^2}{L_1+L_2+2M} = j\omega L$$

$$\therefore L = \frac{L_1L_2 - M^2}{L_1+L_2+2M}$$

$$L=\frac{L_1L_2-M^2}{L_1+L_2+2M}$$

$\because$ $L_1L_2-M^2\geq 0$ $\therefore$ $M\leq\sqrt{L_1L_2}$

$$M_{max}=\sqrt{L_1L_2}$$

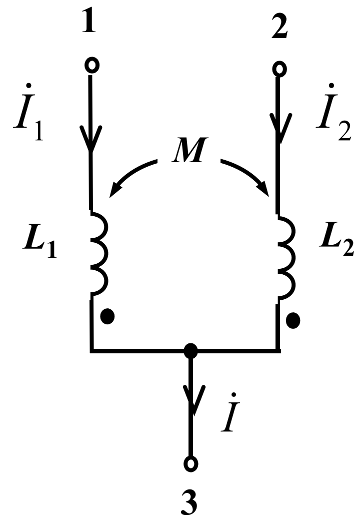
耦合系数: $K=\frac{M}{M_{max}}=\frac{M}{\sqrt{L_1L_2}}$ $0\leqslant K\leqslant 1$

K=1时 ，称全耦合。

K接近1时，称紧耦合，K较小时称松耦合。

3. 两线圈有一端相联

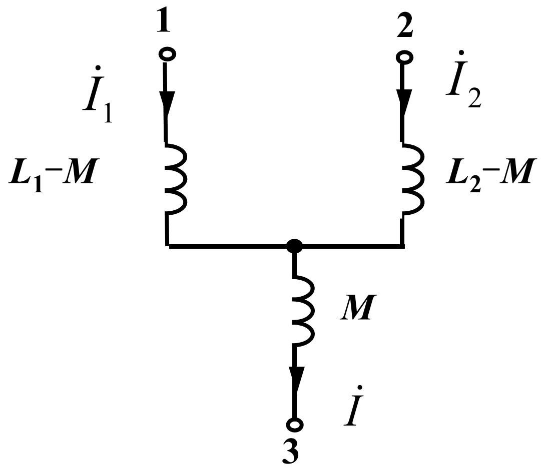
a. 同名端相联



$$\dot{U}_{13} = j\omega L_1\dot{I}_1 + j\omega M\dot{I}_2$$

$$\dot{U}_{23} = j\omega L_2\dot{I}_2 + j\omega M\dot{I}_1$$

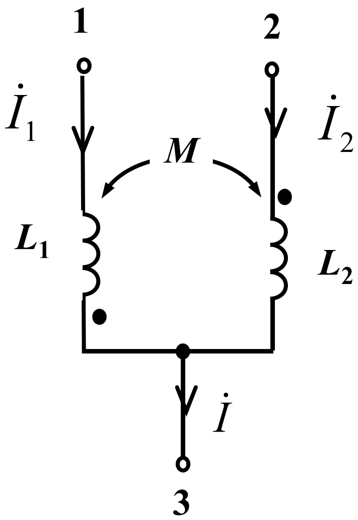
$$\dot{I} = \dot{I}_1 + \dot{I}_2$$



$$\dot{U}_{13} = j\omega(L_1 - M)\dot{I}_1 + j\omega M\dot{I}$$

$$\dot{U}_{23} = j\omega(L_2 - M)\dot{I}_2 + j\omega M\dot{I}$$

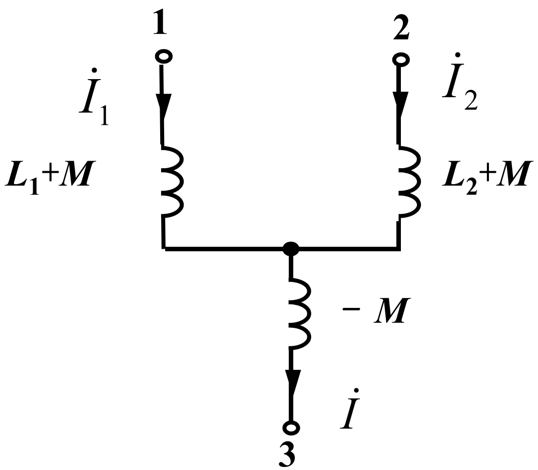
b. 异名端相联：



$$\dot{U}_{13} = j\omega L_1 \dot{I}_1 - j\omega M \dot{I}_2$$

$$\dot{U}_{23} = j\omega L_2 \dot{I}_2 - j\omega M \dot{I}_1$$

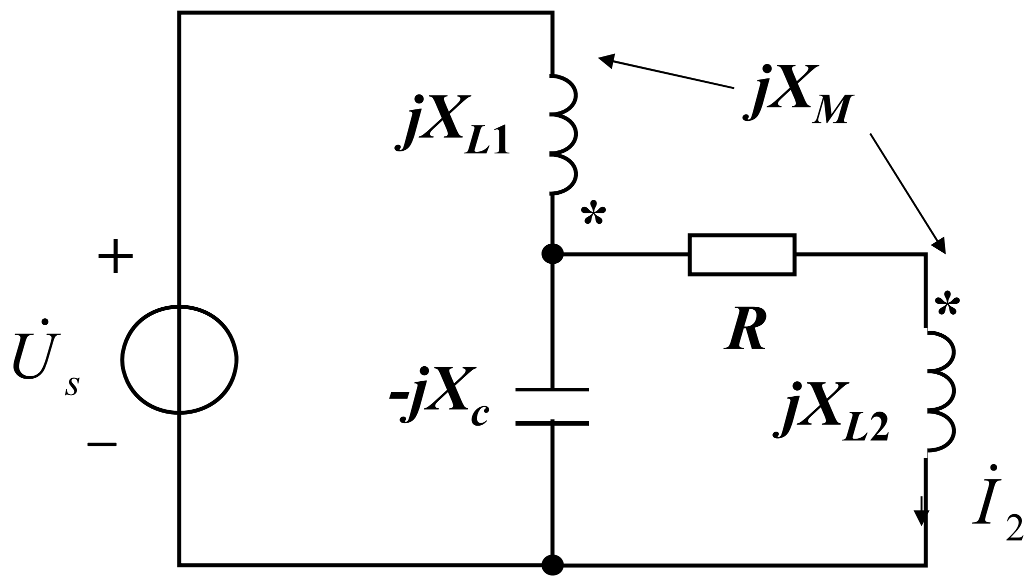
$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

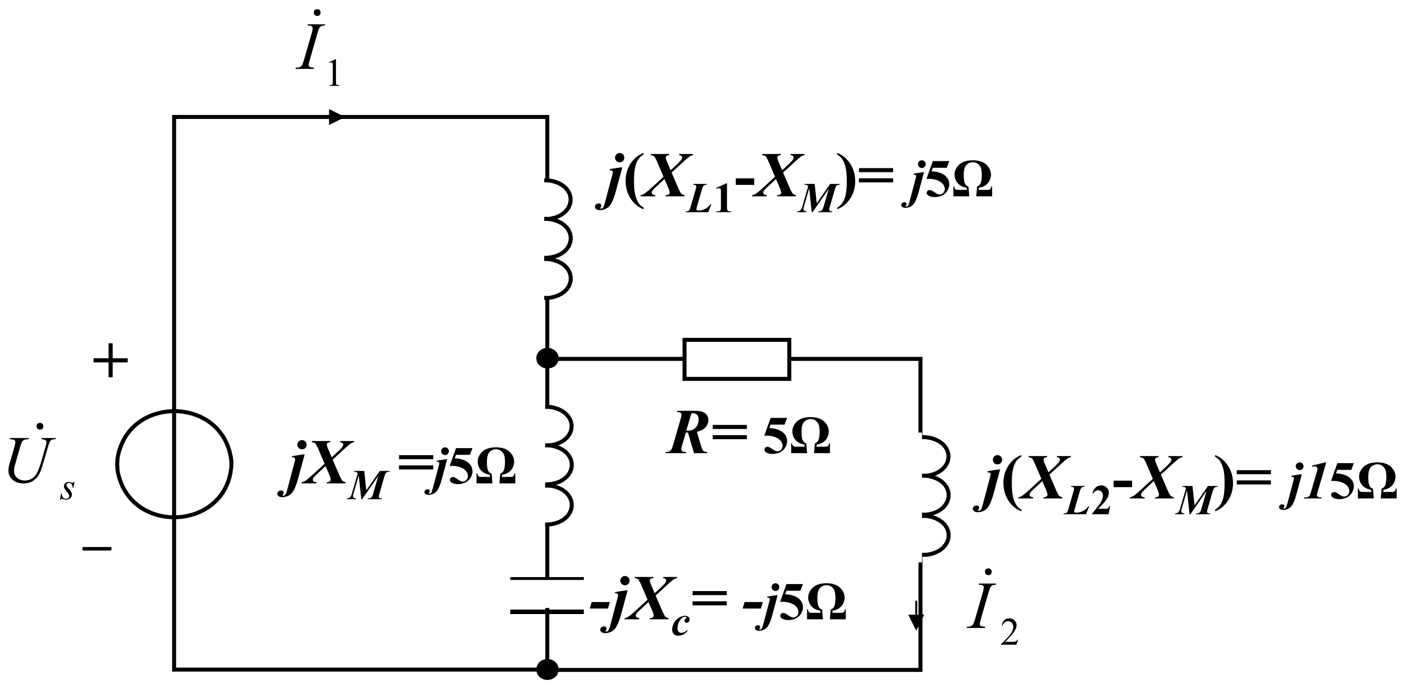


$$\dot{U}_{13} = j\omega(L_1 + M)\dot{I}_1 - j\omega M\dot{I}$$

$$\dot{U}_{23} = j\omega(L_2 + M)\dot{I}_2 - j\omega M\dot{I}$$

例2　图示电路。已知 $\dot{U}_s = 50\angle 0° V$，$X_{L1} = 10\Omega$，$X_{L2} = 20\Omega$，$X_M = X_C = R = 5\Omega$。求 $\dot{I}_2$





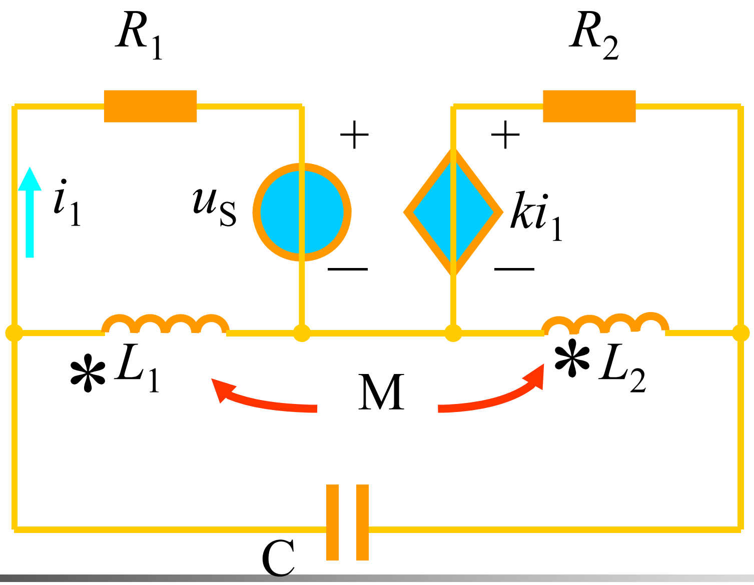
$$\dot{I}_2 = 0$$

$$\dot{I}_1 = \frac{50}{j5} = 10\angle -90^\circ A$$

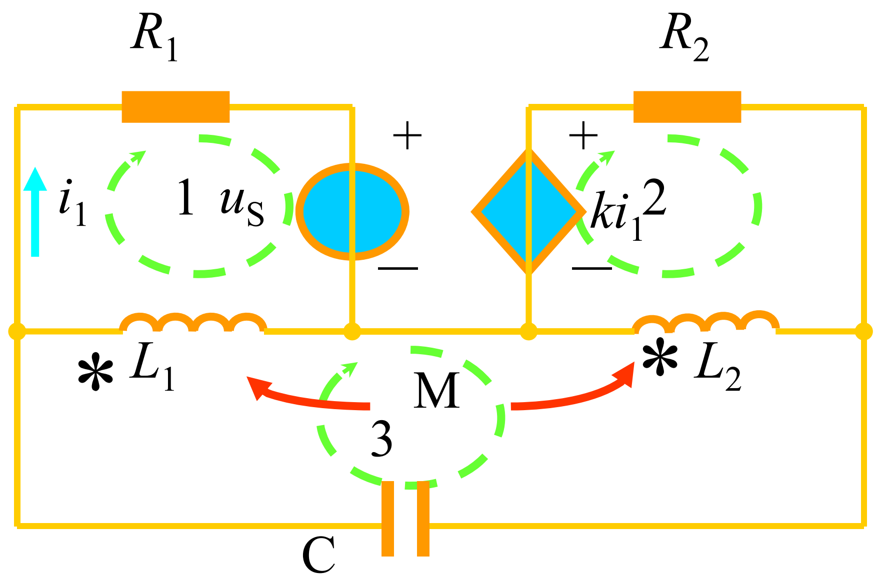
有互感的电路的计算

(1) 有互感的电路的计算仍属正弦稳态分析，前面介绍的相量分析的方法均适用。
(2) 注意互感线圈上的电压除自感电压外，还应包含互感电压。
(3) 一般采用支路法和回路法计算。

例1 列写下图电路的回路电流方程。



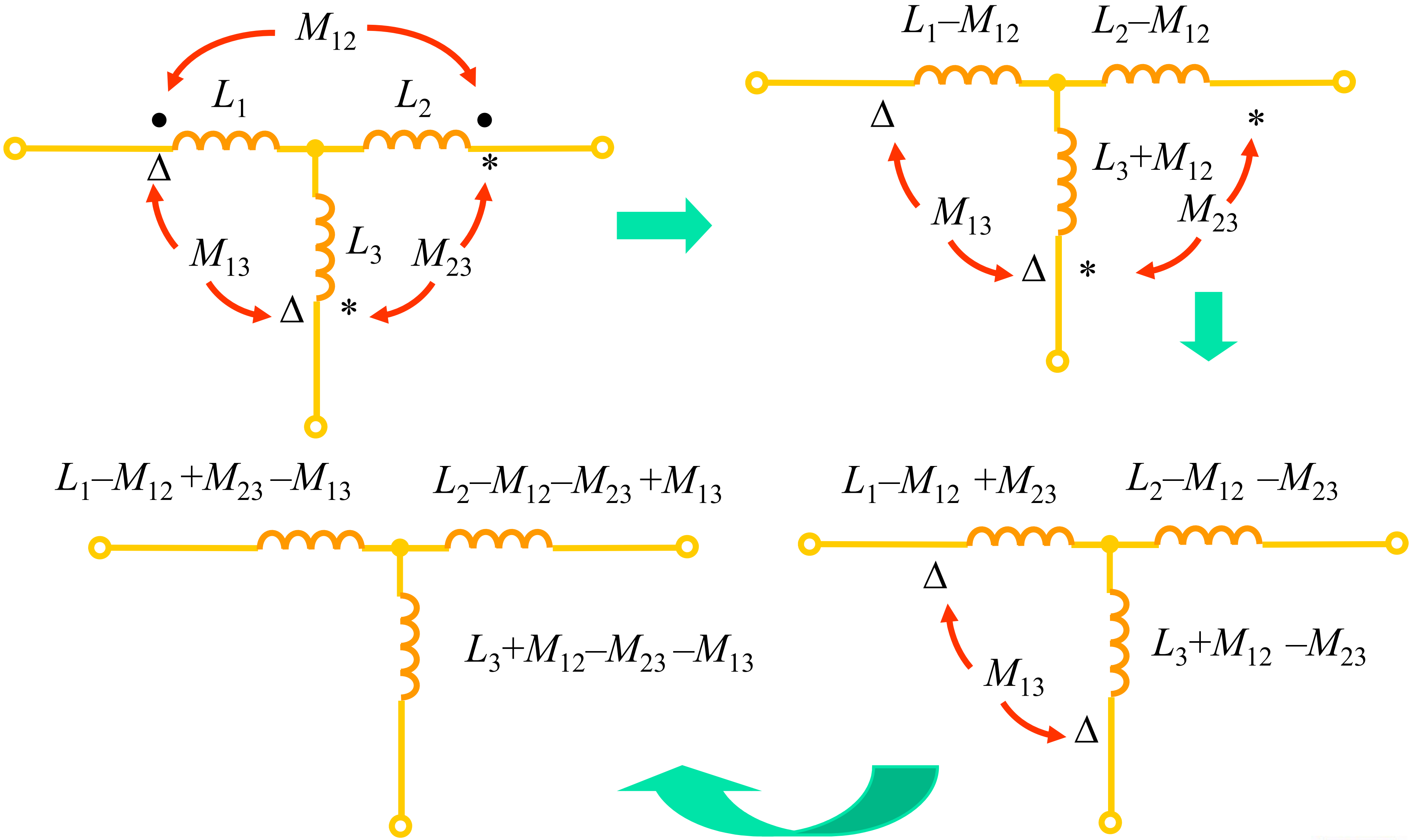
解



$$\begin{cases} R_1\dot{I}_1 + j\omega L_1(\dot{I}_1 - \dot{I}_3) + j\omega M(\dot{I}_2 - \dot{I}_3) = -\dot{U}_S \\ R_2\dot{I}_2 + j\omega L_2(\dot{I}_2 - \dot{I}_3) + j\omega M(\dot{I}_1 - \dot{I}_3) = k\dot{I}_1 \\ j\omega L_1(\dot{I}_3 - \dot{I}_1) + j\omega L_2(\dot{I}_3 - \dot{I}_2) - j\dfrac{1}{\omega C}\dot{I}_3 \\ +j\omega M(\dot{I}_3 - \dot{I}_1) + j\omega M(\dot{I}_3 - \dot{I}_2) = 0 \end{cases}$$

例2

求去耦等效电路，（一对一对消）：



§7-3 空心变压器

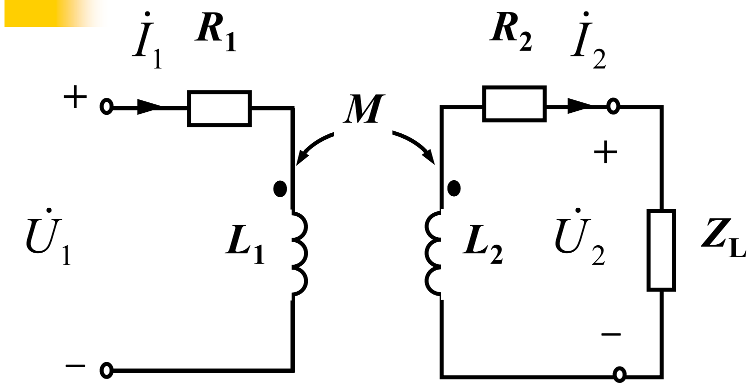
变压器：

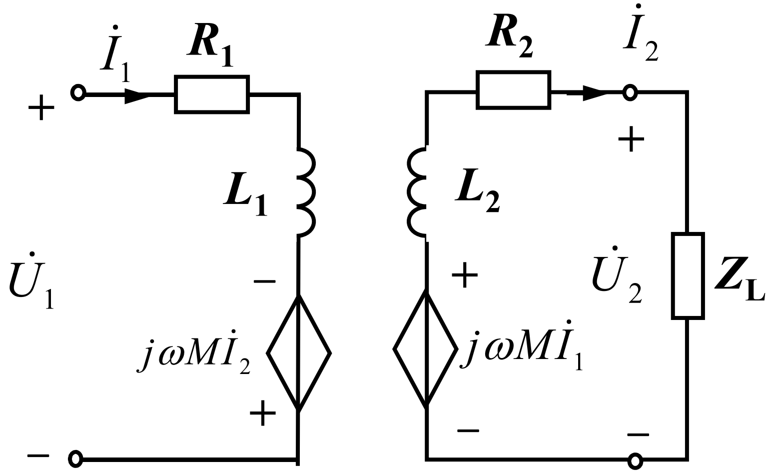
一个线圈接电源——初级线圈或原边

另一个线圈接负载——次级线圈或副边

K较大属紧耦合；K较小，属松耦合。

心子为非铁磁材料称空心变压器，





$$\begin{cases}(R_1 + j\omega L_1)\dot I_1 - j\omega M\dot I_2 = \dot U_1 \\ -j\omega M\dot I_1 + (R_2 + j\omega L_2 + Z_L)\dot I_2 = 0\end{cases}$$

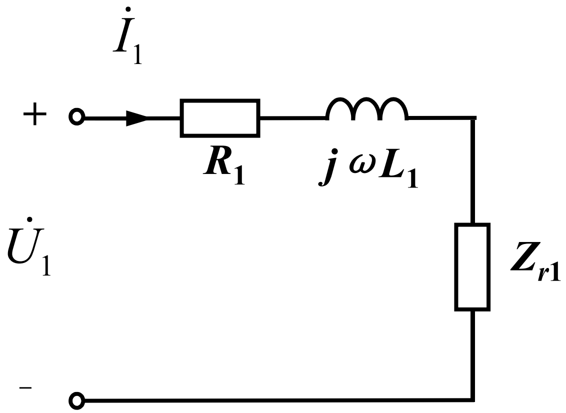
$Z_{11} = R_1 + j\omega L_1$ 初级网络总阻抗

$Z_{22} = R_2 + j\omega L_2 + Z_L$ 副边网络总阻抗

$Z_M = -j\omega M$ 互阻抗

$$\dot{I}_1 = \frac{Z_{22}\dot{U}_1}{Z_{11}Z_{22} - Z_M^2} = \frac{\dot{U}_1}{Z_{11} + \dfrac{(\omega M)^2}{Z_{22}}}$$

$$\dot{I}_2 = \frac{j\omega M \dfrac{\dot{U}_1}{Z_{11}}}{Z_2 + \dfrac{(\omega M)^2}{Z_{11}}}$$



$$Z_{r1} = \frac{(\omega M)^2}{Z_{22}}$$

称为副边对原边的反映阻抗（反射或引入阻抗）

求副边等效电路:

从Z_L看过去的副边开路电压:

$$\dot{I}_2 = 0. \qquad \dot{I}_1 = \frac{\dot{U}_1}{R_1 + j\omega L_1} = \frac{\dot{U}_1}{Z_{11}}$$

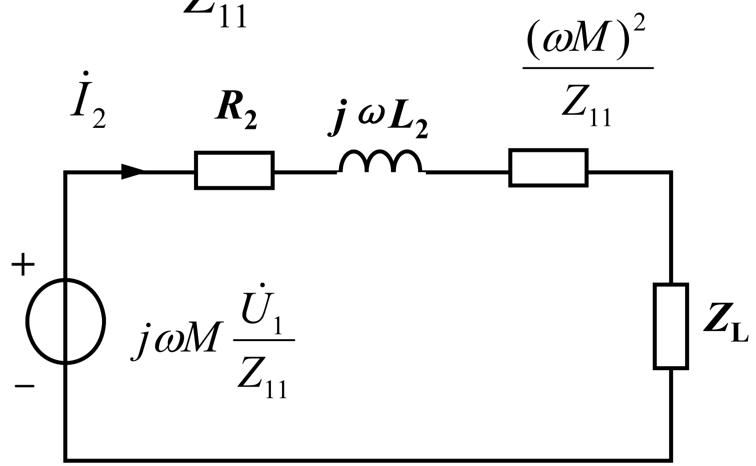
∴ 副边开路电压为: $$j\omega M\,\dot{I}_1 = j\omega M\,\frac{\dot{U}_1}{Z_{11}}$$

等效阻抗: $$R_2 + j\omega L_2 + \frac{(\omega M)^2}{Z_{11}}$$

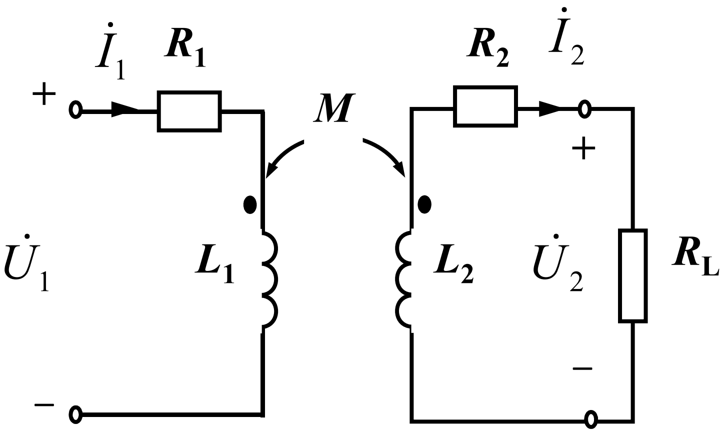
其中 $$Z_{11} = R_1 + j\omega L_1$$

$$Z_{r2} = \frac{(\omega M)^2}{Z_{11}}$$

得副边等效电路:



例7-4　已知空心变压器参数R_1=20Ω，L_1=5H，R_2=2Ω，L_2=1H，M=2H，负载电阻R_L=30Ω，外加电压 $u_1=110\sqrt{2}\cos 10tV$ ，求副边电流i_2及变压器的效率。

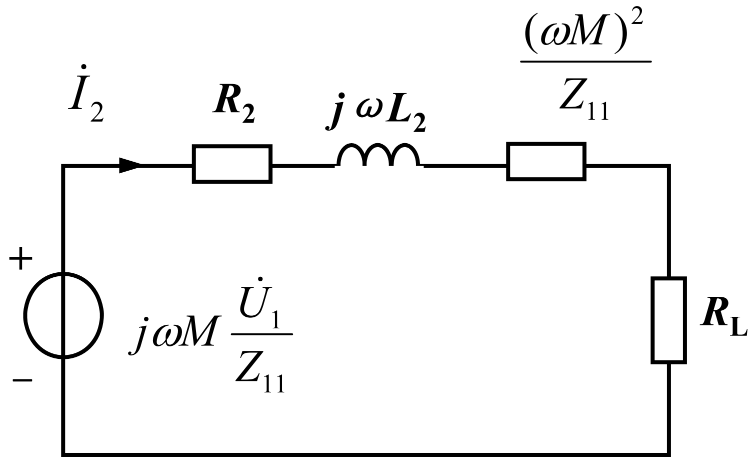


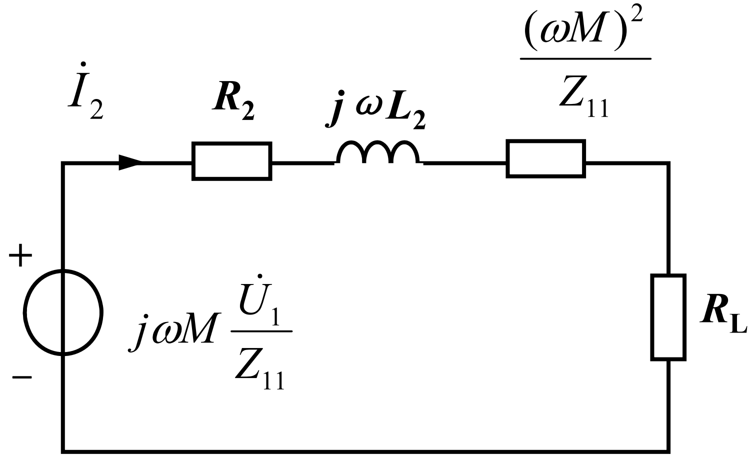
解：　$\dot{U}_1=110\angle 0^\circ\ \ V$

根据副边等效电路求 $\dot{I}_2$

原边回路总阻抗

$$Z_{11}=R_1+j\omega L_1=20+j50\Omega$$





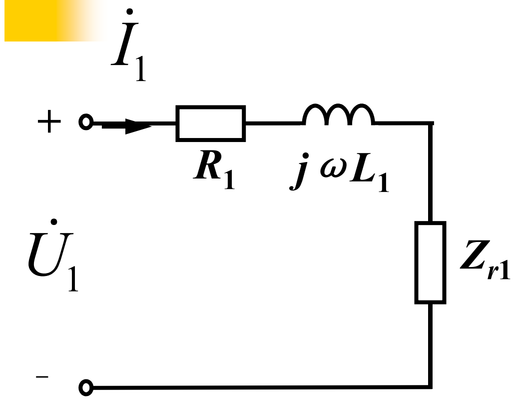
$$\dot{I}_2=\frac{j\omega M\dfrac{\dot{U}_1}{Z_{11}}}{R_2+j\omega L_2+\dfrac{(\omega M)^2}{Z_{11}}+R_L}=1.17\angle 16.7^\circ\ A$$

$$i_2=1.17\sqrt{2}\cos(10t+16.7^\circ)\quad A$$

效率：

负载R_L吸收的功率： $P_2=I_2^2R_L=1.17^2\times 30=41.067W$

电源提供的功率：先利用原边等效电路求 $\dot{I}_1$



副边回路总阻抗:

$$Z_{22} = R_2 + j\omega L_2 + R_L = 32 + j10\Omega$$

$$\dot{I}_1 = \frac{\dot{U}_1}{R_1 + j\omega L_1 + \dfrac{(\omega M)^2}{Z_{22}}} = 1.962\angle -55.946^\circ \ A$$

电源提供的功率

$$P_1 = U_1 I_1 \cos\varphi_1 = 110 \times 1.962 \cos 55.946^\circ = 120.85W$$

变压器的效率

$$\eta = \frac{P_2}{P_1} = \frac{41.067}{120.85} = 0.3398 = 33.98\%$$

§7-4 全耦合变压器
与理想变压器

一、全耦合变压器

1. 耦合系数：　$\Phi_{21}=\Phi_{11}\ ,\Phi_{12}=\Phi_{22}$

$$K=\frac{M}{\sqrt{L_1L_2}}=\sqrt{\frac{M^2}{L_1L_2}}=\sqrt{\frac{N_2\Phi_{21}}{i_1}\cdot\frac{N_1\Phi_{12}}{i_2}\Bigg/\frac{N_1\Phi_{11}}{i_1}\cdot\frac{N_2\Phi_{22}}{i_2}}=1$$

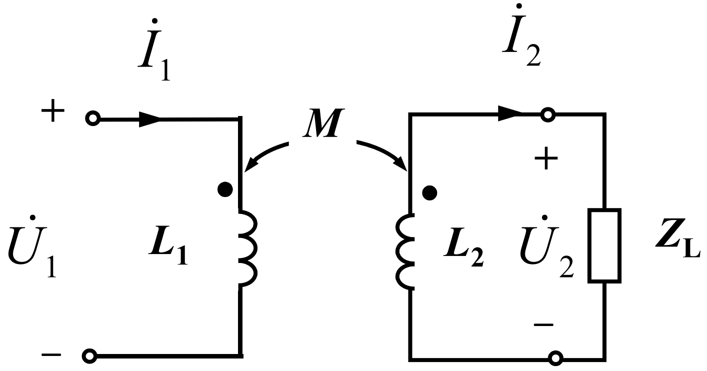
∴ ***K*=1**

2. $$\frac{L_1}{L_2}=\frac{N_1\Phi_{11}}{i_1}\Bigg/\frac{N_2\Phi_{22}}{i_2}=\frac{N_1}{N_2}\cdot\frac{N_2\Phi_{21}}{i_1}\Bigg/\frac{N_2}{N_1}\cdot\frac{N_1\Phi_{12}}{i_2}$$

$$=\frac{N_1}{N_2}M_{21}\Bigg/\frac{N_2}{N_1}M_{12}=\left(\frac{N_1}{N_2}\right)^2=n^2$$

原副边匝数比：　$n=\frac{N_1}{N_2}=\sqrt{\frac{L_1}{L_2}}$

3. 等效电路



$$\begin{cases} \dot{U}_1 = j\omega L_1 \dot{I}_1 - j\omega M \dot{I}_2 \cdots\cdots(1) \\ \dot{U}_2 = j\omega M \dot{I}_1 - j\omega L_2 \dot{I}_2 \cdots\cdots(2) \end{cases}$$

将$M=\sqrt{L_1L_2}$ 代入

$$\begin{cases} \dot{U}_1 = j\omega(L_1\dot{I}_1 - \sqrt{L_1L_2}\dot{I}_2) = j\omega\sqrt{L_1}\left(\sqrt{L_1}\dot{I}_1 - \sqrt{L_2}\dot{I}_2\right) \\ \dot{U}_2 = j\omega(\sqrt{L_1L_2}\dot{I}_1 - L_2\dot{I}_2) = j\omega\sqrt{L_2}\left(\sqrt{L_1}\dot{I}_1 - \sqrt{L_2}\dot{I}_2\right) \end{cases}$$

$$\frac{\dot{U}_1}{\dot{U}_2} = \sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2} = n$$

即 $\dot{U}_1 = n\dot{U}_2$

由式子（1）$\dot{U}_1 = j\omega L_1\dot{I}_1 - j\omega M\dot{I}_2$ 得

$$\dot{I}_1 = \frac{\dot{U}_1}{j\omega L_1} + \frac{M}{L_1}\dot{I}_2 = \frac{\dot{U}_1}{j\omega L_1} + \sqrt{\frac{L_2}{L_1}}\dot{I}_2 = \frac{\dot{U}_1}{j\omega L_1} + \frac{1}{n}\dot{I}_2 = \dot{I}_{10} + \dot{I}_1{}'$$

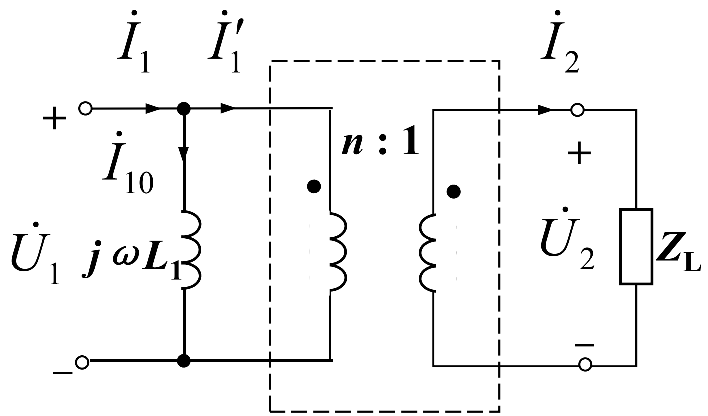
$$\therefore \qquad \dot{I}_1{}' = \frac{1}{n}\dot{I}_2$$

可得等效电路

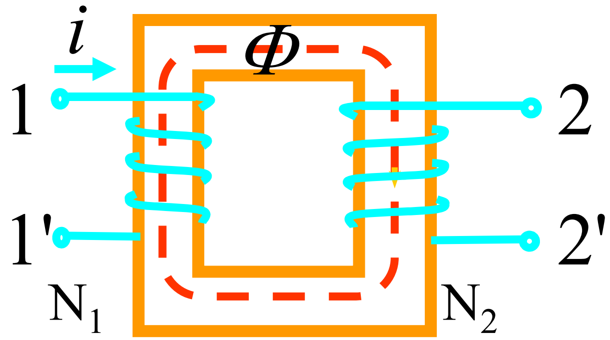
$$\dot{U}_1 = n\dot{U}_2$$

$$\dot{I}_1' = \frac{1}{n}\dot{I}_2$$

$$\dot{I}_1 = \dot{I}_{10} + \dot{I}_1'$$



2. 理想变压器的主要性能



（1）变压关系

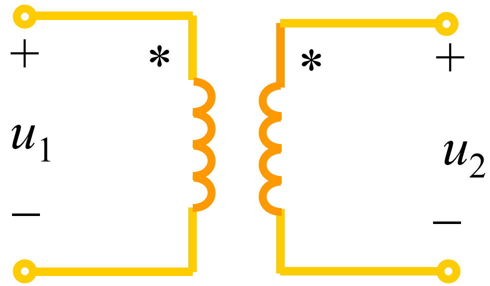
$$k=1 \longrightarrow \varphi_1=\varphi_2=\varphi_{11}+\varphi_{22}=\varphi$$

$$u_1=\frac{d\psi_1}{dt}=N_1\frac{d\varphi}{dt}$$

$$u_2=\frac{d\psi_2}{dt}=N_2\frac{d\varphi}{dt}$$

$$\longrightarrow \frac{u_1}{u_2}=\frac{N_1}{N_2}=n$$

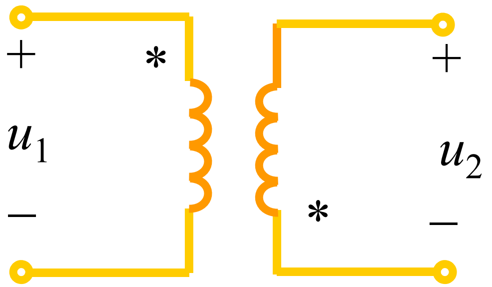
$n:1$



理想变压器模型

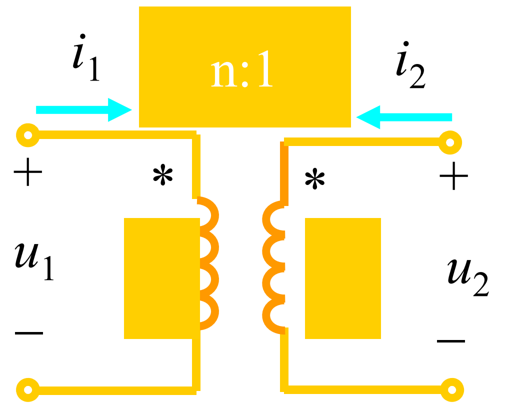
若

$n:1$



$$\longrightarrow \frac{u_1}{u_2}=-\frac{N_1}{N_2}=-n$$

（2）变流关系



$$u_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$i_1(t) = \frac{1}{L_1}\int_0^t u_1(\xi)d\xi - \frac{M}{L_1}i_2(t)$$

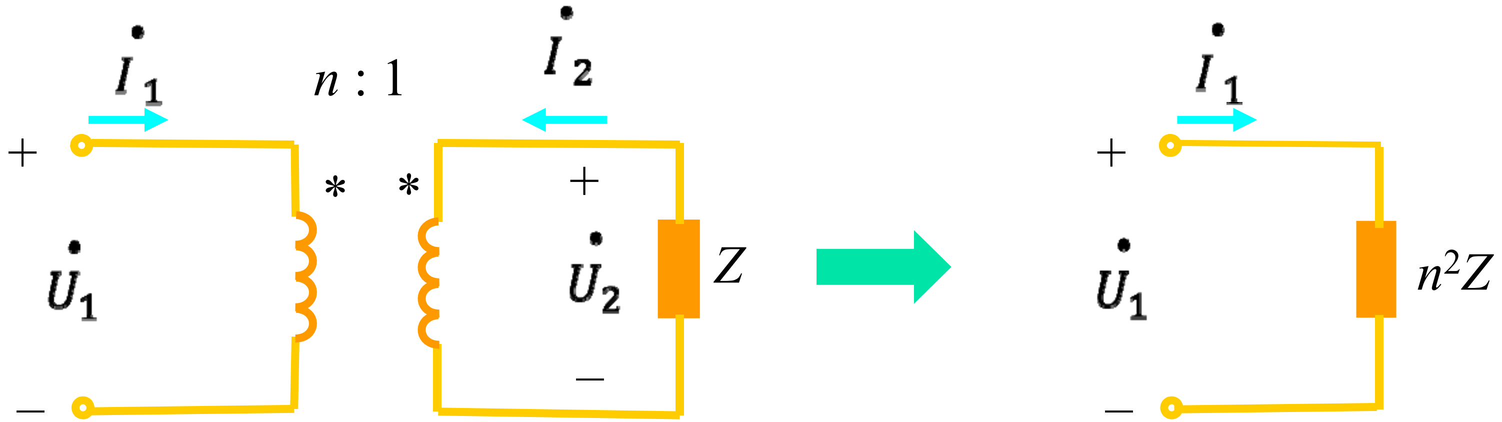
考虑到理想化条件： $k=1 \Rightarrow M=\sqrt{L_1L_2}$

$$L_1 \Rightarrow \infty, \sqrt{L_1/L_2} = \mathrm{N}_1/\mathrm{N}_2 = n$$

$$\frac{M}{L_1} = \sqrt{\frac{L_2}{L_1}} = \frac{1}{n} \quad \Rightarrow \quad \boldsymbol{i_1(t) = -\frac{1}{n}i_2(t)}$$

若i_1、i_2一个从同名端流入，一个从同名端流出，则有： $\boldsymbol{i_1(t) = \frac{1}{n}i_2(t)}$

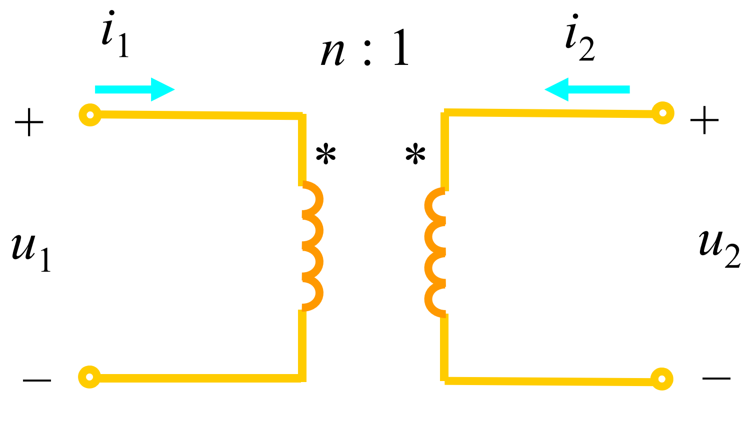
（3）变阻抗关系



$$\frac{\dot{U}_1}{\dot{I}_1}=\frac{n\dot{U}_2}{-1/n\dot{I}_2}=n^2(-\frac{\dot{U}_2}{\dot{I}_2})=n^2Z$$

注　　理想变压器的阻抗变换性质只改变阻抗的大小，不改变阻抗的性质。

（4）功率性质



$$u_1 = nu_2$$

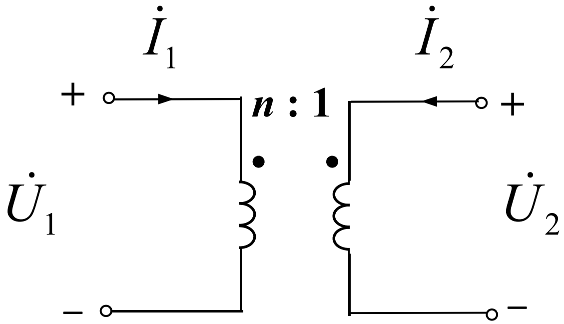
$$i_1 = -\frac{1}{n} i_2$$

$$p = u_1 i_1 + u_2 i_2 = u_1 i_1 + \frac{1}{n} u_1 \times (-n i_1) = 0$$

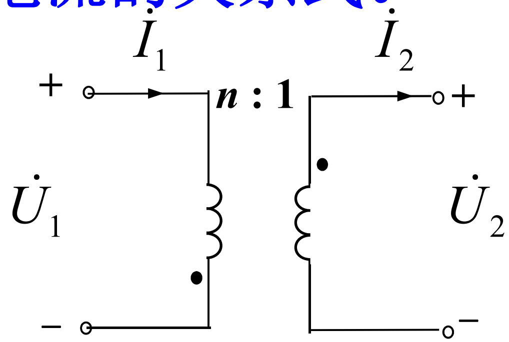
表明：（a）理想变压器既不储能，也不耗能，在电路中只起传递信号和能量的作用。

（b）理想变压器的特性方程为代数关系，因此它是无记忆的多端元件。

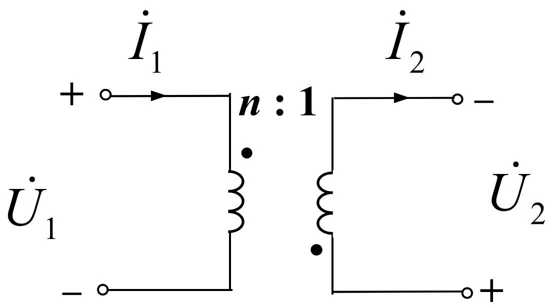
例1：写出下列电路端口电压、电流的关系式。



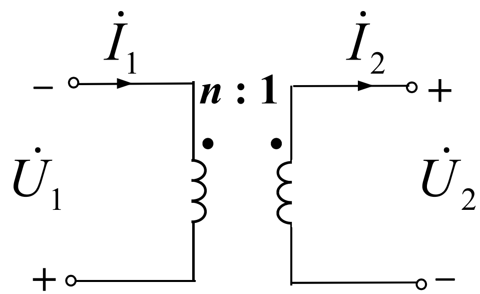
$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n}\dot{I}_2 \end{cases}$$



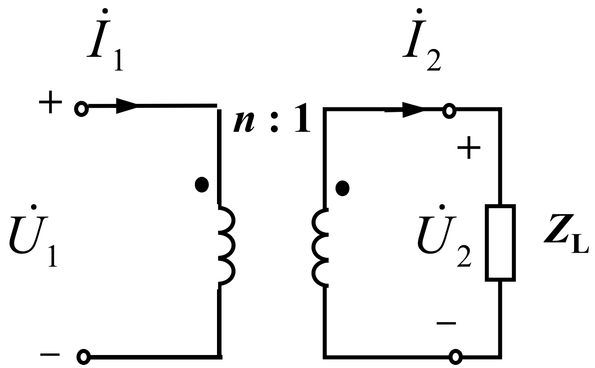
$$\begin{cases} \dot{U}_1 = -n\dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n}\dot{I}_2 \end{cases}$$

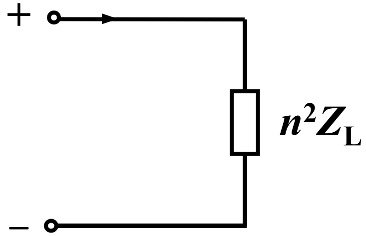


$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n}\dot{I}_2 \end{cases}$$



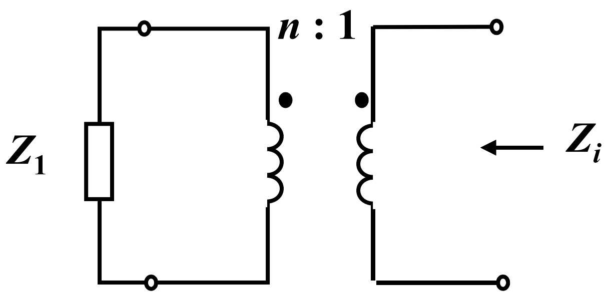
$$\begin{cases} \dot{U}_1 = -n\dot{U}_2 \\ \dot{I}_1 = \dfrac{1}{n}\dot{I}_2 \end{cases}$$

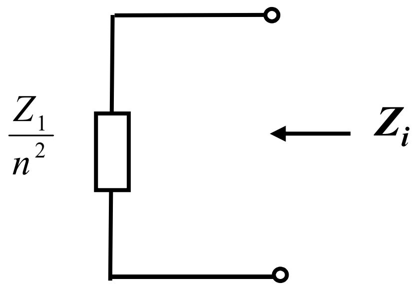




$$\frac{\dot{U}_1}{\dot{I}_1}=\frac{n\dot{U}_2}{\frac{1}{n}\dot{I}_2}=n^2\frac{\dot{U}_2}{\dot{I}_2}=n^2Z_L$$

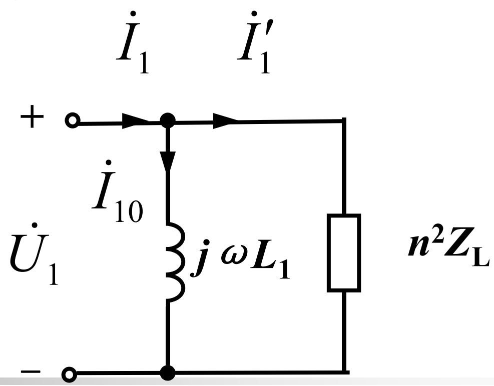
如原边接阻抗Z_1，从副边看过去的等效阻抗为





$$Z_i=\frac{Z_1}{n^2}$$

全耦合变压器原边的等效电路：

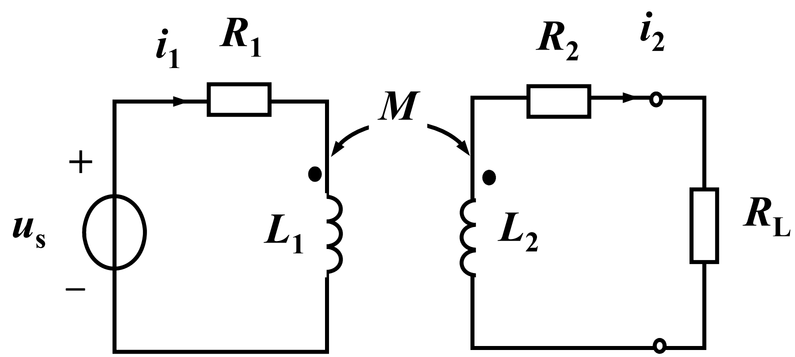


$$\dot{U}_2=\frac{\dot{U}_1}{n} \qquad \dot{I}_2=n\dot{I}_1'$$

例2　已知 $R_1=20\Omega, L_1=0.9H, R_2=10\Omega,\ \ L_2=0.1H,\ M=0.3H,$

$R_L=10\Omega, u_s=100\sqrt{2}\sin 100t\ V$,求$i_1$和$i_2$

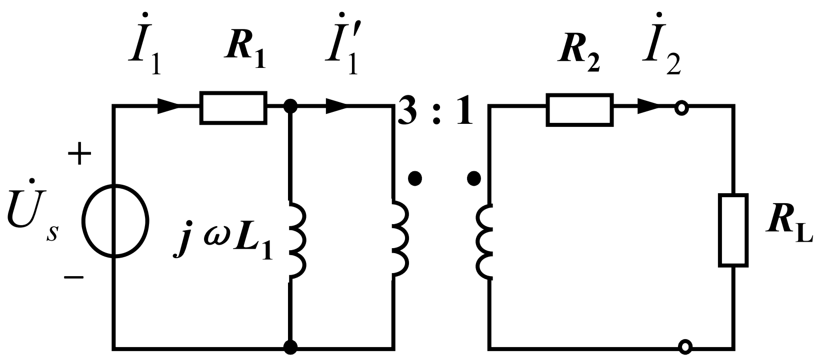
解：$\because\ \dfrac{M}{\sqrt{L_1L_2}}=\dfrac{0.3}{\sqrt{0.9\times0.1}}=1=K$

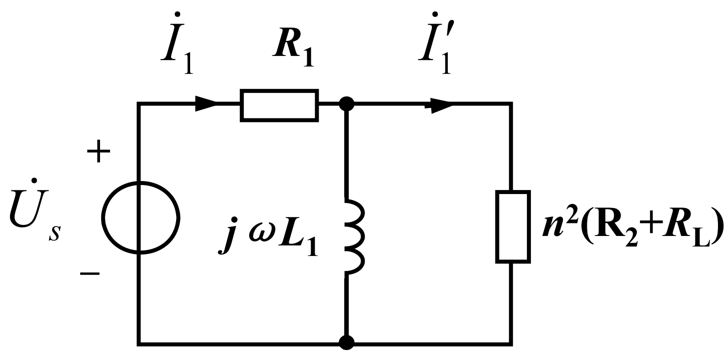


$\therefore$为全耦合变压器

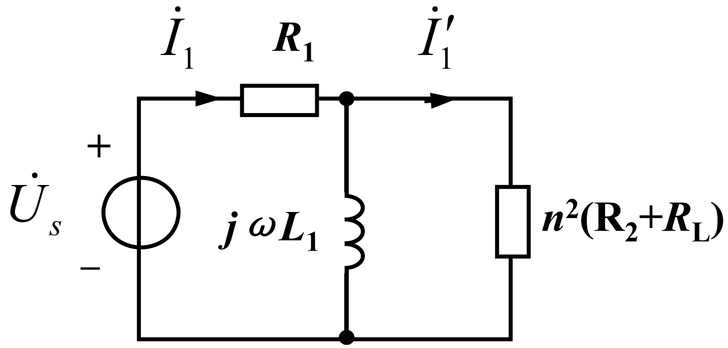
$$n=\frac{N_1}{N_2}=\sqrt{\frac{L_1}{L_2}}=\sqrt{\frac{0.9}{0.1}}=3$$

等效电路如图：





$$\dot{I}_1 = \frac{100}{20 + \dfrac{180 \times j90}{180 + j90}} = 1.096\underline{/-52.125°}\ \text{A}$$



$$\dot{I}_1' = \frac{j90}{180 + j90}\dot{I}_1 = 0.49\underline{/11.305°}\ \text{A}$$

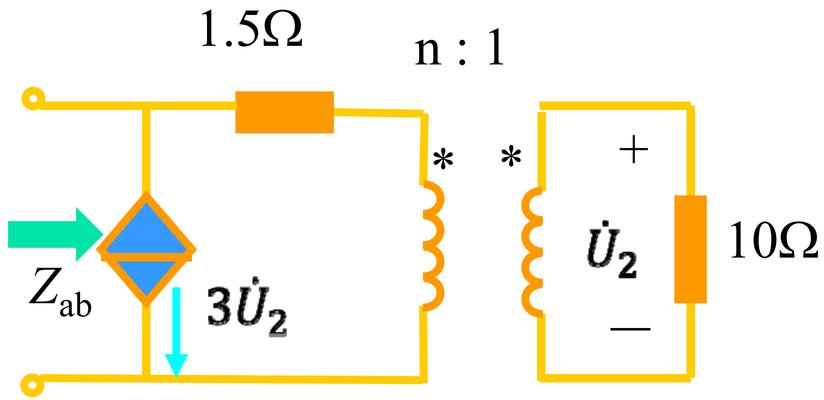
$$\dot{I}_2 = n\dot{I}_1' = 1.47\underline{/11.305°}\ \text{A}$$

$$i_1 = 1.096\sqrt{2}\sin(100t - 52.125°) \quad A$$

$$i_2 = 1.47\sqrt{2}\sin(100t + 11.305°) \quad A$$

例4

已知图示电路的等效阻抗Z_{ab}=0.25Ω，求理想变压器的变比n。



解

应用阻抗变换

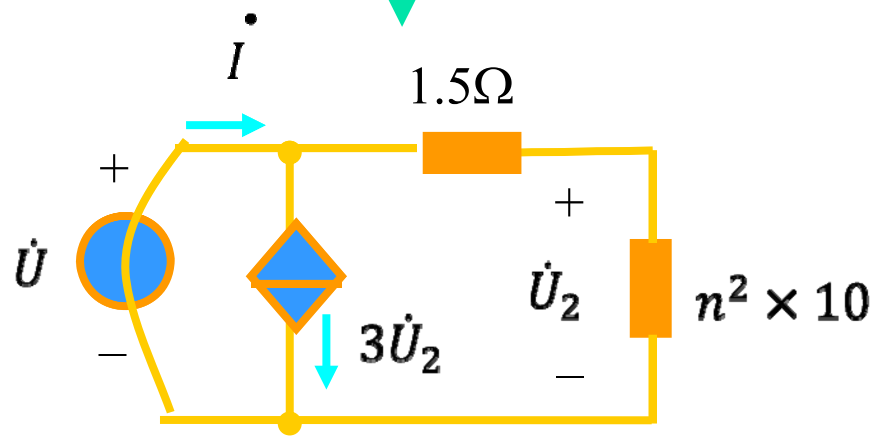
外加电源得：

$$\dot{U}=(\dot{I}-3\dot{U}_2)\times(1.5+10n^2)$$

$$\because \dot{U}_1=(\dot{I}-3\dot{U}_2)\times 10n^2$$

$$\dot{U}_1=n\dot{U}_2$$

$$\dot{U}_2=\frac{10n\dot{I}}{30n+1}$$



$$Z_{ab}=0.25=\frac{\dot{U}}{\dot{I}}=\frac{1.5+10n^2}{30n+1}$$

n=0.5 or n=0.25