THE ZERO PARADOX

ZP-A: Lattice Algebra

Initial: April 2026 | Current: 1.39, November 2026

This document is self-contained within abstract algebra. No topology, probability, or Hilbert space is imported. Every claim proved here is provable using only the tools of semilattice theory; the status of OQ-A1 points to other documents. Cross-framework connections are deferred to ZP-E.

Illustrated Companion: A paired ZP-A Illustrated Companion document provides concrete examples and visual intuitions for the results in this document. Examples are kept separate from the formal layers to distinguish illustrative material from proofs. The companion is a reading aid; no proof-critical judgements should be drawn from examples alone.

Metatheoretic Declaration — ZFC^{-} + AFA

This document is stated over ZFC^{-} + AFA: Zermelo–Fraenkel set theory with Choice, with Aczel's Anti-Foundation Axiom (AFA) in place of the classical Axiom of Foundation. It is Aczel's ZFC^{-} + AFA (Aczel 1988, p. xviii). Barwise and Moss's ZFA (Barwise & Moss 1996, p. 72) is the same swap of Foundation for AFA over a ZFC^{-} that also has urelements (a proper class of them, by Strong Plenitude, p. 28); Choice is among its axioms (p. 28). AFA permits self-containing sets — in particular, sets satisfying $x = \{x\}$.

Scope: This declaration affects only CC-2 (Section V), the Quine atom, whose set-theoretic occupant is a set satisfying $x = \{x\}$. All algebraic results in Sections I–IV are independent of AFA and hold in standard ZF.

Standard concrete models (power sets ordered by inclusion, real intervals ordered by max, etc.) satisfy A1–A4, and no element of them is asserted to be a set $x = \{x\}$. This is expected — they are models of the algebraic structure, not instantiations of the ZFC^{-} + AFA metatheory. Self-containment is stated as a role that each structure occupies or not by its own theorem (Section V), not as an algebraic property of $\bot$.

Choice is kept: it is an axiom of ZFC^{-}. Which results use it is measured result by result (#print axioms); the located cases where it is needed, given a stated premise, are collected in ZeroParadox/Category/ChoiceCannotBe.lean § IV; most footprints are unclassified. T-SNAP depends on no axioms at all (t_snap_derived, ZeroParadox/AxiomProfile.lean). Foundation-freeness is forced (Section V); which anti-foundation axiom hosts the Quine-atom role is not, and AFA is adopted as the canonical host — see ZP-E Remark R-AFA.

I. Primitives and Axioms

1.1 Signature

The algebraic signature of the Zero Paradox state space is a triple: $(L, \vee, \bot)$

L is a non-empty set (the carrier set of states). $\vee : L \times L \to L$ is a binary operation called join. $\bot \in L$ is a distinguished constant called the bottom element.

Axiom Block A — Join-Semilattice with Bottom
A1 — Associativity: $(x \vee y) \vee z = x \vee (y \vee z)$ for all $x, y, z \in L$
A2 — Commutativity: $x \vee y = y \vee x$ for all $x, y \in L$
A3 — Idempotency: $x \vee x = x$ for all $x \in L$
A4 — Identity (Additive): $\bot \vee x = x$ for all $x \in L$

A4 is the load-bearing axiom. It makes $\bot$ the additive identity of the algebra: the element that contributes nothing to a join and is therefore present in every state as the neutral constituent.

II. The Induced Partial Order

2.1 Definition of $\leq$

Definition D1 — Lattice Order
For $x, y \in L$, define the relation $\leq$ by:
$x \leq y \iff x \vee y = y$

Proposition T1 — $\leq$ is a Partial Order
Reflexivity: $x \leq x$ — by A3, $x \vee x = x$. ✓
Antisymmetry: if $x \leq y$ and $y \leq x$, then $x \vee y = y$ and $y \vee x = x$. By A2, $y = x \vee y = y \vee x = x$. ✓
Transitivity: if $x \leq y$ and $y \leq z$, then $x \vee z = x \vee (y \vee z) = (x \vee y) \vee z = y \vee z = z$, so $x \leq z$. ✓

2.2 $\bot$ is the Least Element

Lemma T2 — $\bot$ is a Global Minimum under $\leq$
For all $x \in L$: $\bot \leq x$
Proof: By A4, $\bot \vee x = x$. By D1, this is the definition of $\bot \leq x$. ✓

T2 is the algebraic statement of the foundational claim: $\bot$ is not a void that states depart from — it is the minimum element that every state sits above. Since $\bot \leq x$ for all x, and join accumulates from the bottom, $\bot$ is algebraically present in every element of L.

III. The Additive Ontology

3.1 No Subtraction Operator

Remark R1 — Join-Semilattice vs. Lattice

A full lattice $(L, \vee, \wedge, \bot, \top)$ includes a meet operator $\wedge$ and a top element $\top$. The Zero Paradox restricts to the join-semilattice with bottom. A top element is neither required nor excluded: $\mathbb{N}$ under max has none (nat_has_no_top), while the two-state carrier MachinePhase has one (the example refuting HasNoTop MachinePhase in ZeroParadox/Order/Snap.lean). The meet operator is excluded because it would allow state reduction — the removal of informational content from a state. The additive ontology requires that no operation decreases informational content.

3.2 Join is the Only State Transition

Definition D2 — State Transition

A state transition is any function $f: L \to L$ such that $x \leq f(x)$ for all $x \in L$.

Equivalently, for each $x \in L$, $f(x) = x \vee \alpha$ for some $\alpha \in L$.

Proof of equivalence:

($\Rightarrow$) If $x \leq f(x)$, then $x \vee f(x) = f(x)$ by D1. Take $\alpha = f(x)$: then $f(x) = x \vee \alpha$. ✓

($\Leftarrow$) If $f(x) = x \vee \alpha$ for some $\alpha \in L$, then $x \vee f(x) = x \vee (x \vee \alpha) = (x \vee x) \vee \alpha = x \vee \alpha = f(x)$ by A1, A3. By D1, $x \leq f(x)$. ✓

IV. Monotonicity of State Sequences

4.1 State Sequences

Definition D3 — State Sequence

A state sequence is a function $S: \mathbb{N} \to L$, written $(S_0, S_1, S_2, \ldots)$, such that:

$S_{n+1} = S_n \vee \alpha_n$ for some $\alpha_n \in L$, for all $n \in \mathbb{N}$

Remark R2 — Terminology: State Sequence and Ascending Chain

In the order-theory literature, a sequence (S_n) satisfying $S_n \leq S_{n+1}$ for all n is called an ascending chain. The term "state sequence" is used here in place of "ascending chain" to align with the state-transition framing of ZP-D and ZP-E, where the same structure is introduced as sequences of system states. The two terms denote the same mathematical object. Readers familiar with order theory should read "state sequence" as "ascending chain". For concrete illustrations, see the ZP-A Illustrated Companion.

Theorem T3 — State Sequences are Monotone

For any state sequence (S_n) satisfying D3: $S_n \leq S_{n+1}$ for all $n \in \mathbb{N}$

Proof: By D3, $S_{n+1} = S_n \vee \alpha_n$. By D1, $S_n \leq S_n \vee \alpha_n \iff S_n \vee (S_n \vee \alpha_n) = S_n \vee \alpha_n$. By A1, $(S_n \vee S_n) \vee \alpha_n = S_n \vee \alpha_n$. By A3, $S_n \vee S_n = S_n$. Therefore $S_n \vee \alpha_n = S_{n+1}$. ✓

Monotonicity is a theorem, not a postulate. It is derived from A1–A3 via D3.

4.2 The Initial State

Every state sequence begins somewhere. T2 establishes $\bot \leq S_0$ for any initialisation — the bottom element is always below the starting point, whatever that starting point is. But T2 does not fix where S_0 sits; a sequence could legitimately begin above $\bot$. CC-1 closes this gap: we commit to initialising at the minimum. This is a modelling choice at ZP-A scope. ZP-J restates it rather than removing it: in any AFAStructure lattice a sequence starts at a Quine atom exactly when it starts at $\bot$ (cc1_derived and t_exec_iff), so the choice of starting point is expressed through the Quine-atom role. It is still not forced: on any lattice with a point above $\bot$ (as AX-B1 supplies) a valid sequence can begin there, as the OntologicalStates example in ZeroParadox/Settheory/OntBridge.lean shows; on a one-point lattice every sequence starts at $\bot$.

Conditional Claim CC-1 — $S_0 = \bot$ | restated as an equivalence in ZP-J, not forced
We commit to initialising every state sequence at the minimum of L: $S_0 = \bot$. This is not derived from A1–A4 — it is a modelling choice.
Under CC-1 and T3: $S_0 = \bot \leq S_1 \leq S_2 \leq \ldots$
Note: By T2, $\bot \leq S_0$ for any initialisation — this holds unconditionally from A4. CC-1 strengthens this to equality: $S_0 = \bot$. The commitment is not needed to establish $\bot \leq S_0$; it is needed to fix the starting point precisely.
Status: CONDITIONAL CLAIM — a modelling commitment at ZP-A scope. ZP-J restates it: given AFAStructure, cc1_derived proves that a state sequence starting at a Quine atom starts at $\bot$, and t_exec_iff gives the converse, so the two starting conditions are one. On a lattice with a point above $\bot$ a state sequence can begin there (T2 fixes only $\bot \leq S_0$; witness in ZeroParadox/Settheory/OntBridge.lean), so the equivalence does not remove the choice of starting point; it expresses it through the Quine-atom role.

V. The Self-Containment of $\bot$

5.1 Foundational Characterisation

The axioms A1–A4 establish $\bot$ as the additive identity and algebraic minimum of L. The following forced metatheoretic commitment concerns the set-theoretic host of the Quine-atom role, which $\bot$ occupies in the lattice under a self-application map. Under CC-2, R3 reads off that $\bot$ admits no external interpreter position. That rules out an external executor, not an inert $\bot$: it does not supply DA-1's precondition, which is what the occurrence commitment asserts and which DA-1 consumes (ZP-E Section IV, Path 1). See R3.

Foundation note: The framework is stated over ZFC^{-} + AFA (Zermelo–Fraenkel set theory with Choice, with Aczel's Anti-Foundation Axiom in place of Foundation). The classical Axiom of Foundation is replaced by AFA, which permits self-containing sets. Dropping Foundation is not an arbitrary modelling choice: a set equal to its own singleton, $x = \{x\}$, is a member of itself ($x \in x$), which the Axiom of Foundation (Regularity) forbids — under a well-founded membership relation no set is self-membered (no_quine_atom, ZeroParadox/Settheory/Wall.lean) — so in a Foundation universe no set occupies the Quine-atom role (zfset_quine_role_unoccupied). Which anti-foundation axiom replaces Foundation is a further choice: AFA is the one this framework adopts, not the only one that admits such a set. R3 and ZP-C L-INF corroborate the non-well-founded character of the role's host rather than independently prove the exclusion. See ZP-E Remark R-AFA for the full cross-framework argument. Only Foundation is replaced; Choice is kept.

Forced Metatheoretic Commitment CC-2 — Self-Containment of ⊥

The Quine atom is a role: a point that steps to itself and to nothing else (Motion.SelfOnlyAt, ZeroParadox/Die.lean). Where a set steps to its members, a set occupies the role exactly when it is its own only member, $x = \{x\}$ (zfset_selfOnlyAt_mem_iff).

Formally, chart by chart (ZeroParadox/Settheory/QuineDichotomy.lean): when the step is a self-application map supplied by AbstractSelfApp, whose only fixed point is ⊥, ⊥ of L is the only occupant, by the class fields alone (selfApp_quine_role_iff); when the step is membership in a QuineAtomHost, whose bottom is its own only member and the only such element, that host's own bottom is the only occupant (quineAtomHost_quine_role_iff), so ⊥ of L occupies it exactly when the host's bottom is ⊥ of L (the host examples after quineAtomHost_quine_role_iff); no set of Mathlib's well-founded ZFSet occupies it (zfset_quine_role_unoccupied). Every ZPSemilattice carries an AbstractSelfApp (trivialSelfApp, the constant map to ⊥) and every inhabited type a QuineAtomHost (trivialQuineAtomHost), so an occupancy says something only with the step named. No equation between ⊥ of L and a set is stated: it would be ill-typed.

Under ZFC^{-} + AFA the set chart is occupied: the Quine atom is a set satisfying $x = \{x\}$, and AFA allows at most one solution of $x = \{x\}$ (Aczel 1988, p. 19), so where the role is occupied there is no multiplicity.

This is a Forced Metatheoretic Commitment, not a freely chosen modeling decision. Foundation cannot host a set $x = \{x\}$: such a set is a member of itself, and Regularity forbids self-membership in ZF; in Lean, Mathlib's ZFSet, a model of ZFC, contains no such set (no_quine_atom). The Foundation-freeness of any host of the Quine-atom role is forced and machine-checked (quineAtomHost_not_wellFounded; see ZP-E Remark R-AFA); what remains a commitment rather than a theorem is that a Quine atom and its uniqueness are the right requirements to demand, and which anti-foundation axiom meeting them to adopt (ZP-E Remark R-AFA) — neither is derived from A1–A4 at the algebraic level. A differently-motivated requirement set that hosts the Quine-atom role without demanding a unique Quine atom would reopen the first; it would not touch the forcing, which is settled.

Status: FORCED METATHEORETIC COMMITMENT — Foundation-freeness of any host of the Quine-atom role is forced and machine-checked (quineAtomHost_not_wellFounded); two things remain commitments rather than derivations: the requirements-choice (a unique Quine atom), and the choice of AFA among the anti-foundation axioms that meet it (ZP-E Remark R-AFA). Algebraic fixed-point content formally verified in ZFC by ZP-J (ScaleBridge); the literal self-membership $x \in x$ is refuted in Mathlib's ZFSet (no_quine_atom) and holds of the bottom of an abstract membership relation on a two-element carrier (OneAtom, oneAtom_quineAtomHost); as membership in a model of ZFC^{-} + AFA it is outside Lean scope (metatheoretic), and no equation makes ⊥ of L such a set.

Cross-framework note: Dropping Foundation is not an arbitrary choice — a set $x = \{x\}$ is a member of itself ($x \in x$), which Regularity forbids in ZF, so ZF + Foundation cannot host it; in Lean, Mathlib's ZFSet, a model of ZFC, contains no such set (no_quine_atom). What is forced runs one way: a host whose bottom is its own only member is not well-founded (quineAtomHost_not_wellFounded). Run backwards it selects no theory, so the anti-foundation axiom is chosen, and AFA is adopted as the canonical host (ZP-E Remark R-AFA). R3 and ZP-C L-INF corroborate the non-well-founded character of the role's host; they are not independent proofs of the exclusion. See ZP-E Remark R-AFA for the full cross-framework argument.

Reading (the die, not a result; the die is the corpus's shared vocabulary of step relations, ZeroParadox/Die.lean, see ZP-E): AFA is one selection that meets the minimum requirements for the host's bottom to be the only occupant of the die's Quine-atom role: the two laws of QuineAtomHost, bot_mem_iff and reflexive_unique, which are exactly that condition (quineAtomHost_quine_role_iff and the example after it, ZeroParadox/Settheory/QuineDichotomy.lean); that AFA meets them is cited, not checked.

Forced Metatheoretic Commitment CC-2 — Self-Containment of ⊥

Lean 4 scope — three distinct layers. (1) Set-theoretic: no set $x = \{x\}$ exists in Mathlib's ZFSet, a model of ZFC, whose membership is well-founded (no_quine_atom: $x \notin x$ for every $x : \text{ZFSet}$; zfset_quine_role_unoccupied). Such a set lives in $\text{ZFC}^{-} + \text{AFA}$, outside Lean scope, and no equation between it and ⊥ of L is stated. (2) AFAStructure context (the typeclass taken as a hypothesis): ZP-J defines IsQuineAtom as the lattice-theoretic analog — IsQuineAtom q := selfMem q ∧ ∀ x, selfMem x → x = q. Within an AFAStructure instance, t_exec_iff proves IsQuineAtom q ↔ q = ⊥ (SetTheoryAFA.lean); ZP-K's da1_closed_concrete closes this concretely on MachinePhase. These results take the AFAStructure typeclass as a hypothesis, which every ZP-A lattice can supply trivially, so the hypothesis carries no AFA set-theoretic content of its own. (3) ZFC-clean (no AFA import): ScaleBridge.lean proves the fixed-point content in standard ZFC — selfMem_eq_singleton_free and z2_selfMem_singleton establish $\{x : \mathbb{Z}_2 \mid 2x = x\} = \{0\}$. This result is loop-free from AFA. What relates the layers is the Quine-atom role, occupied or not chart by chart; no equation across them is stated.

Remark R3 — CC-2 Leaves No External Interpreter Position for ⊥

A self-containing object has no external interpreter by structure: an occupant of the Quine-atom role steps to itself and to nothing else, so it is its own interpretation. A description requires a describer distinct from the thing described; CC-2 admits no such distinction for the occupant, which in the lattice under a self-application map is ⊥ (selfApp_quine_role_iff). That rules out an external executor, not an inert ⊥: it does not settle whether the machine configuration reaching P_0 (ZP-E) is there in Sense B, a running machine's current configuration, rather than Sense A, an inert string.

Dependency note: R3 is the structural motivation of ZP-E's Path 1 and derives neither DA-1 nor its precondition. That the configuration reaching P_0 is there in Sense B is DA-1's precondition, what the occurrence commitment asserts, and DA-1 consumes it (ZP-E Section IV).

VI. OQ-A1 — Sufficiency of Monotonicity

OQ-A1 — Sufficiency of Monotonicity [OQ-A1b CLOSED by A1–A4; OQ-A1a: no reason to restrict to join-irreducibles (well-founded carriers); placement in the ordinal tower, for a monotone map sending the tower's stages to c_0, conditional on the alignment hypothesis $h\varepsilon_0$ (ε_0 sent to c_1)]

Is the monotonicity constraint (T3) sufficient to characterise all valid state sequences, or are additional axioms required?

OQ-A1a: Is there algebraic reason to restrict α_n to join-irreducible elements (not expressible as joins of strictly smaller elements)?

OQ-A1b: Does a semilattice with no top element ⊤ (the optional hypothesis HasNoTop; R1) permit unbounded ascending chains?

Status: OQ-A1b CLOSED by A1–A4: yes. The natural numbers under max are a model, and every n has a first distinct state $n + 1$ there, so first steps do not give the ascending chain condition. (On the two-state carrier every chain is bounded; that is a fact about that carrier.)

OQ-A1 — Sufficiency of Monotonicity [OQ-A1b CLOSED by A1–A4; OQ-A1a: no reason to restrict to join-irreducibles (well-founded carriers); placement in the ordinal tower, for a monotone map sending the tower's stages to c_0, conditional on the alignment hypothesis $h\varepsilon_0$ (ε_0 sent to c_1)]

OQ-A1a: no algebraic reason to restrict α_n to join-irreducible elements. On a chain the restriction is vacuous (every element above $\bot$ is join-irreducible); on a branching carrier it is not (the top of the subsets of a two-element set is the join of two smaller states); and in a well-founded carrier every increment is a finite join of join-irreducibles (Birkhoff 1937; see Chakir–Pouzet, arXiv:0812.2300, Thm 2.1(c); exists_supIrred_decomposition), so restricting changes the number of steps, not the set of reachable states. Beyond well-founded carriers this is not measured.

Separately, where the step is placed in the ordinal tower is conditional on the alignment hypothesis: for a monotone map φ sending the tower's stages to c_0, given $h\varepsilon_0$ (ε_0 sent to c_1), nothing below ε_0 fires, because ε_0 is the supremum of the stages, and every fixed point of $\alpha \mapsto \omega^\alpha$ fires, because ε_0 is the least of them (snap_unconditional, hfp_from_epsilon_zero, epsilon0_min_eq_max). $h\varepsilon_0$ ($\varphi\ \varepsilon_0 = c_1$; in the $\mathbb{Z}_2$ chart, snapEmbed ($\varphi\ \varepsilon_0$) = 0, where snapEmbed is ZP-M's map sending c_0 to 1 and c_1 to 0) is the snap's occurrence at ε_0, taken as a hypothesis: of the infinitely many admissible firing points that monotonicity and tower alignment (every tower stage sent to c_0) leave open, $h\varepsilon_0$ selects the least, and so fixes φ uniquely (ZeroParadox/Ordinal/Incompleteness.lean § II). Whether Classical.choice is forced by the metric collapse is a separate open question (ZeroParadox/Ordinal/SyntacticCollapse.lean).

No step is guaranteed. A state sequence moves only upward (T3) and never returns (R1, t_snap_irreversible). Where the next increment is already absorbed ($S_n \vee \alpha_n = S_n$), the state stays the same, which T-SNAP permits (tsnap_holds_but_nothing_moves); different runs need not reach the same height. If from some step x on every new increment is already absorbed, then S_x is the supremum of the run: the value that run reaches in the limit. Short of a top, reaching it cannot be confirmed at any finite step; a run that reaches a top state (c_1 on the two-state carrier) is known there to stay.

Reading: ε_0 is both at once for the ordinal tower: the least fixed point of $\alpha \mapsto \omega^\alpha$ and the supremum of its stages (epsilon0_min_eq_max).

Note on minimal element above $\bot$: Bounded chains do not in themselves guarantee a minimal element above $\bot$. In a dense structure, for any $\varepsilon > 0$, $\varepsilon/2$ also exists — chains may be bounded yet have no first step. That is not a hypothetical: it is ZP-F, where the halving argument is proved for every field carrying a compatible linear order (f_snap_impossible, ZeroParadox/Reals/OrderedField.lean), and is stated there as the direct negation of AX-B1 (axb1_fails_in_ordered_field). The closure of OQ-A1b does not bound ascending chains, and a bound would not establish that ε_0 exists as a specific minimal element above $\bot$ either. In the lattice chart that existence rests on the same binary existence commitment (AX-B1); in the ordinal chart ε_0 is proved to be both the least fixed point of $\alpha \mapsto \omega^\alpha$ and the supremum of the tower's stages (epsilon0_min_eq_max), and a monotone map φ that sends those stages to c_0 and ε_0 to c_1 fires nowhere below ε_0, by the supremum face, and at every fixed point, by the least-fixed-point face (snap_unconditional, hfp_from_epsilon_zero). $h\varepsilon_0$ ($\varphi\ \varepsilon_0 = c_1$; in the $\mathbb{Z}_2$ chart, snapEmbed ($\varphi\ \varepsilon_0$) = 0) is the snap's occurrence at ε_0, taken as a hypothesis: of the infinitely many admissible firing points that monotonicity and tower alignment (every tower stage sent to c_0) leave open, $h\varepsilon_0$ selects the least, and so fixes φ uniquely (ZeroParadox/Ordinal/Incompleteness.lean § II). Whether Classical.choice is forced by the metric collapse is a separate open question (ZeroParadox/Ordinal/SyntacticCollapse.lean). ZP-B's 2-adic structure places 0 at infinite valuation and every non-zero element at finite valuation; what it proves there is that the gap at 0 is clopen (T3) and the return across it irreversible (C3). The non-zero elements still come arbitrarily close to 0 ($\|2^n\|_2 = 2^{-n}$), so there is no closest non-zero element to serve as a first step. This is outside the scope of A1–A4.

VII. Boundary Conditions

Export	Status / Receiving Document
(L, ∨, ⊥) as join-semilattice	Derived (A1–A4) — ZP-D: algebraic structure of state space
≤ partial order (D1, T1)	Derived — ZP-D: ordering on states
Monotonicity of state sequences (T3)	Derived from A1–A3 — ZP-D: state layer ordering
⊥ as global minimum (T2, CC-1)	Derived (T2) / Conditional (CC-1) — ZP-E: ontological grounding claim. CC-1 is restated in ZP-J as an equivalence (cc1_derived, t_exec_iff), not forced.
Quine-atom role, self-containment (CC-2, R3)	Forced Metatheoretic / Remark — ZP-E: Path 1's structural motivation (rules out an external executor, not an inert ⊥). Fixed-point content verified in ZFC by ZP-J.
No subtraction / additive ontology (R1)	Structural — ZP-C: no operation may reduce informational content
OQ-A1 — increment selection	OQ-A1b CLOSED by A1–A4; OQ-A1a: no reason to restrict to join-irreducibles (well-founded carriers); placement in the ordinal tower, for a monotone map sending the tower's stages to c_0, conditional on the alignment hypothesis h_{ε_0} (ε_0 sent to c_1)

VIII. Validation Status

Component	Status / Notes
A1–A4 join-semilattice axioms	Valid — Axioms; self-contained
≤ partial order (D1, T1)	Valid — Derived from A1–A3
⊥ as least element (T2)	Valid — Derived from A4 and D1
Additive ontology / no subtraction (R1)	Valid — Structural; signature restriction
State transition as join (D2)	Valid — Defined; consistent with signature
Monotonicity of state sequences (T3)	Valid — Derived from A1–A3 and D3
CC-1: $S_0 = \bot$	CONDITIONAL CLAIM — modelling commitment at ZP-A scope. ZP-J restates it as an equivalence (a Quine-atom start is a ⊥ start: cc1_derived, t_exec_iff); the starting-point choice is not forced.
CC-2: the Quine atom (a role)	Forced Metatheoretic Commitment about the host — occupancy proved chart by chart (QuineDichotomy.lean); Foundation-freeness forced (QuineAtomHost); AFA adopted as the host, not selected; fixed-point content verified in ZFC by ScaleBridge
ZFC^- + AFA foundation (Foundation replaced by AFA, Choice kept)	Meta-theoretic — framework-wide; the host adopted for CC-2 (Aczel 1988, p. xviii; Barwise & Moss 1996, p. 72, whose ZFC^- on p. 28 also has urelements)

Component	Status / Notes
OQ-A1: Sufficiency of monotonicity	OQ-A1b CLOSED by A1–A4; OQ-A1a: no reason to restrict to join-irreducibles (well-founded carriers); placement in the ordinal tower, for a monotone map sending the tower's stages to c_0, conditional on the alignment hypothesis $h\varepsilon_0$ (ε_0 sent to c_1)