* ϵ = any string length '0'.

Unit-2 Regular Expression

Eg:- $L_1 = < aa, ab, ba, bb >$; $\Sigma = <a,b>$

Sol^n: $\Rightarrow$ $aa + ab + ba + bb$

$= a(a+b) + b(a+b)$

$= (a+b)(a+b)$

$\hookrightarrow$ (string length exactly '2')

Eg2:- $L = < aa, ab, ba, bb, aaa, aba, abb, bbb \ldots >$

Sol^n: $\Rightarrow$ $(a+b)(a+b)(a+b)^*$

$\hookrightarrow$ (string length atleast '2' or more)

$\phi = \{ \}$,

$\epsilon = \{\epsilon\}$.

$a = \{a\}$.

$a^* = \{ \epsilon, aa, aaa, a \ldots \}$

$a^+ = \{ a,$

$(a+b)^* = \{ \epsilon, a, b, aa, ab, bb \ldots \}$

Eg 3 :-> For string length atmost 2.

$L \to < \epsilon, a, b, aa, ab, ba, bb >$

Sol^n = $(a+b+\epsilon)(a+b+\epsilon)$

$a^* = <\epsilon, a, aa, aaa, \ldots>$

$a^+ = <a, aa, aaa, \ldots>$.

$(a+b)^* = <\epsilon, a, b, aa, ab, ba, bb, aaa, \ldots>$.

Q. Even length string over $<a,b>$.

$L = <\epsilon, aa, ab, ba, bb, aaaa, \ldots>$

Regular expression $= [(a+b)(a+b)]^*$

$= [(a+b)^2]^*$

$= (a+b)^{2*}$

$=$

$= \{(a+b)^{2n}\}, n \geq 0$

- Identities of Regular Expression.

1) $\phi + R = R + \phi = R$
2) $\phi \cdot R = R \cdot \phi = R$
3) $\epsilon \cdot R = R \cdot \epsilon = R$
4) $\epsilon^* = \epsilon$
5) $\phi^* = \epsilon$
6) $\epsilon + RR^* = RR^* + \epsilon = R^*$ $\{R^1, R^2 + R^3, R^4, \ldots\}$.

Example: 1) String exactly 2. $= (a+b)(a+b)$

2) at least 2 $:= (a+b)(a+b)(a+b)^*$

3) at most 2 $:= <\epsilon, a, b, aa, ab, ba, bb>$.

$= (a+b+\epsilon)(a+b+\epsilon)$

- $a+b = b+a$
  ↳ Union (+).

- $a \cdot b \neq b \cdot a$. ← concatenation.

- ~~ab ba~~ Here * means 0 or more occurences.
  + means one or more occurences.

Q. Start with a $\Sigma = \{a, b\}$.
→ $a \cdot (a+b)^*$

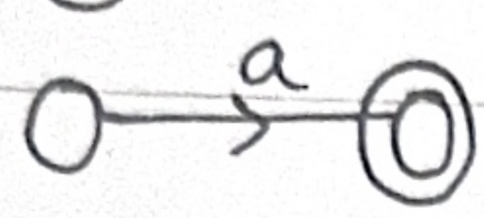
Q. End with a
→ $(a+b)^* a$.

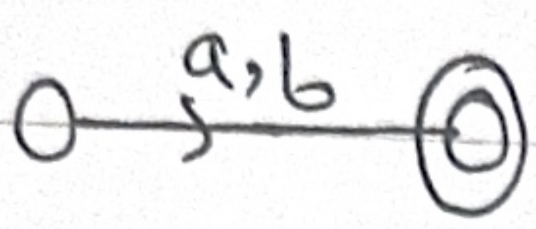
Q. Containing a.
→ $(a+b)^* a (a+b)^*$

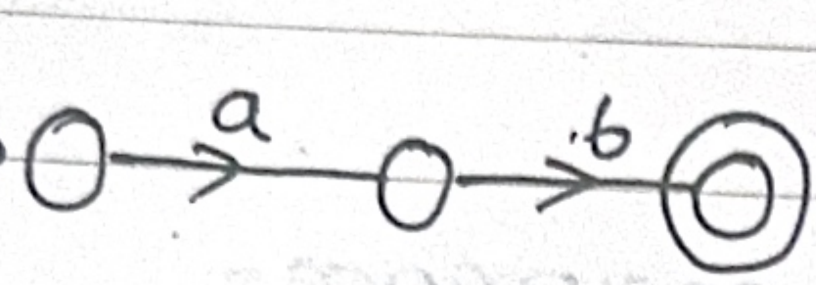
Q. Start and End with different symbol.
→ L = < ab, ba, aaab, bbbaa, ...>
$a(a+b)^*b + b(a+b)^*a$

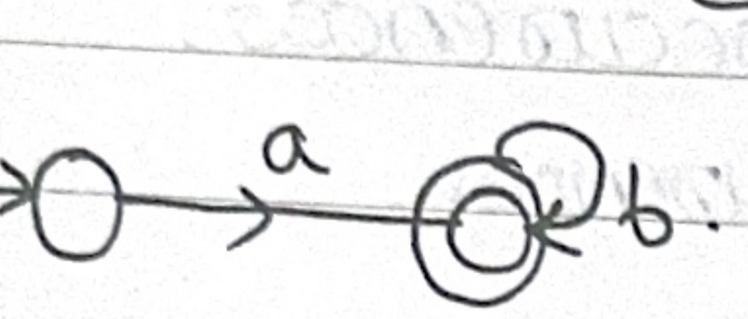
Q. Start and End with same symbol
→ L = < ϵ, a, b, aa, bb, ab, aba, bbab, ...>
$\epsilon + a + b + a(a+b)^*a + b(a+b)^*b$.

Finite automata graph.

- $\phi \rightarrow$ 
- $\epsilon \rightarrow$ 
- $a \rightarrow$ 

- $a+b \rightarrow$ 

- $a \cdot b \rightarrow$ 

- $ab^* \rightarrow$ 

(I) Arden's Theorem:

It states that if P, Q & R are Regular expression and P does not contain the empty string (ϵ), then the equation
$R = Q + RP$ has unique solution,
$R = QP^*$.

1) If $R = P + RQ$, $R = RQ + P$ same, then $R = PQ^*$.

2) If $R = P + QR$ then, $R = Q^*P$

Proof of Theorem (1):

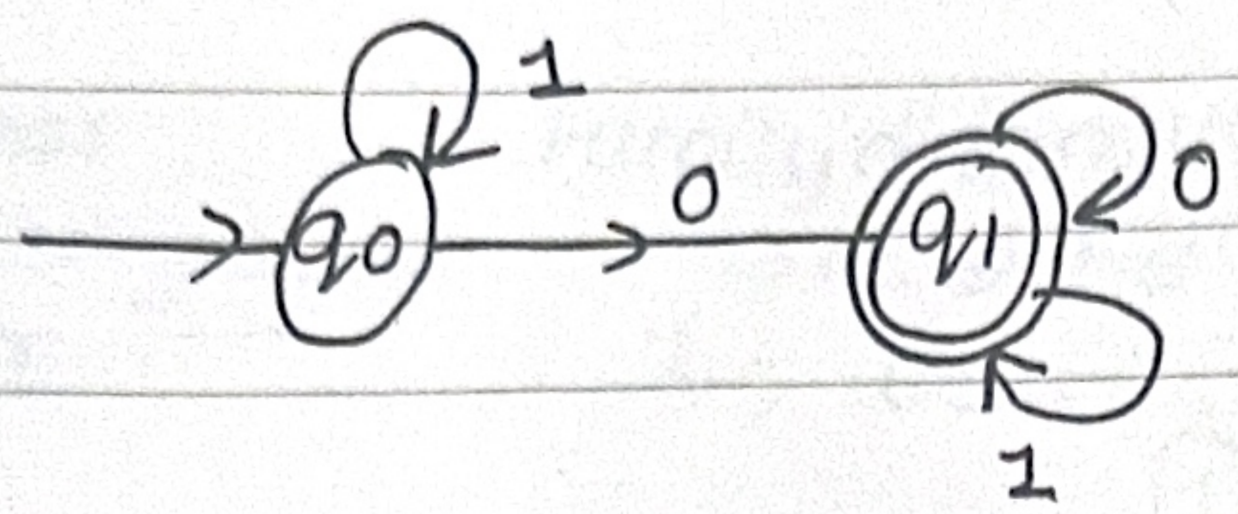
$$R = P + RQ \quad \text{——— (1)}$$

$$R = P + [P + RQ]Q$$
$$R = P + PQ + RQ^2$$
$$R = P + PQ + [P + RQ]Q^2$$
$$R = P + PQ + PQ^2 + RQ^3.$$
$$\vdots$$
$$R = P + PQ + PQ^2 + PQ^3 + PQ^4 + PQ^5 + \ldots$$
$$R = P(\epsilon + Q^2 + Q^3 + Q^4 + \ldots) -$$

if in beginning state, we get final state we add ϵ

$\boxed{R = PQ^*}$

Example 1:



Check incoming edges on particular state.

$q_0 = q_0 \cdot 1$. $\qquad q_0 = q_0 \cdot 1$

$q_0 = \epsilon + q_0 \cdot 1$ ——— (1) $\qquad R = P + RQ$

$q_1 = q_0 \cdot 0 + q_1 \cdot 0 + q_1 \cdot 1$.

$= q_0 \cdot 0 + q_1(0+1)$ ——— (2).

Consider Arden's Theorem.

Let $R = q_0$. $P = \epsilon$, $Q = 1$ } from eq^n (1).

$\therefore R = PQ^*$

$\therefore q_0 = \epsilon \cdot 1^*$

$\Rightarrow q_0 = 1^*$ ——— (3)

$\therefore$ Regular expression

$R = q_1$

$\because$ substitute eq^n (3) in eq^n (2).

$\therefore q_1 = q_0 \cdot 0 + q_1(0+1)$

$q_1 = 1^* \cdot 0 + q_1(0+1)$ ——— (4)

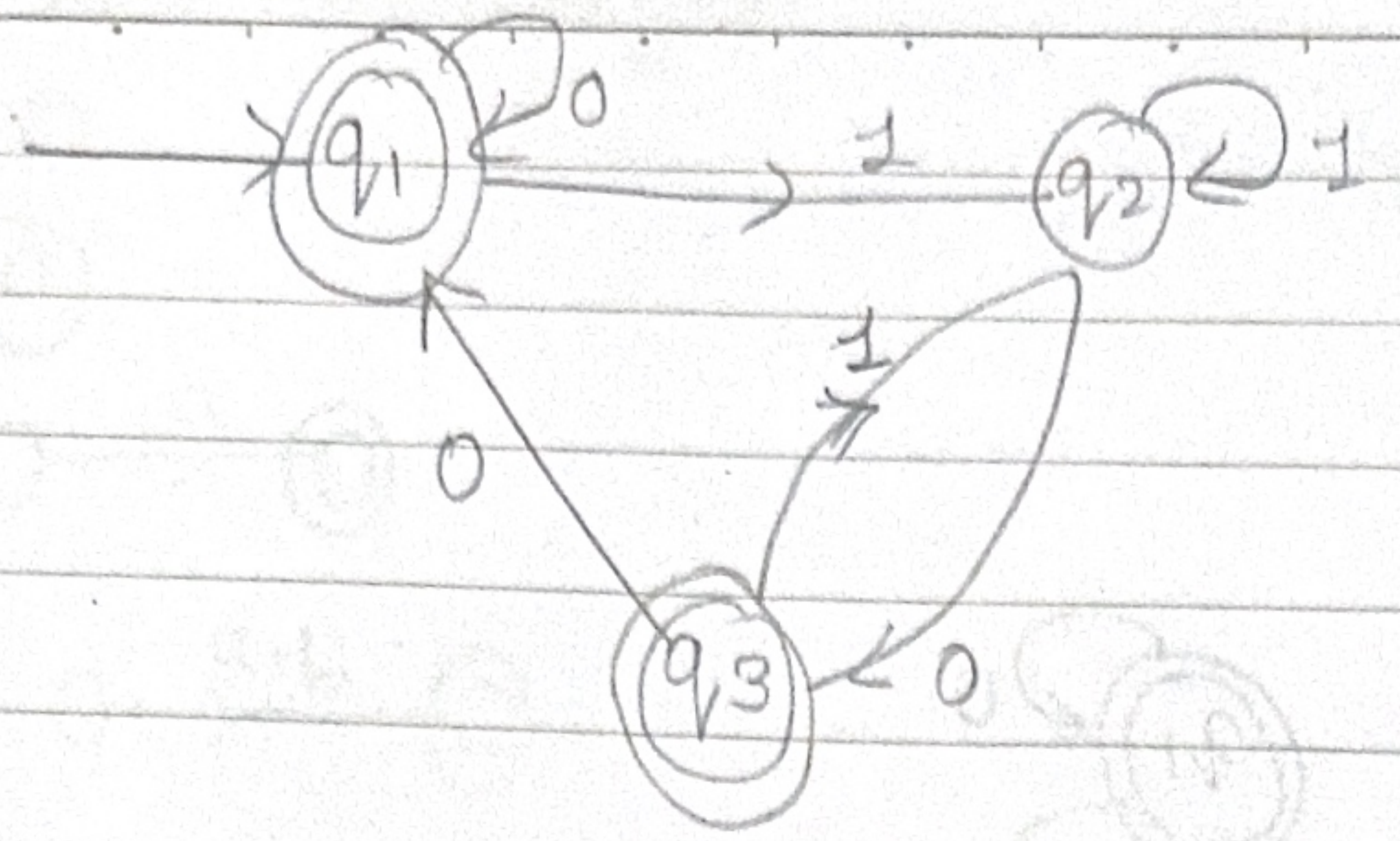
$R = q_1$

$P = 1^* 0$.

$Q = (0+1)$. $\longrightarrow \boxed{R = q_1 = 1^* 0(0+1)^*}$

Q. Based on given final automata, derieve regular expression using Arden's Theorem.

Eg 2 :-



$q_1 = q_1 \cdot 0 + q_3 \cdot 0$ —— (1)

$q_2 = q_1 \cdot 1 + q_2 \cdot 1 + q_3 \cdot 1$ —— (2)

$q_3 = q_2 \cdot 0$ —— (3)

now, final state

$q_1 = q_1 \cdot 0 + q_2 \cdot 00 + \epsilon$ —— (4) .... (substitute eqn (3) in eqn (1)).

now, from eqn (2) & eqn (3),

$q_2 = q_1 \cdot 1 + q_2 \cdot 1 + q_2 0 \cdot 1$.

$\rightarrow (q_3 \cdot 1 \Rightarrow q_2 \cdot 0 \cdot 1)$.

$\Rightarrow q_2 = q_1 \cdot 1 + q_2 (1 + 01)$.

now,

$\boxed{R = P + RQ}$, $\Rightarrow R = PQ^*$

$R = q_2$

$P = q_1 \cdot 1$

$Q = (1 + 01)$

$\boxed{q_2 = q_1 \cdot 1 (1 + 01)^*}$ —— (5)

now, substitute eqn (5) in eqn (4).

$q_1 = q_1 \cdot 0 + q_1 \cdot 1 (1 + 01)^* \cdot 00 + \epsilon$

$\boxed{q_1 = q_1 (0 + 1(1 + 01)^* \cdot 00) + \epsilon}$ —— (6),

from eqn (6) we get.

$P = \epsilon$, $R = q_1$, $Q = 0 + 1(1 + 01)^* 00$.

* Vivek kulkarni - Pg 116, Pg 117.

Chomsky Hierarchy.

* Could be asked for Theory.

• **Family of Formal Languages.**

Family of Grammar
- ↳ Unrestricted grammar
- ↳ Context free grammar
- ↳ context sensitive grammar
- ↳ Regular grammar

Family of languages
- ↳ RE sets $(a^p / p = 1)$
- ↳ CSL $(a^n b^n c^n / n \geq 1)$
- ↳ CFL $(a^n b^n / n \geq 1)$
- ↳ RL $(a^n / n \geq 0)$

Family of Machine.
- ↳ Turing machine.
- ↳ Linear Bounded Automata.
- ↳ Pushdown Automata
- ↳ Finite Automata.

Previous question,

$\Rightarrow R = PQ^*$

$q_1 = \epsilon(0+1(1+01)^* 00)^*$

$\boxed{q_1 = (0+1(1+01)^* 00)^*}$ —— for first final state answer

now,

from eqn (5), we can write eqn (3) as,

$q_3 = q_1 \cdot 1(1+01)^* \cdot 0$

$\boxed{q_3 = (0+1(1+01)^* 00)^* \cdot 1(1+01)^* \cdot 0}$

Final Regular Expression,

$R = q_1 + q_3$

$R = [[0+1(1+01)^* 00]^* + [(0+1(1+01)^* 00)^* \cdot 1(1+01)^* \cdot 0]]$

$R = [0+1(1+01)^* 00]^* [\epsilon + 1(1+01)^* \cdot 0]$