THE ZERO PARADOX

ZP-G: Category Theory

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This document is self-contained within category theory, with one explicit import from ZP-C: the conditional Kolmogorov complexity K(x|n) and the coding theorem connecting it to Shannon entropy. That import is named and labelled; it replaces Bridge Axiom BA-G1 (Leinster categorical entropy characterization) with a native derivation. All other content from ZP-A, ZP-B, ZP-D, and ZP-E remains excluded from this document. Cross-framework connections are deferred to ZP-H.

Honest labelling is the governing discipline. Every claim is marked as Axiom, Definition, Derived, Import, Design Commitment, or Remark. Nothing slides between categories.

Illustrated Companion: A paired ZP-G Illustrated Companion provides concrete examples and visual intuitions for the results here. Examples are kept separate from the formal layers to distinguish illustrative material from proofs.

I. Categorical Primitives

Definitions D1 through D6 and results T1 through T5 are reproduced here for completeness and self-reference.

1.1 The Definition of a Category

Definition D1 — Category
Status: Definition — foundational
A category C consists of:
(i) Objects: A collection ob(C), written A, B, X, 0, ...
(ii) Morphisms: For each ordered pair (A, B), a collection hom(A, B) of morphisms, written f: A → B.
(iii) Composition: ∘: hom(B,C) × hom(A,B) → hom(A,C), written g ∘ f.
(iv) Identity: For each A, a morphism id_A: A → A.
Associativity: h ∘ (g ∘ f) = (h ∘ g) ∘ f.
Unit laws: $id_B \circ f = f = f \circ id_A$.

Definition D2 — Morphism Uniqueness Notation
Status: Definition

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A morphism f: A → B is unique if for any g, h: A → B, g = h. Written: ∃! f: A → B.

1.2 Initial and Terminal Objects

Definition D3 — Initial Object

Status: Definition — load-bearing

An object 0 ∈ ob(C) is initial if for every X ∈ ob(C), there exists a unique morphism ι_X: 0 → X.

Proposition T1 — Uniqueness of the Initial Object

Status: Derived — standard category theory

If 0 and 0′ are both initial in C, ∃! isomorphism 0 ≅ 0′. The initial object is unique up to unique isomorphism.

Proof: ∃! f: 0 → 0′ and ∃! g: 0′ → 0. Initiality forces g ∘ f = id_0 and f ∘ g = $\mathrm{id}_{0'}$. Therefore f is a unique isomorphism. ✓

Definition D4 — Terminal Object

Status: Definition — defined for exclusion

An object 1 ∈ ob(C) is terminal if for every X ∈ ob(C), ∃! morphism τ_X: X → 1. The Zero Paradox framework is not a zero object category (0 ≇ 1).

II. The Foundational Axioms of ZP-G

Axiom AX-G1 — Asymmetry Axiom

Status: Axiom — foundational structural commitment

The category C possesses an initial object 0 and no terminal object.

Formally: ∃ 0 ∈ ob(C) satisfying D3. ¬∃ 1 ∈ ob(C) satisfying D4.

Correspondence: in ZP-A, a join-semilattice with ⊥ and without ∧; a carrier with no top is an optional hypothesis there, and ZP-A's two-state carriers have a top, so the no-terminal half is ZP-G's own commitment. In ZP-B's stand-in, the ℕ-indexed ball-depth category that ZP-H's fb_functor (ZeroParadox/Multihomed/CategoricalBridge.lean) lands in, there is no terminal object because the depth index has no maximum; in TopCat, where the realization fB_functor (ZeroParadox/Valuation/TopFunctor.lean) lands, the one-point space is a terminal object, so AX-G1 fails there (ZP-H C-H2). The present axiom is the categorical generalization.

Axiom AX-G2 — Source Asymmetry

Status: Axiom — foundational

For any non-initial object X ≠ 0: hom(X, 0) = ∅.

Axiom AX-G2 — Source Asymmetry

Motivation: The categorical expression of irreversibility. Morphisms $\iota_X: 0 \to X$ exist for all X. Their reversal does not exist. AX-G2 is consistent with AX-G1 but not derivable from it.

Remark R-AX — On the Non-Triviality of AX-G1 and AX-G2

Status: Remark

AX-G2 holds in many categories — it is a structural condition, not an exotic one. AX-G2 is the standard notion of a strict initial object (Carboni, Lack and Walters, 1993): an initial object into which every morphism is an isomorphism. Strict initial objects are the norm in well-behaved categories — the initial object of any topos, extensive category, or cartesian closed category is strict — so AX-G2 names a recognised structural property, not a ZP-specific stipulation. Together with AX-G1 (no terminal object) it places C at a non-trivial strict initial object: a strict initial that is also terminal (a zero object) forces every object to be isomorphic to it, so AX-G1 rules that degenerate case out — but not conversely, since a non-trivial category can have a strict initial object and still have a terminal one (in the category of sets, the empty set is strict initial and the one-point set is terminal). What distinguishes ZP-G from a trivial application of initial-object asymmetry is that the initial object here is not an abstract placeholder: it is ⊥, the algebraically minimal element of ZP-A's lattice. This identification is not asserted in ZP-G; it is demonstrated in ZP-H via four concrete domain functors (F_A, F_B, F_C, F_D), each of which maps the natural number depth hierarchy into a stand-in category for its domain and preserves the initial object. The axioms are not postulated in isolation; they are shown to hold in each of four ℕ-indexed depth categories, one per domain (ZP-H), and are not claimed for the three Mathlib categories ZP-H also targets (TopCat, KleisliCat PMF, ModuleCat ℂ) — each of which has a terminal object, so AX-G1 fails in every one of them. ModuleCat ℂ's zero module fD_functor.obj 0 is a zero object that every object maps into, so it is initial but not strict. The domain facts behind the stand-ins (ZP-A T2, ZP-B C3, ZP-C T1b, ZP-D T4) are proved separately in their own layers; the categorical check is the same ℕ-order instance each time.

III. Universal Constituent and Unreachability

Lemma T2 — Universal Constituent

Status: Derived — from D3 and AX-G1

For every $X \in ob(C)$, $\exists! \iota_X: 0 \to X$. The initial object 0 is the universal categorical source.

Proof: Immediate from D3 and AX-G1. ✓

Lemma T3 — Unreachability of 0

Status: Derived — from AX-G2

For any $X \neq 0$: $hom(X, 0) = \varnothing$. The initial object 0 is unreachable from any non-initial object.

Proof: Direct from AX-G2. ✓

Remark R1 — Structural Inversion — The Categorical Zero Paradox

Status: Remark

Remark R1 — Structural Inversion — The Categorical Zero Paradox

T2 and T3 together constitute the categorical Zero Paradox. 0 reaches every object in exactly one way (T2); no non-initial object reaches 0 (T3). This is not a logical contradiction. It is a structural inversion: the unique universal source is the unique object with no incoming non-trivial morphisms.

Remark R2 — Categorical Expression of Self-Containment

Status: Remark — connecting note to ZP-A CC-2

R1 frames the structural inversion of 0. This remark connects that structure to ZP-A CC-2, the Quine atom: a role, a point that steps to itself and to nothing else (Motion.SelfOnlyAt, ZeroParadox/Die.lean). Where a set steps to its members, a set occupies it exactly when $x = \{x\}$ (zfset_selfOnlyAt_mem_iff); in a ZP-A lattice under a self-application map supplied by AbstractSelfApp, $\bot$ of L is its only occupant (selfApp_quine_role_iff). An occupant has no external interpreter by structure; it IS its own interpretation.

In categorical terms, this corresponds to two conditions together: (1) AX-G2: $\hom(X, 0) = \emptyset$ for all $X \neq 0$ — no morphism can reach inside 0 from outside; and (2) T2: $\exists!\ \iota_X: 0 \to X$ for all X — 0 is structurally present in every object. Together these are the categorical image of undifferentiated self-containment: unreachable from without, yet constitutive of everything.

0 "points in all directions" (T2) because it is the undifferentiated ground from which all differentiation proceeds — not because it selects a direction. The uniqueness of each ι_X is not a choice among alternatives; it is the absence of internal structure that would allow differentiation among morphisms.

This remark bridges ZP-G to ZP-A CC-2. The formal correspondence between the categorical initial object structure and the Quine-atom role (ZP-A CC-2) is made explicit in ZP-H.

IV. Monotone Structure and the Additive Ontology

Definition D5 — Morphism Chain

Status: Definition

A morphism chain of length n from 0 is a sequence:

$$0 = X_0 \to X_1 \to \ldots \to X_n$$

where each $X_k \to X_{k+1}$ is a morphism in C.

Proposition T4 — Chains are Forward-Only

Status: Derived — from AX-G2

No morphism chain from 0 can return to 0 through non-initial objects.

Proof: A return morphism $X_n \to 0$ for $X_n \neq 0$ would contradict AX-G2. ✓

Definition D6 — Functor

Status: Definition — standard category theory

A functor F: C → D consists of an object map F: ob(C) → ob(D) and a morphism map F: $\hom_C(A,B) \to \hom_D(F(A), F(B))$, preserving composition and identity.

Proposition T5 — Functors Preserve Initial Objects

Status: Derived — [OQ-G2 closed in ZP-H T-H1]

For each instantiation functor $F \in \{F_A, F_B, F_C, F_D\}$, F(0) is an initial object in the codomain (for F_B, in its stand-in). Verified by direct universal property check in ZP-H T-H1. ✓

V. The Kolmogorov Import from ZP-C

This section contains the single import from outside category theory that ZP-G requires. It is named, scoped, and labelled explicitly. It replaces BA-G1 (the Leinster Bridge Axiom) as the informational foundation of D7′.

Import I-KC — Conditional Kolmogorov Complexity from ZP-C

Status: Import — from ZP-C D1 and standard algorithmic information theory

What is imported: The conditional Kolmogorov complexity K(x|y), defined as the length of the shortest program p such that U(p, y) = x, where U is a fixed universal Turing machine:

$$K(x|y) = \min \{ |p| : U(p, y) = x \}$$

Key property — The Coding Theorem: For any computable probability measure P, Kolmogorov complexity and Shannon entropy are related up to an additive constant c (depending only on U, not on x or y):

$$K(x|y) \approx -\log_2 P(x|y) + O(c)$$

The coding theorem is a standard result of algorithmic information theory (Li and Vitanyi, An Introduction to Kolmogorov Complexity and Its Applications). It is not derived within ZP-G. It is imported as a named result.

Scope of import: I-KC imports K(x|y) and the coding theorem only. No other content from ZP-C is imported into ZP-G. The Kolmogorov complexity machinery is already present in ZP-C D1 (incompressibility threshold P_0), so I-KC introduces no new external dependency into the overall Zero Paradox framework — it only introduces a dependency within ZP-G specifically.

Computability note: K(x|y) is not computable in general (it is approximable from above by standard results). This is not a defect for the present purposes: the framework requires that K(x|y) be well-defined, not that it be computable. The ontological claims of ZP-G do not depend on computability.

Status: This is an import, not a bridge axiom. A bridge axiom is a claim derivable from neither side, and so one that must be assumed. K(x|y) is a defined mathematical object with a complete internal theory. I-KC is a decision to use that object within ZP-G, not an assumption about it.

VI. Categorical Information Theory

6.1 Native Categorical Surprisal — Definition D7'

Categorical surprisal is defined via conditional Kolmogorov complexity, imported via I-KC (Definition D7′), rather than the Shannon entropy functor H of BA-G1. The definition is native to the morphism structure of C.

Definition D7' — Native Categorical Surprisal

Status: Definition — from D5, I-KC [replaces D7]

Let f: A → B be a morphism in C. Represent A and B as binary strings x_A and x_B via any injective encoding consistent with the morphism structure of C. The categorical surprisal of f is:

$I(f) = K(x_B \mid x_A)$

the conditional Kolmogorov complexity of the target given the source.

Interpretation: I(f) measures the minimum description length of B given knowledge of A. It is the irreducible informational content added by the transition f: A → B, independent of any probability distribution.

Well-definedness: $K(x_B|x_A)$ depends on the encoding of objects as strings. Different encodings yield values differing by at most an additive constant c (the coding theorem constant of I-KC). All claims in ZP-G are stated in terms of whether I(f) is finite, zero, or undefined — qualitative properties that are invariant under any additive constant. This invariance is a consequence of what ZP-G chooses to claim, not a general property of K-complexity (additive constants can matter for precise K-complexity comparisons in AIT).

Relationship to D7: By the coding theorem (I-KC), $K(x_B|x_A) \approx -\log_2 P(x_B|x_A) + O(c)$ for any computable measure P. Therefore D7′ and D7 are equivalent up to O(c). The choice of D7′ over D7 is not a change in what is being measured; it is a change in how the measure is defined — natively versus via import.

6.2 Properties of the Native Surprisal

Lemma T6-a — Surprisal of the Identity Morphism is Zero

Status: Derived — from D7′ and I-KC

Claim: $I(id_A) = K(x_A|x_A) = 0$ up to the additive constant c.

Proof: The shortest program producing x_A given x_A is the empty program (output the input). Therefore $K(x_A|x_A) = 0$ up to c. The identity morphism adds no informational content. ✓

Lemma T6-b — Surprisal is Non-Negative for Forward Morphisms

Status: Derived — PDF-level (from D5, D7′, AX-G2); NOT Lean-verified — see Lean scope remark below

Claim: For any morphism f: A → B in a forward morphism chain from 0, $I(f) \geq 0$ up to c, with strict inequality when $A \neq B$.

Lemma T6-b — Surprisal is Non-Negative for Forward Morphisms

Proof: $K(x_B|x_A) \geq 0$ by definition (program length is non-negative). Strict inequality holds when x_B cannot be computed from x_A by the empty program — i.e., when A and B are distinct objects encoding distinct states. In a forward morphism chain (D5), each step adds content by the additive ontology (AX-G2). ✓

Proposition T6-c — Surprisal Accumulates Along Chains

Status: Derived — PDF-level (from D5, T6-b, subadditivity of K); NOT Lean-verified — see Lean scope remark below

Claim: For a morphism chain $0 = X_0 \to X_1 \to \ldots \to X_n$, the total surprisal $\sum I(X_k \to X_{k+1}) \geq 0$, with monotone accumulation as n increases.

Proof: By subadditivity of Kolmogorov complexity: $K(x_n|x_0) \leq \sum K(x_{k+1}|x_k) + O(n \cdot c)$. Each term is ≥ 0 by T6-b. Adding distinct objects strictly increases the total. ✓

Remark — Lean Scope of T6-b and T6-c

Status: Scope note — T6-b and T6-c are NOT Lean-verified for their mathematical claims

T6-b and T6-c are not Lean-verified. The Category.lean proofs for these results compile without error, but they verify nothing about Kolmogorov complexity. The ZPSurprisal typeclass defines surp : hom → ℕ (surprisal as a natural number) and the proofs reduce to Nat.zero_le _, which states that any natural number is ≥ 0. This is trivially true by type for any ℕ-valued function, regardless of its mathematical content. A compiling Lean proof here does not mean the K-theoretic claims have been verified.

What the Lean proofs do NOT establish: (1) T6-b strict inequality — $K(x_B|x_A) > 0$ when $A \ncong B$ (distinct objects cannot have zero description length). (2) T6-c subadditivity — $K(x_n|x_0) \leq \sum K(x_{k+1}|x_k) + O(n \cdot c)$ (total surprisal along a chain). These are standard and correct results from algorithmic information theory, but they require a K-formalization this corpus does not have: none was located in the pinned Mathlib as of 2026-10-04 (search record in ZeroParadox/Information/Disjunctive.md), Lean developments outside Mathlib that define K are named at the ZPSurprisal entry of LEAN_CUSTOM_REGISTRY.md, and leaving K unformalized here is a scope decision recorded there.

What IS Lean-verified: T6-a (identity morphism has zero surprisal — from surp_id), T6 Part II (inward surprisal is undefined — from AX-G2 and the absence of morphisms to 0), and T7 insofar as it depends on Parts II, III, V, VI. T6 Part I (outward accumulation) and T7 Part IV rely on T6-b and T6-c and are therefore also PDF-level only.

Readers citing "the Lean-verified ZP-G framework" should note this boundary. The PDF-level arguments for T6-b and T6-c are mathematically valid — these are standard K-theoretic results (Li and Vitanyi). The gap is Lean scope, not mathematical correctness.

VII. The Informational Singularity of 0

This is the central theorem of the information-theoretic section. It is proved from D7′ and AX-G2 alone, with I-KC as the only external dependency.

Theorem T6 — Informational Singularity of 0

Status: Derived — from AX-G2, D7′, I-KC [OQ-G1 closed]

Setup: Let 0 be the initial object of C (AX-G1). Let $I(f) = K(x_B|x_A)$ be the categorical surprisal (D7′). Let I-KC provide the Kolmogorov framework.

Part I — Outward surprisal accumulates (from T6-b, T6-c): For any morphism chain $0 = X_0 \to \ldots \to X_n$, $\sum I(X_k \to X_{k+1}) \geq 0$, with strict accumulation as n increases. ✓

Part II — Inward surprisal is undefined (from AX-G2): For any $X \neq 0$, $\hom(X, 0) = \emptyset$ (AX-G2). Therefore D7′ cannot be applied to any morphism $f: X \to 0$ from outside 0 — no such morphism exists. $I(X \to 0)$ is undefined not because K diverges to infinity, but because there is no morphism to apply D7′ to. ✓

Part III — The singularity: 0 is the unique object in C for which outward surprisal is defined and accumulates (Part I) while inward surprisal is undefined by absence of morphisms (Part II). This is the informational singularity: the initial object is informationally accessible in the outward direction and categorically inaccessible in the inward direction. The singularity is structural, not numerical. It does not require K to diverge — it requires only AX-G2 and D7′. ✓

Comparison with ZP-C (to be reconciled in ZP-H T-H2): ZP-C establishes that the discrete surprisal DF diverges (numerically, to ∞) along infinite sequences approaching 0 in Q_2. T6 Part II establishes that $I(X \to 0)$ is undefined (domain-absent) for any $X \neq 0$ in C. These are compatible: ZP-H T-H2 shows that the two hold together, a conjunction; neither is derived from the other (th2_singularity_compatibility).

Status: DERIVED. Depends on AX-G1, AX-G2, D3, D5, D7′, I-KC, T6-a, T6-b, T6-c. OQ-G1 is closed. BA-G1 is no longer a premise of T6. ✓

7.1 Compatibility with Shannon Entropy — BA-G1 Demoted to Remark

Remark R-BA — Compatibility of D7' with the Shannon Entropy Functor

Status: Remark — BA-G1 demoted from Bridge Axiom to Compatibility Remark

BA-G1 was originally introduced as a bridge axiom: it imported Leinster's categorical characterization of Shannon entropy (naturality, maximality, chain rule) to define the surprisal functor. BA-G1 was originally the only bridge axiom in ZP-G and the source of OQ-G1.

BA-G1 is no longer a premise of any theorem; it is retained here as a compatibility remark: the coding theorem (I-KC) guarantees that D7′ and the Shannon functor of BA-G1 are equivalent up to an additive constant c. Specifically:

$$K(x_B|x_A) \approx H(F(B)) - H(F(A)) + O(c)$$

for any computable probability measure P consistent with the morphism structure of C. This means all quantitative results originally derived from BA-G1 remain valid under D7′ — they differ only by the additive constant c, which does not affect any structural (finite/zero/undefined) claim.

BA-G1 is not false. It is not retired. It is now a derived compatibility result rather than an assumed premise. Any reader who finds the Shannon characterization more intuitive than Kolmogorov complexity may use BA-G1 as an equivalent formulation, knowing that I-KC and the coding theorem connect them.

VIII. The Categorical Zero Paradox — Formal Statement

Theorem T7 is the closing theorem of ZP-G. Part IV (informational singularity) rests on T6, which does not depend on BA-G1.

Theorem T7 — The Categorical Zero Paradox
Status: Derived — Closing Theorem
Setup: Let C satisfy AX-G1 and AX-G2. Let I be the categorical surprisal from D7′. Let I-KC provide the Kolmogorov framework.
Part I — Universal Constituent (T2): $\forall X \in ob(C), \exists!\ \iota_X : 0 \to X$. The initial object 0 is the universal categorical source.
Part II — Unreachability (T3): $\forall X \neq 0, hom(X, 0) = \varnothing$. No non-initial object reaches 0.
Part III — Forward Irreversibility (T4): No morphism chain from 0 can return to 0 through non-initial objects.
Part IV — Informational Singularity (T6, rebuilt): $I(X \to 0)$ is undefined for all $X \neq 0$ (no such morphism exists, AX-G2). Outward surprisal from 0 accumulates along any morphism chain (T6-b, T6-c). 0 is an informational singularity: undefined inward, accumulating outward. This part no longer depends on BA-G1.
Part V — The Structural Inversion: Parts I and II together constitute the paradox. 0 is the unique universal source of all objects, and simultaneously the unique object unreachable from outside. The foundation is the one object the morphism machinery cannot return to.
Part VI — Resolution: The paradox is not a logical contradiction. It is a structural inversion. The correct tools for characterizing 0 are the universal property (D3) and D7′ applied to outward morphisms from 0. Under these tools, 0 is fully characterized. The paradox is the precise boundary between what can reach 0 and what cannot.
Status: DERIVED — Closing Theorem. Depends on D3, D5, D7′, AX-G1, AX-G2, I-KC, T2, T3, T4, T6. BA-G1 is not a dependency. ✓

IX. Open Items Register

Item	Status	Description
OQ-G1	Closed — D7′, T6	Native categorical derivation of surprisal without importing Shannon entropy. Closed by replacing D7 with D7′ (conditional Kolmogorov complexity K(B|A)). BA-G1 demoted from Bridge Axiom to Compatibility Remark R-BA. The single remaining external dependency is I-KC (Kolmogorov framework from ZP-C), which is an import, not a bridge axiom.
OQ-G2	Closed — ZP-H T-H1	Left adjoint verification for instantiation functors. Resolved in ZP-H v1.0 by direct universal property verification for each functor.
OQ-G3	Closed — ZP-H C-H1 through C-H4	Explicit construction of four instantiation functors. Resolved in ZP-H v1.0.

Item	Status	Description
OQ-G4	Closed — ZP-H T-H2	Reconciliation of categorical and ZP-C singularity characterizations. Resolved in ZP-H v1.0. Undefined domain (ZP-G) and infinite accumulation (ZP-C) shown to hold together, a conjunction; neither is derived from the other.
I-KC	Import — named dependency	Conditional Kolmogorov complexity K(x|y) and the coding theorem, imported from ZP-C D1 and standard algorithmic information theory. This is an import, not a bridge axiom: K(x|y) is a fully defined mathematical object. ZP-G is no longer purely categorical; this dependency is explicitly stated.
AX-G1	Axiom — intentional	Asymmetry: initial object 0, no terminal object. Foundational structural commitment. Not a gap.
AX-G2	Axiom — intentional	Source asymmetry: hom(X, 0) = ∅ for X ≠ 0. Foundational irreversibility commitment. Not a gap.
R-BA	Remark — BA-G1 demoted	Leinster Shannon entropy characterization is now a compatibility remark, not a bridge axiom premise. Derivable from D7′ and I-KC via the coding theorem. Not a gap.

X. Validation Status

Component	Status / Notes
D1: Category	Valid — Definition. Standard; foundational. Unchanged.
D2: Uniqueness notation	Valid — Definition. Unchanged.
D3: Initial object	Valid — Definition. Load-bearing for T2, T7. Unchanged.
D4: Terminal object	Valid — Definition. Defined for exclusion. AX-G1 prohibits it. Unchanged.
D5: Morphism chain	Valid — Definition. Native to C. Unchanged.
D6: Functor	Valid — Definition. Standard. Unchanged.
D7′: Native categorical surprisal	Valid — Definition. $K(x_B\|x_A)$ via I-KC. Replaces D7. Well-defined up to additive constant c. Structurally invariant.
I-KC: Kolmogorov import	Import — named. K(x|y) and coding theorem from ZP-C. Not a bridge axiom. Introduces explicit ZP-C dependency into ZP-G.
AX-G1: Asymmetry Axiom	Axiom — intentional. Unchanged.
AX-G2: Source Asymmetry	Axiom — intentional. Unchanged.
R-BA: BA-G1 compatibility remark	Remark — [BA-G1 demoted from Bridge Axiom]. Shannon entropy functor compatible with D7′ up to O(c) by coding theorem. No longer a premise of any theorem.
Proposition T1: Uniqueness of initial object	Valid — Derived. Relabelled Proposition (subsidiary uniqueness result). Unchanged. ✓

Component	Status / Notes
Lemma T2: Universal constituent	Valid — Derived. Relabelled Lemma (stepping-stone result). Unchanged. ✓
Lemma T3: Unreachability of 0	Valid — Derived. Relabelled Lemma (stepping-stone result). Unchanged. ✓
R1: Structural inversion	Valid — Remark. Unchanged.
R2: Categorical expression of self-containment	Valid — Remark. Connects T2 + AX-G2 to ZP-A CC-2 (the Quine atom, a role). No new derivation; explanatory bridge note.
Proposition T4: Chains are forward-only	Valid — Derived. Relabelled Proposition. Unchanged. ✓
Proposition T5: Functors preserve initial objects	Valid — Conditional on ZP-H T-H1 (closed). Relabelled Proposition. Unchanged. ✓
Lemma T6-a: Identity surprisal is zero	Valid — Derived. Relabelled Lemma. $K(x_A \mid x_A) = 0$ up to c. ✓
Lemma T6-b: Non-negative outward surprisal	Valid (PDF-level) — Derived from D5, D7′, AX-G2. $K \geq 0$; strict inequality for distinct objects. NOT Lean-verified: Lean proof reduces to Nat.zero_le _ (trivially true for any ℕ-valued function; verifies nothing about K).
Proposition T6-c: Surprisal accumulates along chains	Valid (PDF-level) — Derived from D5, T6-b, subadditivity of K. NOT Lean-verified: Lean proof reduces to Nat.zero_le _. Subadditivity of K is a standard AIT result but requires a formalization of K, none located in the pinned Mathlib as of 2026-10-04 (the search is recorded in ZeroParadox/Information/Disjunctive.md).
Theorem T6: Informational singularity	Valid — Derived. Does not depend on BA-G1. Part II: undefined domain (AX-G2) — fully Lean-verified. Parts I, III: accumulation via T6-b, T6-c — PDF-level only (T6-b/T6-c not Lean-verified).
Theorem T7: Categorical Zero Paradox	Valid — Derived. All six parts derived. BA-G1 not a dependency. ✓
OQ-G1: Native surprisal derivation	Closed — D7′, T6. No bridge axiom remains as a theorem premise.

Zero Paradox ZP-G: Category Theory | T6-b and T6-c: PDF-level only; Lean proofs verify non-negativity by type only (Nat.zero_le _), not K-theoretic content | T6 Part II: Lean-verified