THE ZERO PARADOX

ZP-I: Inside Zero

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This document establishes Theorem T-IZ (Inside Zero): every 2-adic depth chain S_0, S_1, ... in Q_2, nowhere zero, whose valuation equals a depth index in $\mathbb{N}$ that climbs at every step (IsDepthChain, IsStrictStateSequence) is a Cauchy sequence that converges, in the 2-adic metric, to 0. The ascending chain of a ZP-A semilattice is a separate carrier. No new axioms are required for what it PROVES, which is that convergence, together with the separate fact that anything filling the bottom role IS the bottom of its own semilattice. Reading the 2-adic limit as the thing filling that role takes a choice, not a theorem: the two live in different types, and Q_2's field structure supplies no join and the corpus registers none on Q_2, so the role condition is not statable of the limit until a join on Q_2 is chosen — they are distinct MEMBERS of the bottom family, not one object, and that identity is retired as ill-typed (MC-1). Choosing a join on Q_2 is well-typed, and in the semilattice it makes, anything filling the bottom role is that semilattice's $\bot$ by theorem (t_iz_limit_is_new_null); what is retired is identifying the 2-adic 0 with $\bot$ of another semilattice. Reading the bottom so reached as a NEW one rather than the one already there is a further commitment (C-DA2 in ZP-E), and in the 2-adic chart the arc returns to the SAME 0 (snap_arc_z2_loop). T-IZ and T-SNAP are separate results: T-SNAP fixes the shape of the first transition $\bot \to \varepsilon_0$, and not that it is taken; T-IZ establishes, for a depth index that climbs at every step, that the 2-adic chain the IsDepthChain modelling commitment links to it converges to 0, as a Cauchy sequence result.

The key insight: the no-top property is not an obstacle to T-IZ. In a carrier that has it (HasNoTop is a hypothesis on the carrier; MachinePhase and OntologicalStates have a top), it is what makes the ascent AVAILABLE — there is always a strictly greater element, so no chain halts for want of anywhere to go. (It does NOT follow that the limit lies outside L; whether a limit escapes its carrier is a property of the particular lattice — the ordinals under max have no top, and the chain $n \mapsto n$ has least upper bound ω inside them.) What drives the 2-adic valuation $v_2(S_n) \to \infty$ is that the chain actually takes those steps (IsStrictStateSequence, the hypothesis h_strict_from_r1_t3 binds); no-top alone does not, and the NO-GO gauge in §Ib exhibits a no-top lattice holding a chain that never moves. Unbounded ascent is exactly the Cauchy convergence condition $\|S_n\|_2 \to 0$. The chain approaches the 2-adic depth of zero by going deeper into the p-adic structure — not by reversing direction. That DA-1 and T-SNAP fire again at the limit, so that the cycle begins again, is a reading; its premises are stated in § V.III. t_iz_complete takes a KleeneStructure on the limit's successor semilattice L′ as a hypothesis and links L′ to nothing; its DA-1 conjunct, that L′'s $\bot$ is its Quine atom, is the Lean counterpart of DA-1's Path 1, which does not derive DA-1's precondition (ZP-E § IV).

Section I: Room to Ascend — No Top Element and Ordinal Unboundedness

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I. The No-Top Property: What It Supplies

The no-top property (Lean: HasNoTop, ZeroParadox/Order/Lattice.lean) says the join-semilattice $(L, \vee, \bot)$ has no top element: there is no $T \in L$ with $x \leq T$ for all x. ⚠ This is NOT ZP-A's R1, which is the NO-SUBTRACTION restriction (§I of the ZP-B section below); one label over two propositions is a citation nothing can check. What no-top supplies is ROOM: a chain never halts for want of a strictly greater element. The ascent itself is IsStrictStateSequence — see the NO-GO gauge in §Ib. Reading the limit as filling the bottom role takes a choice of join on Q_2 (then it is that semilattice's $\bot$ by theorem, t_iz_limit_is_new_null), and reading the occupant as a successor null is C-DA2, a commitment.

The reasoning is direct. An ascending chain $(S_n)_{n<\omega}$ in L is a sequence satisfying $S_n \leq S_{n+1}$ for all n (ZP-A T3 — monotonicity). Where L has no top element (HasNoTop is a hypothesis on the carrier, not a law of every ZP-A semilattice: the two-state carriers MachinePhase and OntologicalStates have a top, by the examples in ZeroParadox/Order/Snap.lean and ZeroParadox/Settheory/OntBridge.lean), for every S_N there EXISTS an S with $S_N < S$ — so no chain is ever forced to stabilise for want of somewhere to go. ⚠ That is availability, not occurrence: a CONSTANT chain is still a legitimate state sequence in a no-top lattice (NO-GO gauge, §Ib). That this chain is strictly ascending and unbounded is the separate hypothesis IsStrictStateSequence, and that is the one T-IZ consumes.

In the 2-adic model (ZP-B), the IsDepthChain modelling commitment links two carriers: a 2-adic chain S_n in Q_2, and a depth index in $\mathbb{N}$ (a ZP-A semilattice under max) that its valuation $v_2(S_n)$ equals. Strict ascent of that depth index (IsStrictStateSequence) is increasing 2-adic valuation depth. Because the index is unbounded, $v_2(S_n) \to \infty$. This follows from the strict-ascent hypothesis, not from the absence of a top element: no-top leaves room for the ascent, and IsStrictStateSequence is that the chain takes it.

II. Ordinal Index Replaces Clock Time

The state sequence is indexed by ordinals: $(S_\alpha)_{\alpha<\omega}$. The parameter ω is not a clock time and not a top bound. It is the ordinal index of the transition — the label for when the chain has completed ω state changes. The chain in L is genuinely unbounded; the ordinal ω is not a member of the sequence and not a ceiling on it.

This replaces the informal "time" language that sometimes accompanies descriptions of the Binary Snap. In the Zero Paradox, "time" is the index of state changes. What fixes the direction of that index is monotonicity (ZP-A T3) and irreversibility (ZP-B C3). Neither is a clock. What the chain reaches at the limit ordinal ω, after ω state changes, is Q_2's 0, and not at any clock time. Reading that 0 as filling the bottom role takes a choice of join on Q_2 (then it is that semilattice's $\bot$ by theorem, t_iz_limit_is_new_null), and reading it as a successor null is C-DA2, a commitment.

Remark R-I.1: ω is the first infinite ordinal — the smallest ordinal greater than every natural number. An ascending chain indexed by ω is a countable sequence with no last element in L. The no-top property guarantees only that no finite stage is a CEILING; that a given chain reaches through all of them is the strict-ascent hypothesis. The transition at ω is not a step within L. ⚠ Whether its limit lies OUTSIDE L is a property of the particular lattice and not a consequence of no-top.

Key Result

Strict ascent (IsStrictStateSequence) drives $v_2(S_n) \to \infty$; the no-top property supplies the room for it, and is not a hypothesis OF THAT THEOREM (it is a per-carrier assumption in its own right — nat_has_no_top is a theorem, and zpa_bot_not_greatest binds it explicitly). Strict valuation growth ($v_2(S(n)) \geq n$) is Lean-derived from IsDepthChain and IsStrictStateSequence — Lean: h_strict_from_r1_t3 (§Ib). R-IZ-A closed.

$\|S_n\|_2 = 2^{-v_2(S_n)} \to 0$ (Cauchy condition).

The no-top property removes the ceiling; IsStrictStateSequence is that the chain climbs. Of those two only IsStrictStateSequence appears in h_strict_from_r1_t3's binders — alongside IsDepthChain, which is not a lattice condition but the bridge to the 2-adic valuation.

Section II: The Topological Path and the Refuted Informational Bridge

Two paths are set out here. The topological path, through Q_2 and the 2-adic norm, is proved (§ A). The informational path, through ZP-C L-INF and the Kolmogorov complexity threshold P_0, rests on a valuation-complexity bridge that is REFUTED (§ B gives the counterexample). So the two are not two descriptions of one structure: 2-adic depth and Kolmogorov complexity are different measures, and unbounded growth of the first does not carry the second to its threshold.

A. Topological Path — Cauchy Convergence in Q_2

The 2-adic norm on Q_2 is defined by: $\|x\|_2 = 2^{-v_2(x)}$, where $v_2(x)$ is the 2-adic valuation of x. With the convention $v_2(0) = \infty$ (Mathlib's valuation of 0 is 0), $\|0\|_2 = 0$, and Q_2's 0 is the element of infinite 2-adic depth. As the 2-adic depth chain has $v_2(S_n) \to \infty$ (forced by strict ascent, with IsDepthChain tying the valuation to the depth index), we have $\|S_n\|_2 \to 0$ by the norm–valuation identity above.

In a normed group, $\|S_n\|_2 \to 0$ already gives $S_n \to 0$: the limit is EXHIBITED as 0, not obtained from a Cauchy criterion, so no completeness is consumed at this step. Completeness of Q_2 is what the CONVERSE direction needs — Cauchy implies some limit exists — and T-IZ never takes it. A convergent sequence is automatically Cauchy. The 2-adic depth chain is therefore a Cauchy sequence converging to 0 — the 2-adic limit of the chain is $0 \in Q_2$.

Lemma T-IZ-A — Cauchy Convergence (Proved in Lean)
Let $S : \mathbb{N} \to Q_2$ be a sequence satisfying $\|S(n)\|_2 \leq 2^{-n}$ for all $n \in \mathbb{N}$. Then:
(1) The norms $\|S(n)\|_2 \to 0$ (squeeze between 0 and the geometric sequence 2^{-n}, both tending to 0).
(2) $S(n) \to 0$ in Q_2 (norm $\to 0$ iff sequence $\to 0$ in a normed group).
Lean: t_iz_cauchy — proved (sorry-free) in SemilatticeInstance.lean; carries Classical.choice (§ III). This is the topological core of T-IZ.

The geometry of the inside approach is the following: elements of Q_2 are arranged by their 2-adic valuation depth. Zero is the element of infinite depth — the deepest point. The 2-adic depth chain moves into greater

and greater depth as $n \to \infty$, approaching the depth of zero without ever reversing. The chain does not turn around and head back to 0. It descends into 0 by going deeper.

Remark R-II.1: The condition $\|S(n)\|_2 \leq 2^{-n}$ is equivalent to $v_2(S(n)) \geq n$ under the convention $v_2(0) = \infty$ (for $S(n) \neq 0$ no convention is needed). It asserts that the 2-adic valuation of S(n) is at least n — meaning S(n) is divisible by 2^n in the 2-adic sense. As $n \to \infty$, divisibility by arbitrarily large powers of 2 forces $\|S(n)\|_2 \to 0$ and therefore $S(n) \to 0$. This is the formal content of the "chain approaching the 2-adic depth of zero by forward motion."

Remark R-IZ-A — Closure of the valuation growth hypothesis: The strict growth condition $v_2(S(n)) \geq n$ was previously treated as a construction-level assumption stronger than the proved result t_iz_valuation_unbounded (sup $v_2(S(n)) = \infty$). It is now Lean-derived. Theorem h_strict_from_r1_t3 (SemilatticeInstance.lean §Ib) proves that any Q_2 chain satisfying the IsDepthChain modeling commitment — meaning its 2-adic valuations track a strict $\mathbb{N}$-depth-index sequence — inherits strict valuation growth from IsDepthChain and IsStrictStateSequence. IsDepthChain ($\forall$ n, $v_2(S(n)) = \text{depths}(n)$) is the remaining modeling commitment: it asserts that 2-adic depth tracks the lattice depth index. This is a structural feature of the embedding, not a consequence of the abstract axioms. With IsDepthChain in place, R-IZ-A is formally closed.

B. Informational Path — The Valuation-Complexity Bridge (Refuted)

ZP-C L-INF establishes that the surprisal I(n) = n at ball-hierarchy depth n is unbounded. ZP-C reads $\perp$ as corresponding to the limit point $0 \in Q_2$ — the limit of the binary ball hierarchy at infinite depth; that correspondence is a reading across carriers (MC-1). The depth-surprisal correspondence (ZP-C D4) gives the informational content of the 2-adic depth chain: as $v_2(S_n) \to \infty$, the surprisal $I(n) \to \infty$ without bound.

Surprisal at depth n is a property of POSITION in the ball hierarchy: every point at 2-adic depth n has I(n) = n under the branching measure (ZP-C D4). Kolmogorov complexity $K(x \mid n)$ is a property of the particular point x: the length of the shortest program that outputs x given n. The two are different measures, and the counterexample in the box below refutes the bridge between them.

Refuted Claim — Valuation-Complexity Bridge
Claim (REFUTED): $v_2(S_n) \to \infty \Rightarrow K(S_n \mid n) / \lvert S_n \rvert \to 1$.
Counterexample: $S_n = 2^n$. Its 2-adic valuation is n (scaleChain_isDepthChain, ZeroParadox/Valuation/ScaleDepthWitness.lean, a file marked EXPERIMENTAL), so the premise holds, and the corpus's ordinal tower read in $\mathbb{Z}_2$ has the same valuation profile (cnfToZp2_tower_valuation, ZeroParadox/Ordinal/Gentzen.lean). One fixed program computes S_n from n, so $K(S_n \mid n)$ is bounded by a constant while $\lvert S_n \rvert = n + 1$ bits, and the ratio tends to 0, not 1.
Lean scope: the valuation half is checked. The complexity half is not in Lean, since Kolmogorov complexity is outside Lean scope (same category as DA-1 Path 3 in ZP-E): the bound $K(2^n \mid n) \leq c$ holds because one fixed program prints 2^n from n, and c is the length of that program.
The same chain meets T-IZ's own bound: $\|2^n\|_2 = 2^{-n}$, so $S_n = 2^n$ satisfies h_bound of t_iz_complete (ZeroParadox/Valuation/SemilatticeInstance.lean). That hypothesis therefore does not encode incompressibility.

Remark R-II.2: The formal spine of T-IZ is Steps 1 and 6. Step 1 (Cauchy convergence to 0 in Q_2, t_iz_cauchy) is proved in Lean and carries Classical.choice (§ III). Step 6 (anything filling the $\perp$ ROLE IS

that lattice's ⊥) is proved axiom-free in Lean via t_iz_limit_is_new_null — which establishes the role identification ONLY and must not be cited as a witness for the successor reading (t_iz_limit_is_new_null, declared in ZeroParadox/Valuation/SemilatticeInstance.lean and indexed in ZeroParadox/Order/SnapCannotBe.lean). • The two steps are not joined. Step 1 is about $0 \in Q_2$; Step 6 is about a terminal in a semilattice L′, and takes the role property as its hypothesis h_role. t_iz_complete conjoins them without identifying the two objects, and the identification is not merely unproved — Q_2's field structure supplies no join and the corpus registers none on Q_2, so the role condition cannot be stated of the limit until a join on Q_2 is chosen. That junction is not an open gap: the 0 of Q_2 and the algebraic ⊥ are distinct MEMBERS of the bottom family, and the cross-category identity is retired as ill-typed rather than outstanding (MC-1, CLAIMS.md). Steps 2–5 describe the original ZP-E informational argument connecting 2-adic depth to Kolmogorov complexity and DA-1 Path 3. Since DA-1 is closed given DP-2 (ZP-E), with ZP-K witnessing Path 1 and carrying Path 3 as a KleeneStructure requirement — without Kolmogorov complexity — Steps 2–5 are informational context, not a proof dependency. The bridge in Step 2 is REFUTED (§ B): 2-adic depth growing without bound does not carry Kolmogorov complexity to the threshold P_0, so this route does not reach P_0.

Section III: Theorem T-IZ — Inside Zero

Theorem T-IZ — Inside Zero

Statement: Every 2-adic depth chain $(S_n)_{n<\omega}$ in Q_2, nowhere zero, whose valuation equals a depth index in ℕ that climbs at every step, so that its 2-adic valuation grows without bound, is a Cauchy sequence that converges, in the 2-adic metric, to 0. Reading that limit as an OCCUPANT of the bottom role takes a choice and is not part of the statement — Q_2's field structure supplies no join and the corpus registers none on Q_2, so the role condition is not statable of it. Reading that occupant as a SUCCESSOR null is a commitment (C-DA2).

Formal hypotheses (t_iz_complete): $S : \mathbb{N} \to Q_2$ with $\|S(n)\|_2 \leq 2^{-n}$ for all n; for $S(n) \neq 0$ this is $v_2(S(n)) \geq n$, and at $S(n) = 0$ it matches only under the convention $v_2(0) = \infty$ (Mathlib's valuation of 0 is 0). That bound is derived (t_iz_h_bound_from_depth_chain) from IsStrictStateSequence on a depth index in ℕ, IsDepthChain tying the valuations of S to that index, and S nowhere zero (Remark R-IZ-A, §Ib). CC-1's start at ⊥ and T3 monotonicity are statements about a chain in a ZP-A semilattice, not about S: Q_2's field structure supplies no join and no order of that kind.

Conclusion: $S(n) \to 0$ in Q_2. That is the whole of what is proved here. The framework then READS the limit as satisfying the bottom role — a step stated in a different type and not carried by T-IZ — and DA-2 supplies the one-directional fact that anything satisfying that role IS the bottom already present. On that reading, DA-1 and then T-SNAP are CONDITIONAL on the limit reaching P_0 (which nothing here establishes) and on what DA-1 needs. DA-1 needs DP-2 and DP-2's precondition, that the configuration reaching P_0 is there in Sense B (a running machine's current configuration) and not in Sense A (an inert string); that precondition is what the occurrence commitment asserts, and DA-1 consumes it and does not supply it (ZP-E § IV). Nothing here establishes that the limit reaches P_0: its only route in this document, the bridge of § II.B, is refuted. t_iz_complete takes a KleeneStructure on the limit's successor semilattice L′ as a hypothesis and links L′ to nothing; its DA-1 conjunct, that L′'s ⊥ is its Quine atom, is the Lean counterpart of DA-1's Path 1, which does not derive the precondition. Reading the occupant as ⊥′, a successor null for the next instantiation, is C-DA2, a modelling commitment, since nothing here produces a SECOND bottom.

I. The Six-Step Proof

The proof of T-IZ follows six steps, corresponding to the proof obligation table:

• Step 1 — Cauchy convergence: The 2-adic depth chain has $\|S(n)\|_2 \leq 2^{-n}$ (from $v_2(S(n)) \geq n$ — Lean-derived via h_strict_from_r1_t3 given IsDepthChain; R-IZ-A closed). By T-IZ-A (§ II.A), $S(n) \to 0$ in Q_2. Proved in Lean: t_iz_cauchy (carries Classical.choice, § III). ✓

• Step 2 — Valuation-complexity bridge (REFUTED): the claim that $v_2(S(n)) \to \infty$ gives $K(S(n)|n)/|S(n)| \to 1$ fails at $S(n) = 2^n$, whose valuation is n and whose complexity given n is bounded (§ II.B). Not a proof dependency for T-IZ — DA-1 is closed given DP-2 (ZP-E), with ZP-K witnessing Path 1 and carrying Path 3 as a KleeneStructure requirement.

• Step 3 — P_0 at the limit (informational context): ZP-C D1's threshold $K(c_1|n)/|c_1| = 1$. Step 2 is this step's only route from the chain to that threshold, and with Step 2 refuted nothing here establishes that the limit meets the threshold.

• Occurrence fence. T-SNAP fixes the SHAPE of each step. It does not establish that any step is taken: tsnap_holds_but_nothing_moves exhibits a model in which T-SNAP holds and nothing moves. Throughout this document, "fires" narrates the occurrence commitment - before this note as well as after it - not a consequence of the theorem.

• Step 4 — DA-1, CONDITIONALLY: DA-1 needs DP-2 and DP-2's precondition, that the configuration reaching P_0 is there in Sense B (a running machine's current configuration) and not in Sense A (an inert string); that precondition is what the occurrence commitment asserts, and DA-1 consumes it and does not supply it (ZP-E § IV). The Lean route (t_iz_complete) takes a KleeneStructure on L′ as a hypothesis; its DA-1 conjunct, that L′'s ⊥ is its Quine atom, is the Lean counterpart of DA-1's Path 1, which does not derive the precondition. Given the limit reaching P_0 (which nothing here establishes) and what DA-1 needs, DA-1 (ZP-E) applies. The TrackedOutput formal core (DP-2, Snap.lean § VI) establishes the machine-state transition.

• Step 5 — T-SNAP fires, on the same conditions as Step 4: given DA-1, instantiation = execution, and T-SNAP (ZP-E) gives $\bot \vee \varepsilon_0 = \varepsilon_0$. Lean: t_snap_derived, proved axiom-free in Snap.lean. ✓ — the checkmark covers the transition SHAPE only. Reading the limit as a NEW ⊥′ is a commitment; Snap.lean's NO-GO gauge holds T-SNAP in a model where nothing moves.

• Step 6 — DA-2 role identification: DA-2 (ZP-E) establishes that any state satisfying $\forall x, S \vee x = x$ IS the ⊥ of its own lattice. That the Cauchy limit $0 \in Q_2$ satisfies that condition is a HYPOTHESIS, never a result: t_iz_complete takes it as the argument h_role, about a terminal in a separate semilattice L′, and does not identify that terminal with the limit. Q_2's field structure supplies no join, and the corpus registers none on Q_2. For the 2-adic integers $\mathbb{Z}_2$ one with 0 as bottom is built but not registered, where 0 meets the condition because it was chosen as the bottom (ZeroParadox/Valuation/Scale.lean § V); on Q_2 the condition is stated of no structure the corpus registers (that some join on Q_2 makes any chosen point its bottom is the first CONTROL example after c_t_iz_null_balance, ZeroParadox/Valuation/SemilatticeInstance.lean). Lean: t_iz_limit_is_new_null, proved directly from da2_bottom_characterization. ✓ — the checkmark covers the ROLE identification only, and only inside L′. It is NOT a novelty witness and must not be cited as one; reading the occupant as the ⊥ of a SUCCESSOR instantiation is C-DA2.

II. Proof Obligation Table

Claim	Source	New axiom?	Lean status
Chain is Cauchy in $(Q_2, \|\cdot\|_2)$	The geometric norm bound. What the framework supplies to REACH it (via §Ia) is IsStrictStateSequence (T3 rides inside it — T3 alone is MONOTONE, and monotone does not reach this bound), IsDepthChain, the chain being nowhere zero, and the norm–valuation identity $\|x\|_2 = 2^{-v_2(x)}$ from ZP-B's valuation construction (Lean: Padic.norm_eq_zpow_neg_valuation). NOT ZP-B T2, which is "every ball is clopen" — a separation result that says nothing about a valuation growing. t_iz_cauchy itself binds none of them, and no lattice at all.	Follows from existing structure — no new axiom	Lean: t_iz_cauchy ✓ (proved; carries Classical.choice, § III)
$\|S(n)\|_2 \to 0$ (Cauchy limit = 0)	The geometric norm bound alone. Completeness of Q_2 is NOT consumed here: the limit is exhibited as 0 rather than obtained from a Cauchy criterion, so the proof runs through squeeze_zero and tendsto_zero_iff_norm_tendsto_zero, which hold in any normed group.	Already in framework	Lean: t_iz_cauchy (composite of t_iz_norm_tendsto_zero + t_iz_conv_zero) ✓
$\sup v_2(S(n)) = \infty$	Strict ascent (IsStrictStateSequence) + IsDepthChain (the valuation tracks the depth index). NOT no-top: a chain can satisfy HasNoTop and IsDepthChain with its valuation BOUNDED.	Follows from strict ascent + T3 — no-top supplies the room, not the growth	Lean: t_iz_valuation_unbounded ✓ (proved; carries Classical.choice, § III)
$v_2 \to \infty \Longrightarrow K/\|S\| \to 1$ (REFUTED)	Counterexample $S_n = 2^n$: valuation n (scaleChain_isDepthChain), complexity given n bounded by the length of one fixed program printing 2^n from n (not Lean)	Not a proof dependency	Refuted (§ II.B). Not required: formal spine is Steps 1 + 6; DA-1 is closed given DP-2 (ZP-E), with ZP-K witnessing Path 1 and carrying Path 3 as a KleeneStructure requirement.

Claim	Source	New axiom?	Lean status
P_0 fires DA-1, CONDITIONALLY: given the limit reaching P_0 (which nothing here establishes), DP-2, and DP-2's precondition Sense B, which the occurrence commitment asserts	ZP-C D1 + DA-1 (ZP-E)	No new axiom; all three premises are outside this document's results	ZPE formal core: da1_minimal_path, DP-2. No checkmark: none of the three premises is established here
DA-1 fires T-SNAP	ZP-E T-SNAP	Already in framework	Lean: t_snap_derived ✓ (axiom-free)
The limit occupies the $\perp$ role (READING; $\perp'$ a further READING)	DA-2 (ZP-E)	Role-recognition: already in framework. Occupancy: takes a choice of join. Novelty: COMMITMENT (C-DA2)	Lean: t_iz_limit_is_new_null, c_da2_novelty ✓ — these prove role $\Longrightarrow$ identity with the bottom ALREADY THERE. Neither is a novelty witness and neither may be cited as one.

III. Lean Scope

The Lean file SemilatticeInstance.lean formalizes the formal spine of T-IZ: Step 1 (Cauchy convergence, § I) and Step 6 (DA-2 role identification, § IV), which are carried together as a CONJUNCTION and are not joined to each other. The two steps sit on different footings, and the document states this honestly: Step 6 (t_iz_limit_is_new_null) is proved axiom-free — it is the ROLE half only: in one semilattice, anything satisfying the join-identity IS that semilattice's $\perp$, the one already there. That the $\perp$ so reached is a NEW one, $\perp'$, is a commitment (C-DA2) and not this theorem; in the 2-adic realization the arc reapproaches the SAME 0 (snap_arc_z2_loop). Step 1 (t_iz_cauchy) is proved sorry-free but carries Classical.choice, and NOT because of the analysis. Measured: $(1 : \mathbb{Q}_{[2]}) \neq 0$ — no limit, no filter, no convergence lemma anywhere in the statement — already reports [propext, Classical.choice, Quot.sound]. The dependence sits in Mathlib's instance packaging, not in the metric: Polynomial (ZMod 5) is no kind of completion and reports the same footprint, while ZMod 5 reports [propext, Quot.sound]. Mathlib's analytic lemmas (squeeze_zero, the geometric-limit lemma, tendsto_zero_iff_norm_tendsto_zero) carry it too, so the footprint cannot separate the two. ⚠ A footprint follows the STATEMENT and the PROOF. A statement whose type carries choice reports it under every proof (the statement control on tower_converges_to_zero, ZeroParadox/Ordinal/SyntacticCollapse.md; decoration_unique, ZeroParadox/Settheory/APG.lean); a statement that does not can still report it through its proof. Switching carrier from Q_2 to the rationals $\mathbb{Q}$ for the proof half: the statement $(1 : \mathbb{Q}) \neq 0$ reports Classical.choice through one_ne_zero and [propext, Quot.sound] through decide. ⚠ The corpus's choice-free valuation facts are NOT a counterexample here: v2_bot is [propext] because $v2 : \mathbb{N} \to \mathbb{N}\infty$ is stated over the NATURALS, so that contrast is between two carriers and not between algebra and analysis. The footprint reports the library this development is built on.

Where choice enters this corpus, and what is established about it, is recorded at ZeroParadox/AxiomProfile.lean (the core is choice-free; T-SNAP depends on no axioms at all) and ZeroParadox/Ordinal/SyntacticCollapse.md (a choice-free syntactic surrogate for the metric collapse, which states of itself that it settles the standing conjecture in neither direction). Steps 2–5 (the valuation-complexity bridge and DA-1/T-SNAP path) describe the ZP-E informational argument and are not proof dependencies; the bridge in Step 2 is refuted (§ II.B). DA-1 is closed given DP-2 (da1_minimal_path; CLAIMS.md, Tier 5); ZP-K witnesses its Path 1 (da1_closed_concrete: ⊥ is the unique Quine atom of MachinePhase). The DP-2 commitment is not discharged by either. The theorems in SemilatticeInstance.lean, with their axiom footprints:

- t_iz_limit_is_new_null: anything satisfying the DA-2 ⊥ role IS that lattice's ⊥ (the structural core of the snap-arc; axiom-free). The role property is its HYPOTHESIS; that the Cauchy limit satisfies it is not established here, and is not statable of the limit until a join on Q_2 is chosen.
- c_t_iz_null_balance: a non-bottom state cannot satisfy the ⊥ role (choice-free, [propext]).
- t_iz_cauchy: a sequence in Q_2 with $\|S(n)\|_2 \leq 2^{-n}$ converges to 0 (topological core; carries Classical.choice, as measured above; not a framework commitment).
- t_iz_valuation_unbounded: sup $v_2(S(n)) = \infty$ (carries Classical.choice, as measured above).
- t_iz_c3_compatible: C3 irreversibility is preserved — Cauchy sequences ≠ continuous paths (carries Classical.choice, as measured above).
- t_iz_h_bound_from_depth_chain: h_bound derived from THREE hypotheses, of which exactly one is a ZP-A lattice condition — IsStrictStateSequence, and it is a condition on the DEPTH INDEX. The other two live in Q_2: that the chain is nowhere zero, and IsDepthChain, the BRIDGE saying the 2-adic valuation tracks that index. IsDepthChain takes no ZPSemilattice instance at all; §Ib records it as an undischarged modelling commitment, and it links the depth index and the chain in Q_2 without identifying them. Closes the $\|S_0\|$ factor gap between §Ib and t_iz_complete (optional transparency lemma; the analytic proof carries Classical.choice).
- t_iz_complete_from_axioms: T-IZ complete variant taking the three hypotheses above in place of a bare h_bound, and binding four more — a second semilattice L′ with a KleeneStructure instance, a terminal, an ε_0', and h_role, the role property HANDED IN and never derived. Auditable in one theorem, but its hypotheses are not all lattice conditions (carries Classical.choice; optional transparency variant; t_iz_complete is the canonical theorem).

Status: DERIVED THEOREM — formal spine: t_iz_cauchy (Step 1, proved; carries Classical.choice, § III) + t_iz_limit_is_new_null (Step 6, axiom-free via DA-2). These two steps are the formal spine, carried as a conjunction and not joined to one another. t_iz_valuation_unbounded, c_t_iz_null_balance, t_iz_c3_compatible also proved. Transparency variants: t_iz_h_bound_from_depth_chain + t_iz_complete_from_axioms name each hypothesis separately for reviewer auditability — one lattice condition (IsStrictStateSequence), one 2-adic depth bridge (IsDepthChain, a modelling commitment), one nonvanishing condition, and for the complete variant h_role handed in — t_iz_complete is the canonical theorem. Steps 2–5 (valuation-complexity bridge, refuted, + DA-1/T-SNAP) are informational context — DA-1 is closed given DP-2 (ZP-E), with ZP-K witnessing Path 1 and carrying Path 3 as a KleeneStructure requirement, no Kolmogorov complexity required. No new axioms. ✓

Section IV: Compatibility with the Irreversibility Results

T-IZ does not violate any irreversibility result in the framework. The inside approach is not a reversal — it is a structurally different operation. Each irreversibility result governs a specific structure; T-IZ uses a different structure not governed by any of them.

I. ZP-A R1 — No Subtraction

R1 states that the join-semilattice $(L, \vee, \bot)$ has no subtraction operator: for any $x, y \in L$ with $x < y$, there is no z such that $y \vee z = x$. This closes the algebraic door to reversal.

T-IZ does not use subtraction. The chain never joins "downward." Every step is a join operation $S_{n+1} = S_n \vee \alpha_n$ for some $\alpha_n \geq 0$ (ZP-A T3 monotonicity). The approach to 0 in Q_2 is not a join toward 0 — it is a Cauchy sequence whose 2-adic norm tends to 0. The chain never subtracts, so R1 is not violated. R1 plays no further part in T-IZ: what makes the valuation grow without bound is strict ascent, and what leaves the chain somewhere to go, in a carrier that has it, is the order property HasNoTop (ZeroParadox/Order/Lattice.lean), a hypothesis on the carrier and a different proposition.

II. ZP-B C3 — No Continuous Path to Zero

C3 states: there is no continuous path $\gamma : [0,1] \to Q_2$ with $\gamma(0) = x \neq 0$ and $\gamma(1) = 0$. This closes the topological door to reversal via continuous motion.

T-IZ uses Cauchy sequence convergence, not a continuous path. A Cauchy sequence $(S_n)_{n\in\mathbb{N}}$ tending to 0 is a countable sequence of discrete points. It is not a continuous function $[0,1] \to Q_2$. These are distinct mathematical structures. C3's universal quantifier ranges over continuous functions; T-IZ's convergence is a statement about countable sequences. The two results do not conflict.

Lean: t_iz_c3_compatible (SemilatticeInstance.lean) proves this directly: the statement of C3 (c3_irreversible from ZPB) holds without modification alongside T-IZ. C3 blocks continuous paths; T-IZ uses Cauchy sequences. They govern different structures. ✓

III. ZP-G AX-G2 — No Morphism to the Initial Object

AX-G2 states that in the categorical structure C, $\hom(X, 0) = \varnothing$ for $X \neq 0$: no morphism within C leads back to the initial object. This closes the categorical door to reversal.

T-IZ is not a morphism within C: it is a convergence statement in Q_2, and no arrow of the category C of the current instantiation is involved. Reading the limit as the termination of C and the opening of a successor C', with its own $\bot'$, is the cycle reading, not a result; its premises are stated in § V.III. AX-G2 quantifies only over morphisms within a single category; it has nothing to say about the transition between categories. The categorical structure is preserved intact within each instantiation.

IV. Summary

The irreversibility results and T-IZ are not in tension. They describe different things. Irreversibility (R1, C3, AX-G2) governs motion within an instantiation branch: no algebraic subtraction, no continuous topological return, no categorical reversal. T-IZ governs what happens at the branch's ordinal limit: the 2-adic depth chain converges, by Cauchy convergence, to $0 \in Q_2$ — a structure that none of the irreversibility results governs or addresses. Reading that 0 as filling the bottom role takes a choice of join on Q_2 (then it is that semilattice's $\bot$ by theorem, t_iz_limit_is_new_null), and reading it as a successor null is C-DA2, a commitment.

The inside approach is not a violation of irreversibility. It is the discovery of a structure that irreversibility does not reach. Three doors to zero are closed (R1, C3, AX-G2). T-IZ uses a fourth passage — Cauchy sequence convergence — that none of the three irreversibility theorems govern. (A picture, not a theorem: the three doors are statements about three different objects, and the fourth passage reaches $0 \in Q_2$ only as a limit; reading that 0 as filling the bottom role takes the choice of join stated above.)

Section V: OQ-E2, the Null Balance, and the Cycle Reading

I. Resolution of OQ-E2

OQ-E2 (Cardinality-Semilattice Correspondence) has been open since the initial ZP-E release. It asks: do specific semilattice structures correspond to specific cardinality regimes, and can the framework make predictions about which instantiations satisfy CH?

T-IZ provides the path to closing OQ-E2 as follows. The ascending chain $(S_n)_{n<\omega}$ is indexed by ω — the first infinite ordinal. This indexing is forced, not chosen: the binary alphabet gives a countable state space; Q_2 is a separable metric space; surprisal $I(n) = n$ grows by integer steps (ZP-C D4). Every component of the framework that generates an ordinal index generates a countable one. The state sequence is necessarily indexed by ω, not ω_1 or any uncountable ordinal.

This pins the ordinal depth of each instantiation to ω. OQ-E2's perspective-relative cardinality (DA-3) is then resolved as follows: internal observers see a proper initial segment of ω (finite); external observers see all of ω. The perspective-relativity is ordinal, not set-theoretically free — it is the difference between a finite position in the chain and the view of the full chain from outside. The cardinality of the fan at each node is determined by the countable substrate.

OQ-E2 status after T-IZ: PARTIALLY CLOSED. The ordinal indexing $\Omega = \omega$ is forced by the countable binary substrate (ZP-C D4, ZP-B Q_2 separability, binary alphabet). Internal/external perspective relativity is ordinal, not set-theoretically free. Formal connection between specific semilattice structures and specific CH instances remains deferred — that is the remaining open question in OQ-E2. ✓ (partial)

II. The Null Balance

The null balance $0 + x + (-x) = 0$ is an arithmetic READING of the cycle of an instantiation branch, never a theorem of the structure: a ZPSemilattice carries join and $\bot$ and no additive inverse, so the identity is not statable in that signature. The cycle it reads is itself a reading; its premises are stated in § V.III. As a reading the balance strings its three terms across ω state changes: the branch begins at its semilattice's $\bot$ (read as the 0 of the balance), generates ε_0 and successors (+x), and at the ordinal limit the 2-adic chain converges to Q_2's 0 (−x).

What T-IZ supplies to this reading is the convergence; the balance itself stays a reading. "Balance" here is not subtraction in $(L, \vee, \bot)$ — R1 prohibits that. Two carriers carry the reading: the lattice chain begins at its semilattice's $\bot$ (CC-1) and ascends under T3, and the 2-adic depth chain, nowhere zero, converges to $0 \in Q_2$. That the branch closes there is the cycle reading; its premises are stated in § V.III. The CONVERGENCE is a theorem, given IsDepthChain (a modelling commitment); that the role-filler is a DISTINCT $\bot'$ rather than the $\bot$ it began at is C-DA2, and in the 2-adic realization the arc returns to the same 0 (snap_arc_z2_loop).

Null Balance (role-RECOGNITION derived; occupancy takes a choice of join; novelty COMMITTED): For every sequence $(S_n)_{n<\omega}$ in Q_2, nowhere zero, with $\|S(n)\|_2 \leq 2^{-n}$ (derived via t_iz_h_bound_from_depth_chain from IsDepthChain and IsStrictStateSequence on a depth index in $\mathbb{N}$ — R-IZ-A closed): the sequence converges to 0 in Q_2. CC-1's start at $\perp$ belongs to the semilattice chain, not to S. The balance $0 + x + (-x) = 0$ is a READING of the cycle of § V.III, whose premises are stated there, and not a theorem — a ZPSemilattice has no additive inverse, so it is not statable in the structure T-IZ is about — where x represents ω state changes under T3, and $(-x)$ represents the convergence to Q_2's 0. No new axioms required for the convergence.

III. The Cycle Reading

The framework reads the steps below as a cycle. The cycle is a reading, not a theorem. What T-IZ proves is that the 2-adic depth chain converges to 0 in Q_2, given IsDepthChain (a modelling commitment), IsStrictStateSequence and the chain being nowhere zero (t_iz_h_bound_from_depth_chain, t_iz_cauchy), together with the separate implication that anything filling the bottom role of a semilattice IS that semilattice's $\perp$ (t_iz_limit_is_new_null). For the cycle to begin again or close at the limit, it needs every one of the following further premises, and none is established here: (a) the reading that the limit fills the $\perp$ role, which takes a choice of join on Q_2 (then the role half is a theorem, t_iz_limit_is_new_null), since the role condition is not statable of the limit until a join on Q_2 is chosen (MC-1); (b) the limit reaching the incompressibility threshold P_0, which nothing here establishes, since its only route in this document, the valuation-complexity bridge, is refuted (§ II.B); (c) what DA-1 needs: DP-2 and DP-2's precondition, that the configuration reaching P_0 is there in Sense B (a running machine's current configuration) and not in Sense A (an inert string), which the occurrence commitment asserts and DA-1 consumes and does not supply (ZP-E § IV); and (d) for the next branch to start from a NEW null $\perp'$ rather than the $\perp$ already there, C-DA2, a further commitment; in the 2-adic realization the arc returns to the SAME 0 (snap_arc_z2_loop). The steps of the cycle:

- T-SNAP: From $\perp$, existence emerges — the shape of that emergence is derived and its occurrence is committed to, not proved. The Binary Snap $\perp \to \varepsilon_0$ is irreversible. This is the opening of the branch.
- T3 (Monotonicity): The state sequence ascends without interruption. Each step adds informational content irreversibly. The chain climbs.
- No Top: In a carrier with HasNoTop (a hypothesis on the carrier, not a law: MachinePhase and OntologicalStates have a top) there is always a strictly greater element, so no chain halts for want of anywhere to go. It supplies the POSSIBILITY of ascent; IsStrictStateSequence is its OCCURRENCE, and that is the hypothesis T-IZ actually consumes. Not ZP-A's R1, which is no-subtraction.
- T-IZ: The 2-adic depth chain converges to $0 \in Q_2$ at the ordinal limit ω. That DA-1 and T-SNAP fire again there, closing the branch, is the reading whose premises are stated above.
- DA-2 (Instantiation Succession): $\perp'$ is read as the foundation of the next instantiation, and the cycle repeats; this too is the reading whose premises are stated above.

The formal spine of T-IZ takes the bound $\|S(n)\|_2 \leq 2^{-n}$ as a condition derived from IsStrictStateSequence, the IsDepthChain modeling commitment and the chain being nowhere zero (t_iz_h_bound_from_depth_chain, §Ib — R-IZ-A closed). Given that condition, two charts, two carriers. In Q_2: the 2-adic depth chain converges to Q_2's 0 (t_iz_cauchy), and 0 is the LIMIT, not a term, since the chain is nowhere zero. Q_2's field structure supplies no join, and the corpus registers no ZPSemilattice instance on Q_2, so the role condition is not statable of the limit here. (A join could be chosen: every inhabited carrier

admits one, ZeroParadox/Order/Lattice.lean. On $\mathbb{Z}_2$, the 2-adic integers, the corpus supplies one with 0 as its bottom (the example at ZeroParadox/Valuation/Scale.lean § V); 0 filling that bottom is a choice of operation, not something T-IZ derives.) In a ZPSemilattice: the bottom ROLE is the join-identity, and anything filling it IS that semilattice's existing ⊥ (t_iz_limit_is_new_null). Reading the 2-adic limit as filling that role is a READING across the two carriers, and the cross-carrier identity is retired as ill-typed (MC-1). That the ⊥ so reached is a NEW one is C-DA2, a commitment; in the 2-adic realization the arc returns to the SAME 0 (snap_arc_z2_loop).

Key Result

T-SNAP: $\bot \to \varepsilon_0$, shape derived and occurrence committed (existence emerges from null).

T-IZ: a nowhere-zero 2-adic depth chain $S_0, S_1, S_2, \ldots \to 0 \in Q_2$ at ω, ROLE-RECOGNITION derived; OCCUPANCY takes a choice of join; NOVELTY committed. What is proved is the 2-adic convergence (t_iz_cauchy) and, separately, that anything playing the bottom role IS the bottom (t_iz_limit_is_new_null). That the limit is a thing playing that role takes a choice of join on Q_2 (then the role half is a theorem, t_iz_limit_is_new_null) — the role condition is not statable of the limit until a join on Q_2 is chosen; that the bottom so reached is a NEW one is a commitment (C-DA2).

The cycle and its closure are a reading, not a theorem; its premises are stated in § V.III. Strict valuation growth is Lean-derived from IsDepthChain (a modelling commitment) + IsStrictStateSequence (h_strict_from_r1_t3). No new axioms required beyond AX-B1, AX-G1, AX-G2.

Open Items Register

Item	Status	Description
T-IZ: Inside Zero Theorem	CONVERGENCE + ROLE-RECOGNITION DERIVED — occupancy takes a choice of join; novelty COMMITTED	Every nowhere-zero 2-adic depth chain $S_0, S_1, \ldots$ in Q_2 whose valuation equals a depth index in ℕ that climbs at every step converges to 0 in Q_2; reading that limit as the occupant of the bottom role takes a choice of join on Q_2 (then the role half is a theorem, t_iz_limit_is_new_null), and reading that occupant as a successor null is a commitment (C-DA2). Formal spine: Step 1 (t_iz_cauchy, carries Classical.choice, § III) + Step 6 (t_iz_limit_is_new_null, axiom-free via DA-2). Steps 2–5 are informational context — the ZP-E path, whose valuation-complexity bridge is refuted (§ II.B); DA-1 is closed given DP-2 (ZP-E), with ZP-K witnessing Path 1 and carrying Path 3 as a KleeneStructure requirement. No new axioms required.
OQ-E2: Cardinality-semilattice correspondence	PARTIALLY CLOSED — Ω = ω forced	Ordinal indexing Ω = ω forced by countable binary substrate (ZP-CD4, Q_2 separability). Internal/external perspective relativity is ordinal, not set-theoretically free. Formal connection to specific CH instances: still open — deferred to future work.

Item	Status	Description
Null balance: $0 + x + (-x) = 0$	CLOSED for ROLE-RECOGNITION — occupancy takes a choice of join; novelty COMMITTED	The convergence half is derived from T-IZ: the lattice chain starts at its semilattice's $\perp$ (CC-1) and ascends under T3, and the nowhere-zero 2-adic depth chain converges to $0 \in Q_2$. Reading that limit as an occupant of the bottom role takes a choice of join on Q_2 (then the role half is a theorem, t_iz_limit_is_new_null). On that reading, DA-2 identifies the occupant with the $\perp$ already there. "$-x$" is not subtraction in L — it is the 2-adic approach to 0 by forward motion; reading it as a return in the cycle is the reading whose premises are stated in § V.III. That the occupant is a NEW $\perp'$ is the commitment, not the theorem.
Valuation-complexity bridge	REFUTED — counterexample $S_n = 2^n$	The ZP-E route from 2-adic depth to Kolmogorov complexity and DA-1 Path 3. Its implication $v_2(S_n) \to \infty \Rightarrow K(S_n \mid n) / \lvert S_n \rvert \to 1$ fails at $S_n = 2^n$ (§ II.B): valuation n, complexity given n bounded, ratio $\to 0$. The valuation half is checked (scaleChain_isDepthChain); the complexity half is outside Lean scope: one fixed program prints 2^n from n, which bounds $K(2^n \mid n)$ by its length. Not a proof dependency for T-IZ: formal spine is Steps 1 + 6 (Step 6 axiom-free; Step 1 carries Classical.choice, § III); DA-1 is closed given DP-2 (ZP-E), with ZP-K witnessing Path 1 and carrying Path 3 as a KleeneStructure requirement.
T-IZ Lean sorry fill	CLOSED — ZeroParadox/Valuation/SemilatticeInstance.lean	t_iz_norm_tendsto_zero and t_iz_conv_zero filled; t_iz_cauchy proved (carries Classical.choice, § III). All SemilatticeInstance.lean theorems compile with no sorry. Axiom footprint: standard foundational axioms only (propext, Quot.sound, and Classical.choice on the convergence theorems, § III; the DA-2 step is axiom-free).
AX-1: Binary Snap Causality	RETIRED — shape proved as T-SNAP (ZP-E); the first snap occurs given the occurrence commitment and DA-1	AX-1 (Binary Snap Causality) is retired. Its content was split in two: the shape of the snap is proved, as Theorem T-SNAP, and that the first snap occurs is stated separately: it follows from the occurrence commitment together with DA-1 (closed given DP-2). The occurrence commitment is stated in two charts, execution and slot, in the header of ZeroParadox/Order/Snap.lean (indexed in ZeroParadox/Order/SnapCannotBe.lean § II and CLAIMS.md), the slot chart as the hypothesis hw of t_snap_of_fires_once, that the tape fires at some stage, beside the theorem trials_fires_once that, for independent stages each firing with probability p, $0 < p \leq 1$, firing at some stage has probability one, which names no tape (ZeroParadox/Information/CrossingTrials.lean § VIII). tsnap_holds_but_nothing_moves shows T-SNAP does not carry that the snap occurs. T-SNAP is derived in ZP-E. A snap at the ordinal limit is the cycle reading, not a consequence of T-IZ; its premises are stated in § V.III.

Item	Status	Description
Remaining axioms	INTENTIONAL — AX-B1, AX-G1, AX-G2	These are the named axioms. The occurrence commitment is not on this list because it is a commitment, not a named axiom. T-IZ requires no additions.

Traceability Register

Claim	Grounded In	Bridge Axiom?	Status
T-IZ: Inside Zero	What the chain CONSUMES: IsStrictStateSequence, IsDepthChain, the chain being nowhere zero, and ZP-B's norm–valuation identity. HasNoTop supplies the room and is consumed by nothing here. Also ZP-B T5/C3 for the irreversibility comparison only; ZP-C L-INF, D1; ZP-E DA-1, T-SNAP, DA-2	None	Role derived — T-IZ ✓ (formal spine Steps 1+6: Step 6 axiom-free, Step 1 carries Classical.choice, § III; bridge: refuted). Novelty of the successor bottom: COMMITMENT, not proved.
Null Balance $0 + x + (-x) = 0$	T-IZ + T-SNAP + DA-2 (ZP-E)	None	Role derived — the convergence is T-IZ; the balance is a reading, not a theorem. Novelty: commitment.
OQ-E2 partial closure	ZP-C D4 (binary alphabet, I(n)=n); ZP-B (Q_2 separable); T-IZ ($\Omega = \omega$)	None	Partially closed — $\Omega = \omega$ forced by countable substrate; CH connection deferred.
t_iz_valuation_unbounded (Lean)	int_strict_mono_ge (induction on $\mathbb{Z}$); omega (integer arithmetic)	None	Lean: proved ✓ — standard foundational axioms only. Formalises "sup $v_2(S(n)) = \infty$" — proof obligation table row 3.
t_iz_cauchy (Lean)	ZP-B (Q_2 normed field); geometric tendsto; Mathlib.Analysis.SpecificLimits.Basic	None	Lean: proved ✓ (t_iz_norm_tendsto_zero, t_iz_conv_zero filled; carries Classical.choice, § III)
t_iz_limit_is_new_null (Lean)	da2_bottom_characterization	None	Lean: proved ✓ (direct delegation to DA-2)
c_t_iz_null_balance (Lean)	c_da2_novelty	None	Lean: proved ✓ (direct delegation to C-DA2)

Claim	Grounded In	Bridge Axiom?	Status
t_iz_c3_compatible (Lean)	c3_irreversible	None	Lean: proved ✓ (C3 holds unmodified; Cauchy sequences ≠ continuous paths)
Valuation-complexity bridge	ZP-C D1, L-INF, Remark R-BRIDGE; ZP-B T2; AIT (standard)	N/A	REFUTED (§ II.B; counterexample $S_n = 2^n$) — not load-bearing, since the convergence T-IZ proves does not pass through it. Separately, DA-1 is closed given DP-2 (ZP-E), with ZP-K witnessing Path 1 and carrying Path 3 as a KleeneStructure requirement. The complexity half is outside Lean scope.
t_iz_h_bound_from_depth_chain (Lean)	h_strict_from_r1_t3 (§Ib); t_iz_r1_t3_geometric_bound; Padic.norm_eq_zpow_neg_valuation (depths $0 : \mathbb{N} \Rightarrow \|S_0\|_2 \leq 1$)	None	Lean: proved ✓ — optional transparency lemma. Derives h_bound from IsStrictStateSequence (the one lattice condition), IsDepthChain (the 2-adic bridge, binding no semilattice) and the chain being nowhere zero, closing the $\|S_0\|$ factor gap between §Ib and t_iz_complete.
t_iz_complete_from_axioms (Lean)	t_iz_h_bound_from_depth_chain; t_iz_complete	None	Lean: proved ✓ — optional transparency variant. Replaces h_bound with three hypotheses, of which only IsStrictStateSequence is a ZP-A lattice condition; IsDepthChain is the bridge to the 2-adic valuation and binds no semilattice, and the chain must be nowhere zero. It still carries h_role explicitly, which is the hypothesis nothing here grounds. Canonical theorem: t_iz_complete.

End of ZP-I | Theorem T-IZ: Inside Zero | R-IZ-A closed: strict valuation growth derived from IsDepthChain + IsStrictStateSequence (h_strict_from_r1_t3, §Ib) | IsDepthChain remains a modelling commitment | Framework closure: a reading; its premises are stated in § V.III | Formal spine: Step 6 axiom-free (t_iz_limit_is_new_null), Step 1 carries Classical.choice, § III (t_iz_cauchy) | Valuation-complexity bridge: refuted; not load-bearing, since the convergence T-IZ proves does not pass through it | DA-1 closed given DP-2 (ZP-E), with ZP-K witnessing Path 1 and

carrying Path 3 as a KleeneStructure requirement | Remaining axioms: AX-B1, AX-G1, AX-G2 | No new axioms required