B.TECH-1st SEMESTER END TERM EXAMINATION – 2018 S₁ (All branch): All

Subject Name: Engineering Mathematics-I

Subject Code: UMA21B04

Full Marks: 100

Time: 3 hours

Group-A

Answer the following questions:

Marks:55

1. a) Using method of variation of parameter, solve: $(D^2+4)y = e^x + sin2x$

   b) State **Raabe's test** for infinite series.

   c) Test the convergence of the series: $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \cdots, (x>0)$

[5 + 2+ 7]

2. a) If $y = \cos(m \sin^{-1} x)$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$$

   b) If $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$, prove that $c \in (a,b)$ of CMVT is the geometric mean between a and b, $a > 0, b > 0$.

   c) State **Leibnitz's Theorem** of successive differentiation.

   d) Expand $f(x) = 2x^3 + 7x^2 + x - 1$ in powers of $(x-2)$.

[5 + 4 + 2 + 4]

3. a) Evaluate: $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}$

   b) State **Maclaurin's series expansion** of the function $f(x)$.

   c) i) If $y = \cos(ax+b)$ find y_n.

   ii) If $y = a^{mx}$ find y_n.

[5 + 2 + (2+ 2)]

4. a) Solve: $(x^2D^2 - 4xD + 6)y = -x^4 sinx$

   b) Solve: $(D^2 - 2D)y = e^x\, sinx$

   c) Solve: $\frac{dx}{dt} = 3x + 8y;\ \frac{dy}{dt} = -x - 3y$

[5 + 5 + 5]

Group-B

Answer the following questions:

Marks:45

1. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$

[6]

2. State and prove Euler's theorem for homogeneous function of two variables.

3. What do you mean by **Saddle Point**? If $u = \frac{x^4+y^4}{x^2y^2} + x^6 \tan^{-1}\left(\frac{x^2+y^2}{x^2+2xy}\right)$, find the value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} \text{ at } x = 1, y = 2.$$

[1+5]

4. Given $x + y + z = a$, find the maximum value of $x^m y^n z^p$ using Lagrange's multiplier.

[5]

5. State Convolution theorem for inverse Laplace transforms & hence find the inverse Laplace transform of $\frac{1}{(s^2+4s+13)^2}$.

[1+5]

6. Solve: $(D^2 - 6D + 9)y = t^2e^{3t}, y(0) = 2$ and $y'(0) = 6$ using Laplace Transform.

[6]

7. Find the Laplace transform of $f(t) = |\sin \omega t|, t \geq 0$.

[5]

8. State Second shifting theorem of Laplace Transform. Find $L\{\frac{1-\cos t}{t^2}\}$.

[1+5]

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No. [redacted]

B.TECH I[st] SEMESTER, END TERM EXAMINATION-2017

SUBJECT NAME: Engineering Mathematics-I

SUBJECT CODE: UCE/UCS/UEC/UEI/UPE/UCH/UBE02C07/UME/UEE02C06

Full Marks: 100 **Time: 3 Hours**

Symbols used here have their usual meanings

Group A

Answer the following questions: **[10 × 5] = 50**

1. a) State **Leibnitz's Theorem.**

b) Using the method of **Variation of Parameter** solve

$$\frac{d^2y}{dx^2} + 4y = 4\tan 2x$$

2. a) State **Rolle's Theorem.**

b) Solve the following system of simultaneous **differential equations:**

$$\frac{dx}{dt} + 5x + y = e^t$$

$$\frac{dy}{dt} - x + 3y = e^{2t}$$

3. Solve: $x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = x^3 + 3x$

4. a) State the necessary condition for **convergence of an infinite series.**

b) Solve: $(D^2 + 2D + 1)y = e^{-x}\log x$

5. Test the convergence of the infinite series

$$\frac{x}{1} + \frac{1}{2}\frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} + \frac{1.3.5}{2.4.6}\frac{x^7}{7} + \cdots, \quad \text{for } x > 0$$

6. If $y = \cos(m\sin^{-1}x)$, prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$, Hence obtain $y_n(0)$.

7. a) State **Gauss Test** for infinite series.

b) Evaluate $\lim_{x\to e}(\log x)^{\frac{1}{1-\log x}}$.

8. a) Solve $(D^4 + 16)y = 0$.

b) Prove that $0 < a < 1$, $o < b < 1$ and $a < b$, then

$$\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$$

9. a) State Taylor's series for the expansion of a function.

b) Expand $\log(1+x)$ in powers of x in infinite series.

10. Test the convergence of the infinite series

$$1+\frac{x}{1!}+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\cdots,\ for\ x>0$$

Group B

Answer all the following questions: Marks: 5/6

1. State **Convolution theorem** of Laplace Transform and use it to find the inverse Laplace Transform of $\frac{1}{s^3(s^2+1)}$.

[1+5=6]

2. Find the inverse Laplace Transform of $\frac{1}{s^3+1}$.

[6]

3. Solve the differential equation (by using Laplace Transform):

$$y''+2y'-3y=6e^{-2t},\ y(0)=2,\ y'(0)=-14.$$

[6]

4. Find a point in the plane $x+2y+3z=13$ nearest to the point $(1,1,1)$ (using the method of Lagrange's multiplier).

[5]

5. State **Euler's theorem** for function of two variables. If $u=cosec^{-1}\left(\sqrt{\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}}\right)$ then prove that

$$x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}=\frac{\tan u}{144}(13+tan^2u).$$

[6]

6. Define **Composite function** of two variables. If $z=f(x,y),\ x=u-av,\ y=u+av$, then prove that $a^2\frac{\partial^2z}{\partial u^2}-\frac{\partial^2z}{\partial v^2}=4a^2\frac{\partial^2z}{\partial x\partial y}$.

[5]

7. If $\frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1$, prove that $(\frac{\partial u}{\partial x})^2+(\frac{\partial u}{\partial y})^2+(\frac{\partial u}{\partial z})^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right)$.

8. Define **saddle point** of a function of two variables. Find the extreme value of the following function: $f(x,y)=x^2y-3x^2-2y^2-4y+3$.

[6]

9. Find the Laplace Transform of $t(\frac{Sin\,t}{e^t})^2$.

[5]

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[5]

S1(All Br...

B.TECH FIRST SEMESTER, END-TERM EXAMINATION-2016

NAME OF THE SUBJECT: Engineering Mathematics-I

CODE NO: UCE/UCS/UEC/UPE/UEI/UCH/UBE-02C07, UME/UEE-02C06

Full Marks: 100 Time: 3 hours

Symbols used here have their usual meanings

GROUP-A

Marks: 50

Answer all the following questions:

1. Solve : $[(x+1)^2D^2 + (x+1)D]y = (2x+3)(2x+4)$ [5]

2. Solve the following simultaneous ordinary differential equations

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

[6]

3. Solve: $(D^2 + 5D + 6)y = e^{-2x}sec^2x(1 + 2tanx)$ [6]

4. Test the convergence of the series: $\frac{2^2}{3^2} + \frac{2^2.\,4^2}{3^2.\,5^2} + \frac{2^2.\,4^2.\,6^2}{3^2.\,5^2.\,7^2} + \cdots$ [5]

5. Test the convergence of the series: $1 + \frac{x}{1!} + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \cdots$, for $x > 0$ [6]

6. i. State Leibnitz's theorem.

   ii. If $y = \sin[\log(x^2 + 2x + 1)]$, prove that

   $(x+1)^2y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$ [2+4]

7. If $0 \le x \le 1$, prove that

   $\sqrt{\frac{1-x}{1+x}} < \frac{\log(1+x)}{sin^{-1}x} < 1$ [6]

8. Expand 5^x up to the first three non-zero terms of the series. [5]

9. Solve: $(2xycosx^2 - 2xy + 1)dx + (sinx^2 - x^2)dy = 0.$ [5]

GROUP-B

Answer all the questions:

$10 \times 5 = 50$

1. If $u = \tan(y + ax) - (y - ax)^{3/2}$ prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

2. If $w = x + 2y + z^2, x = \frac{u}{v}, y = u^2 + e^v, z = 2u$ show that $u\frac{\partial w}{\partial u} + v\frac{\partial w}{\partial v} = 12u^2 +$

   $2ve^v$.

3. If $u = x\log(xy)$ and $x^2 + y^2 + 3xy - 1 = 0$, find $\frac{du}{dx}$.

4. If $u = x^3y^2\sin^{-1}(\frac{y}{x})$ prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 25u$.

5. Find the shortest distance from the origin to the surface $xyz^2 = 2$

6. Define **Unit step function**. Find Laplace transform of $f(t) = \begin{cases} \sin t, 0 < t < \pi \\ t, \ t > \pi \end{cases}$

7. State **Convolution theorem**. Find inverse Laplace transform of $\frac{s^2}{(s^2+1)(s^2+4)}$

8. Solve $y'' + y = t\cos 2t, y(0) = 0, y'(0) = 0$

9. Define **Periodic function**. Find the Laplace transformation of

$$f(t) = t, 0 < t < a$$
$$= \pi - t, a < t < 2\pi, \text{ where } f(t + 2\pi) = f(t)$$

10. A rectangular box is open at the top is to have volume of 32 cubic units. Find the dimensions of the box requiring least material for its construction.

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B.TECH 1st SEMESTER, END-TERM EXAMINATION-2015 S_1(All), BRANCH: All Branch

NAME OF THE SUBJECT: Engineering Mathematics-I

CODE NO: UCE/UCS/UEC/UPE/UEI/UCH/UBE-02C07, UME/UEE-02C06

ull Marks: 100

Time: 3 hours

Symbols used here have their usual meanings

GROUP-A

Answer any eleven (11) from the following questions: $11 \times 5 = 55$

1. Solve the differential equation $(4x\ \ D^2 + 16D + 9/x)y = 0$.

2. Test the convergence of the series: $1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\frac{4!}{5^4}x^4+\cdots$

3. Solve: $p^2 + 2py\cot x = y^2$, where $p = \frac{dy}{dx}$.

4. Solve the simultaneous ordinary differential equations

$$(D^2+4)x - 3Dy = 0;\ 3Dx + (D^2+4)y = 0.$$

5. Test the convergence of the series: $\frac{14}{1^3}+\frac{24}{2^3}+\frac{34}{3^3}+\cdots$

6. Prove that, $\frac{d^{2n}}{dx^{2n}}(x^2-1)^n = (2n)!$

7. Test the convergence of the series: $1+\frac{1}{2.8}+\frac{1.3}{2.4.5}+\frac{1.3.5}{2.4.6.7}+\cdots$

8. If $y = \frac{x^3}{x^2-1}$, then prove that $(y_n)_0 = \begin{cases} -(n)!, & if\ n\ is\ odd \\ 0, & if\ n\ is\ even \end{cases}$

9. If k is a real constant, prove that the equation $x^3 - 6x^2 + k = 0$ cannot have distinct roots in [0,4].

10. Find the point on the curve $y = \log x$, where the tangent is parallel to the chord joining the points $(1,0)$ and $(e,1)$.

11. Test the convergence of the series: $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\cdots$

12. Prove that $\lim_{x\to\infty}\frac{e^x}{[(1+\frac{1}{x})^x]^x} = e^{\frac{1}{2}}$

Please Turn Over

GROUP-B

Answer any nine (9) from the following questions: $9 \times 5 = 45$

13. If $u = \log(x^3 + y^3 - x^2y - xy^2)$, prove that $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = -\frac{4}{(x+y)^2}$.

14. If $z = log\,(x^2 + y^2) + \frac{x^2+y^2}{x+y} - 2log\,(x + y)$, by Euler's theorem find the value of $(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y})$.

15. Find the points on the surface $z^2 = xy + 1$ nearest to the origin. Also find the distance.

16. If $u = x^2 + y^2 + z^2$, where $x = e^t$, $y = e^t sin\, t$, $z = e^t cos\, t$, find $\frac{dy}{dt}$.

17. a) If $u = e^{xyz}$, show that $\frac{\partial^3 u}{\partial y \partial x \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$.

b) If $z = x^y + y^x$, show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$

18. A rectangular box open at the top is to have volume of 32 cubic units. Find the dimensions of the box requiring least material for its construction.

19. Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4e^{2t}$ given $y(0) = -3$ and $y'(0) = 5$.

20. Obtain the inverse Laplace transform of $\frac{s+4}{s(s-1)(s^2+4)}$.

21. Find the Laplace transform of the following

a) $t^2 u(t-2)$

b) $u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$

22. Evaluate Laplace transform of the following periodic functions

$$f(t) = \begin{cases} a \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

MT+(Sequence, ...

UCE/ME/EE/CSE/ECE/PE/ECH/BIOT/BRANCH ALL
B. TECH 1st SEMESTER, END TERM EXAMINATION 2014
Engineering Mathematics - I
UCE/ME/EE/CSE/ECE/PE/ECH/BL01C04
Full Marks: 50 Time: 3 Hour
Symbols used here have their usual meanings

5×10=50

Answer any five from the following questions:

1. a)State Raabe's test for convergence of an infinite series and hence test the convergence of $\sum (n+1)^{\frac{1}{3}} - n^{\frac{1}{3}}$.

b)Find the n^{th} order derivative of $y = (a + bx)^{-m}$, m is any positive integer.

c) If $u = \frac{x^2+y^2}{x+y}$ then show that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$.

d) If $x^2 + y^2 + z^2 - 2xyz = 1$, show that $\frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} + \frac{dz}{\sqrt{1-z^2}} = 0$.

(3+1+3+3)

2. a)Solve the simultaneous ordinary differential equations

$$\frac{dx}{dt} + 5x + y = e^t;\quad \frac{dy}{dt} - x + 3y = e^{2t}$$

b) Find the values of a and b such that $\lim_{x\to 0} \frac{a\sin^2 x + b\log\cos x}{x^4} = \frac{1}{2}$

c)State Convolution theorem and hencefind inverse Laplace Transform of $\frac{s+4}{s(s-1)(s^2+4)}$

3. a) Solve $(x-1)^3\frac{d^3y}{dx^3} + 2(x-1)^2\frac{d^2y}{dx^2} - 4(x-1)\frac{dy}{dx} + 4y = 4\log(x-1)$ (3+2+5)

b)State Leibnitz's theorem. If $y = \tan^{-1}x$, Prove that

(i) $(x^2+1)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$

(ii) $y_n(0) = \begin{cases} 0, if n is even \\ (-1)^{\frac{n-1}{2}}(n-1)!, if n is odd \end{cases}$

$\frac{S+4}{S(S-1)}$

c) If $u = 3(ax+by+cz)^2 - (x^2+y^2+z^2)$ and $a^2+b^2+c^2 = 1$ then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

4. a)Using Lagrange's Mean value theorem prove that $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$ and (3+3+4)
hence deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$

b) If $x = e^u\tan v, y = e^u\sec v, z = e^{-2u}f(v)$, then show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + 2z = 0$.

c)Solve $(D^2+4)y = 4\tan 2x$ by using variation of parameter method.

5. a)Prove that between any two roots of $e^x\sin x = 1$ there exist at least one root of $e^x\cos x + 1 = 0$.

b)Test the convergence of the following series $1 + \frac{3}{7}.x + \frac{3.6}{7.10}.x^2 + \frac{3.6.9}{7.10.13}.x^3 + \dots$ (4+4+2)

c) Find the extreme values of $u = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$.

6. a)If $x^3 + y^3 + xy - 1 = 0$ then prove that $y = 1 - \frac{x}{3} - \frac{26}{81}x^3 - \dots$

b) Solve $(D^2+3D+2)y = e^{e^x}$ by finding complementary function and particular integral (3+3+4)

c) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$ then prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$

7. a) Solve $(2+3x)^2\frac{d^2y}{dx^2} + 3(2+3x)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$.

b) Prove that $\lim_{x\to 0}\sin x\log x = 0$ (3+3+4)

c)Solve $\{tD^2 + (1-2t)D - 2\}y = 0$ using Laplace Transform, $D \equiv \frac{d}{dt}$ given $y(0) = y'(0) = 2$.

(3+2+5)