THE ZERO PARADOX

ZP-C: Information Theory

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This document is self-contained within information theory and discrete analysis on Q_2, with two named cross-layer dependencies. The topological structure of Q_2 — total disconnectedness (ZP-B T5), the clopen ball hierarchy, and the binary existence axiom (AX-B1) — is imported from ZP-B. The Candidate Theorem T-BUF (Section V) additionally invokes ZP-A's state-transition definition D2 ($f(x) = x \vee \alpha$) to identify the resulting state as the Binary Snap. T-BUF is conditional on execution: its Step 2, that the configuration at P_0 is executing, is not derived here or in ZP-E; it is the occurrence commitment (ZP-E, DA-1 insert § IV), which DA-1 consumes and does not supply. Every claim is marked as Derived, Axiomatic, Defined, or Candidate.

Encoding commitment. The two ontological states of AX-B1 are represented here as point-mass (Dirac) distributions over {0, 1} — the minimum-sufficient probabilistic encoding (RP-1, Section II). Under this encoding, the information-theoretic separation between $\bot$ and ε_0, the atom above $\bot$ of the two-state carrier, is exactly 1 bit (T1b). The 1-bit result is conditional on this encoding; RP-1 declares and justifies the commitment. Separately, the primary descriptive tool of information theory, surprisal, is unbounded above in the depth of the ball hierarchy, each value finite (L-INF, Section III), and c_0, $\bot$ of MachinePhase, corresponds to that hierarchy's limit point $0 \in Q_2$, where the 2-adic additive valuation is $\top$ (R-BRIDGE); from these the framework argues that no finite external description can contain c_0. These two results — the 1-bit cost of the transition and the infinite descriptive cost of the origin — are the information-theoretic expression of the same structural constraint, read on two bottoms: $\bot$ of the two-state encoding of AX-B1, and c_0, $\bot$ of MachinePhase.

Illustrated Companion: A paired ZP-C Illustrated Companion provides concrete examples and visual intuitions for the results here. Examples are kept separate from the formal layers to distinguish illustrative material from proofs.

I. Kolmogorov Complexity and the Incompressibility Threshold

Let x be a binary string of length n. The conditional Kolmogorov complexity is:

$K(x|n) = \min \{ |p| : U(p, n) = x \}$

where U is a fixed universal Turing machine and |p| is the length of program p.

Definition D1 — Incompressibility Threshold

$P_0 = \lim_{n\to\infty} K(x|n) / n$

P_0 is the algorithmic entropy rate of x. When P_0 reaches its maximum, x has exhausted its capacity for self-description.

Remark R1 — Scope of P_0

What P_0 establishes (Derived): the point at which x becomes incompressible.

Remark R1 — Scope of P_0

What P_0 does not establish: that incompressibility causes the Binary Snap. The generative claim is a Candidate Theorem (T-BUF), not a bare axiom, and it is conditional on execution at P_0. See Section V.

II. Informational Work: State Representations and JSD

2.1 The Representation Principle

Principle RP-1 — Minimum Sufficient Probabilistic Representation

The minimum sufficient probabilistic representation of a binary ontological state is a point-mass (Dirac) distribution over {0,1}: all probability mass is assigned to the value the state occupies, and none to the other value.

Justification: A point-mass distribution is the unique distribution that is (a) faithful — assigns non-zero probability only to the ontologically actual value, and (b) non-redundant — carries no uncertainty about which state is occupied. Any distribution with mass at both values represents partial existence and partial non-existence simultaneously. AX-B1 excludes this.

Status: PRINCIPLE — representational commitment parallel to MP-1 in ZP-B.

Theorem T1 — State Representations are Uniquely Derived

Given AX-B1 and RP-1, the distributions are unique:

$P = (1, 0)$ (the bottom state $\perp$: all mass at 0 — non-existence)

$Q = (0, 1)$ (ε_0, the atom above $\perp$ of the two-state carrier: all mass at 1 — existence)

Proof: AX-B1 establishes two ontological states. RP-1 requires point-mass representation of each. The bottom state $\perp$ occupies value 0: $P = (1,0)$. ε_0, the atom above $\perp$ of the two-state carrier, occupies value 1: $Q = (0,1)$. No distribution $(p, 1-p)$ with $0 < p < 1$ is consistent with AX-B1. P and Q are unique. Status: DERIVED from AX-B1 and RP-1. ✓

Corollary T1b — JSD = 1 bit

For $P=(1,0)$ and $Q=(0,1)$ with $M=(1/2, 1/2)$:

$D_{KL}(P\|M) = 1$ bit, $D_{KL}(Q\|M) = 1$ bit, $JSD(P\|Q) = 1$ bit

$E = JSD(P\|Q) = 1$ bit [information-theoretic, dimensionless]

Status: Derived from AX-B1 and RP-1 via T1. E is not physical energy in joules. Dimensional bridge belongs to ZP-E as BA-1.

Definition D3 — Dirac Measure δ_0

$\int_\Omega f \, d\delta_0 = f(0)$

δ_0 places unit mass at 0. Compatible with the discrete topology of {0,1} and with the totally disconnected topology of Q_2 (ZP-B T5). δ_0 governs behaviour exactly at $x = 0$.

Remark R2 — Hamming Cross-Validation

$d_H(P, Q) = 1$ [Hamming, on {0,1}]

The agreement between $d_H = 1$ and $E = 1$ bit is a consistency check, not a proof that Hamming distance and JSD are equivalent in general. Both are computed by independent methods on the same AX-B1-derived distributions.

III. The Discrete Surprisal Field on Q_2

Remark R3 — Why the Smooth Embedding Remains Retired

ZP-B T5 establishes Q_2 is totally disconnected. ZP-B T2 establishes every ball is clopen. Importing smooth calculus operators onto a totally disconnected space imports a smoothness assumption that contradicts ZP-B. The smooth embedding, MO-1, and P1 from v1.1 remain retired.

Definition D4 — Discrete Surprisal Function I

For $x \in Q_2$ with $x \neq 0$ and $P(x) > 0$:

$I(x) = -\log_2 P(x)$

As $v_2(x) \to +\infty$ (x approaches 0 in the 2-adic metric), $I(x) \to +\infty$: states 2-adically close to 0 are informationally extreme.

The probability measure P on $Q_2 \setminus \{0\}$ is the branching measure induced by the binary ball hierarchy of Q_2 (RP-2 below): at each level k, the two sub-balls of $B(0, 2^{-k})$ each receive half the probability mass of their parent ball.

Principle RP-2 — Branching Measure on $Q_2 \setminus \{0\}$

The probability measure P on $Q_2 \setminus \{0\}$ is the canonical branching measure induced by the binary ball hierarchy: at each level k, each sub-ball receives half the probability mass of its parent. This is a representational commitment — a specific measure choice not uniquely forced by AX-B1 alone.

Justification: The branching measure assigns equal weight to each binary branch, consistent with AX-B1's symmetric treatment of existence and non-existence. Alternative measures (e.g. non-uniform branching weights) would depart from this symmetry without additional motivation.

Parallel: RP-1 fixes the point-mass representation of ontological states. RP-2 fixes the probability measure on the 2-adic domain. Both are representational commitments sitting alongside, not derived from, AX-B1.

Status: PRINCIPLE — representational commitment. Required by T2 and L-INF.

Definition D5 — Discrete Surprisal Difference Operator DF

For $x, y \in Q_2 \setminus \{0\}$:

$DF(x, y) = I(y) - I(x) = \log_2[P(x) / P(y)]$

DF is antisymmetric: $DF(x,y) = -DF(y,x)$. No smoothness is assumed; DF is defined entirely pointwise.

Definition D6 — Discrete Circulation (Extended)

FINITE CASE: A discrete loop $\gamma_n = (x_0, x_1, \ldots, x_n, x_0)$ in $Q_2 \setminus \{0\}$ returning to its start. Circulation: $C(DF, \gamma_n) = \sum_{i=0}^{n} DF(x_i, x_{i+1})$ where $x_{n+1} = x_0$.

Note: By telescoping, $C(DF, \gamma_n) = I(x_0) - I(x_0) = 0$ for all finite loops. DF is conservative on all finite loops.

INFINITE CASE: An infinite sequence $\sigma = (x_1, x_2, \ldots)$ in $Q_2 \setminus \{0\}$ where $v_2(x_i) = i$ for all $i \geq 1$. Partial circulation: $C_n(DF, \sigma) = I(x_{n+1}) - I(x_1)$.

Circulation of σ: $C(DF, \sigma) = \lim_{n\to\infty} C_n(DF, \sigma)$ if the limit exists or diverges.

An infinite sequence σ exhibits non-conservative behaviour if $C(DF, \sigma)$ diverges.

Theorem T2 — DF Exhibits Non-Conservative Behaviour on Infinite Sequences Approaching 0

DF is conservative on all finite loops ($C = 0$ by telescoping). On infinite sequences through the ball hierarchy approaching 0, the circulation diverges: $C(DF, \sigma) \to +\infty$.

Proof: Let $\sigma = (x_1, x_2, \ldots)$ with $x_i = 2^i$. Then $P(x_i) = 2^{-i}$, so $I(x_i) = i$. Partial circulation $C_n = I(x_{n+1}) - I(x_1) = n$. As $n \to \infty$: $C(DF, \sigma) = +\infty$.

Divergence arises from the unbounded depth of the 2-adic ball hierarchy (ZP-B). Finite conservation and infinite divergence are distinct regimes; no contradiction arises.

Status: DERIVED from ZP-B ball hierarchy structure and branching measure on Q_2. OQ-C1 closed. ✓

III-B. Unbounded Surprisal of the Bottom Element

Lemma L-INF — Unbounded Surprisal of ⊥

The surprisal $I(n) = n$ at ball-hierarchy depth n is unbounded above: for any finite bound M, there exist depths n with $I(n) > M$.

c_0, ⊥ of MachinePhase, corresponds to the limit point $0 \in Q_2$ — the limit of the binary ball hierarchy at infinite depth. The binary branching measure assigns equal probability mass at each branch level (D4), and the surprisal diverges without bound as depth increases (T2), each value finite. Surprisal is defined at each depth, not at the limit point itself.

Proof: Let $M \in \mathbb{R}$. By the Archimedean property, $\exists\, n \in \mathbb{N}$ with $n > M$. Then $I(n) = n > M$. Since M was arbitrary, surprisal is unbounded above. ✓

Formal content: surprisal is not bounded above by any real M.

Lemma L-INF — Unbounded Surprisal of ⊥

Semantic content: c_0, ⊥ of MachinePhase, corresponds to the limit point 0 of the binary ball hierarchy under the 2-adic metric, approached by sequences of increasing depth along which surprisal exceeds every finite bound; at that point the 2-adic additive valuation is ⊤ (R-BRIDGE). From these two charts the framework argues that no finite external interpreter can hold c_0 as a static description. This is one argument for DA-1's precondition (ZP-E, DA-1 insert § IV, Path 2), and it does not derive it. ZP-A CC-2 (the Quine atom, a role: in a ZP-A lattice under a self-application map supplied by AbstractSelfApp, ⊥ is its only occupant, selfApp_quine_role_iff), itself a commitment, gives a second, structural argument for the same conclusion: a self-containing object has no external interpreter by structure (ZP-A R3). The informational argument from the ball hierarchy and the structural argument from self-containment are independent; neither makes the configuration at P_0 executing.

Note: L-INF supplies unbounded surprisal; the step from there to executing is not a consequence of L-INF and is not DA-1. It is a bridge principle of its own (ZP-E, DA-1 insert § IV, Path 2), which the framework does not adopt, which is not the occurrence commitment, and which is not a premise of the Snap.

Status: DERIVED from D4 and T2. Structural corroboration: ZP-A CC-2 (the Quine atom, a role) and R3. Lean: l_inf (purity check: no non-Mathlib axioms).

Remark R-BRIDGE — On the Relationship Between K and 2-Adic Surprisal

ZP-C uses two complexity measures: Kolmogorov complexity $K(x|n)/|x|$ (Section I) and 2-adic surprisal $I(n) = n$ (Section III). These measure related but distinct structures. Their relationship warrants explicit statement.

What each measures. $K(x|n) \geq |x|$ is a property of a specific binary string x: no program shorter than x generates it. It is a pointwise statement about a particular configuration at the incompressibility threshold. $I(n) = n$ is a property of position in the ball hierarchy: the surprisal of any string at 2-adic depth n is n, by the branching measure RP-2. It is a structural property of depth, not of any particular string.

The bridge between them is refuted. The claim that the two meet at P_0, that 2-adic depth growing without bound carries $K(x|n)/|x|$ to 1, is refuted (counterexample $S_n = 2^n$, ZP-I § II.B); T-IZ never depended on it (it goes through t_iz_cauchy). The measures are not interchangeable: $K(2^n) = O(\log n)$, so the n-fold power of 2 is compactly described by a short program, yet $v_2(2^n) = n$ grows without bound. A string can be 2-adically deep without being Kolmogorov-incompressible, and vice versa.

How ZP-C uses them. ZP-C uses K and I as two separate arguments, not as a single unified measure and not as two routes to one threshold. The 2-adic surprisal argument has two charts. Depth chart, L-INF: $I(n) = n$ is unbounded in the depth n with each value finite (l_inf). Point chart: at 0 of Q_2, the one point in every ball $B(0, 2^{-n})$ of the chain centred at 0 (fB_bottom_is_limit), the 2-adic additive valuation is ⊤ (padic_addVal_bot; the floor-and-⊤ coincidence is infinitude_forces_infinite_complexity at q2InfinitudeFloor); c_0, ⊥ of MachinePhase, corresponds to that point. From these the framework argues that no finite interpreter can hold c_0. DA-1 Path 3 (ZP-E) is the K-incompressibility argument: $K(x|n) \geq |x|$ rules out a shorter external generator, not an inert string, so it does not derive executing from incompressibility. Neither derives DA-1's precondition.

IV. Execution as State: The Hardware Lemma

Definition D7 — Machine Configuration

A machine configuration c is a complete description of a Turing machine at a given moment: the current state of the control unit, the position of the read/write head, and the contents of all tape cells.

c_0: the initial configuration — the machine exists but has not yet begun execution (no instruction has been fetched).

c_1: the first running configuration — the machine has fetched and begun executing its first instruction.

Note: D7 does not import physics. It is a structural distinction between two discrete machine states within the standard Turing model.

Lemma L-RUN — Execution is a State Change off ⊥

The transition from c_0 (initial: not yet running) to c_1 (running: first instruction fetched) is a discrete, irreducible state change in the machine configuration.

Proof:

Step 1 — c_0 and c_1 are distinct configurations by D7: the control unit is in a different state (idle vs. executing). $c_0 \neq c_1$.

Step 2 — By AX-B1, a state either exists or it does not. The machine at c_1 occupies a configuration distinct from c_0. The transition $c_0 \to c_1$ is a binary state change in the sense of AX-B1.

Step 3 — The transition is irreducible: there is no intermediate configuration between c_0 and c_1. The first instruction fetch is the minimal unit of execution.

Step 4 — c_0 is modeled as corresponding to ⊥ in the semilattice (ZP-C CC-2 — see below). c_1 is strictly above ⊥: the machine configuration has gained content (an active execution context) not present in c_0.

Conclusion: $c_1 \neq \bot$ (in Lean, l_run, with ⊥ of MachinePhase = c_0). The act of execution is itself a non-bottom state, regardless of what the output tape contains. ✓

Status: DERIVED from AX-B1 and D7. No Coding Theorem required. No output-tape contents required.

Conditional Claim CC-2 (ZP-C) — $c_0 = \bot$ (Parallel to CC-1 in ZP-A; not ZP-A CC-2, the Quine atom)

We model the initial machine configuration c_0 as corresponding to ⊥ in the semilattice (L, ∨, ⊥). This is not derived from D7 or from A1–A4 — it is a modeling choice.

Motivation: c_0 is the machine state prior to any execution: no instruction has been fetched, no content has accumulated. ⊥ is the algebraic element below all others — the state of no accumulated content. The identification is motivated by structural correspondence, not derivation.

Parallel: CC-1 in ZP-A commits to $S_0 = \bot$ for state sequences. ZP-C CC-2 makes the same commitment for machine configurations. Both are modeling choices, not derivations from the core axioms.

Effect: Step 4 of L-RUN depends on ZP-C CC-2. The conclusion $c_1 \neq \bot$ follows from L-RUN's proof; the identification of c_0 with ⊥ is the additional modeling commitment supplied by ZP-C CC-2.

Conditional Claim CC-2 (ZP-C) — $c_0 = \perp$ (Parallel to CC-1 in ZP-A; not ZP-A CC-2, the Quine atom)

Status: CONDITIONAL CLAIM — modeling commitment; not derived from D7 or A1–A4.

Remark R4 — Configuration and Output Are Independent

D-TQ-2 (Output Definition): A computation's output is defined by the contents of its tape/register at termination.

L-RUN establishes that configuration state and output state are independent. A program may terminate with null output while still having passed through non-bottom configuration states during execution.

The question "can a program output $\perp$ without any non-bottom intermediate state?" is answered by looking at the execution trace, not the output tape. This distinction is load-bearing for T-BUF and TQ-IH.

V. The Test Question and the Buffer Overflow Theorem

Test Question TQ-IH — Can a Program Output $\perp$ Without a Non-Bottom Intermediate State?

Question: Does there exist a program p such that $U(p) = \perp$ and the execution trace $\tau(p)$ contains no configuration c_i with $c_i \neq \perp$?

Answer: No.

Proof:

Step 1 — Any program that executes must pass through c_1 (by definition of execution, D7).

Step 2 — By L-RUN, $c_1 \neq \perp$.

Step 3 — Therefore the execution trace $\tau(p) = (c_0, c_1, \ldots)$ contains c_1, which is a non-bottom configuration state, regardless of what appears on the output tape at termination.

Step 4 — The output of p being $\perp$ does not imply that all configurations in $\tau(p)$ are $\perp$. Output state and configuration state are distinct (R4).

Conclusion: No program can produce $\perp$ as output without passing through a non-bottom intermediate configuration state. TQ-IH answered negatively. ✓ The quantification over programs is Step 1's, from D7; what Lean proves is the configuration fact, tq_ih: $c_1 \neq c_0$ (Ne.symm l_run).

Status: DERIVED from L-RUN, D7, and R4. No Kolmogorov machinery required. TQ-IH is one of the results from which ZP-E derives the shape of the Snap as Theorem T-SNAP.

Remark R-TQ — External Confirmation of TQ-IH (April 2026)

TQ-IH was independently reviewed by an external PhD mathematician.

Confirmation: "As long as you have $c_1 \neq c_0$, this implies $c_i \neq c_0$ for some $1 \leq i < n$, at least for $n \geq 2$. This has nothing to do with computation or anything at all, it's a triviality."

Remark R-TQ — External Confirmation of TQ-IH (April 2026)

This confirms two things. First, TQ-IH holds. Second, and more significantly, the result is domain-independent: it does not depend on any Turing-specific machinery, any Kolmogorov complexity argument, or any property of the semilattice. It holds in any setting where a first step $c_1 \neq c_0$ is given. The derivation from L-RUN alone is complete and unassailable.

Status: EXTERNALLY CONFIRMED — April 2026.

Candidate Theorem T-BUF — Execution at P_0 Forces a Non-Bottom State

Statement: At the incompressibility threshold P_0, the Binary Snap $\bot \to \varepsilon_0$ is a structural consequence of execution, not an external trigger.

Step 1 — P_0 identifies the configuration x at which $K(x|n) \geq n = |x|$: the configuration string is incompressible, and, since $K(x|n) \leq |x| + c$ for every x, $K(x|n)/n$ is 1 up to an additive constant in K (bounds and sources: ZP-E, T-SNAP Step 1). (D1)

Step 2 — The configuration at P_0 is executing: it is a running machine's current configuration (Sense B, ZP-E, DA-1 insert § II), not an inert string. This step is not derived: it is the occurrence commitment, and T-BUF is conditional on it. Arguments for it, none deriving it: the 2-adic surprisal argument in its two charts (depth chart, L-INF: surprisal unbounded in the depth n with each value finite, l_inf; point chart: the 2-adic additive valuation $\top$ at 0 of Q_2, the point to which c_0 corresponds, padic_addVal_bot; R-BRIDGE), and the step from unbounded surprisal to executing is a bridge principle of its own, which the framework does not adopt (ZP-E, DA-1 Path 2); it is incompressible (D1), which rules out a shorter external generator and not an inert string (ZP-E, DA-1 Path 3). DA-1 (ZP-E, DA-1 insert § IV, closed given DP-2) consumes this step and does not supply it. In ZP-A's order, given CC-1, its first-step form is the hypothesis hocc of t_snap_given (ZP-E, Premises of T-SNAP).

Step 3 — Any execution passes through c_1 (L-RUN). $c_1 \neq \bot$ (L-RUN conclusion).

Step 4 — In $(L, \vee, \bot)$, this non-bottom configuration state is $c_1 = \bot \vee \varepsilon_0$. By ZP-A D2, this is the Binary Snap.

Conclusion: Given Step 2, execution at P_0 guarantees a non-bottom configuration state (L-RUN), and that state is ε_0 in the semilattice. The derivation pathway is P_0 + L-RUN + TQ-IH + ZP-A D2, given Step 2. ✓

Status: CANDIDATE THEOREM — conditional on execution. Step 1 is a definition (D1) and Steps 3 and 4 are derived within ZP-C; Step 2 is the occurrence commitment (ZP-E, DA-1 insert § IV), not a derived step, and DA-1 does not close it. ZP-E derives the Snap's shape as Theorem T-SNAP; that the Snap occurs follows from the occurrence commitment together with DA-1 (closed given DP-2). The occurrence commitment is stated in two charts, execution and slot, in the header of ZeroParadox/Order/Snap.lean (indexed in ZeroParadox/Order/SnapCannotBe.lean § II and CLAIMS.md), the slot chart as the hypothesis hw of t_snap_of_fires_once, that the tape fires at some stage, beside the theorem trials_fires_once that, for independent stages each firing with probability p, $0 < p \leq 1$, firing at some stage has probability one, which names no tape (ZeroParadox/Information/CrossingTrials.lean § VIII).

Remark R5 — Updated Status of AX-1

AX-1 (Binary Snap Causality) was previously labeled Axiomatic in ZP-C.

Remark R5 — Updated Status of AX-1
AX-1 was a Candidate Theorem in ZP-C; it is now retired: its shape is proved as T-SNAP (ZP-E), and that the Snap occurs is stated separately: it follows from the occurrence commitment together with DA-1 (closed given DP-2). The derivation pathway ZP-C gave it: P_0 (D1) identifies the threshold. L-RUN establishes that execution at the threshold constitutes a state change off ⊥. TQ-IH, read with D7, establishes that no executing program avoids this. ZP-A D2 establishes that a state change off ⊥ is the Binary Snap.
That a configuration at P_0 is an execution event (a running machine's current configuration, not an inert string) is the occurrence commitment, DA-1's precondition. DA-1 (ZP-E, a Derived Proposition closed given DP-2) consumes it and does not supply it: given it, $c_0 \to c_1$ is D7's. Both bear on whether the Snap occurs, not on its shape.
Status label: RETIRED in the ZP-E DA-1 insert, as stated above. AX-1 was a Candidate Theorem in ZP-C, with its gap identified: T-BUF Step 2, which is the occurrence commitment.

VI. Open Items Register

Item	Status	Description
S1: Distribution stipulation	Closed — T1	T1 derives P and Q from AX-B1 and RP-1.
OQ-C1: Non-conservation	Closed — T2 rebuilt	Telescoping critique resolved; infinite divergence proven within extended D6.
Smooth embedding, MO-1, P1	Retired	Retired. Inconsistent with ZP-B.
RP-1: Representation principle	Principle — explicit	Bridge between AX-B1 and probabilistic tools.
RP-2: Branching measure on $Q_2 \setminus \{0\}$	Principle — explicit	Canonical branching measure; representational commitment; required by T2 and L-INF.
ZP-C CC-2: $c_0 = \perp$	Conditional Claim	Modeling commitment — c_0 identified with ⊥ in semilattice. Parallel to CC-1 in ZP-A. Required by L-RUN Step 4.
D7: Machine configuration	Defined	Foundation for L-RUN and TQ-IH.
L-RUN: Hardware Lemma	Derived — Lemma	Execution is a state change off ⊥. Derived from AX-B1 and D7.
TQ-IH: Test Question	Closed — Confirmed	No program can output ⊥ without a non-bottom intermediate configuration state, given D7 (every executing program passes through c_1); L-RUN supplies $c_1 \neq c_0$ (Lean: tq_ih). Externally confirmed April 2026 (R-TQ): domain-independent, requires no Turing-specific or Kolmogorov machinery.
T-BUF: Buffer Overflow Theorem	Candidate Theorem	Conditional on execution: given that the configuration at P_0 is executing (Step 2, the occurrence commitment), its state is non-bottom and is ε_0. DA-1 consumes Step 2 and does not supply it.

Item	Status	Description
AX-1: Binary Snap Causality	Retired (was a Candidate Theorem)	Derivation pathway formalized. AX-1 retired in the ZP-E DA-1 insert: shape proved as T-SNAP; the snap occurs given the occurrence commitment and DA-1.

VII. Validation Status

Component	Status / Notes
K(x|n) and P_0 (D1)	Valid — standard algorithmic IT
R1: Scope of P_0	Valid — updated: AX-1 was a Candidate Theorem, not a bare axiom; it is now retired (shape proved as T-SNAP; the snap occurs given the occurrence commitment and DA-1)
RP-1: Representation Principle	Valid — Principle; explicit bridge; resolves reviewer gap in T1
T1: Distributions from AX-B1 + RP-1	Valid — Derived; reviewer gap closed
D2: JSD definition	Valid — standard
T1b: E = JSD = 1 bit	Valid — Derived from AX-B1 + RP-1
D3: Dirac measure δ_0	Valid — standard; compatible with discrete Q_2 topology
R3: Smooth embedding retired	Valid — Retired; inconsistent with ZP-B
D4: Discrete surprisal I(x)	Valid — pointwise on $Q_2 \setminus \{0\}$; branching measure stated
RP-2: Branching measure	Valid — Principle; explicit representational commitment; canonical binary branching measure
ZP-C CC-2: $c_0 = \perp$	Valid — Conditional Claim; modeling commitment; parallel to CC-1 in ZP-A; load-bearing for L-RUN Step 4
D5: Difference operator DF	Valid — antisymmetric; no smoothness assumed
D6: Circulation (extended)	Valid — finite and infinite cases both defined; finite conservation acknowledged
T2: Non-conservation rebuilt	Valid — Derived; telescoping critique addressed; divergence at $\sigma \to 0$ established
R2: Hamming cross-validation	Valid — Consistency check
D7: Machine configuration	Valid — Defined; standard Turing model; no physics imported
L-RUN: Hardware Lemma	Valid — Derived from AX-B1 and D7
R4: Configuration vs. output independence	Valid — load-bearing distinction for TQ-IH and T-BUF
TQ-IH: Test question answered	Valid — Derived from L-RUN and D7 (Lean: tq_ih, $c_1 \neq c_0$); no Kolmogorov machinery required

Component	Status / Notes
T-BUF: Candidate Theorem	Candidate — conditional on execution; Step 1 defined (D1), Steps 3 and 4 derived in ZP-C; Step 2 is the occurrence commitment (ZP-E), not closed by DA-1
R5: AX-1 status updated	Valid — AX-1 was a Candidate Theorem in ZP-C (prior Axiomatic status corrected); it is now retired: shape proved as T-SNAP (ZP-E); the snap occurs given the occurrence commitment and DA-1
R-BRIDGE: K vs. 2-adic surprisal	Refuted (counterexample $S_n = 2^n$, ZP-I § II.B); T-IZ never depended on it (it goes through t_iz_cauchy)