Completeness Thresholds for Memory Safety of Array Traversing Programs

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Workshop on the State Of the Art in Program Analysis
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What This Work Is About

- Connection between bounded & unbounded proofs
- Ideas to increase trust in bounded model checking

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- Connection between bounded & unbounded proofs
- Ideas to increase trust in bounded model checking
- When is a bounded “proof” a proof?

Model Checking: Easy Off-by-1 Error

- WHILE language with pointer arithmetic
- Targeted property: Memory safety
- Memory assumption $\texttt{array}(a, s)$:
  $a[0]$ … $a[s{-}1]$ allocated

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for i in [0 : s-1] do
    !a[i+1]
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for i in [0 : s-1] do
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Which bounds should we choose for s?

- $s = 0$: No error
- $s = 1$: Error

Model Checking: “Harder” Off-by-N Error

Memory assumption:
$\texttt{array}(a, s)$

```
for i in [0 : s-2] do
    !a[i+2]
```

Which bounds should we choose for s?

Model Checking: “Harder” Off-by-N Error

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Which bounds should we choose for s?

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- $s = 1$: No error
- $s = 2$: Error

Model Checking: No Off-by-N Error

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for i in [0 : s-1] do
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Which s can convince us?

Model Checking: No Off-by-N Error

Memory assumption: $\texttt{array}(a, s)$	for i in [0 : s-1] do !a[i]

Which s can convince us?

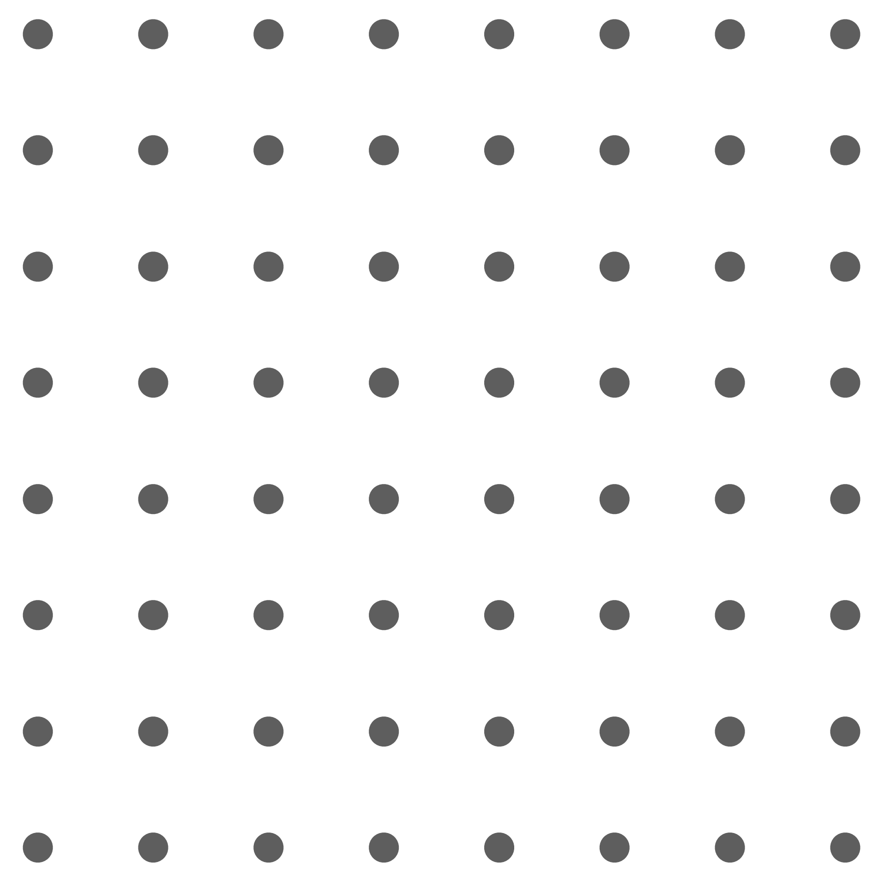
- $s = 0$: No error
- $s = 1$: No error
- $s = 2$: No error
- $s = 3$: No error

$\vdots$

$\Rightarrow$ Which size bound is large enough?

Model Checking Finite Systems

- Finite state transition system T
- Prove property Gp
  G ≈ globally ≈ p holds in every state
- Approach:
  Prove Gp for all paths up to length k
  $T \models_k \mathrm{G}p$



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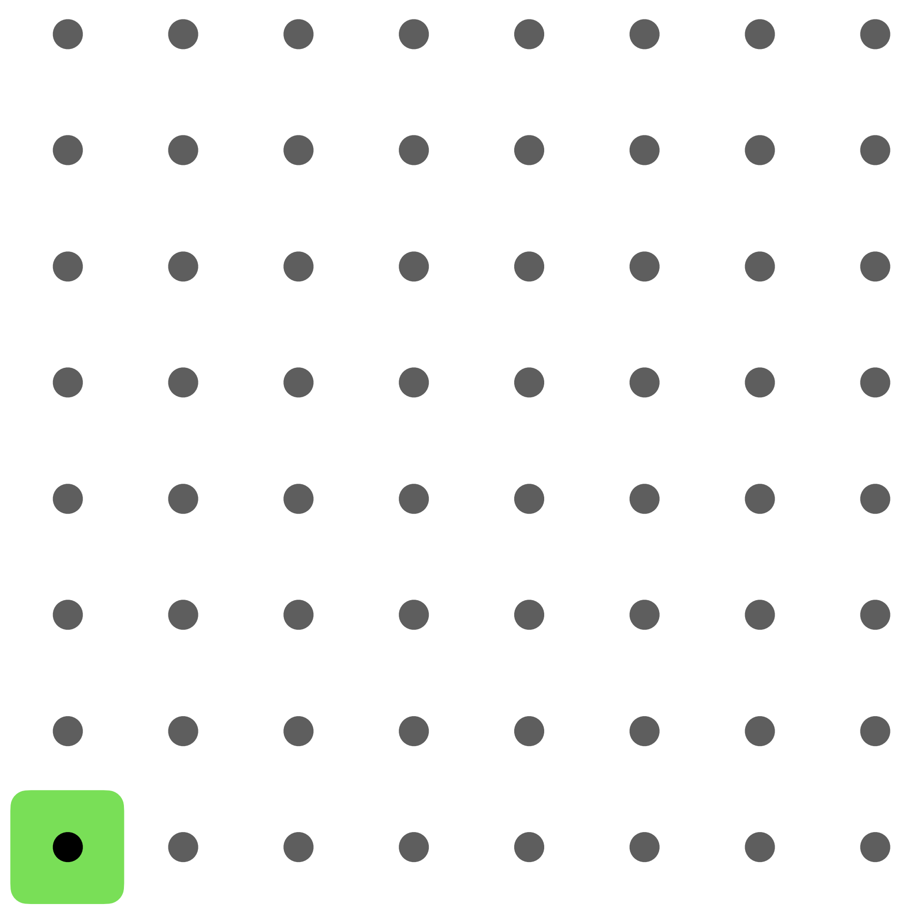

$T \vDash_0 \mathrm{G}p$

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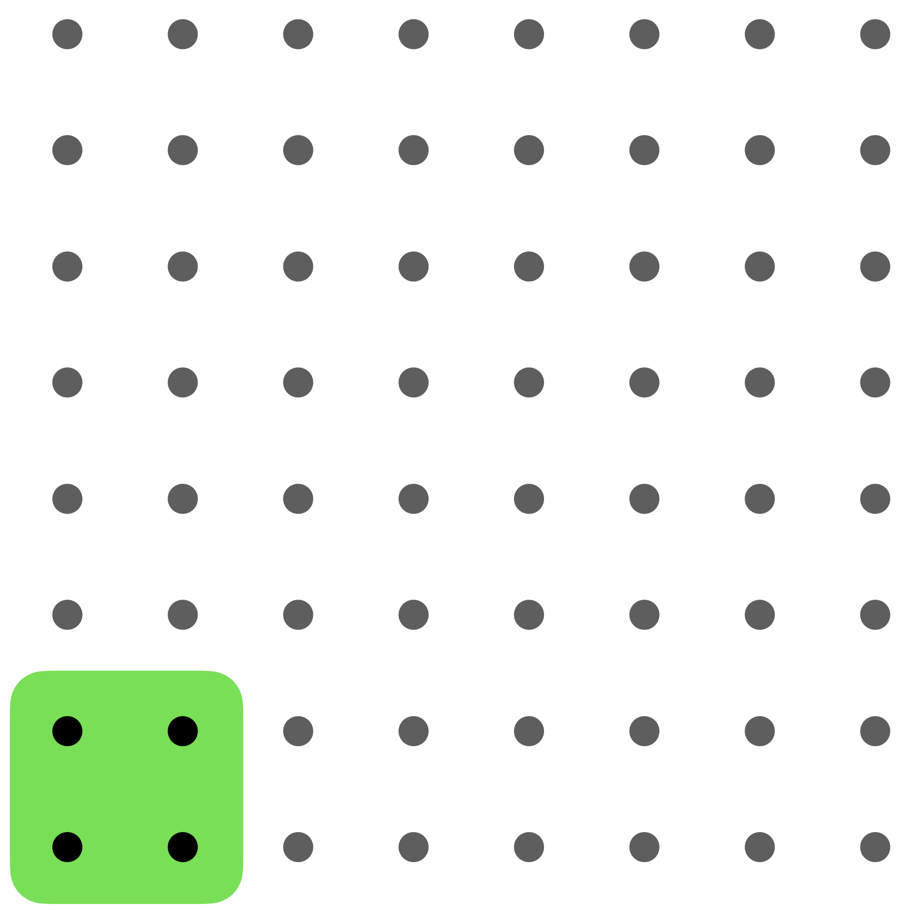

$T \vDash_1 \mathrm{G}p$

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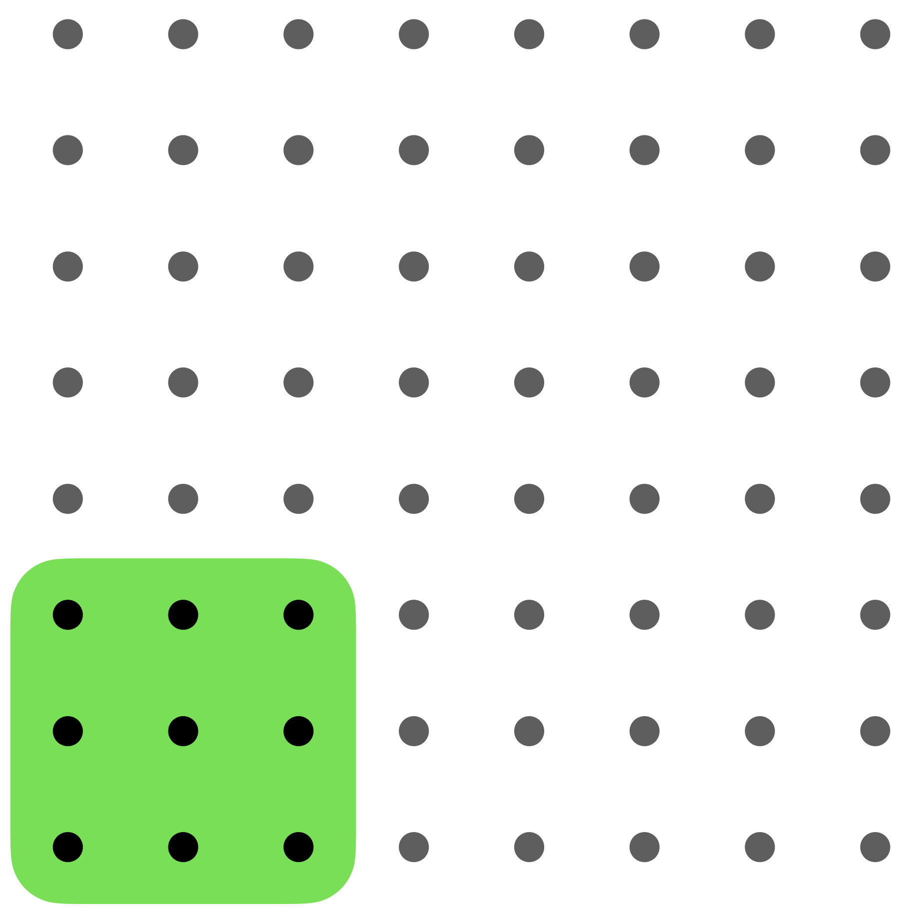


$T \vDash_2 \mathrm{G}p$

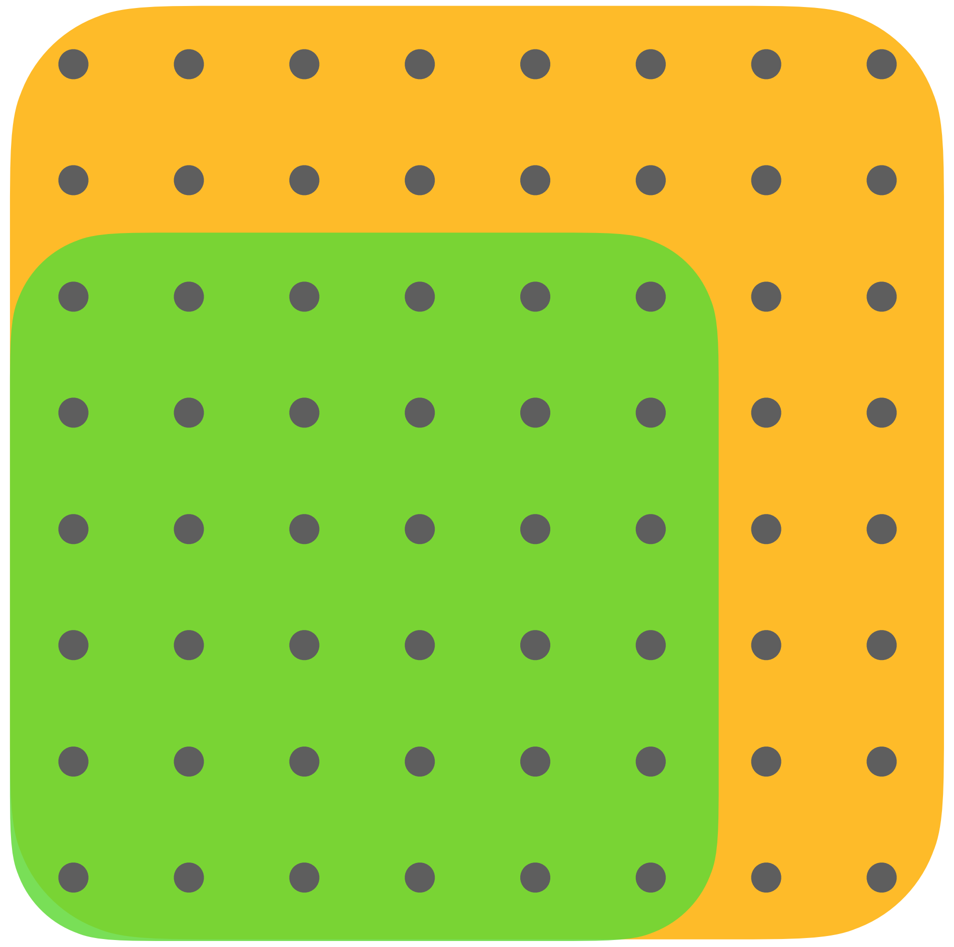
When should we stop?

Completeness Thresholds for Finite Systems

- k is completeness thresholds (CT) iff

$$\boxed{T \vDash_k \phi} \Rightarrow \boxed{T \vDash \phi}$$

- For specific ϕ:
  Can over-approximate CT via of key props of T



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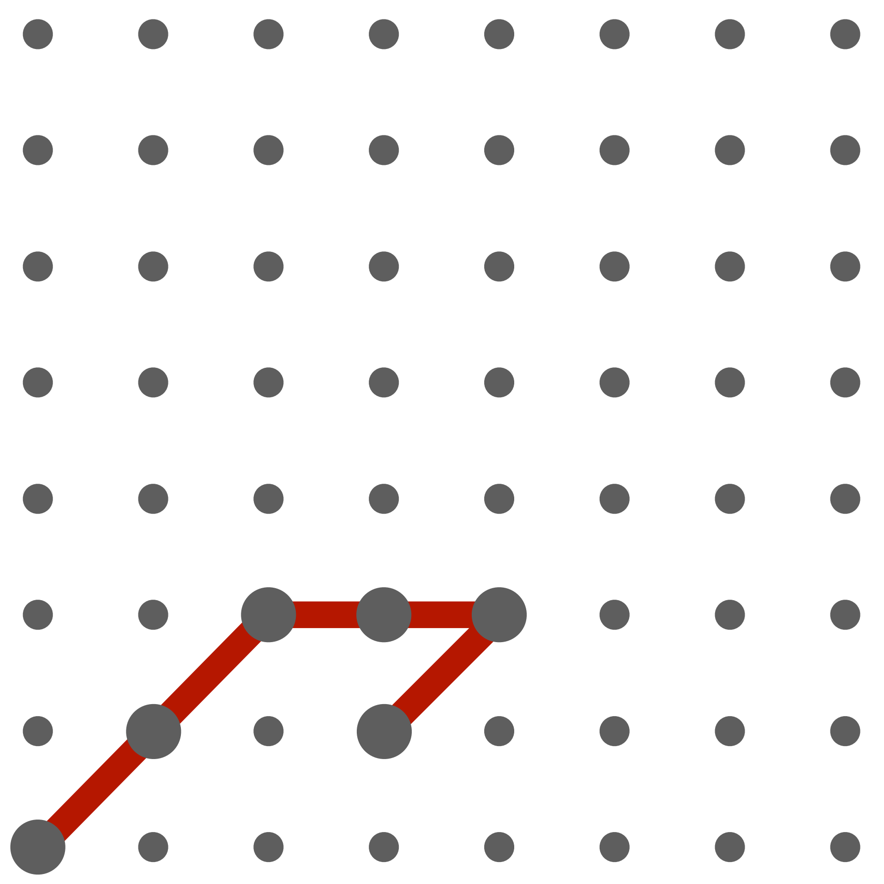
  (longest distance between any states)



diameter$(T) = 5$

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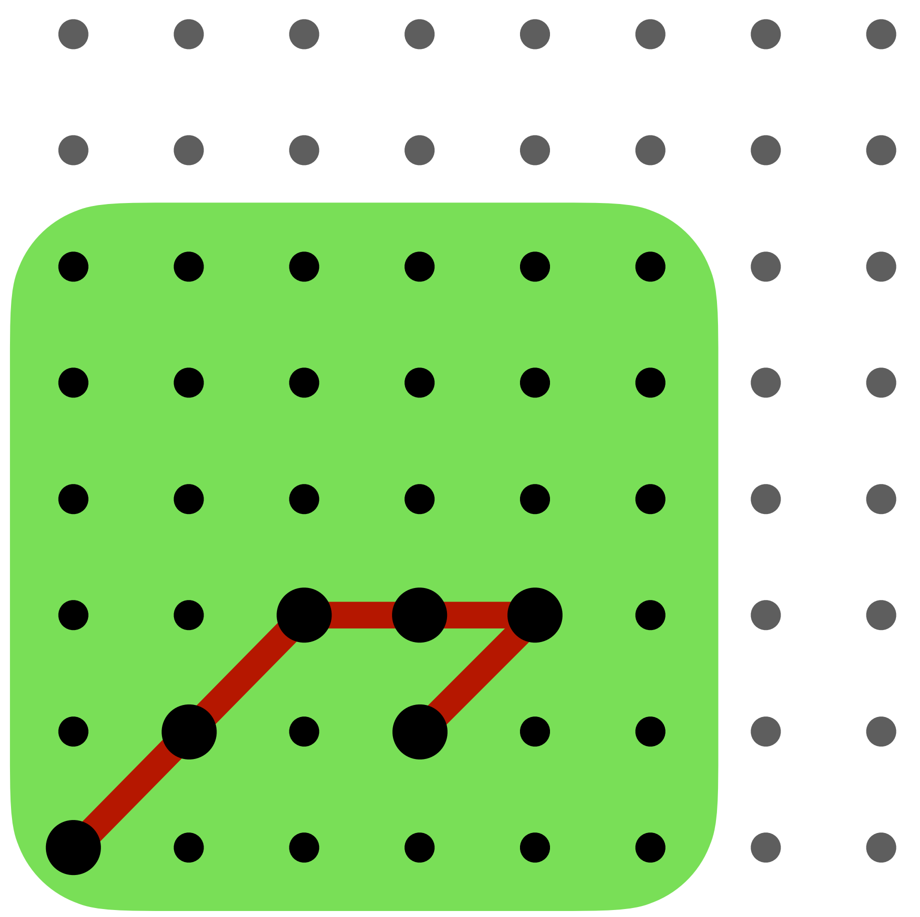

diameter$(T) = 5$

$T \vDash_5 \mathrm{G}p$

Completeness Thresholds for Finite Systems

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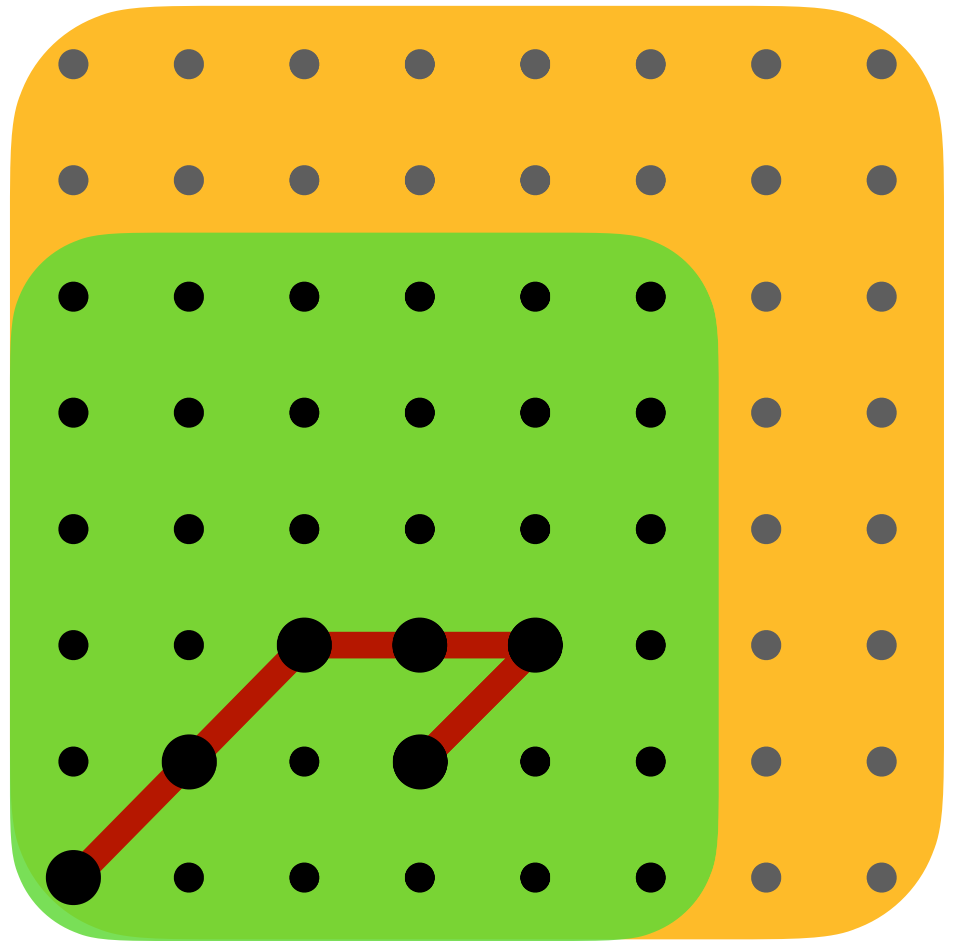


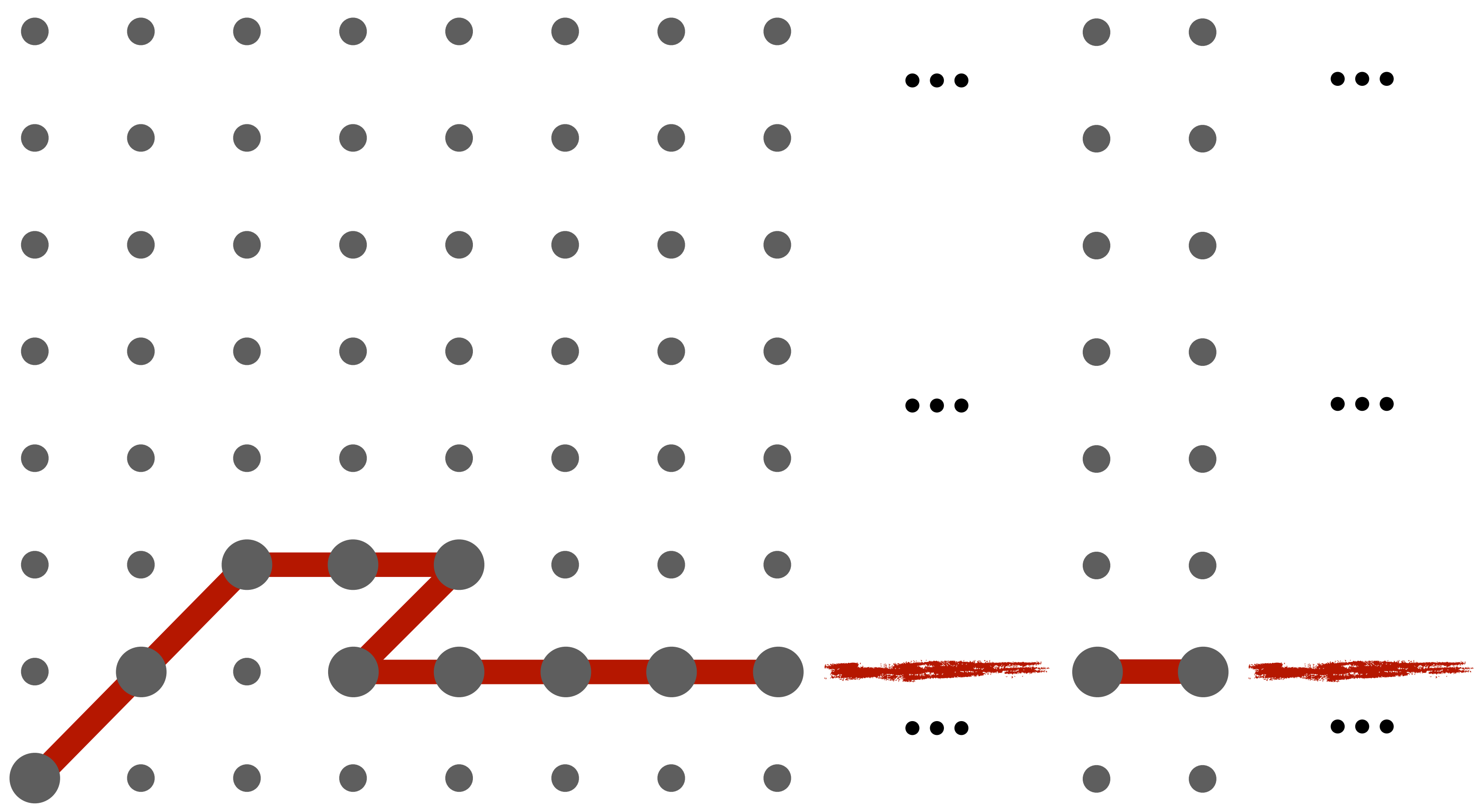
$\mathrm{diameter}(T) = 5$

$T \vDash_5 \mathrm{G}p \longrightarrow T \vDash \mathrm{G}p$

CTs for Infinite Systems?

Problem

Key properties used to describe CTs may be ∞



diameter$(T) = \infty$

CTs for Infinite Systems?

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Our Approach

Analyse program's *verification conditions*
instead of transition system

Verification Conditions

- Logical formula vc is VC for any spec $Spec(c)$ iff

$$\models vc \quad \Rightarrow \quad \models Spec(c)$$

- Can verify VC instead of program

- In general: VCs are over-approximations, i.e.,

  possible that $\not\models vc$ but $\models Spec(c)$

Completeness Thresholds

- Program variable x with domain X
- Specification $\forall x \in X \,.\, Spec(c)$

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- Program variable x with domain X
- Specification $\forall x \in X \,.\, Spec(c)$
- Subdomain $Q \subseteq X$ is a CT for x in $\forall x \in X \,.\, Spec(c)$ iff

$$\models \forall x \in Q \,.\; Spec(c) \quad \Rightarrow \quad \models \forall x \in X \,.\; Spec(c)$$

- For us: CT are subdomains, not depths

How to Prove CTs

- Generate VC: $Spec(c) \rightsquigarrow \forall x \in X \,.\, vc(x)$

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- Generate VC: $Spec(c) \rightsquigarrow \forall x \in X \,.\, vc(x)$

- Identify subdomain $Y \subseteq X$ where choice $x \in Y$ does not influence validity of $vc(x)$

$$\Big(\models vc(x) \quad \Leftrightarrow \quad \models vc' \quad \text{with } \; x \notin \text{free}(vc') \Big)$$

$$\Longrightarrow \quad \text{Found CT:} \quad (X \backslash Y) \cup \{y\} \quad \text{(for any choice of } y \in Y\text{)}$$

How Does the Array Size Affect Memory Safety?

Memory assumption: $\texttt{array}(a, s)$	for i in [L : s-R] do !a[i+Z]

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Generate VC

(fully automated)

VC $vc_0 := \forall s\,.\, \texttt{array}(a, s) \rightarrow \forall i \in \{L, \ldots, s - R\}\,.\, a[i+Z]$ alloc

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Range L, ..., s-R empty?

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Range L, …, s-R empty?

Yes

$s^- < L + R$

Simplify VC!

$$vc_0 \ \equiv \ \forall s^-.\ \ldots \to \forall i \in \varnothing\,.\ \ldots$$

$$\equiv \ \text{True}$$

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No

$s^+ \geq L + R$

$$vc_0 \;\equiv\; \forall i\,.\,(L \leq i < s^+ - R) \to (0 \leq i{+}Z < s^+)$$

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$$vc_0 \; \equiv \; \forall i \, . \, (L \leq i < \cancel{s^+} - R) \rightarrow (0 \leq i{+}Z < \cancel{s^+})$$

$$\equiv \; \forall i \, . \, (L \leq i \rightarrow 0 \leq i{+}Z)$$

$$\wedge \; (i \leq -R) \rightarrow i{+}Z < 0)$$

$\Rightarrow$ Validity does not depend on size

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Can check for any

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Found CT: $\{s^+\}$

Workflow: How to Find CTs

Mem spec
+ program

Workflow: How to Find CTs

Mem spec + program $\xrightarrow{\text{Generate VC}}$ VC $\forall x \in X \,.\, vc(x)$

Workflow: How to Find CTs

Mem spec + program

Generate VC

VC $\forall x \in X\,.\ vc(x)$

Select subdomain

$Y \subseteq X$

Restricted VC

$vc(x)$ with $x \in Y$

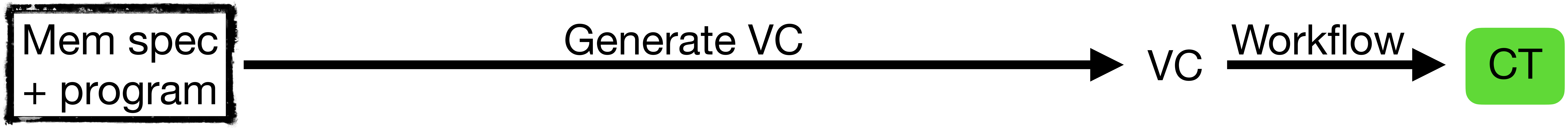
Try to eliminate x

$vc'(x)$ with $x \in Y$

Workflow: How to Find CTs

Scalability

Program Slicing



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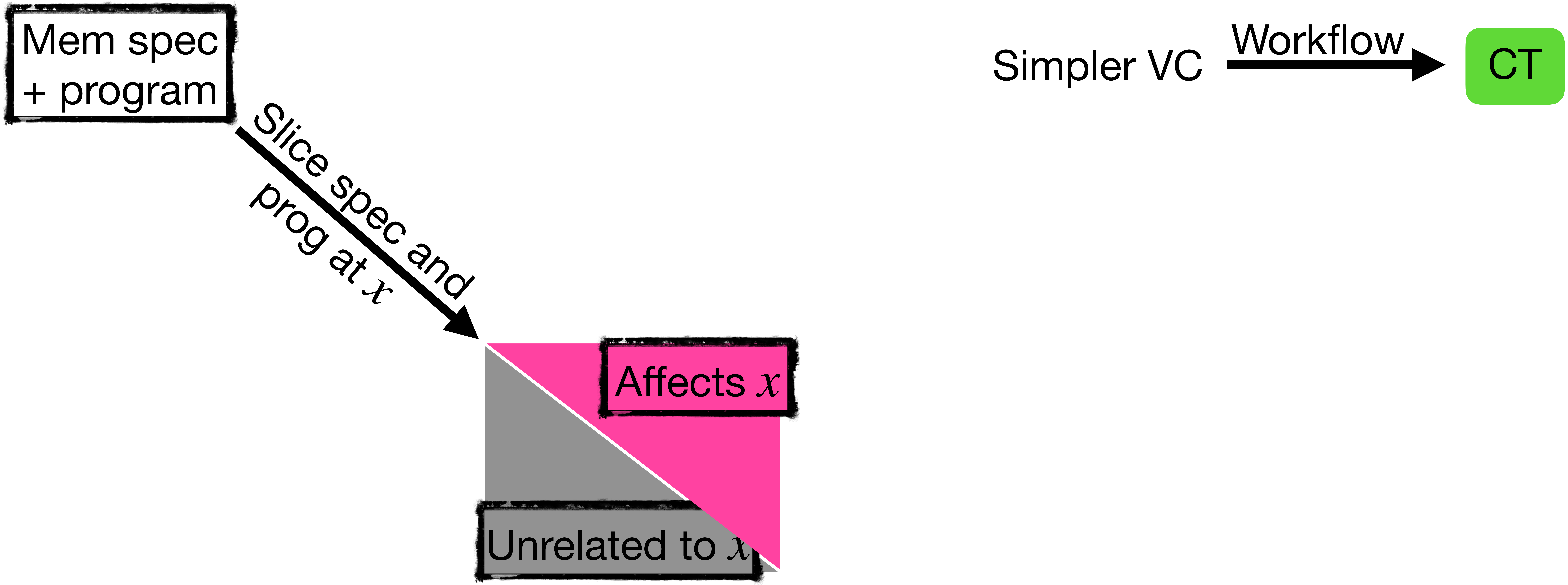

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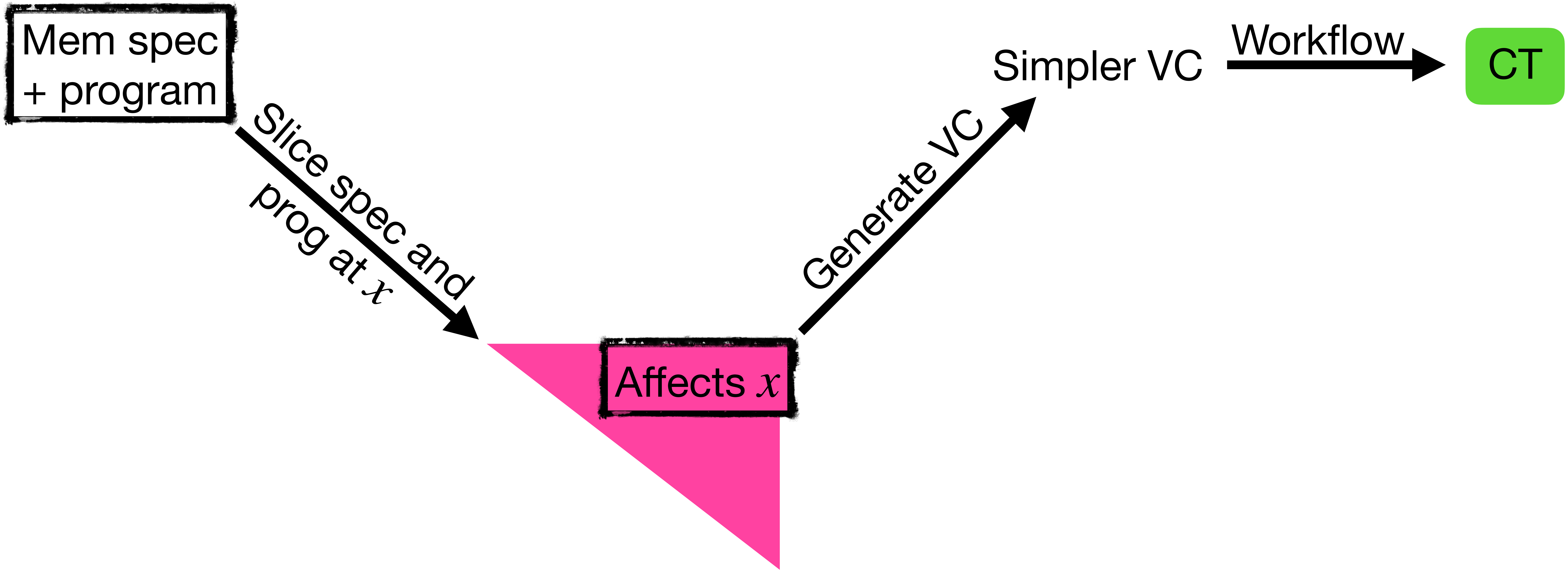

Scalability

CT Combinators

Sequencing

$c_1; c_2$ CTs Q_1, Q_2

$$\downarrow$$

$Q = Q_1 \cup Q_2$

Scalability

CT Combinators

Sequencing

$c_1; c_2$

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$$Q = Q_1 \cup Q_2$$

Branching

if e then c_1 else c_2

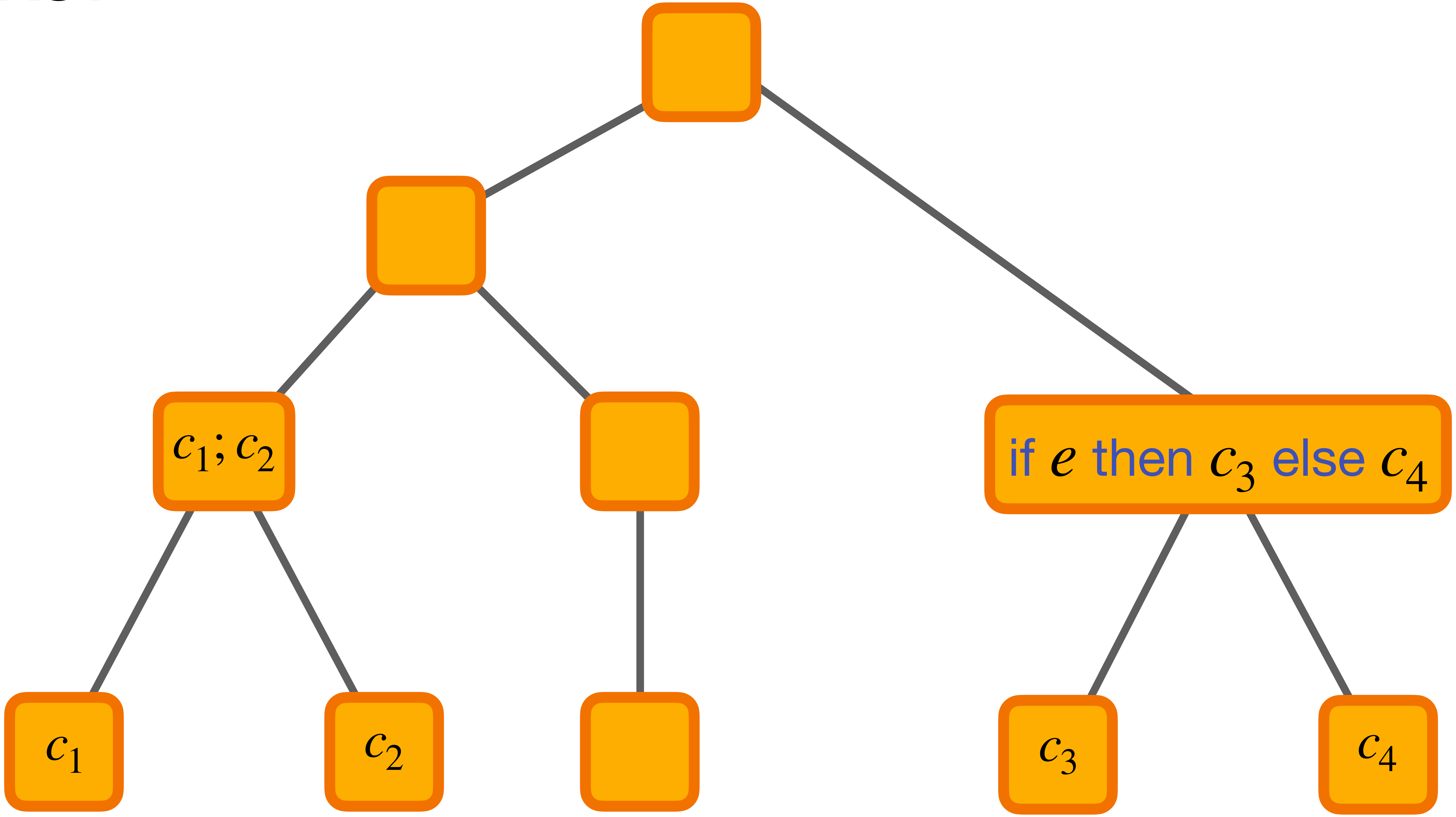
CTs as contraints

$Q_i \sim K_i$

$$Q \ \sim \ (e \wedge K_1) \vee (\neg e \wedge K_2)$$

Scalability

Follow AST



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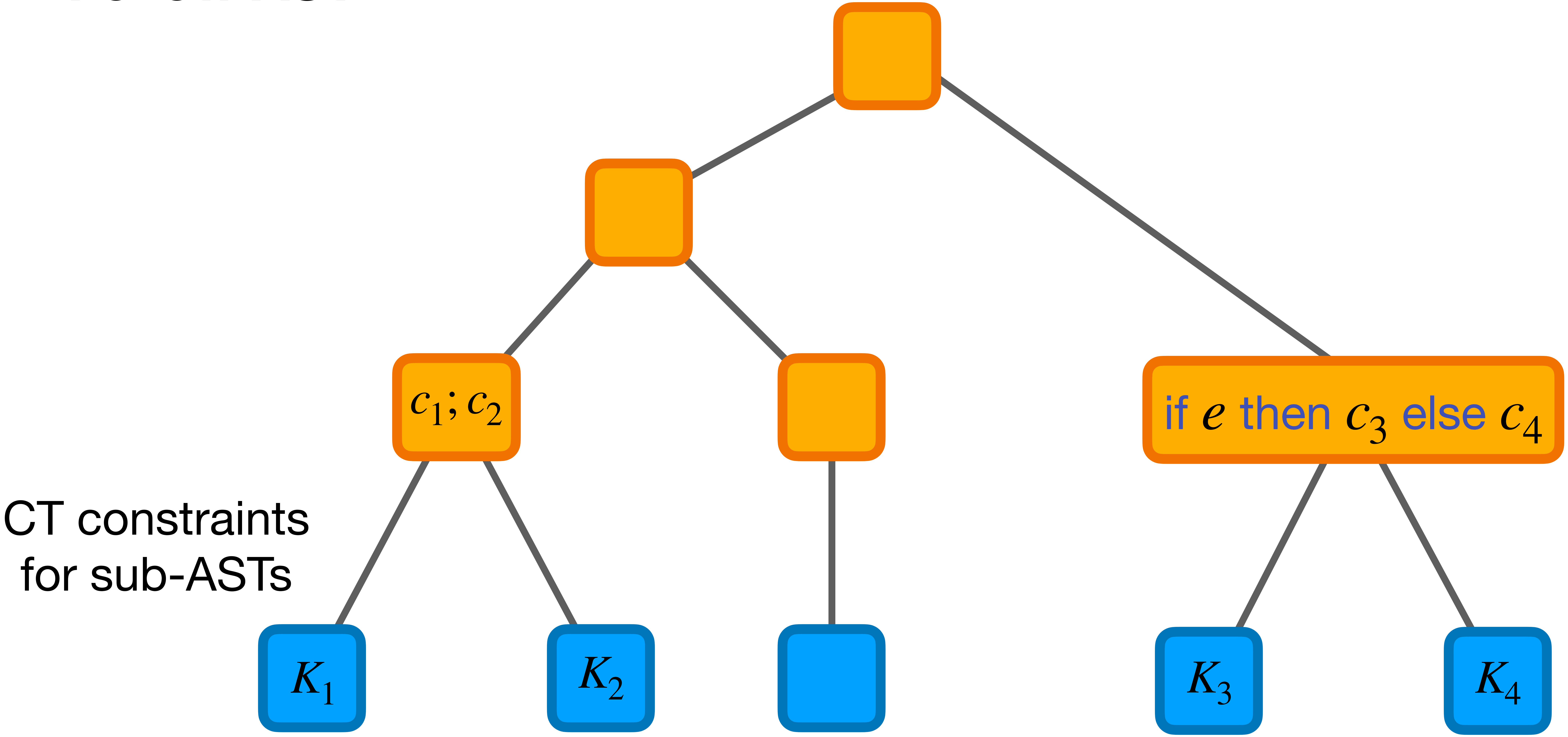

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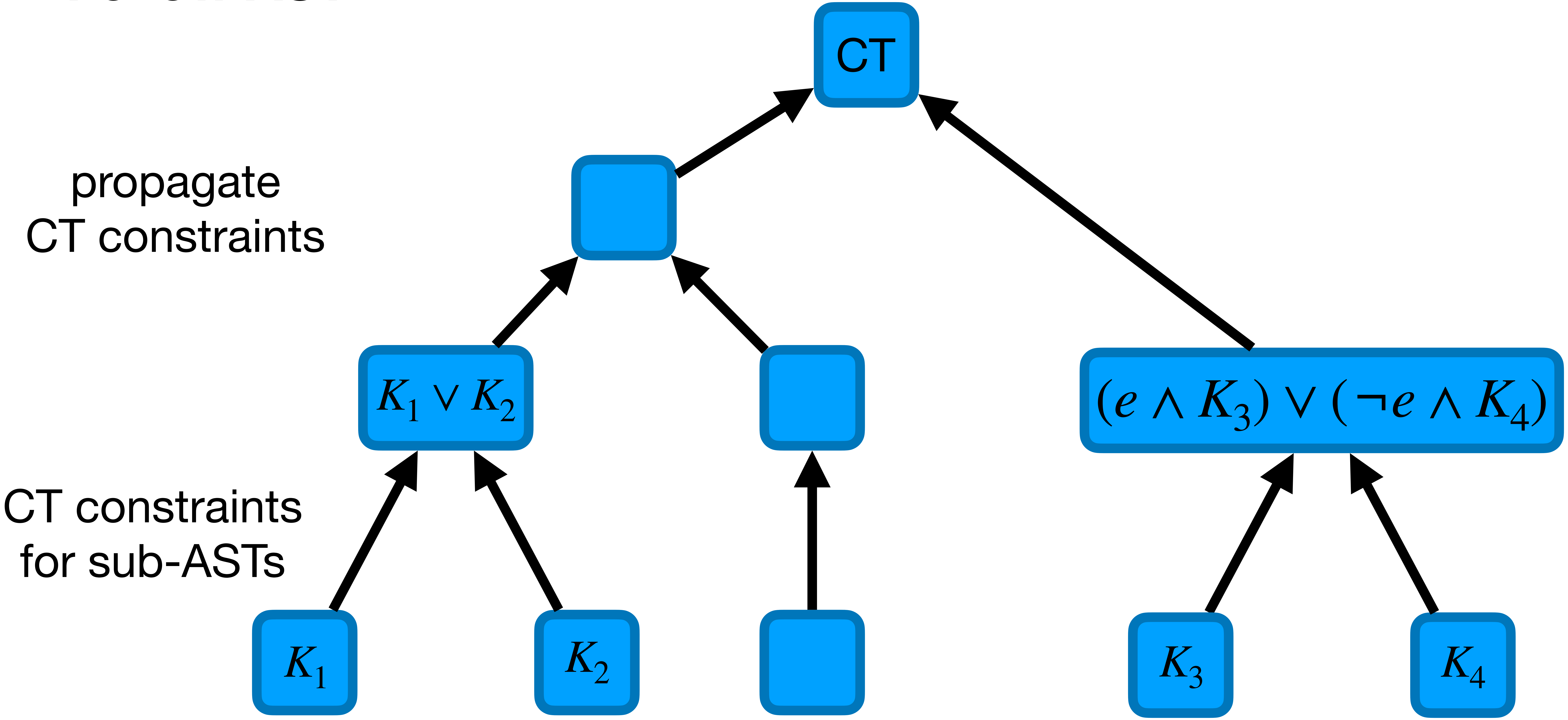

Outlook: Challenges

- Automation, e.g., automatic VC rewriting
- Demo scalability: Complex programs & data (e.g. lists, trees)

Outlook: Increase Trust in BMC

- Turn bounded into unbounded proof

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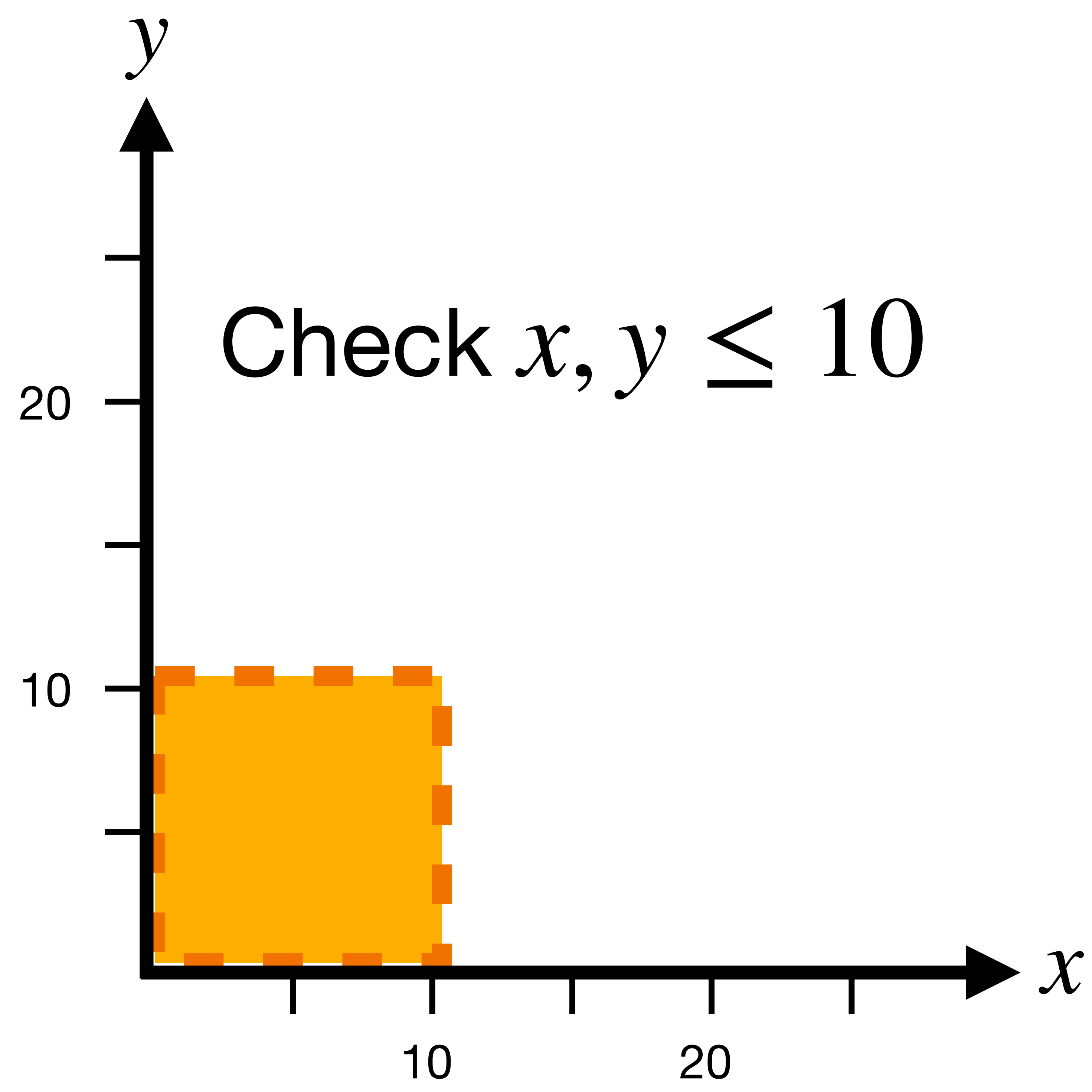
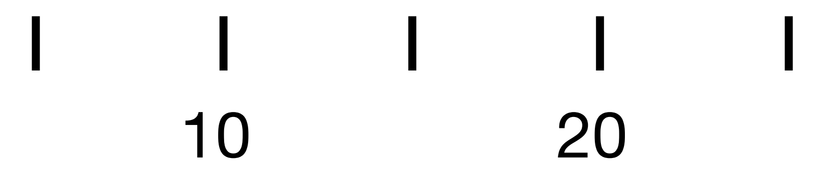
- Turn bounded into unbounded proof
- Shift resources to critical bounds





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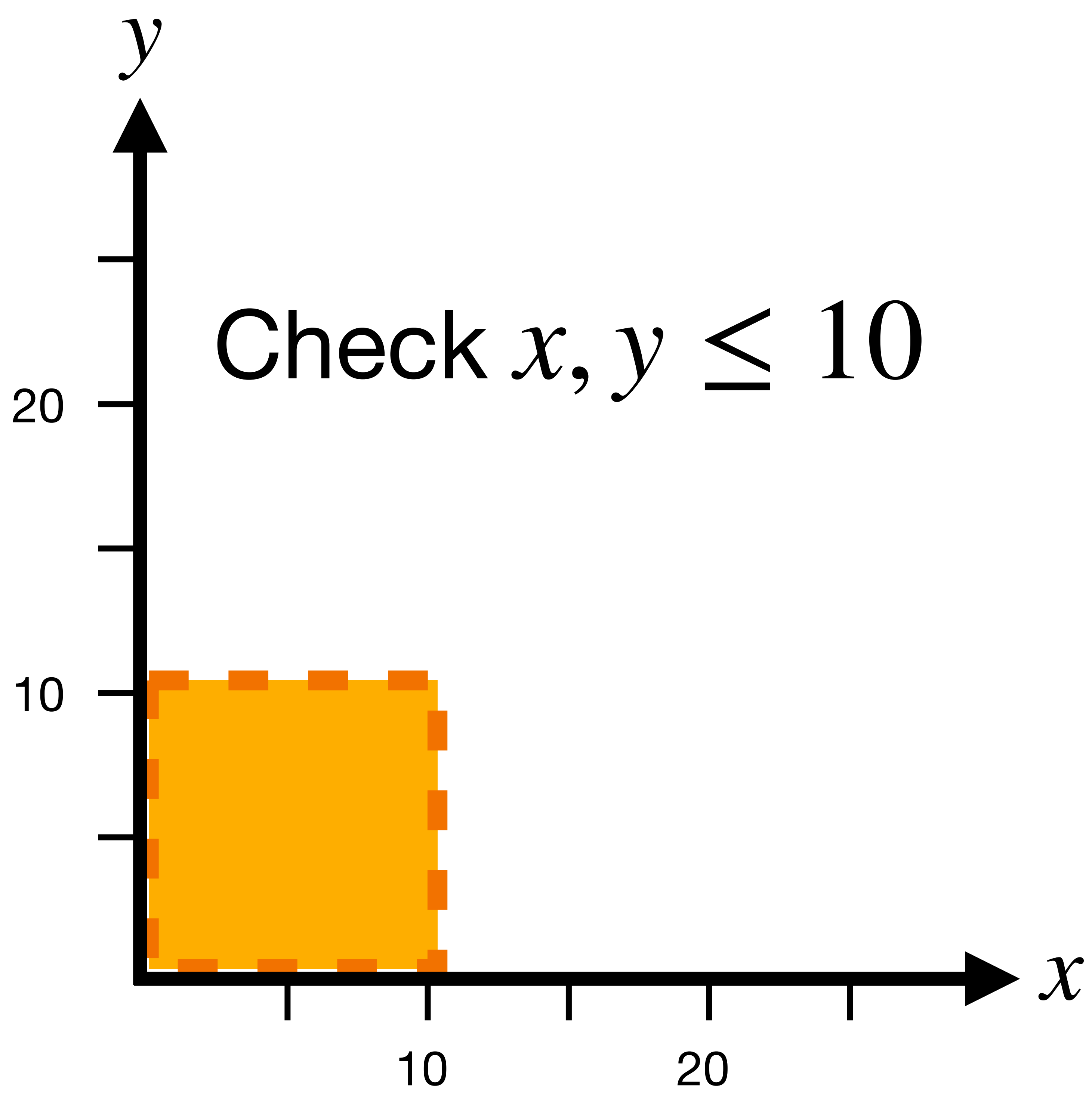
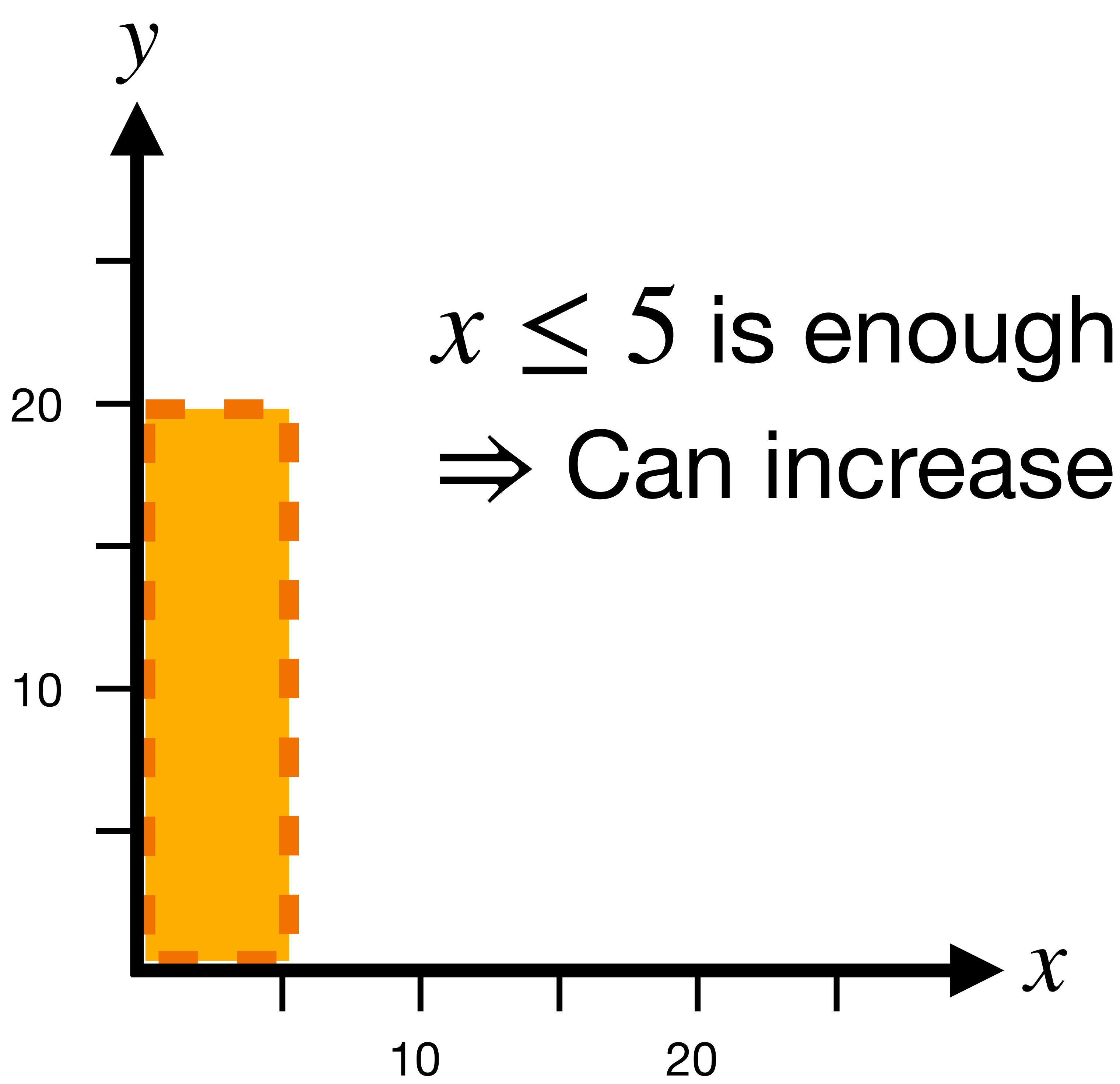


CT for x:
$\{0,\ldots,5\}$



$x \leq 5$ is enough
$\Rightarrow$ Can increase y bound

Conclusion

- First generalisation of CTs to infinite state systems
- Connection between bounded & unbounded proofs in program verification
- Foundational research but potential for integration into BMC

Backup Slides

Precise VCs

- VC vc is *precise* for x in $Spec$ iff

  $$\forall v\,.\ \Big(\ \models Spec[x \mapsto v] \ \ \Rightarrow\ \ \models vc[x \mapsto v]\ \Big)$$

  Intuition: vc does not over-approximate wrt. x

- Q is CT $vc\ \wedge\ vc$ is precise $\ \Rightarrow\ Q$ is CT $Spec$

Precise VCs

$$
\begin{tikzcd}
\forall x\,.\,Spec \arrow[r, "\text{Unbounded proof}"] \arrow[d] & \vDash \forall x\,.\,Spec \\
\forall x\,.\,vc \arrow[d] & \\
Q \text{ is CT for } vc \arrow[d, "vc \text{ precise for } x"] & \\
Q \text{ is CT for } Spec \arrow[r, "\text{Bounded proof}"] & \vDash \forall x \in Q\,.\,Spec \arrow[uuu, "\text{CT } Q"']
\end{tikzcd}
$$