Construct OBST (a1, a2, a3) = {do, if, while}

Let p(1:3) = (0.5, 0.1, 0.05)

q(0:3) = (0.15, 0.1, 0.05, 0.05)

Let Tij → OBST for $a_{i+1}, a_{i+2}, \ldots, a_j$ $i<j$

Cij → Cost of Tree Tij

rij → root of Tij

Wij → weight of Tij

where

$$w_{ij} = q_i + \sum_{k=i+1}^{j} (p_k + q_k)$$

$C_{ii} = 0$

By def^n $r_{ii} = 0$, $w_{ii} = q_i$ $0 \le i \le n$

$$C_{ij} = \min_{i<k\le j} \{C_{i,k-1} + C_{kj}\} + w_{ij}$$

Step 1 :

$w_{00} = 0.15$, $C_{00} = 0$, $r_{00} = 0$

$w_{11} = 0.1$ $C_{11} = 0$, $r_{11} = 0$

$w_{22} = 0.05$ $C_{22} = 0$ $r_{32} = 0$

$w_{33} = 0.05$ $C_{33} = 0$ $r_{33} = 0$

Step 2 :

$$w_{01} = q_0 + \sum_{k=0+1}^{1} (p_1 + q_1)$$

$= 0.15 + (0.5 + 0.1) = 0.75$

$$C_{01} = \min_{0<k\le 1} \{C_{00} + C_{11}\} + w_{01}$$

$= 0.75 + \min\{0 + 0\} = 0.75$

$R_{01} = 1$

$$w_{12} = q_1 + \sum_{k=1+1}^{2} (p_2 + q_2)$$

$= 0.1 + (0.1 + 0.05) = 0.25$

$$C_{12} = \min_{1<k\le 2} \{C_{11} + C_{22}\} + w_{12}$$

$= \{0 + 0\} + 0.25 = 0.25$

$R_{12} = 2$

$$W_{23} = q_2 + \sum_{k=2+1}^{3} (p_3 + q_3)$$

$$= 0.05 + (0.05 + 0.05) = 0.15$$

$$C_{23} = \min_{2<k\leq 3} \{C_{22} + C_{33}\} + W_{23}$$

$$= \{0 + 0\} + 0.15 = 0.15$$

$$r_{23} = 3$$

Step 3:

$$W_{02} = q_0 + \sum_{k=0+1}^{2} (p_1 + q_1 + p_2 + q_2)$$

$$= 0.15 + (0.5 + 0.1 + 0.1 + 0.05)$$

$$= 0.15 + 0.75 = 0.9$$

$$C_{02} = \min_{0<k\leq 2} \{\underbrace{C_{00} + C_{12}}_{k=1}, \underbrace{C_{01} + C_{22}}_{k=2}\} + W_{02}$$

$$= \{0 + 0.25, 0.75 + 0\} + 0.9$$

$$= 0.25 + 0.9 = 1.15 \quad \text{at } k = 1$$

$$r_{02} = 1$$

$$W_{13} = q_1 + \sum_{k=1+1}^{3} (p_2 + q_2 + p_3 + q_3)$$

$$= 0.1 + (0.1 + 0.05 + 0.05 + 0.05)$$

$$= 0.35$$

$$C_{13} = \min_{1<k\leq 3} \{\underbrace{C_{11} + C_{23}}_{k=2}, \underbrace{C_{12} + C_{33}}_{k=3}\} + W_{13}$$

$$= \{0 + 0.15, 0.25 + 0\} + 0.35$$

$$= 0.15 + 0.35 = 0.5 \quad \text{at } k = 2$$

$$r_{13} = 2$$

Step 4:

$$W_{03} = q_0 + \sum_{k=0+1}^{3} (p_1 + q_1 + p_2 + q_2 + p_3 + q_3)$$

$$= 0.15 + (0.5 + 0.1 + 0.1 + 0.05 + 0.05 + 0.05)$$

$$= 1$$

$$C_{03} = \min_{0<k\leq 3} \{ \underbrace{C_{00}+C_{13}}_{k=1}, \underbrace{C_{01}+C_{23}}_{k=2}, \underbrace{C_{02}+C_{33}}_{k=3} \} + w_{03}$$

$$= \{0 + 0.5,\ 0.75 + 0.15,\ 1.15 + 0\} + 1$$

$$= \{0.5,\ 0.9,\ 1.15\} + 1$$

$$= 1.5 \quad \text{at} \quad k = 1$$

$$r_{03} = 1$$