

Intrinsic attenuation and scattering

Why coda Q does not tell the full story

Using seismic envelopes to separate the effects of source,
site and path

Tom Eulenfeld



@trichter on GitHub and in the ObsPy forum



Framework for calculation
of receiver functions

Deconvolution, moveout,
piercing points, ...

obspyh5

Quick & dirty IO of
waveforms preserving
metadata, HDF5



ObsPy

A Python Framework for Seismology



dv/v with stretching technique

CLI

configuration in JSON file

easy definition and house-keeping of
different correlation and stretching schemes

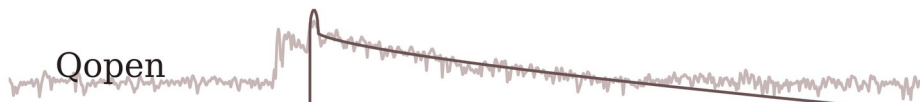
cc shorter than 1d possible

obspycsv

Quick & dirty IO of
earthquake catalogs to
CSV format

read EVENTTXT

flatten ObsPy catalogs to
NumPy arrays



Introduction

Motivation

Qopen method

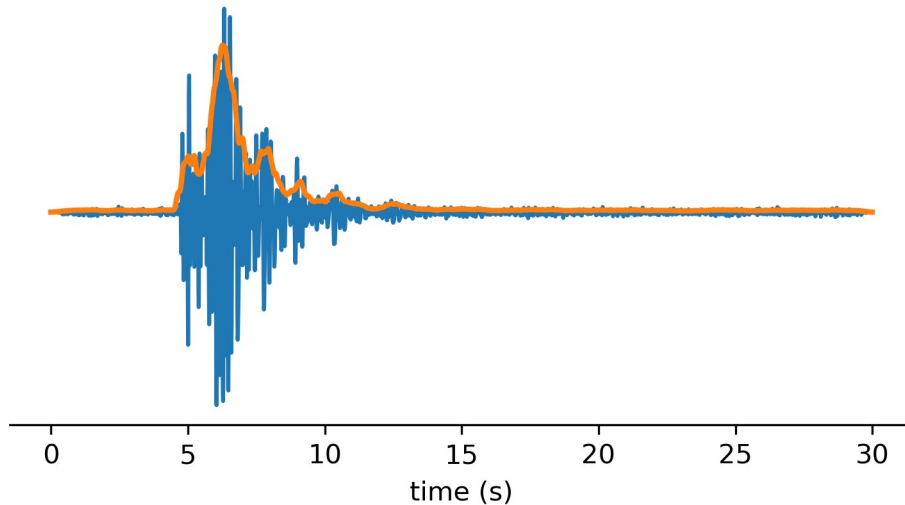
Applications

- USArray
- Helsinki geothermal stimulation

Introduction

Envelope and radiative transfer

- Phase information in the coda cannot be modeled easily
- Only coda amplitude (resp. energy) is of interest
- Convert waveforms to envelopes (Hilbert transform)
- Transition of wave equation to equation of radiative transfer
- Opens field for Monte-Carlo particle simulations



Intrinsic attenuation vs scattering, about Quality factors

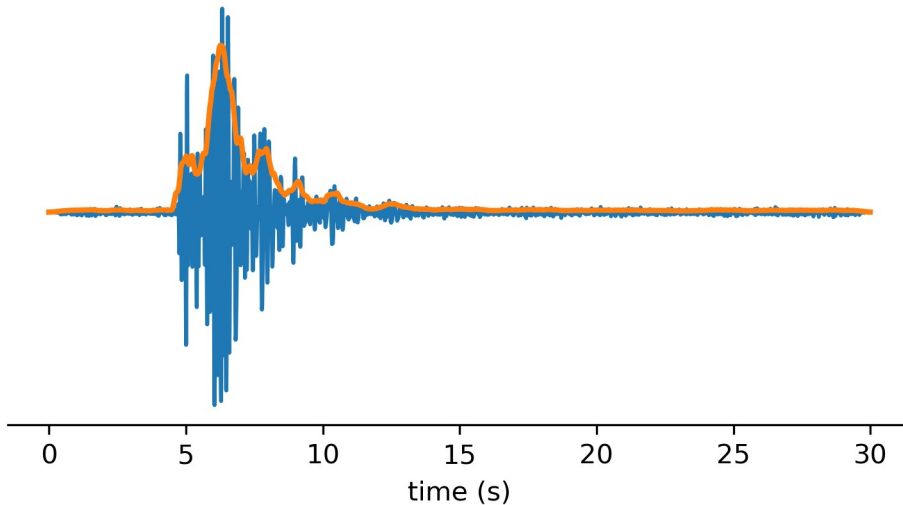
Definition Quality factor

$$Q := 2\pi \frac{\text{total energy}}{\text{energy loss per cycle}}$$

For direct wave:

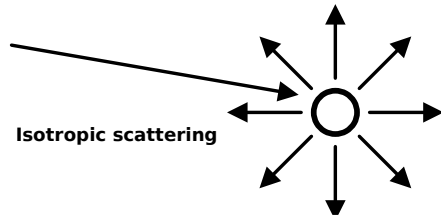
$$Q^{-1} = Q_{\text{intr}}^{-1} + Q_{\text{scatt}}^{-1}$$

$$Q_{\text{intr}}^{-1} = \frac{v}{2\pi fl_a} \quad Q_{\text{scatt}}^{-1} = \frac{v}{2\pi fl}$$

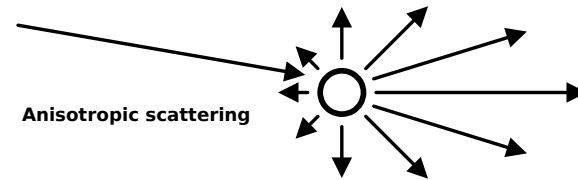


Isotropic vs nonisotropic scattering – transport mean free path

- Mean free path l_0 : Length in which 63% of the wave energy is scattered, mean length between two scattering events
- Transport mean free path l^* : Length in which the propagation direction of 63% of the wave energy becomes independent from its original propagation direction—the wave “forgets” its initial direction due to scattering



$$l^* = l_0 \quad t^* = t_0$$



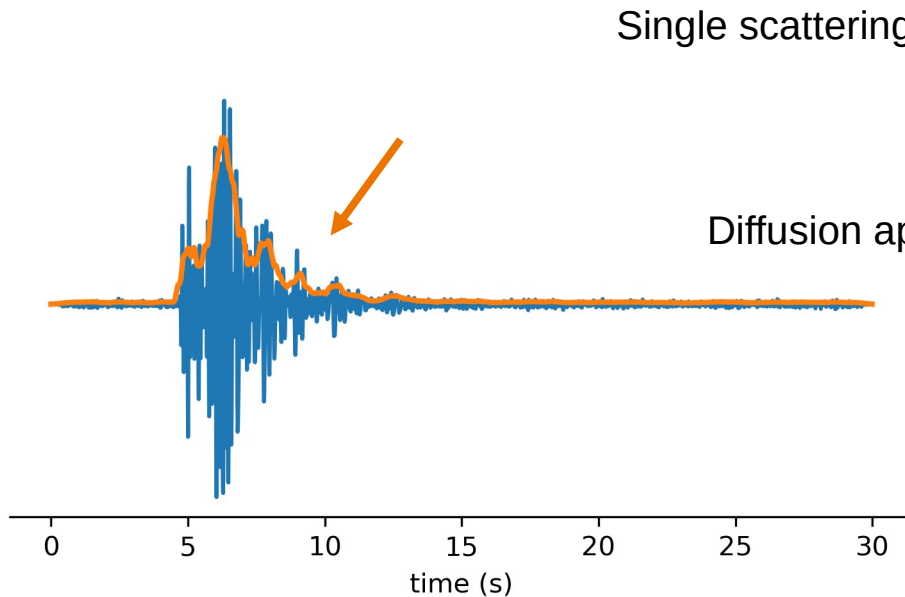
$$l^* > l_0 \quad t^* > t_0$$

$$t_0 = \frac{l_0}{v_s} \quad t^* = \frac{l^*}{v_s}$$

What about coda Q?

Obviously coda Q is not simply the sum of intrinsic and scattering Q as for the direct wave.

The interpretation of coda Q depends on the scattering regime in the coda!



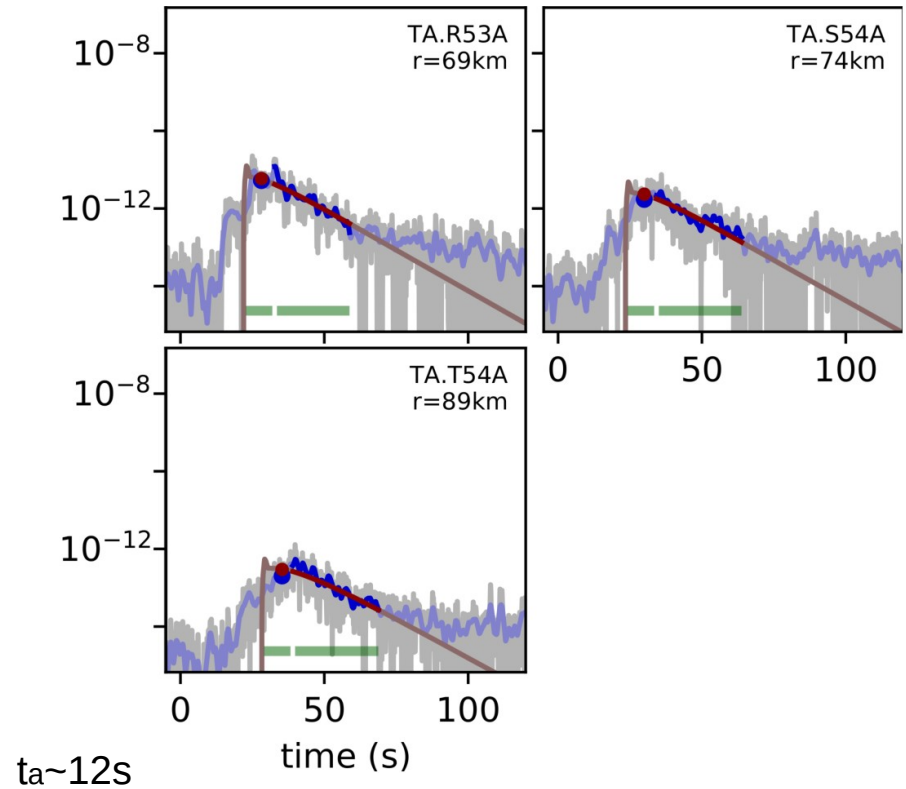
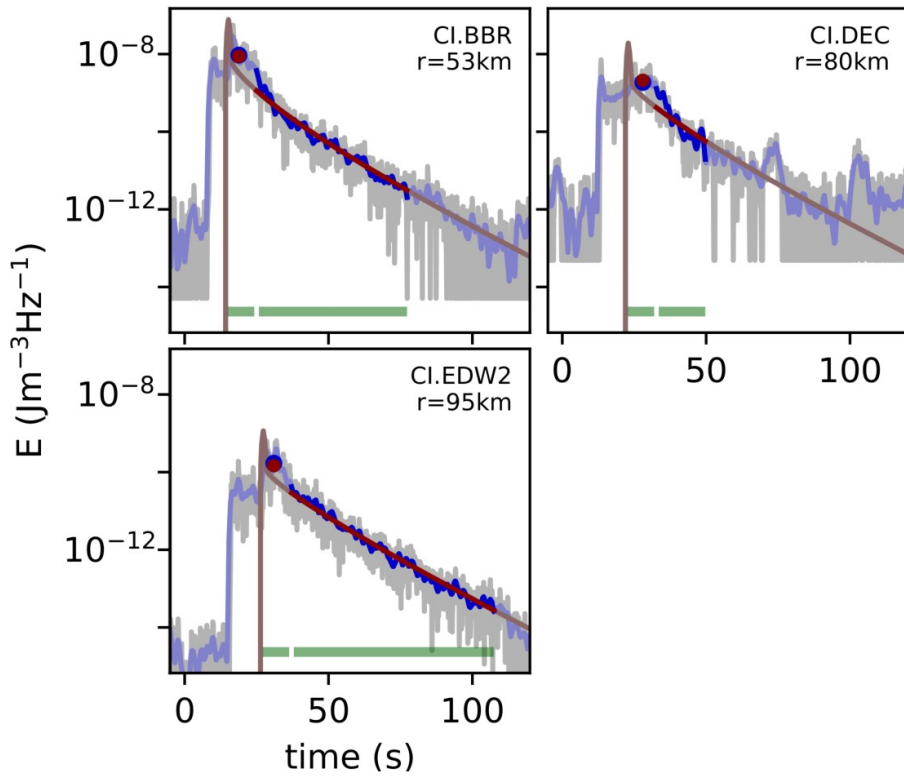
Single scattering approximation:

$$Q_{\text{coda}}^{-1} = Q_{\text{intr}}^{-1} + Q_{\text{scatt}}^{-1}$$

Diffusion approximation:

$$Q_{\text{coda}}^{-1} = Q_{\text{intr}}^{-1}$$

Scattering regime in the coda – transport mean free time



$t^* \sim 40s \Rightarrow$ single and multiple scattering

$t^* \sim 7s \Rightarrow$ diffusion approximation valid

$$Q_{\text{coda}}^{-1} = Q_{\text{intr}}^{-1} + Q_{\text{scatt}}^{-1} ?$$

$$Q_{\text{coda}}^{-1} = Q_{\text{intr}}^{-1}$$

\Rightarrow scattering regime can be determined with the shape of the envelope

Motivation

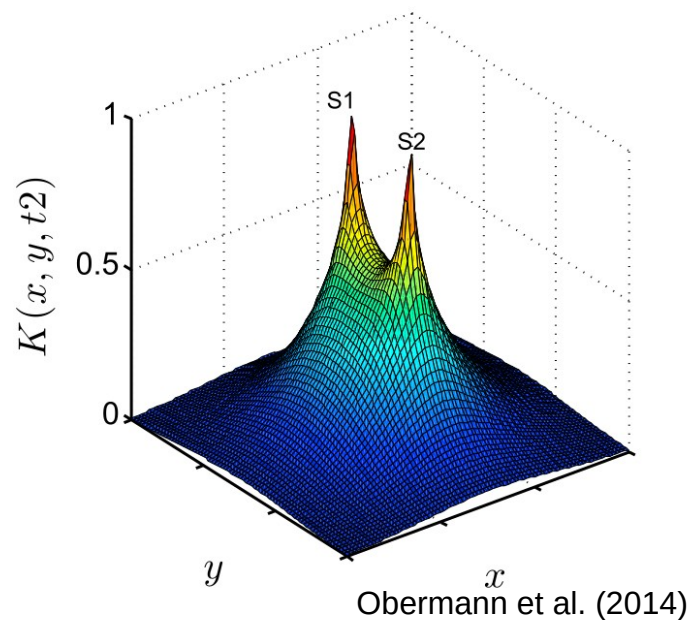
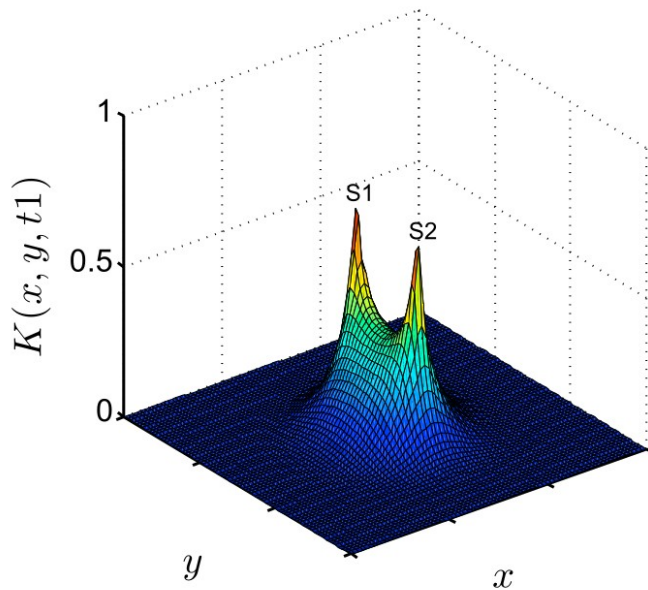
Kernels for tomography of coda Q and dv/v observations

Observations of relative velocity change (dv/v) often use the coda

Coda Q can be determined for each station-earthquake pair (similar to first arrivals) and is therefore predestined for tomography.

=> Need for travel time kernel of the coda

=> Estimate of transport mean free path can confine the shape of the kernel
(and check validity of assumptions leading to kernel estimate)



Source, Site, Path

Seismogram is convolution of

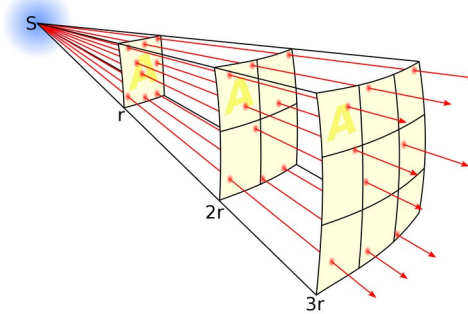
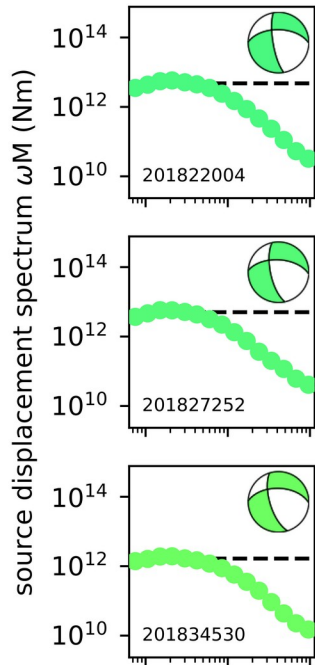
source function

x

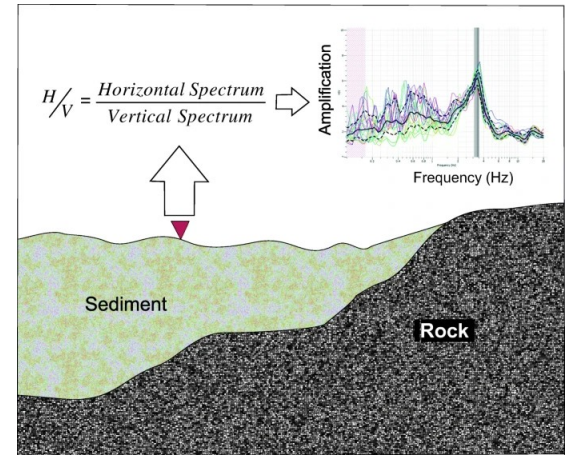
propagation filter

x

site response



- Geometrical spreading
- Attenuation
- Scattering
- Reflections, conversions, ...



Yilmaz al. 2021

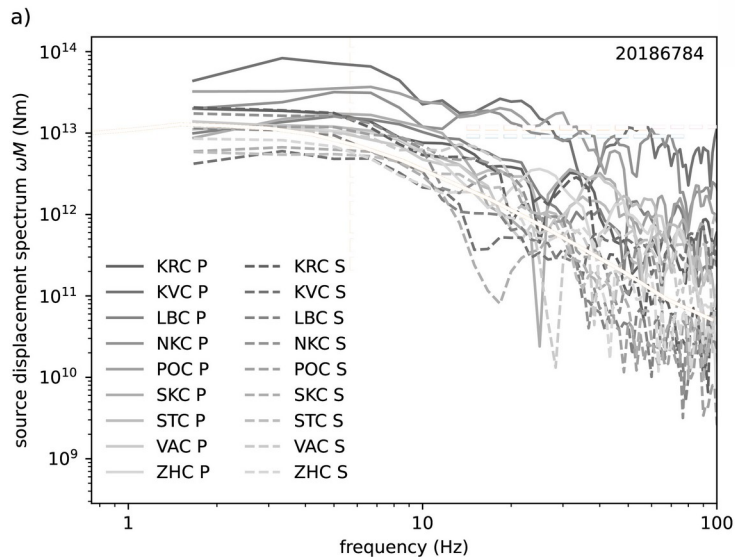
- H/V
- Vs30
- kappa

- Moment tensor
- Moment rate function / source displacement spectrum
- Slip distribution

Conventional method to calculate source spectrum

- Take spectra of waveforms around onset
- Correct for geometrical spreading and radiation pattern
- Optimize seismic moment M_0 , corner freq f_c and attenuation Q

$$\Omega(f) = \frac{\Omega_0 e^{-(\pi f t / Q)}}{\left[1 + (f / f_c)^m\right]^{1/\gamma}} \quad \text{Abercrombie 1995}$$



- Tradeoff between Q and f_c
- Q can be a function of frequency

Spectrum can be used to calculate stress drop.
Self-similarity of differently sized earthquakes?

Qopen method

Separation of intrinsic and scattering
Q by envelope inversion

Idea: Intrinsic attenuation and scattering strength can be separated and quantified with the temporal and spatial shape of the envelope!

Open method for shear waves

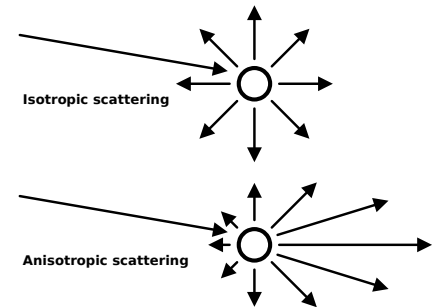
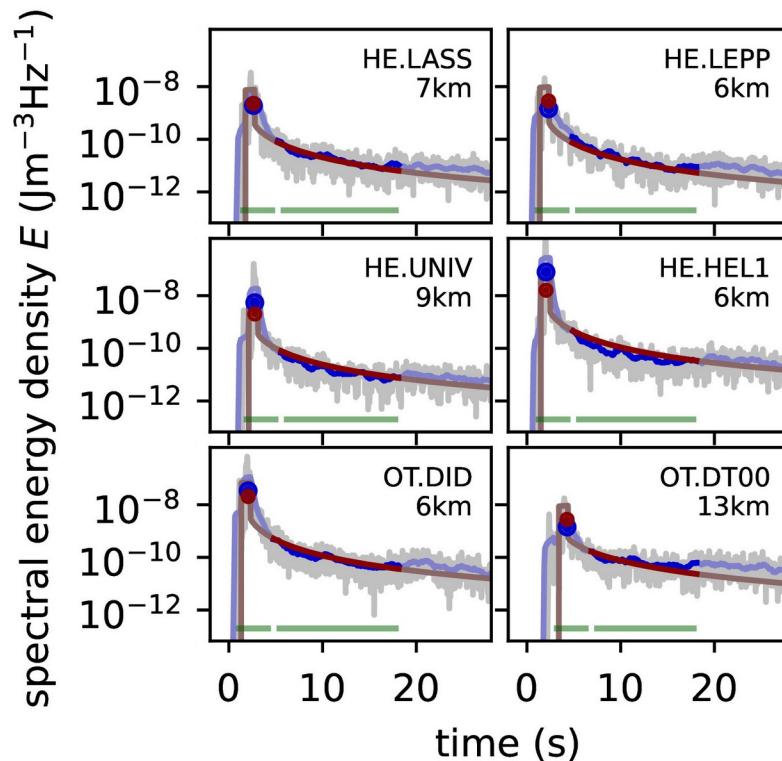
$$\begin{array}{c}
 \text{Modelled energy} \\
 \text{density envelope} \\
 \hline
 E_{\text{mod}ij} = \underbrace{R_i}_{\substack{\text{station index} \\ \text{sample index} \\ \text{site amplification factor}}} \underbrace{W}_{\text{spectral source energy}} \underbrace{G(g_0, r_i, t_j)}_{\substack{\text{Scattering} \\ \text{Green's function} \\ \text{scattering parameter} \\ \rightarrow Q_{sc}}} \underbrace{e^{-bt_j}}_{\substack{\text{Constant} \\ \text{damping} \\ \text{absorption constant} \\ \rightarrow Q_i}}
 \end{array}$$

- G accounts for geometrical spreading and scattering => here G is analytic
- Compare with observed envelopes of S wave + coda
- Invert for R_i , W , g_0 and b (optimization in g_0 + least squares log fit)
- Repeat the steps for all frequency bands
- Repeat with different earthquakes
- Assumptions:
 - homogeneous half space
 - point source (small EQ)
 - moment tensor ignored

Sens-Schönfelder & Wegler 2006, Eulenfeld & Wegler 2016

Imprint of anisotropic scattering

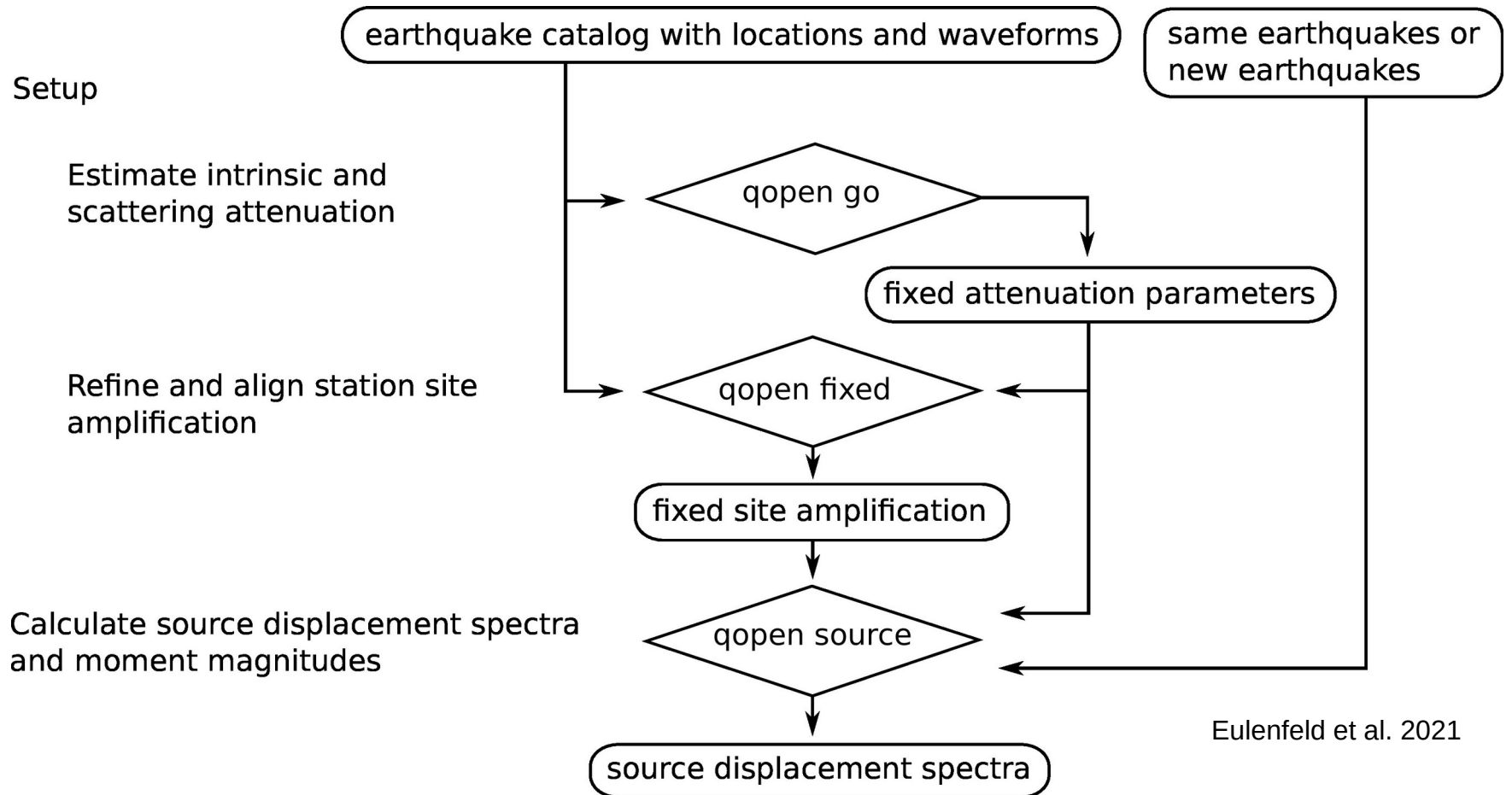
- Qopen assumes isotropic scattering, this is often a bad assumption
- In an anisotropic scattering environment the scattering strength estimated with Qopen relates to the transport mean free path (Gaebler et al. 2015)
- Model cannot predict correct envelope directly after the S body wave
=> In the inversion the envelope inside the direct wave window needs to be averaged



Estimation of site response and source spectra

$$E_{\text{mod}ij} = R_i W G(g_0, r_i, t_j) e^{-bt_j}$$

Modelled energy density envelope: $E_{\text{mod}ij}$
 station index: i
 sample index: j
 site amplification factor: R_i
 spectral source energy: W
 Scattering Green's function: $G(g_0, r_i, t_j)$
 scattering parameter: g_0
 distance: r_i
 Constant damping: e^{-bt_j}
 absorption constant: $b \rightarrow Q_i$



Eulenfeld et al. 2021

Source spectra and seismic moments

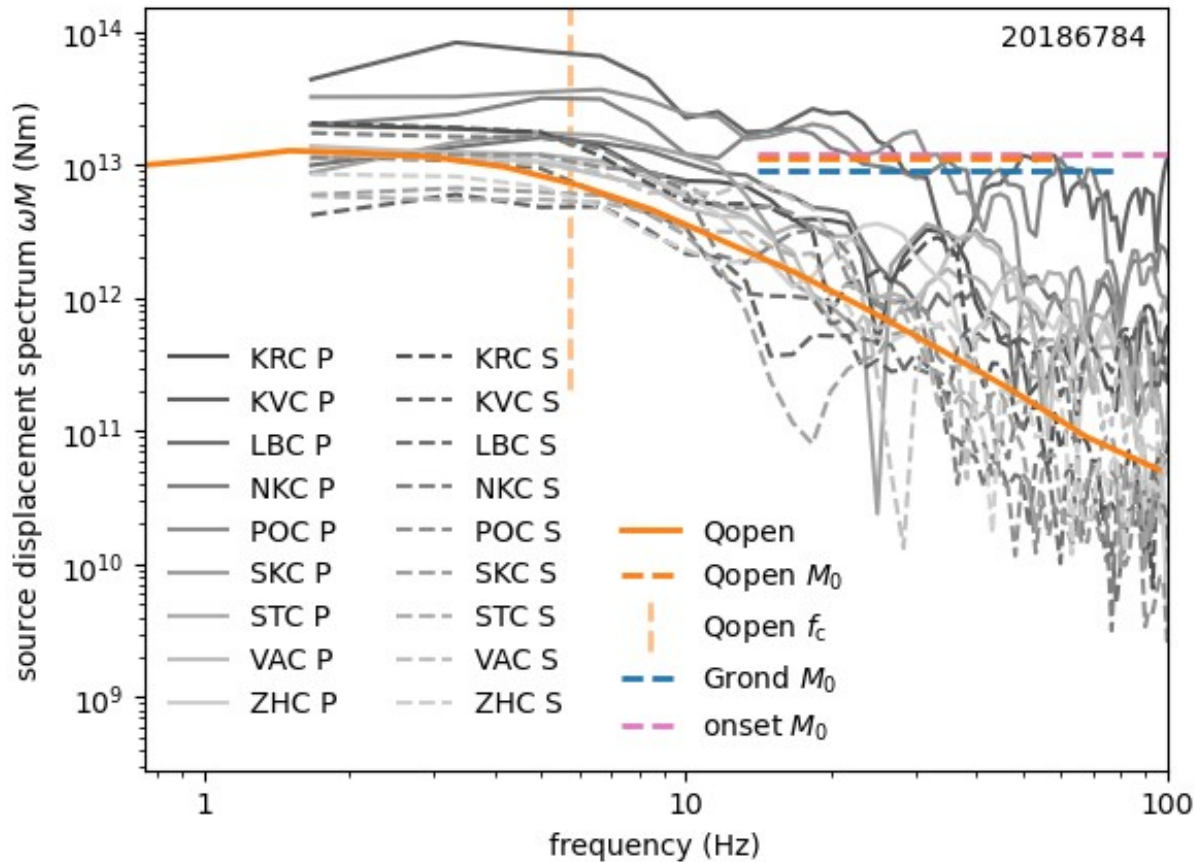
Qopen inversion
 vs Grond moment tensor inversion
 vs spectra from Fourier transform
 of body waves

Abercrombie 1995

$$\omega M(f) = M_0 \left(1 + \left(\frac{f}{f_c} \right)^{\gamma n} \right)^{-\frac{1}{\gamma}}$$

source displacement spectrum
seismic moment
high frequency fall-off
(2 for omega-square model)

corner sharpness
corner frequency

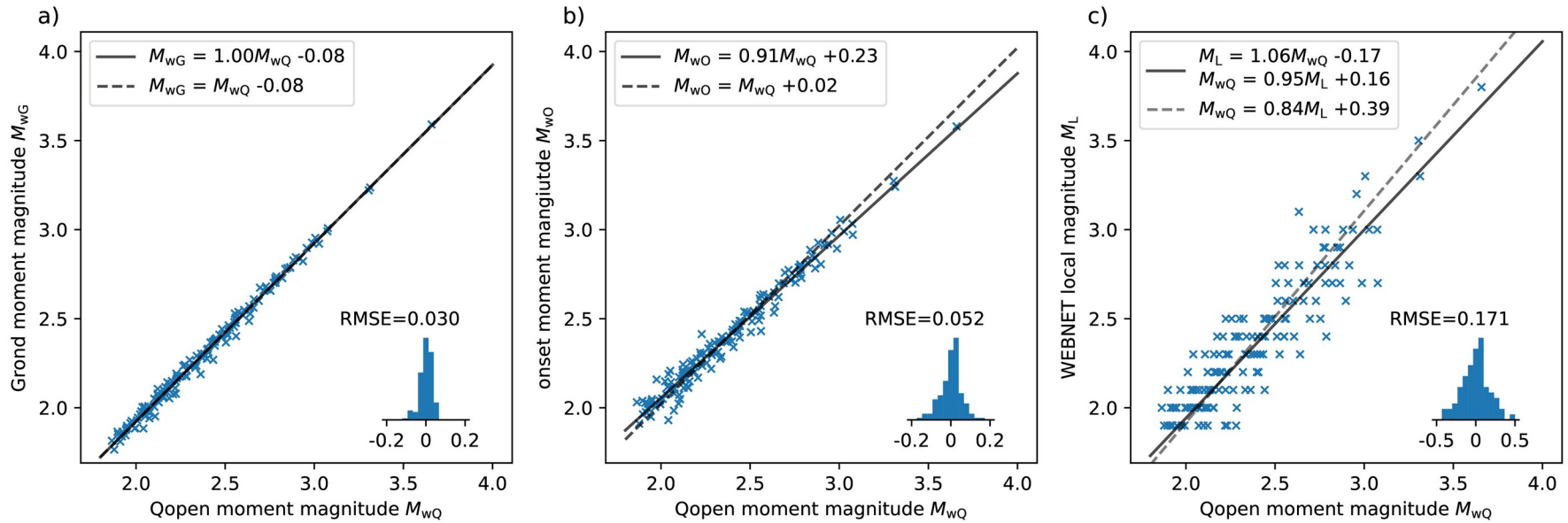


Czech 2018 EQ swarm – moment magnitudes

Qopen inversion

vs Grond moment tensor inversion

vs spectra from Fourier transform of body waves



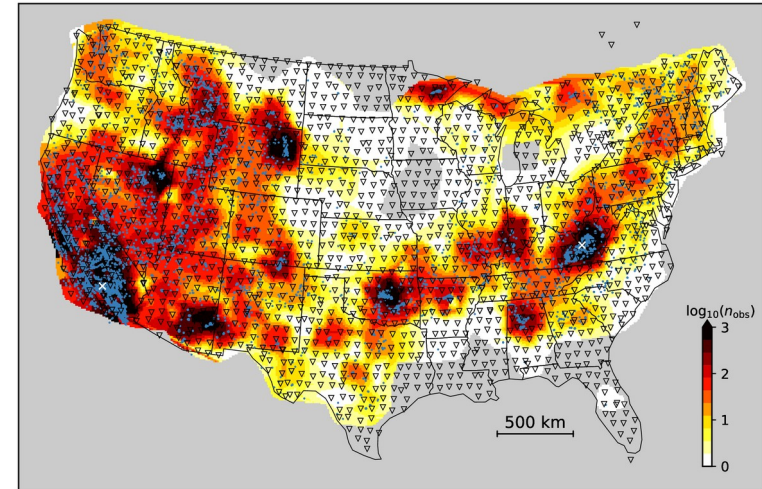
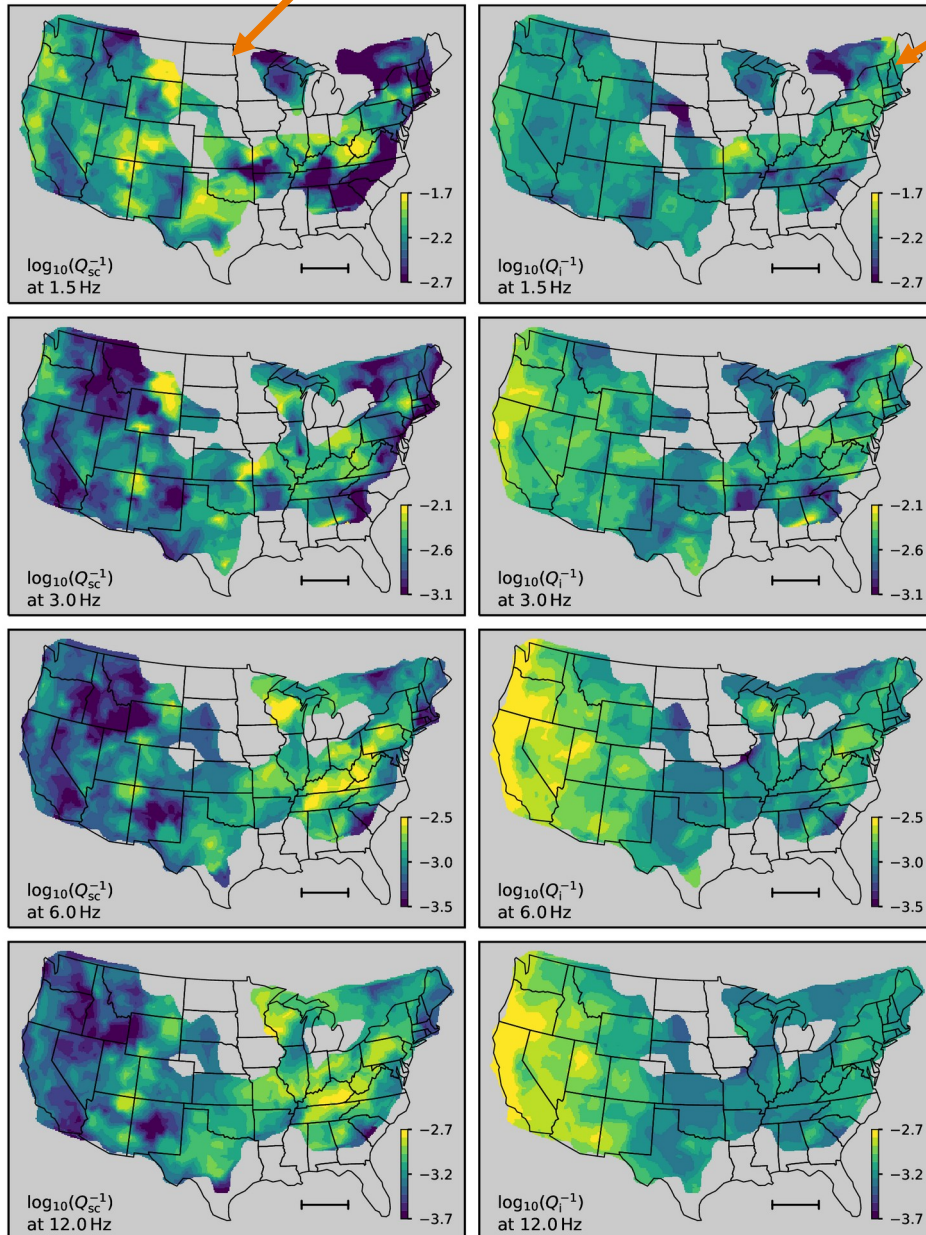
=> Robust estimation of moment magnitudes for small earthquakes

=> Can be used in high scattering environments with a lack of impulsive onsets

Code available at github.com/trichter/qopen

Applications

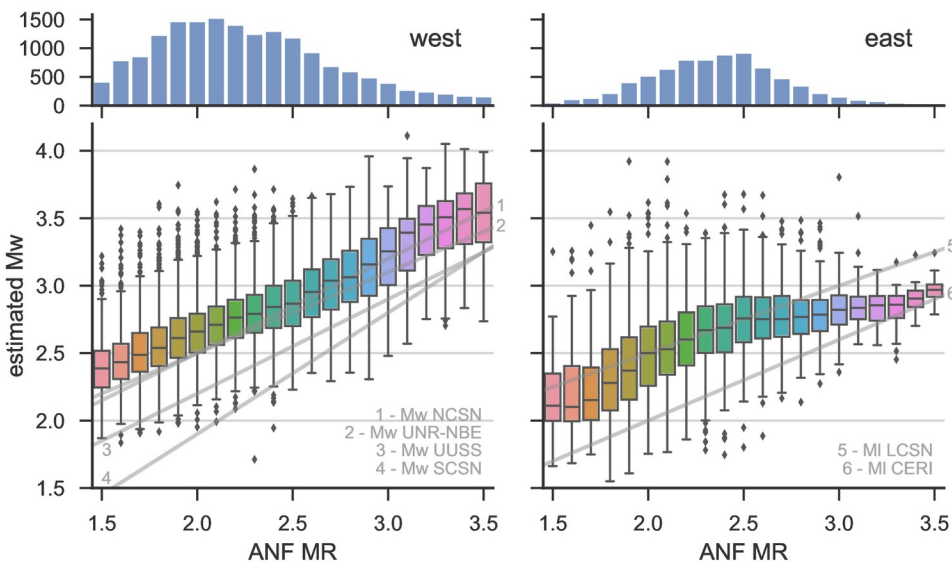
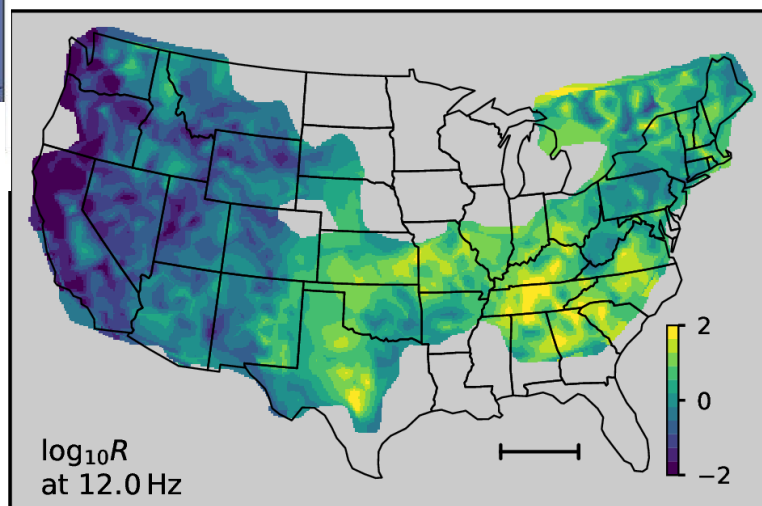
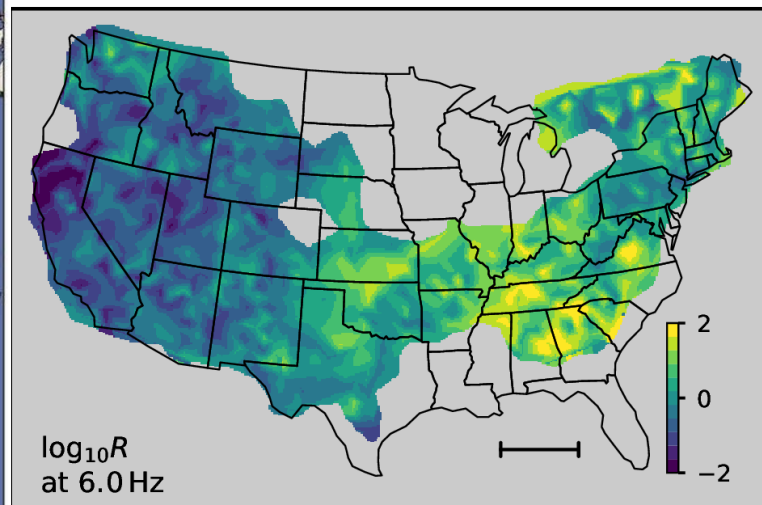
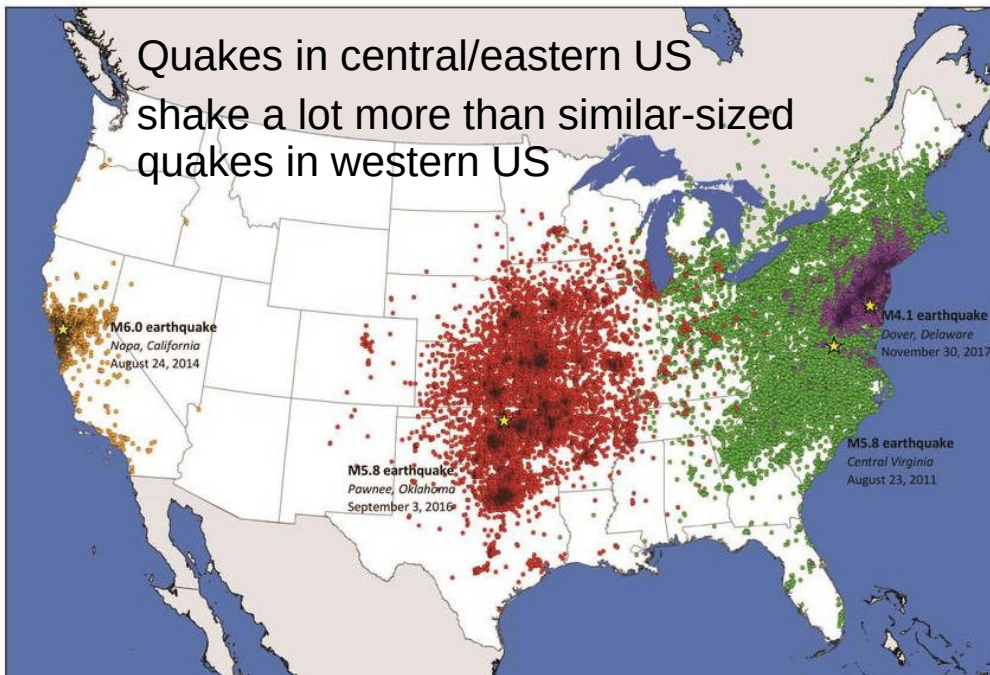
USArray – scattering strength (left) versus intrinsic attenuation (right)



Eulenfeld & Wegler 2017

Application USArray – high freq site amplification, magnitudes

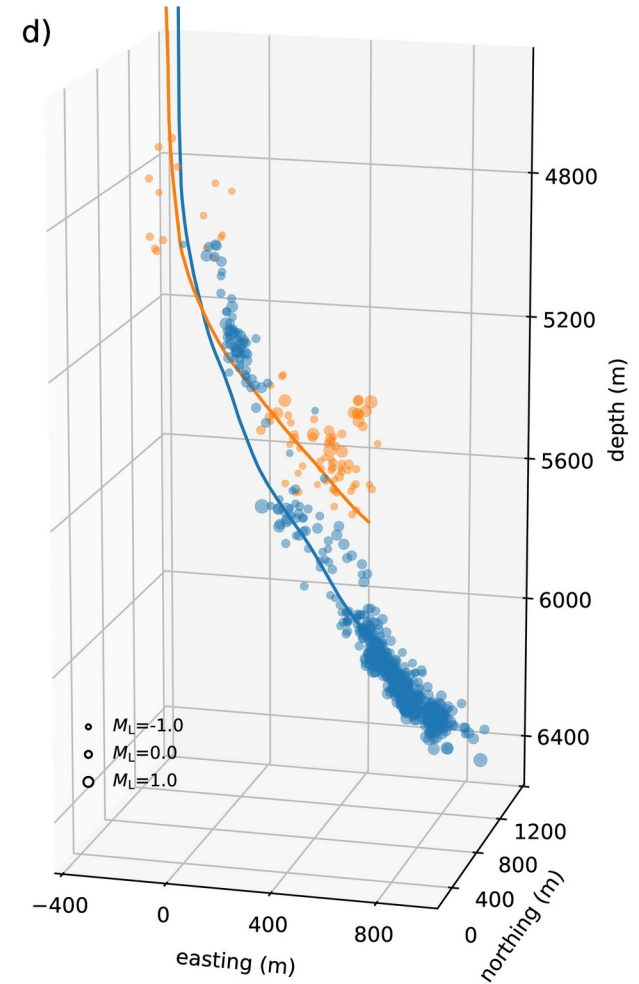
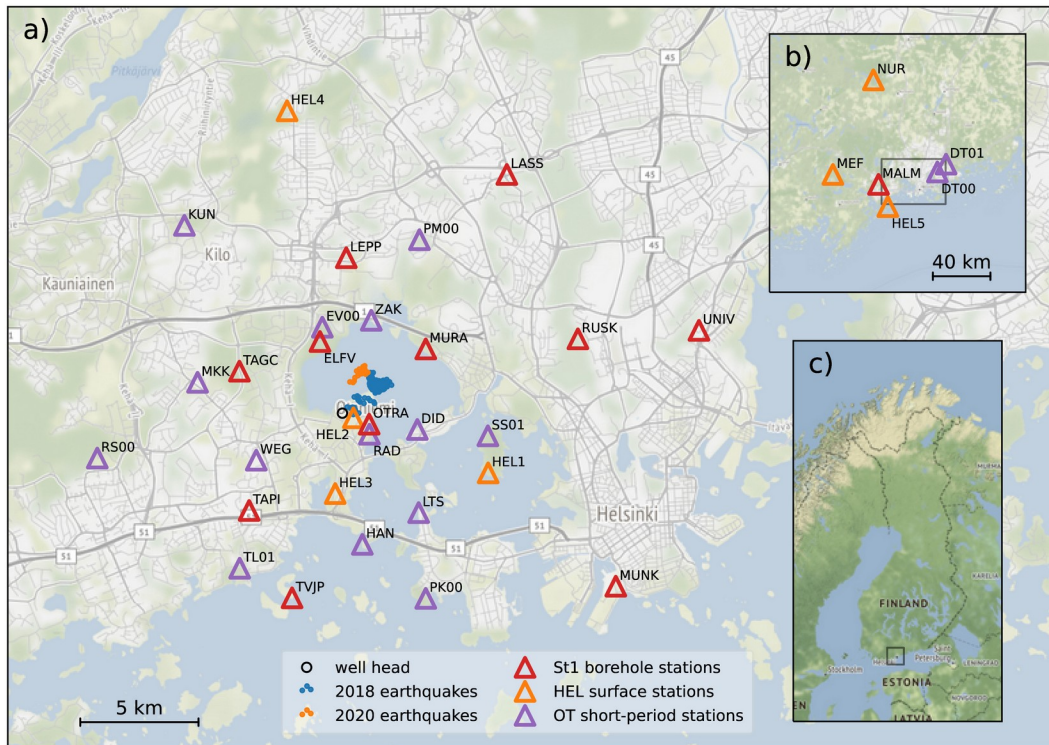
Quakes in central/eastern US
shake a lot more than similar-sized
quakes in western US



Eulenfeld & Wegler 2017

Helsinki 2018 and 2020 stimulation

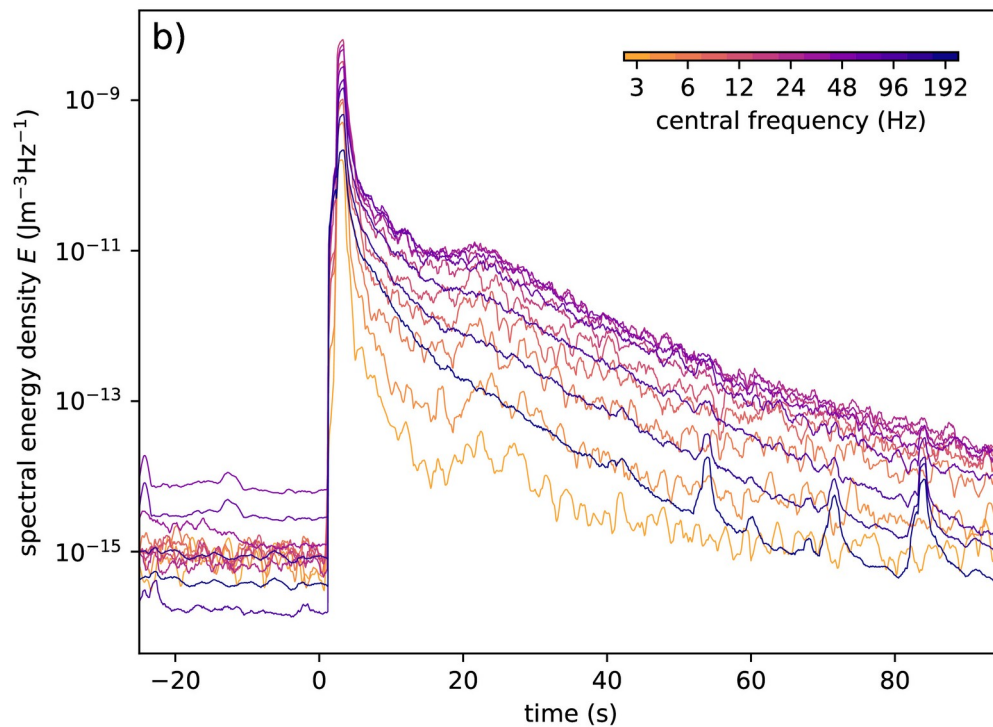
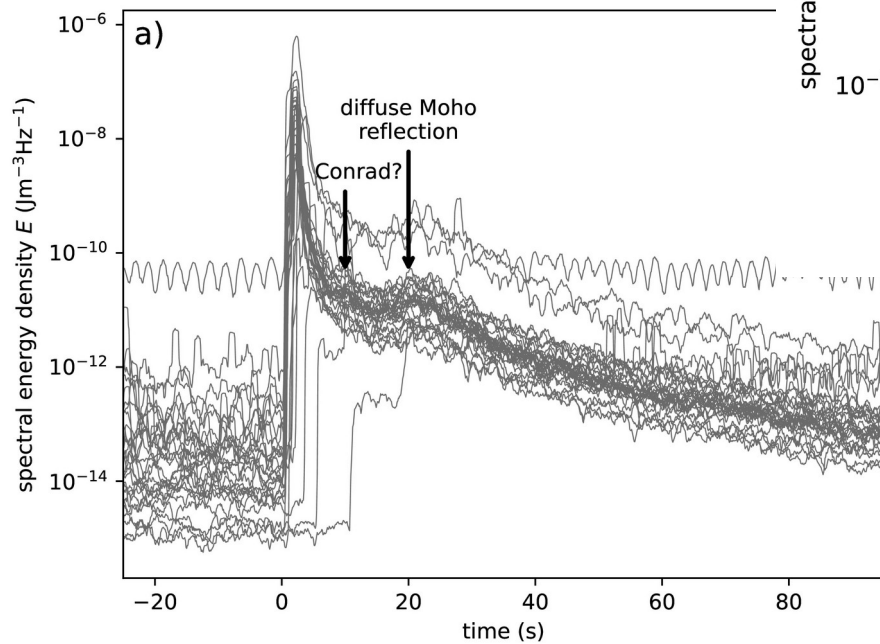
- 2018 stimulation induced ~450 earthquakes (blue) with $0 \leq M_L \leq 1.8$, 90 MPa peak well-head pressure, 18 000 m³ volume
- 2020 stimulation induced ~25 earthquakes (orange) with $0 \leq M_L \leq 1.8$, 70 MPa, 2 900 m³ volume



Eulenfeld et al. 2023

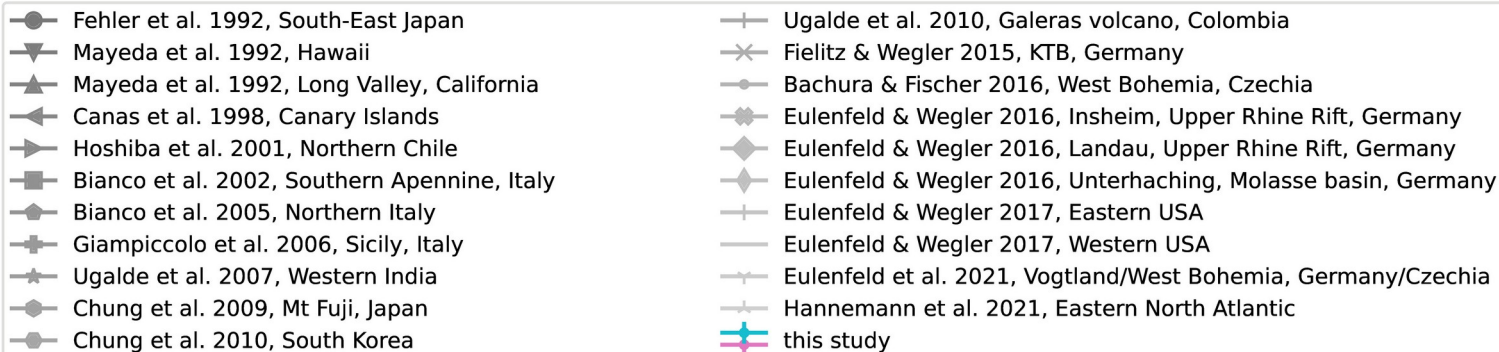
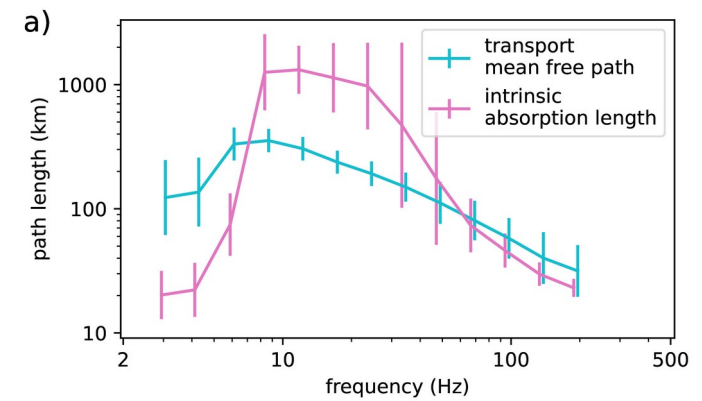
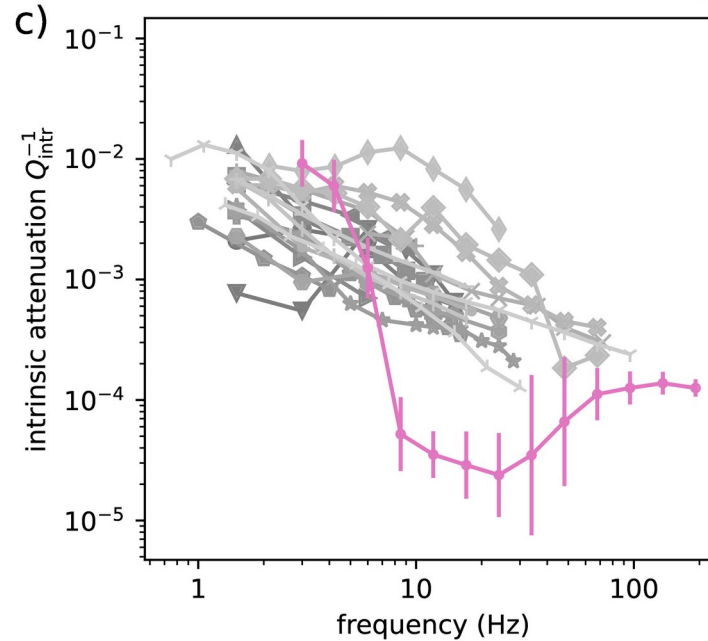
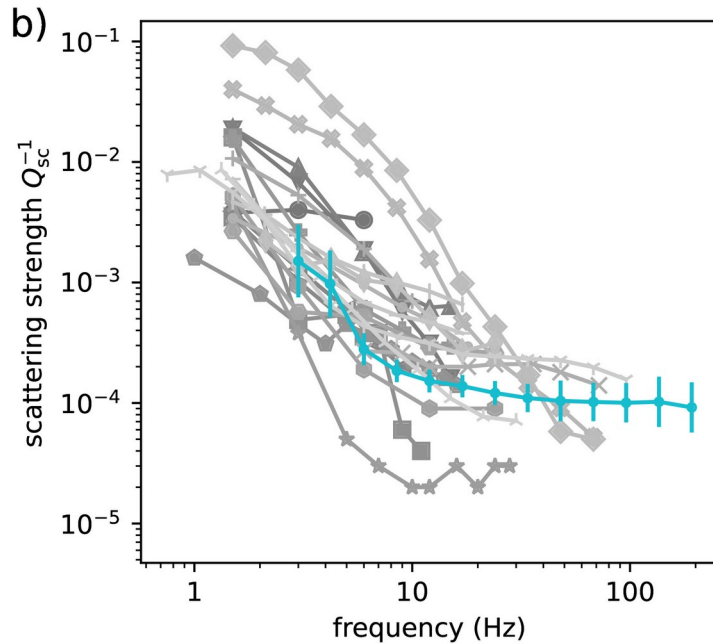
Helsinki 2018 stimulation – example envelopes for 1 event

All stations 8 Hz – 16 Hz

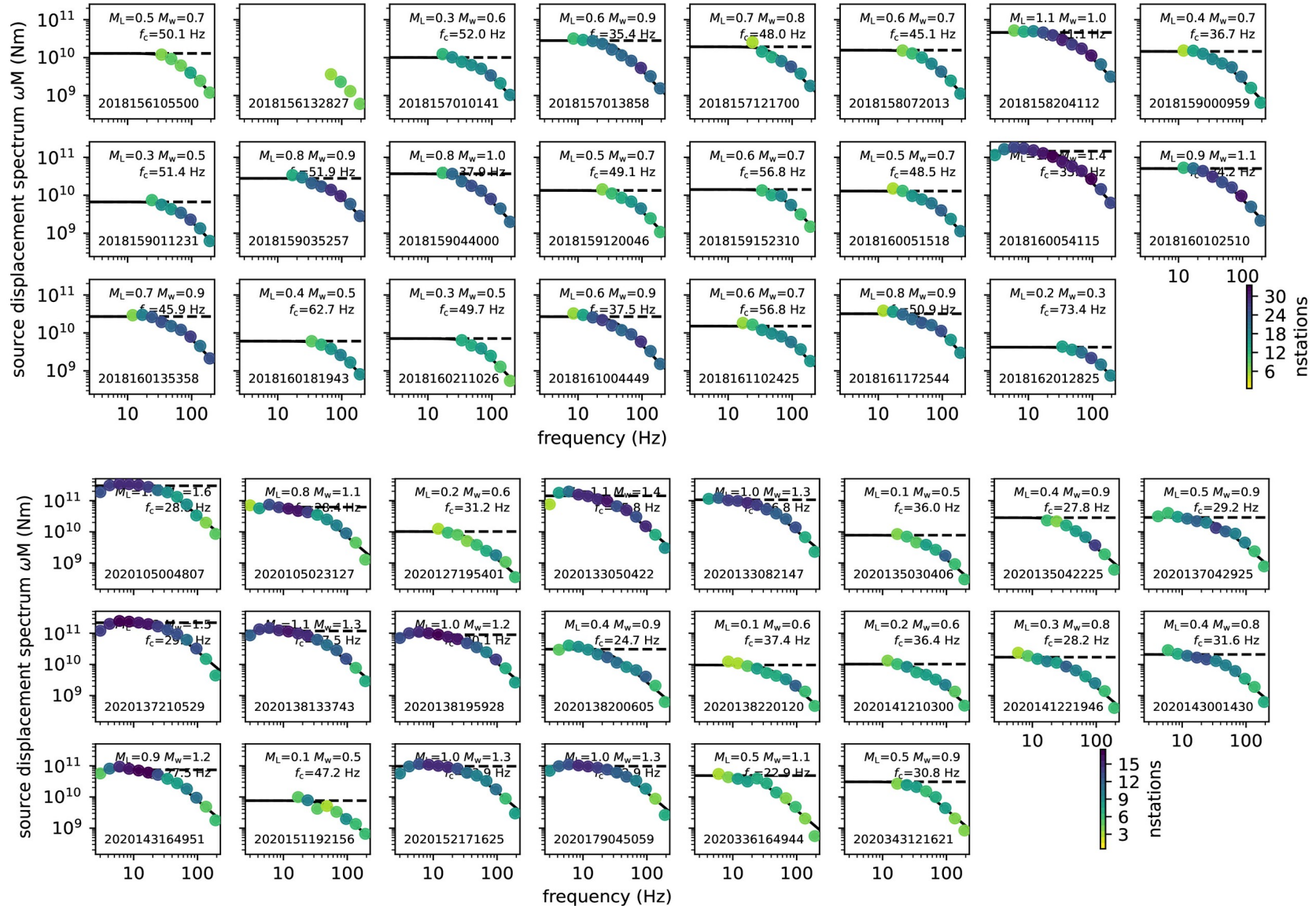


Single station,
different frequency bands

Helsinki 2018 stimulation – Q values

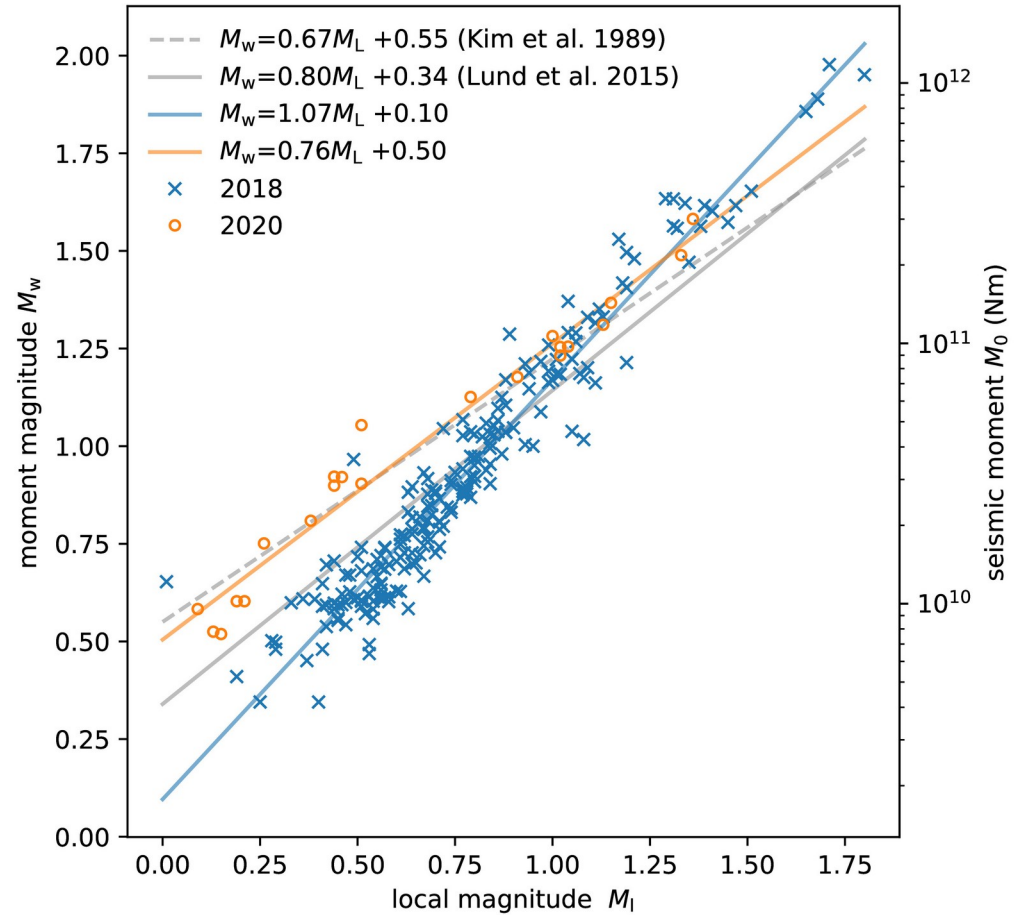


Helsinki 2018/2020 – source displacement spectra

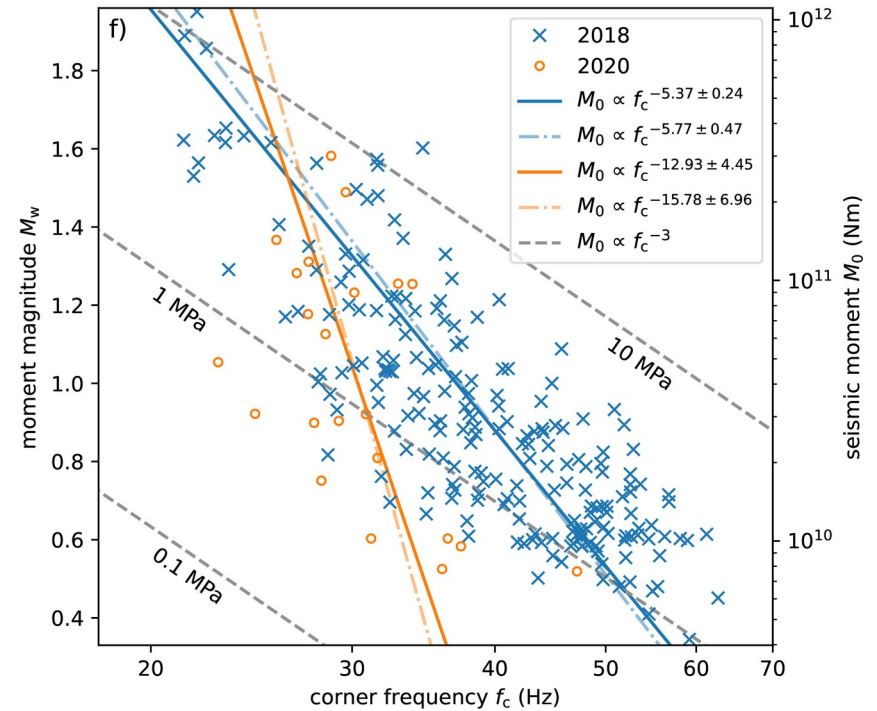
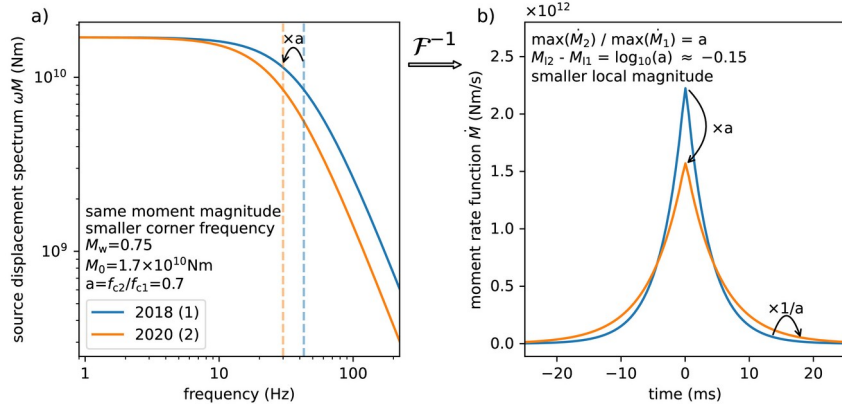


Helsinki 2018/2020 – moment magnitudes

- M_w versus M_L relationship for the two stimulations
- 2020 events have systematically smaller M_L for same M_w compared to 2018 events



- M_w versus f_c relationship for the two stimulations
- 2020 events have systematically smaller f_c for same M_w compared to 2018 events
- Consistent with M_w - M_L relationship



Summary

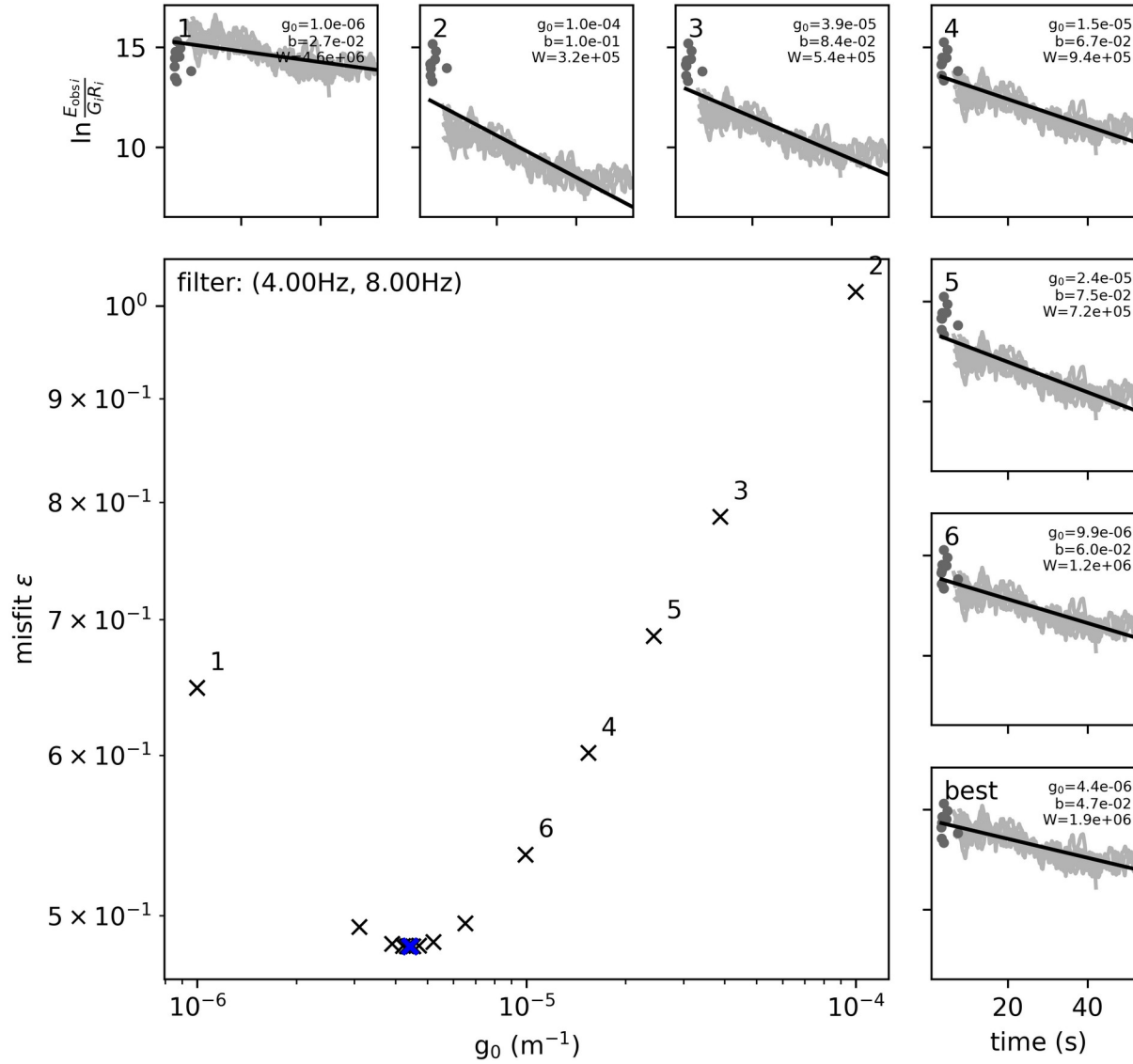
- Quickly estimate scattering and intrinsic attenuation parameters for your local data set
- Estimation of site responses (relative)
- Robust determination of moment magnitude and other source parameters

Thanks!

References

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- Sens-Schönfelder C & Wegler U (2006), Radiative transfer theory for estimation of the seismic moment, Geophysical Journal International, doi: 10.1111/j.1365-246X.2006.03139.x
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- Abercrombie R E (1995), Earthquake source scaling relationships from -1 to 5 ML using seismograms recorded at 2.5 km depth, Journal of Geophysical Research, doi: 10.1029/95JB02397

Open optimization



Eulenfeld & Wegler 2016

Helsinki 2018 stimulation – envelope fits example 16 – 32 Hz

