

Enrolment No. 11899681

S<sub>1</sub> (All branch): All

B.TECH 1<sup>st</sup> SEMESTER, MID TERM EXAMINATION-2018

SUBJECT NAME: Engineering Mathematics-I

SUBJECT CODE: UMA11B04/UMA21B04

Full Marks: 50

Time: 2 Hours

Symbols used here have their usual meanings

**Group A**

Answer the following questions

Marks: 25

1. Solve:  $(xy \sin xy + \cos xy)y dx + (xy \sin xy - \cos xy)x dy = 0$
2. Solve the following ordinary differential equation

$$(x+y+1) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{x+y+1}$$

3. (a) Define Bernoulli's equation.

$$(b) \text{ Solve: } (xy^2 - e^{\frac{1}{x^3}}) dx - x^2 y dy = 0$$

$$4. \text{ Solve: } (D^2 + 3D + 2)y = e^{e^x}$$

5. Solve the following differential equation by finding Complementary Function and Particular Integral  $(D^2 + 9)y = xe^{2x} \cos x$ .

6. Find the general solution of homogeneous linear differential equation:

$$2D^2 y + Dy - 6y = 0$$

7. Find the general solution of non-homogeneous linear differential equation:

$$(D^3 - D^2 - 6D)y = 1 + x^2$$

$$(xy^2 - e^{\frac{1}{x^3}}) dx - x^2 y dy$$

$$\frac{du}{dy} = x+y+1$$

$$[4 + 3 + (1 + 3) + 4 + 5 + 2 + 3 = 25]$$

**Group B**  
Answer the following questions

$$\frac{dy}{dx} + P y = Q y^n$$

Marks: 25

1. By using the definition of Laplace Transform, evaluate Laplace Transform of (i)  $\cosh at$  (ii)  $t^n$ . [2.5+2.5]

2. (a) State First Shifting theorem of Laplace Transform. Using it, evaluate  $L\{e^t \cos^2 t\}$ .

$$(b) \text{ Find } L\{G(t)\}, \text{ where, } G(t) = \begin{cases} \sin(t - \frac{\pi}{3}), & t > \frac{\pi}{3} \\ 0, & t < \frac{\pi}{3} \end{cases}$$

3. State Change of Scale property of Laplace Transform. Evaluate:  $L\{(t^2 - 3t + 2)\sin 3t\}$ . [1+2+2]  
[1+4]

4. Prove that,  $L\{\frac{\sin^2 t}{t}\} = \frac{1}{4} \log(\frac{s^2 + 4}{s^2})$ .

$$\frac{1}{2} \left[ y \tan xy + \frac{1}{x} \right] dx +$$

$$\text{OR} \\ L\left\{\int_0^t \frac{1 - e^{-2x}}{x} dx\right\} = \frac{1}{s} \log(1 + \frac{2}{s})$$

$$\frac{1}{2} \left[ x \tan xy - \frac{1}{y} \right] dy$$

[5]

5. Evaluate  $\int_0^\infty t^3 e^{-t} \sin t dt$ , by using Laplace Transform.

[5]

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$$L\{f(t)\} = \frac{f(s)}{s}$$

$$\frac{1}{(D+a)} Q = e^{ax} \int e^{-ax} Q dx$$

$$\frac{m}{y} = 2xy \sin xy + xy^2 \cos xy$$

$$2D^2 + D - 6 = 0$$

$$4 \times 2 \times 6$$

$$\frac{N}{x} = 2xy \sin xy - xy^2$$

C.F.

$$2m^2 + m - 6 = 0$$

$$-1 \pm \sqrt{1 - 24}$$



GROUP - A

Answer the following questions:

1. (a) Define Exact Differential Equation.

(b) Solve:  $(1 + xy)y dx + (1 - xy)x dy = 0$

2. (a) Write the general form of non-homogeneous ordinary differential equation of order n.

(b) Solve:  $x^2 y - x^3 \frac{dy}{dx} = y^4 \cos x$

3. (a) Define Clairaut's Equation.

(b) Solve:  $y + px = x^4 p^2$

4. (a) Define Particular Integral.

(b) Solve:  $(D^3 - 4D^2 + 9D - 10)y = 24e^x \sin 2x$

5. Solve:  $(D^2 + 1)y = x - \cot x$

Answer the following questions:

Group B

1. State First Shifting theorem of Laplace Transform. Evaluate  $e^{-3t} \cosh 5t \sin 4t$  by using the definition of Laplace Transform. [1+4]

2. Evaluate:  $\int_0^\infty e^{-t} \left( \frac{1}{t} \int_0^t e^{-u} \sin u du \right) dt$  by using Laplace Transform. [5]

3. Define Heaviside's unit step function and then use it to find the Laplace Transform of  $f(t)$ , where [1+4]

$$\begin{aligned} f(t) &= \cos t, 0 < t < \pi \\ &= \cos 2t, \pi < t < 2\pi \\ &= \cos 3t, t > 2\pi \end{aligned}$$

4. (a) Explain Periodic Function.

(b) Find the Laplace Transform of the periodic function  $f(t)$ , where [1+4]

$$\begin{aligned} f(t) &= t, 0 < t < a \\ &= 2a - t, a < t < 2a, \text{ and also } f(t + 2a) = f(t). \end{aligned}$$

(a) Find the Laplace Transform of  $f'(t)$ , where [1+4]

$$\begin{aligned} f(t) &= t + 1, 0 \leq t \leq 2 \\ &= 3, t > 2 \end{aligned}$$

(b) Deduce that  $L\{t^2 f(t)\} = (-1)^2 \frac{d^2}{ds^2} F(s)$ , where  $F(s) = L\{f(t)\}$ . [3+2]

Group A

Answer any five from following questions:

1. Solve the differential equation  $x \frac{dy}{dx} - 3y = x^4(e^x + \cos x) - 2x^2$  given that  $y = \pi^3 e^\pi + 2\pi^2$  when  $x = \pi$ . [Full]
2. (i) Write down the general form of a Bernoulli's equation.  
(ii) Solve:  $(x - y \cos x) dx - \sin x dy = 0$  [1+4]
3. (i) Write down the necessary condition for an equation to be an exact differential equation.  
(ii) Solve:  $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$  [1+4]
4. (i) Write down the general form of a linear non-homogeneous ordinary differential equation with constant coefficients.  
(ii) Find the general solution of Ordinary Differential Equation  $(D^2 + 4)y = \tan 2x$  [1+4]
5. Solve:  $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$  [5]
6. (i) Solve:  $(x^2 + y^2 + 1)dx - 2xy dy = 0$   
(ii) Solve:  $x \frac{dy}{dx} + \frac{y^2}{x} = y$ . [2.5+2.5]

Group B

Answer any five from following questions:

7. (i) State the first shifting theorem of Laplace Transformation.  
(ii) Find the Laplace Transformation of  $\frac{\cos 2t \sin t}{e^t}$ . [25]
8. (i) Find  $L\left\{\frac{e^{-t} \sin t}{t}\right\}$ .  
(ii) Find  $L\{g(t)\}$  where  $g(t) = (t-1)^3$ , when  $t > 1$   
 $= 0$ , when  $t < 1$  [2+3]
9. Solve by using Laplace Transform:  
 $(D^2 + 2D + 5)y = e^{-t} \sin t, y(0) = 0, y'(0) = 1$ . [3+2]
10. (i) Find the Laplace Transform of  $\int_0^t t e^{-3t} \sin^2 t dt$   
(ii) Find the inverse Laplace Transform of  $\frac{(s+2)}{(s^2+4s+8)(s^2+4s+13)}$  [5]
11. (i) Find the inverse Laplace Transform of  $\frac{2s+2}{(s^2+2s+10)}$   
(ii) Find the Laplace Transform of  $t\left(\frac{\sin t}{e^t}\right)^2$  [2.5+2.5]



Answer any three from the following questions:

1. a) Solve  $(x\sqrt{1-x^2y^2} - y)dy + (x + y\sqrt{1-x^2y^2})dx = 0$

b) Solve  $(D^3 - 2D^2 - 5D + 6)y = 0$ ,  $y(0) = 0$ ,  $y'(0) = 0$ ,  $y''(0) = 1$

c) If  $L[f(t)] = \log\left(\frac{s+3}{s+1}\right)$  then find  $L[f(2t)]$

d) Evaluate  $L\left[\frac{\cos 2t \sin t}{e^t}\right]$

2. a) Solve  $(1 + x + xy^2)dy + (y + y^3)dx = 0$

b) Solve  $(D^2 - 2D - 1)y = e^x \cos x$

c) Find the Laplace transform of  $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \sin t & t > \pi \end{cases}$

d) Find the inverse Laplace transform of  $\frac{1}{(s^2 + a^2)(s^2 + b^2)}$  using convolution theorem.

3. a) Solve  $x \sin x \frac{dy}{dx} + y(x \cos x - \sin x) = 2$

b) Solve  $(D^2 + 3D + 2)y = 2x^2 - 3x + 1$

c) Evaluate  $\int_0^x e^{-t} \int_0^t \frac{\sin u}{u} du dt$

d) Evaluate  $L^{-1}\left(\frac{3s+7}{s^2-2s-3}\right)$

4. a) Solve  $(D^2 - 3D + 2)y = 0$

b) Solve  $(D^2 - 1)y = x \sin x + x^2 e^x$

c) Solve the ODE by using Laplace transform  $(D - 2)x + 3y = 0$

$2x + (D - 1)y = 0$ ;  $t > 0$  and  $D \equiv \frac{d}{dt}$  given that  $x(0) = 8$  and  $y(0) = 3$ .

d) Evaluate  $L[t^2 u(t - 2)]$

5. a) Find the particular integral of  $(D^2 + a^2)y = \tan ax$

b) Solve  $x \frac{dy}{dx} + y \log y = x y e^x$

c) Solve the ODE by using Laplace transform

$\frac{d^2 x}{dt^2} + 9x = \cos 2t$  if  $x(0) = 1$  and  $x\left(\frac{\pi}{2}\right) = -1$

d) Show that  $L[t^n e^{at}] = \frac{n!}{(s-a)^{n+1}}$

Group A

[25]

Answer any five from following questions:

1. Solve the differential equation  $x \frac{dy}{dx} - 3y = x^4(e^x + \cos x) - 2x^2$  given that  $y = \pi^3 e^\pi + 2\pi^2$  when  $x = \pi$ .  
[5]
2. (i) Write down the general form of a Bernoulli's equation.  
(ii) Solve:  $(x - y \cos x) dx - \sin x dy = 0$  [1+4]
3. (i) Write down the necessary condition for an equation to be an exact differential equation.  
(ii) Solve:  $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$  [1+4]
4. (i) Write down the general form of a linear non-homogeneous ordinary differential equation with constant coefficients.  
(ii) Find the general solution of Ordinary Differential Equation  $(D^2 + 4)y = \tan 2x$  [1+4]
5. Solve:  $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$  [5]
6. (i) Solve:  $(x^2 + y^2 + 1)dx - 2xy dy = 0$   
(ii) Solve:  $x \frac{dy}{dx} + \frac{y^2}{x} = y$ . [2.5+2.5]

Group B

Answer any five from following questions:

[25]

7. (i) State the first shifting theorem of Laplace Transformation.  
(ii) Find the Laplace Transformation of  $\frac{\cos 2t \sin t}{e^t}$ . [2+3]
8. (i) Find  $L\left\{\frac{e^{-t} \sin t}{t}\right\}$ .  
(ii) Find  $L\{g(t)\}$  where  $g(t) = (t-1)^3$ , when  $t > 1$   
 $= 0$ , when  $t < 1$  [3+2]
9. Solve by using Laplace Transform:  
 $(D^2 + 2D + 5)y = e^{-t} \sin t, y(0) = 0, y'(0) = 1$ . [5]
10. (i) Find the Laplace Transform of  $\int_0^t t e^{-3t} \sin^2 t dt$   
(ii) Find the inverse Laplace Transform of  $\frac{(s+2)}{(s^2+4s+8)(s^2+4s+13)}$  [2.5+2.5]
11. (i) Find the inverse Laplace Transform of  $\frac{2s+2}{(s^2+2s+10)}$   
(ii) Find the Laplace Transform of  $t\left(\frac{\sin t}{e^t}\right)^2$  [2.5+2.5]



Symbols used here have their usual meanings

Group A

Answer any five from following questions:

1. Solve the differential equation  $x \frac{dy}{dx} - 3y = x^4(e^x + \cos x) - 2x^2$  given that  $y = \pi^3 e^\pi + 2\pi^2$  when  $x = \pi$  [2.5]
2. (i) Write down the general form of a Bernoulli's equation. [5]  
 (ii) Solve:  $(x - y \cos x) dx - \sin x dy = 0$  [1+4]
3. (i) Write down the necessary condition for an equation to be an exact differential equation. [1+4]  
 (ii) Solve:  $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$  [1+4]
4. (i) Write down the general form of a linear non-homogeneous ordinary differential equation with constant coefficients. [1+4]  
 (ii) Find the general solution of Ordinary Differential Equation  $(D^2 + 4)y = \tan 2x$  [5]
5. Solve:  $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$  [2.5+2.5]
6. (i) Solve:  $(x^2 + y^2 + 1)dx - 2xy dy = 0$   
 (ii) Solve:  $x \frac{dy}{dx} + \frac{y^2}{x} = y$

Group B

Answer any five from following questions:

7. (i) State the first shifting theorem of Laplace Transformation. [2.5]  
 (ii) Find the Laplace Transformation of  $\frac{\cos 2t \sin t}{e^t}$
8. (i) Find  $L\left\{\frac{e^{-t} \sin t}{t}\right\}$ . [2+3]  
 (ii) Find  $L\{g(t)\}$  where  $g(t) = (t-1)^3$ , when  $t > 1$   
 $= 0$ , when  $t < 1$
9. Solve by using Laplace Transform: [3+2]  
 $(D^2 + 2D + 5)y = e^{-t} \sin t, y(0) = 0, y'(0) = 1$
10. (i) Find the Laplace Transform of  $\int_0^t t e^{-3t} \sin^2 t dt$  [5]  
 (ii) Find the inverse Laplace Transform of  $\frac{(s+2)}{(s^2+4s+8)(s^2+4s+13)}$
11. (i) Find the inverse Laplace Transform of  $\frac{2s+2}{(s^2+2s+10)}$  [2.5+2.5]  
 (ii) Find the Laplace Transform of  $t\left(\frac{\sin t}{e^t}\right)^2$  [2.5+2.5]

Full Marks: 50

Symbols used here have their usual meanings Time: 2 Hours

Answer all the following questions:

1. (a) Solve:  $(x + y + 1) \frac{dy}{dx} = 1$
- (b) Solve:  $3x^2y^4dx + 4x^3y^3dy = 0, y(1) = 1$
- (c) Solve:  $(x\sqrt{1-x^2y^2} - y)dy + (x + y\sqrt{1-x^2y^2})dx = 0$
- (d) Solve:  $(xy^2 - e^{1/x^3})dx - x^2ydy = 0$
- (e) Solve:  $(D^3 - 2D^2 - 5D + 6)y = 0, y(0) = 0, y'(0) = 0, y''(0) = 1$ , without using Laplace Transform.

(f) Solve the Ordinary Differential Equation  $(D^4 + 4)y = 0$

(g) Find the general solution of Ordinary Differential Equation  $(D^2 - 2D - 1)y = e^x \cos x + x^2 e^{3x}$   
 Or

Solve:  $(D^2 + 3D + 2)y = xe^x \sin x + x^2$

[3+4+4+4+3+2+5]

2. Answer all the following questions:

- (a) Evaluate the Laplace transform of  $\int_0^t t \cosh t dt$
- (b) Evaluate the following integral using Laplace transform  
 $\int_0^\infty e^{-2t} \sin(t + \frac{\pi}{4}) \cos(t - \frac{\pi}{4}) dt$
- (c) Evaluate inverse Laplace transform using convolution theorem

$$\frac{1}{s(s^2 + a^2)}$$

(d) Evaluate Laplace transform of the following periodic functions  
 $f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 1 \end{cases}$

(e) Find the Laplace transform of the following unit step function  
 $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$

OR

$$f(t) = \sin t u(t - \frac{\pi}{2}) - u(t - \frac{3\pi}{2})$$

(f) Solve the differential equation using Laplace transform  
 $\frac{d^2x}{dt^2} + 9x = \cos 2t$  if  $x(0) = 1$  and  $x(\frac{\pi}{2}) = -1$

[3+3+5+5+4+5]