Stochastic downscaling using Gaussian random fields

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Downscaling to convective scales

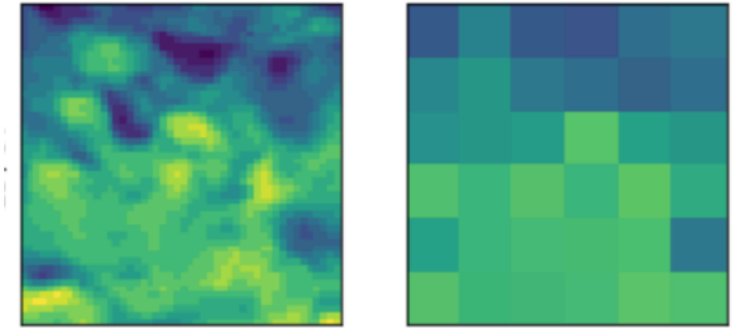
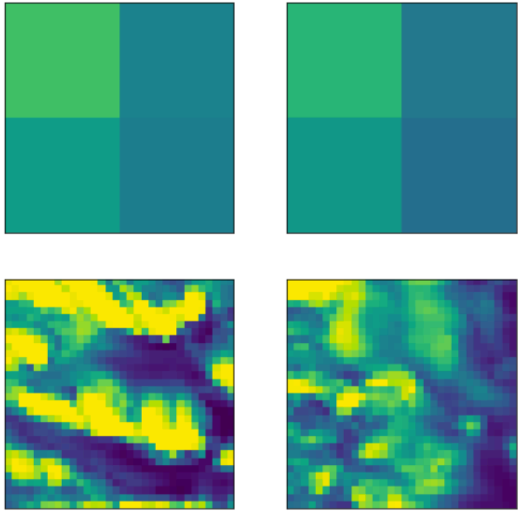


Figure 1: Original wet bulb potential temperature field together with synthetic synoptic scale field (right) formed by block averaging with block size 8×8.

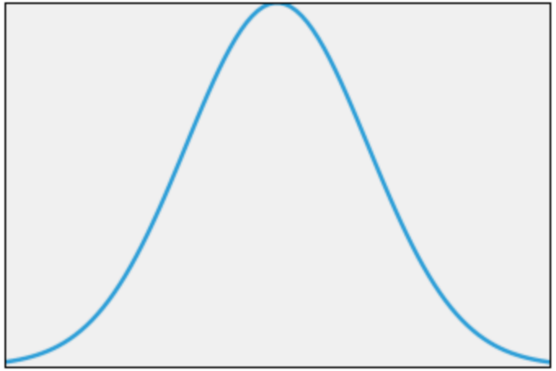
Downscaling to convective scales



The goal:
approximate this
distribution

Gaussian random fields

Gaussian random fields



$$X \sim \mathcal{N}(\mu, \sigma^2)$$

$$X = \mu + \sigma R \qquad R \sim \mathcal{N}(0, 1)$$

$$\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \mathrm{C})$$

$$\mathbf{X} = \boldsymbol{\mu} + \mathrm{C}^{\frac{1}{2}}\mathbf{R} \qquad \mathbf{R} \sim \mathcal{N}(\mathbf{0}, \mathrm{I})$$

Gaussian distributions

C =

Variance of A	Cov(A,B)
Cov(B,A)	Variance of B

$$\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \mathrm{C})$$

$$\mathbf{X} = \boldsymbol{\mu} + \mathrm{C}^{\frac{1}{2}}\mathbf{R} \qquad \mathbf{R} \sim \mathcal{N}(\mathbf{0}, \mathrm{I})$$

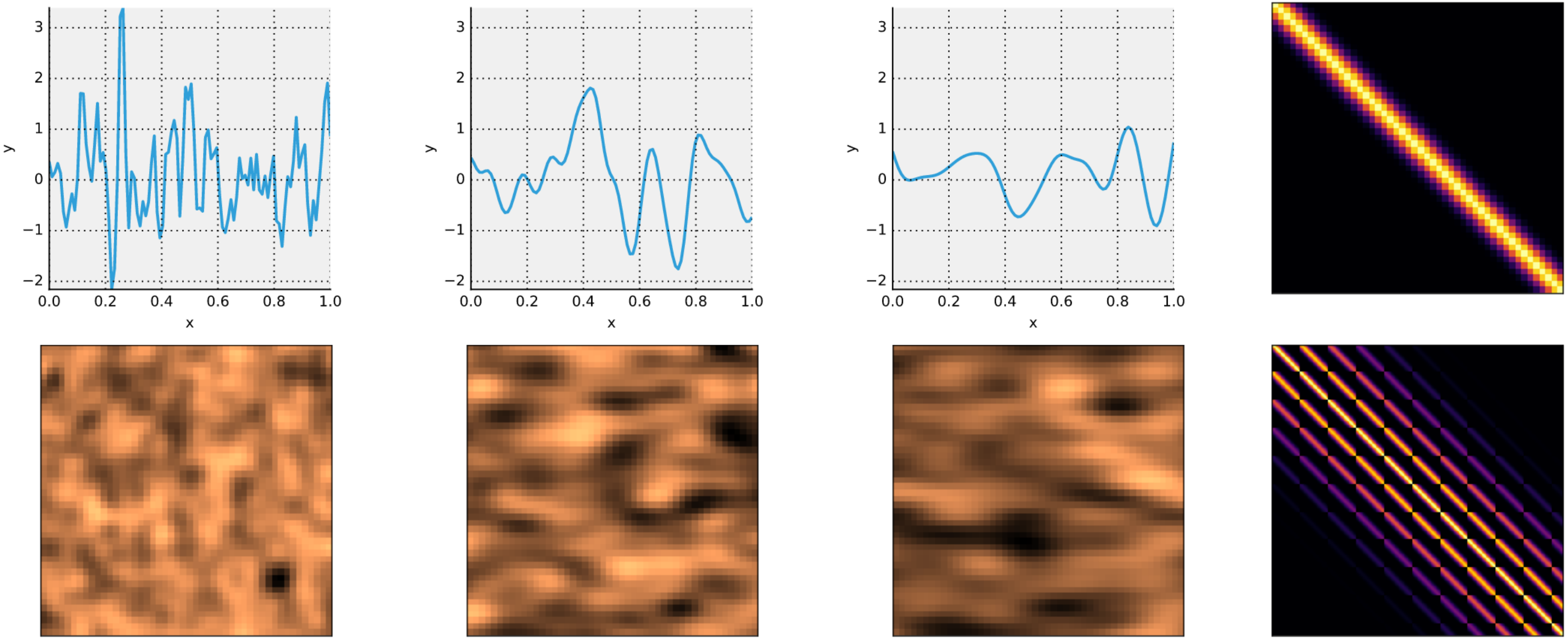
Gaussian random fields

Samples

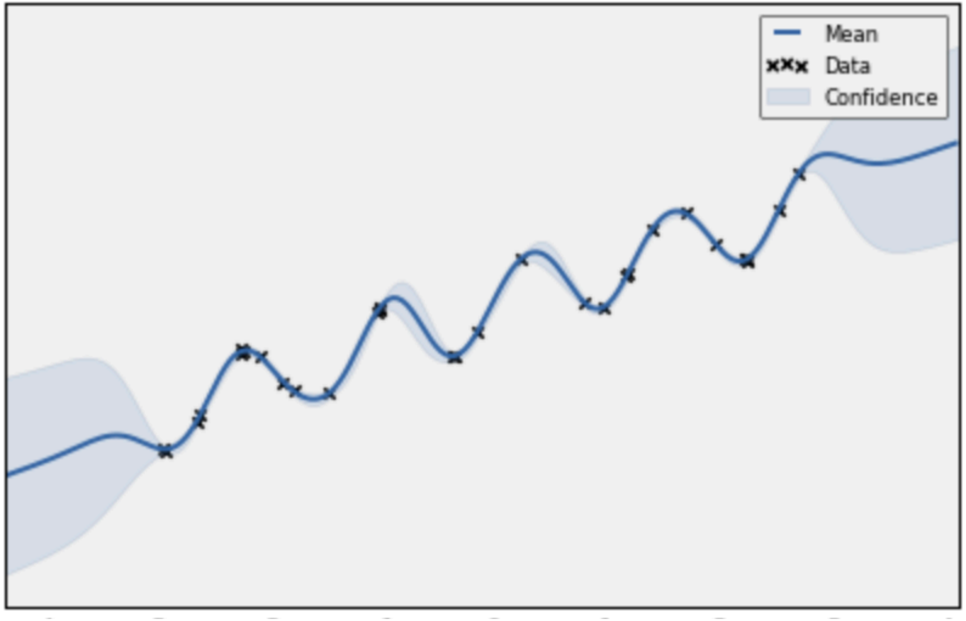
Example covariance

Gaussian process

Gaussian field



Conditioning



$$\mu_{t|o} = \mu_t + C_{t,o}^T \cdot C_o^{-1} \cdot (\text{obs} - \mu_o)$$

$$C_{t|o} = C_t - (C_{t,o}^T \cdot C_o^{-1} \cdot C_{t,o})$$

Likelihood and inference

Log likelihood: $$\log p(\mathbf{x}|\theta) = -\frac{1}{2}\left(n \log(2\pi) + n \log|K| + \sum_{i=1}^{n} \mathbf{x}_i^T K^{-1} \mathbf{x}_i\right)$$

Fit parameters (e.g. kernel length scale) via maximum likelihood estimation or similar.

Simple 1D optimisation, gradients easy to compute.

Sampling

$$\mathbf{X} = \boldsymbol{\mu} + \mathrm{C}^{\frac{1}{2}}\mathbf{R} \qquad \mathbf{R} \sim \mathcal{N}(\mathbf{0}, \mathrm{I})$$

In practice, use the Cholesky decomposition instead of the true square root.

How to apply this to downscaling?

Conditioning on spatial averages

Given an observed low resolution field, what is our new distribution?

Covariance is linear:

$$cov\left(\sum_i \mathbf{X}_i, \mathbf{Y}\right) = \sum_i cov(\mathbf{X}_i, \mathbf{Y}))$$

Use to find covariances:

$$\mathrm{C}_{high} \rightarrow \mathrm{C}_{low}, \mathrm{C}_{high,low}$$

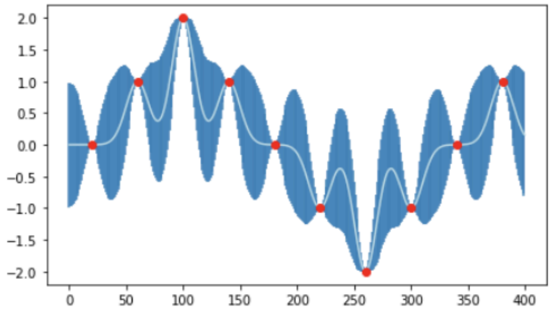
Condition on low resolution fields:

$$\mathrm{C}_{h|l} = \mathrm{C}_h - (\mathrm{C}_{h,l}^T \cdot \mathrm{C}_l^{-1} \cdot \mathrm{C}_{h,l})$$

$$\boldsymbol{\mu}_{h|l} = \mathrm{C}_{h,l}^T \cdot \mathrm{C}_l^{-1} \cdot \mathbf{obs}_l$$

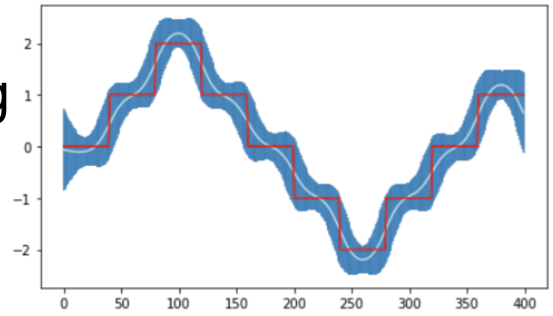
Spatial conditioning

Point

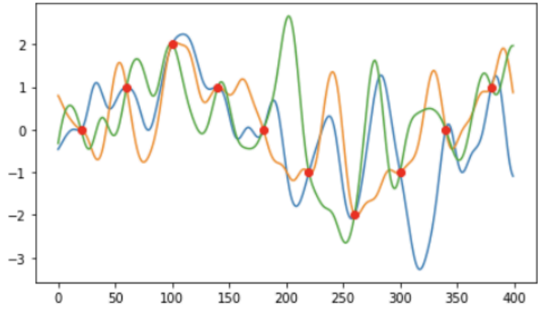


(a)

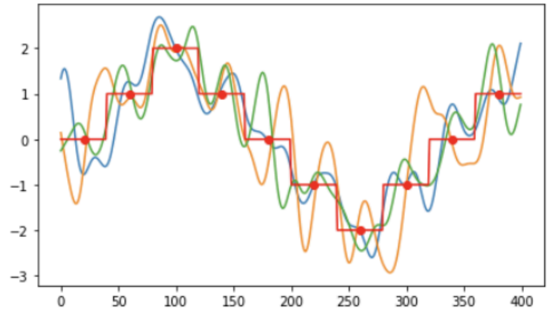
Spatial avg



(b)

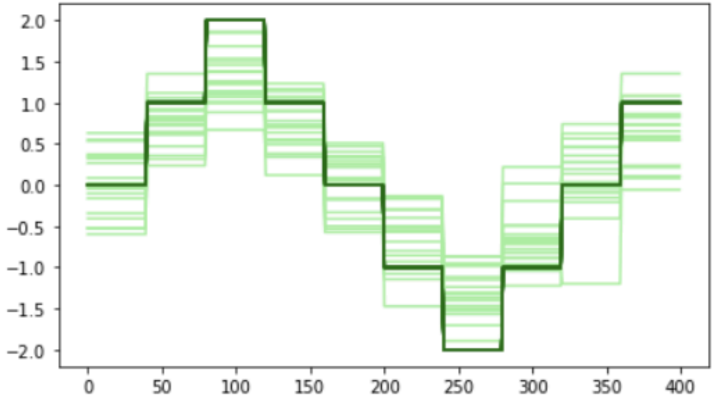


(c)

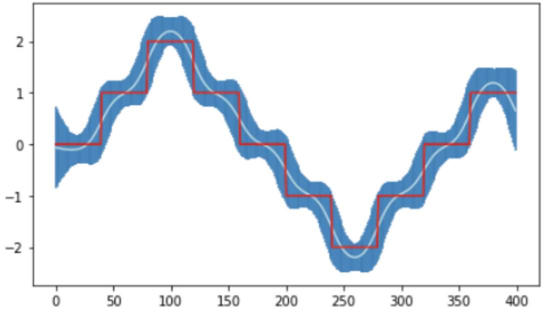


(d)

Spatial conditioning



Spatial avg



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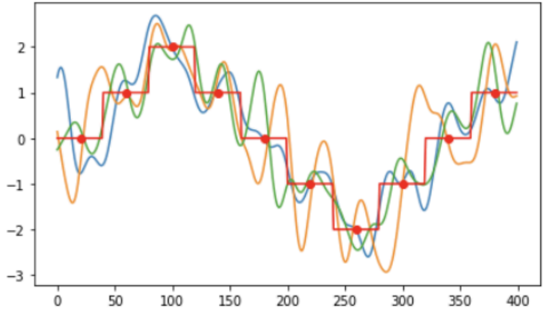

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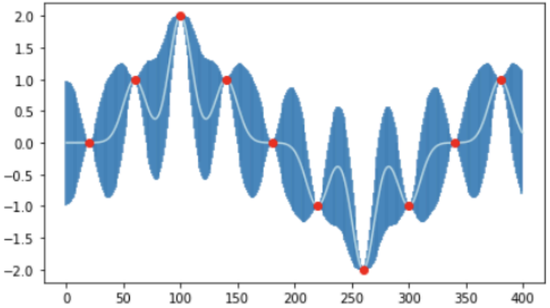
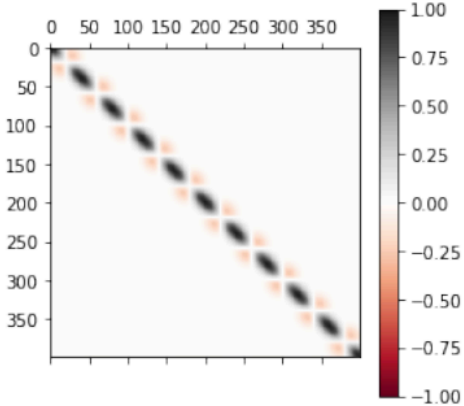
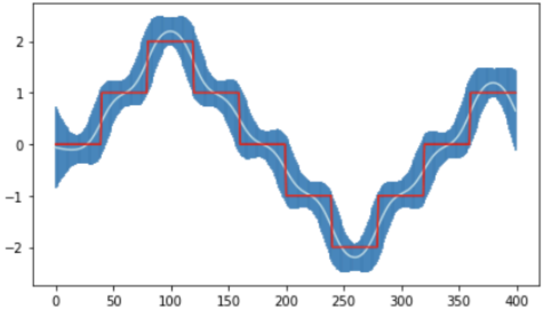
Spatial conditioning

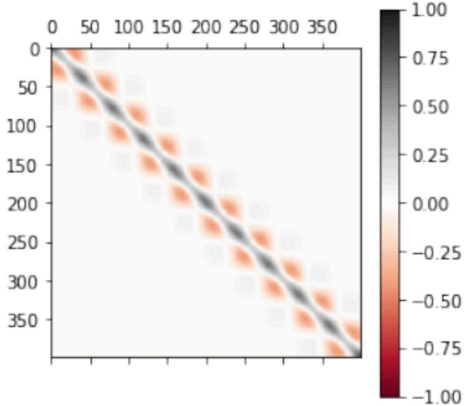
Point





Spatial avg





Spatial conditioning

Joint distribution of high res and low res fields:

Estimate by maximum likelihood

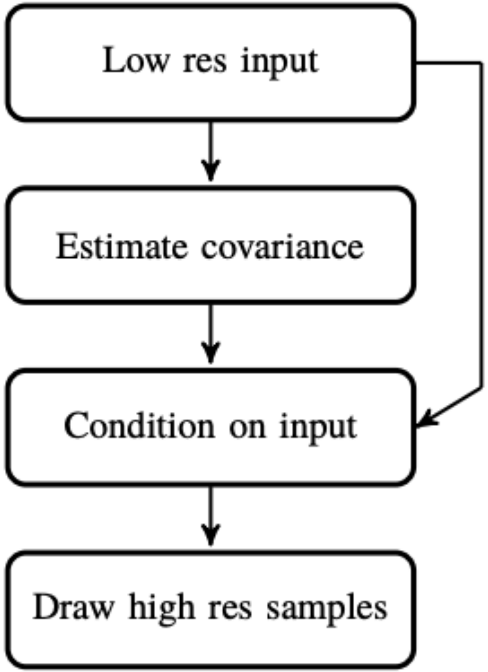
$$\mathbb{E}\begin{pmatrix} \mathbf{x} \\ A\mathbf{x} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu}_t \\ \boldsymbol{\mu}_o \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu}_t \\ A\boldsymbol{\mu}_t \end{pmatrix} \qquad \mathrm{Cov}\begin{pmatrix} \mathbf{x} \\ A\mathbf{x} \end{pmatrix} = \begin{pmatrix} C_t & C_{o,t} \\ C_{t,o} & C_o \end{pmatrix} = \begin{pmatrix} \boxed{C_t} & C_t A^T \\ AC_t & AC_t A^T \end{pmatrix}$$

Conditional distribution of high res conditioned on low res:

$$p(\mathbf{x} | A\mathbf{x} = \bar{\mathbf{x}})$$

$$\boldsymbol{\mu}_{t|o} = \boldsymbol{\mu}_t - C_t A^T (AC_t A^T)^{-1} (\bar{\mathbf{x}} - A\boldsymbol{\mu}_t)$$

$$C_{t|o} = C_t - C_t A^T (AC_t A^T)^{-1} AC_t.$$


Low res input
Estimate covariance
Condition on input
Draw high res samples

Experiment on model data

Synthetic coarse-graining

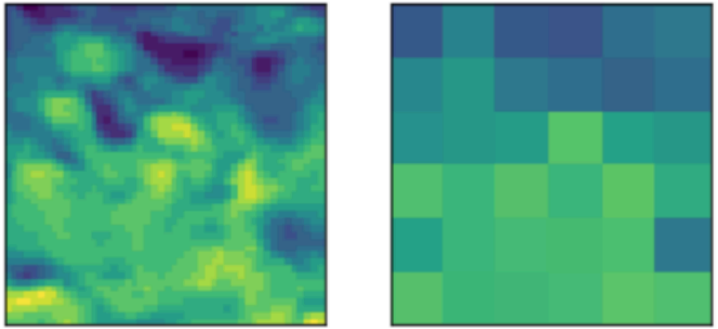
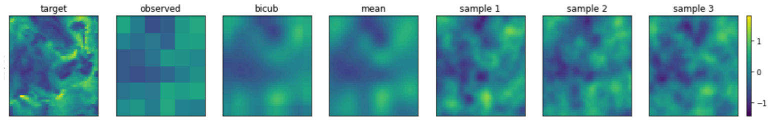


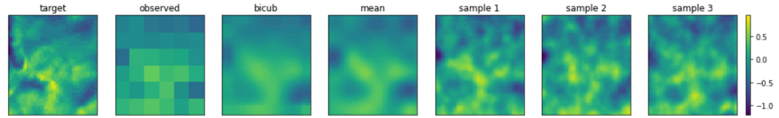
Figure 1: Original wet bulb potential temperature field together with synthetic synoptic scale field (right) formed by block averaging with block size 8×8.

mesoscale (4x4 pixels, ~10km)
synoptic (8x8 pixels, ~20km)

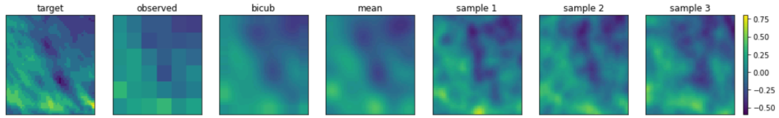
Results



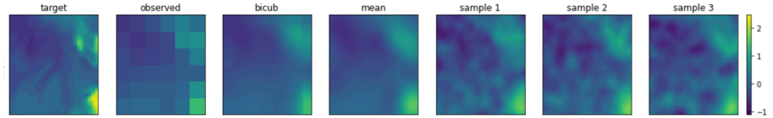
(a)



(b)



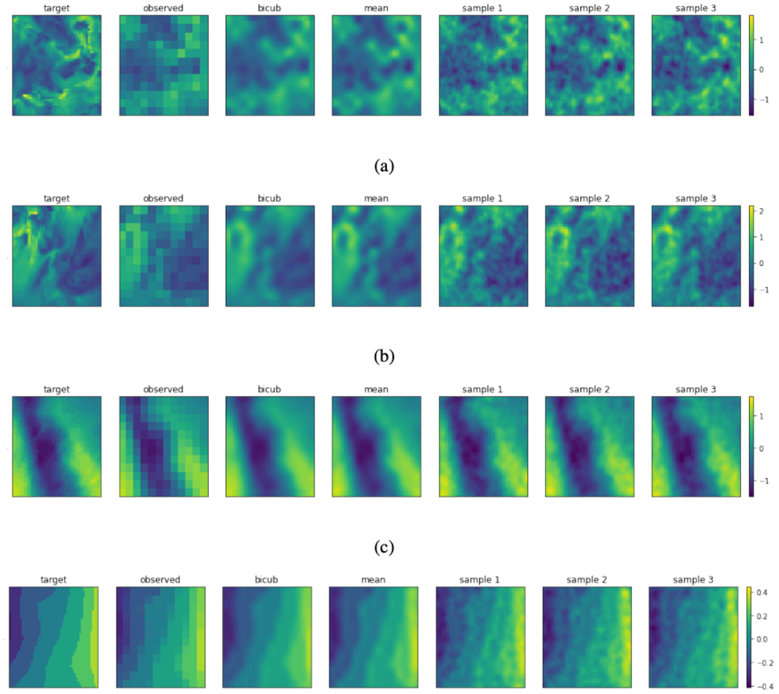
(c)



(d)

convolution	model	MSE	PSD Wass	Nd Wass (4)
8×8	lres	0.034	1.65	0.048
	bicub	0.025	2.56	0.047
	GRF-E mean	0.036	1.69	0.042
	GRF-E samples	0.036	1.69	0.042
	GRF-S mean	**0.023**	2.82	0.038
	GRF-S samples	0.039	1.65	0.039
	GRF-T mean	**0.023**	2.85	0.038
	GRF-T samples	0.037	**1.50**	**0.036**

Results



(a)

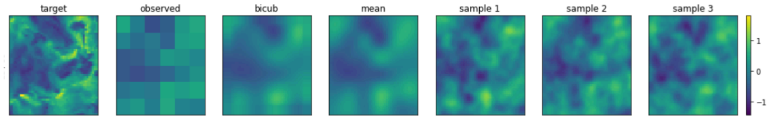
(b)

(c)

(d)

convolution	model	MSE	PSD Wass	Nd Wass (4)
4×4	lres	0.015	0.76	0.030
	bicub	0.010	1.64	0.036
	GRF-E mean	0.014	0.77	0.032
	GRF-E samples	0.014	0.77	0.032
	GRF-S mean	**0.008**	1.57	**0.027**
	GRF-S samples	0.015	0.70	0.030
	GRF-T mean	**0.008**	1.59	**0.027**
	GRF-T samples	0.015	**0.60**	0.029

Results



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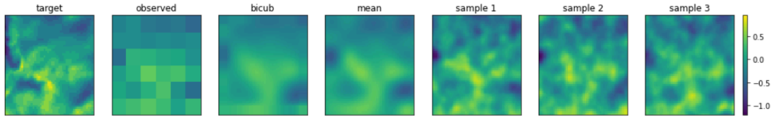

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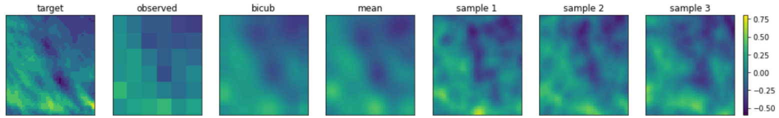

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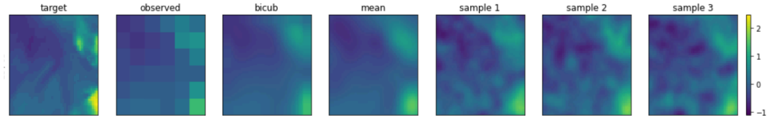

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Take-aways

- stochastic / generative model
- using GRFs in a non-standard way
- (essentially) no training

Ongoing work

- anisotropy (straightforward)
- spatial non-stationarity
- non-Gaussian variables
- speed

Stochastic Downscaling to Chaotic Weather Regimes using Spatially Conditioned Gaussian Random Fields with Adaptive Covariance

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