The Few-Get-Richer:
A Surprising Consequence of Popularity-Based Rankings

Fabrizio Germano, Vicenç Gomez, Gaël Le Mens

Universitat Pompeu Fabra, Barcelona

The Problem
Ranking algorithms systematically affect the information people access

Motivation
Address theoretical gap

Theoretical Gap
Interaction of ranking algorithms and what people access as a result still poorly understood

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Universitat Pompeu Fabra Barcelona

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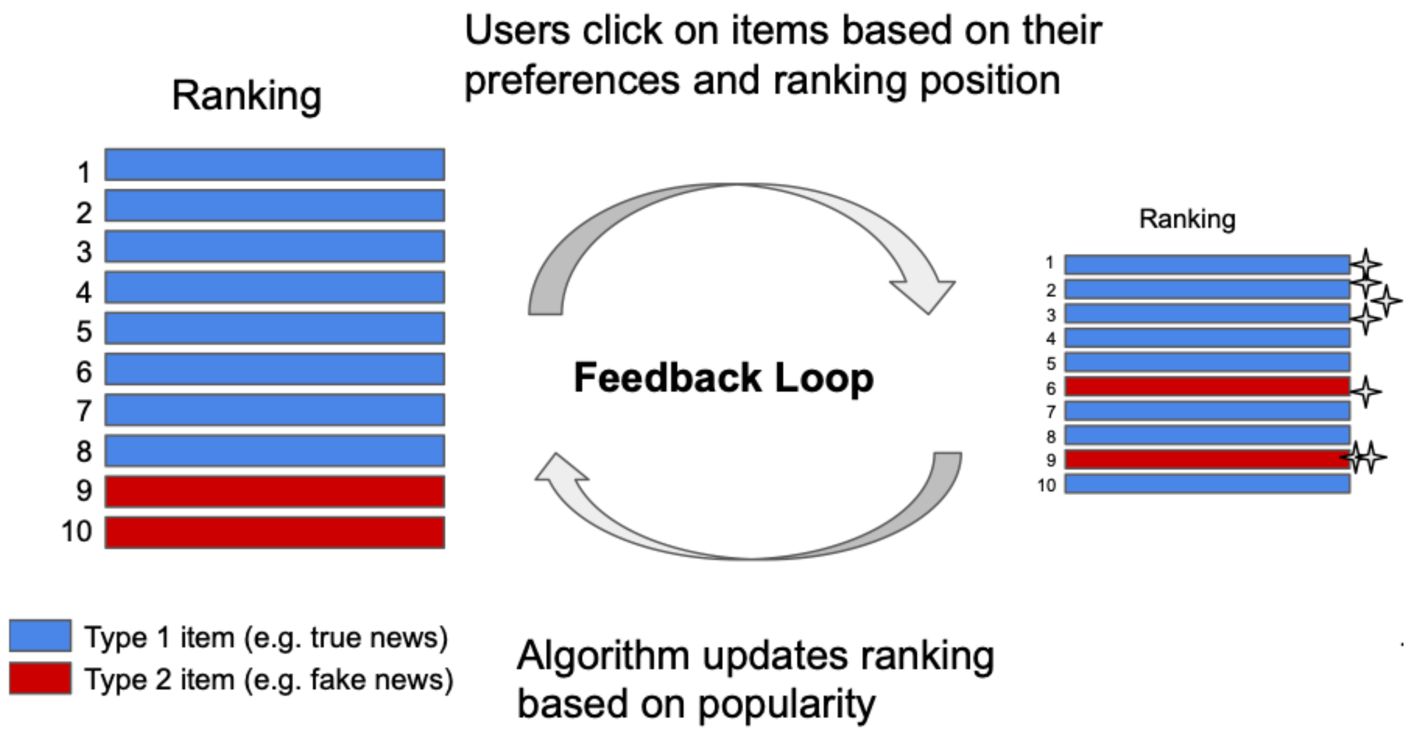
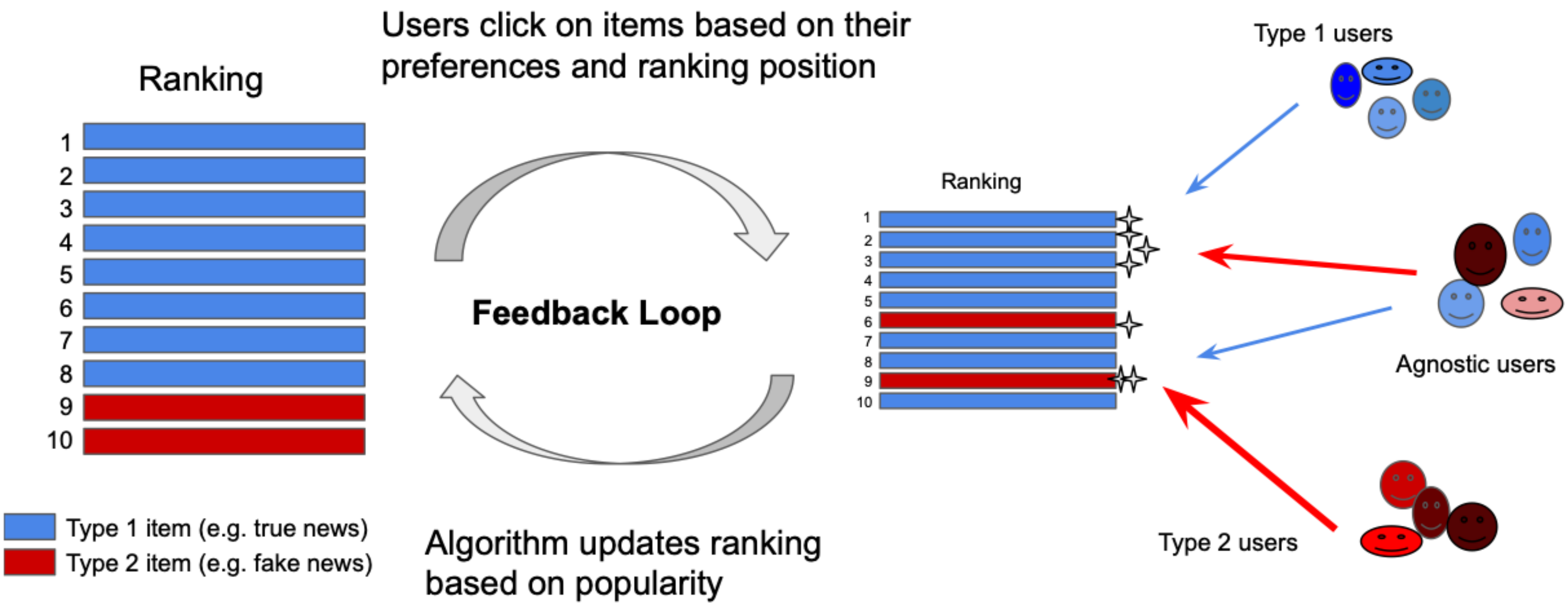
Gaël Le Mens: Funding for 2-3 postdocs (3.5 years)

The Setting

1. Search query (e.g., “should I vaccinate my child…”)
2. Two classes of items (“yes, vaccinate”, “no, don’t vaccinate”)



1. Search query (e.g., “should I vaccinate my child…”)
2. Two classes of items (“yes, vaccinate”, “no, don’t vaccinate”)
3. Items are ranked based on their popularity (number of clicks)
4. Users search sequentially, they:
   - have **heterogeneous** preferences for (visible) classes of items
   - are more likely to click on **higher-ranked** items.



A surprising consequence of popularity-based rankings:

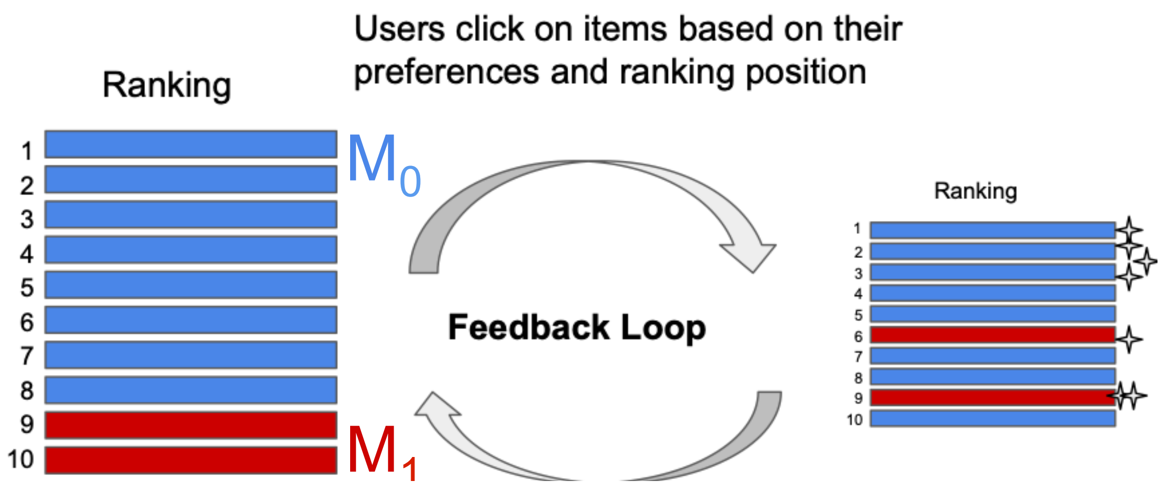
The fewer the items of a given class,

the greater the share of the overall traffic

they collectively attract.

The Model

- The **search environment** consists of a **ranking algorithm** that ranks M **items** of two types $k \in \{0, 1\}$ that get accessed by N **users** who sequentially use the ranking to decide which item to click on.
- $r_{n,m} \in \{1, \ldots, M\}$ is the rank of item m observed by user $n \in \{1, \ldots, N\}$, which depends on the **number of clicks received**.



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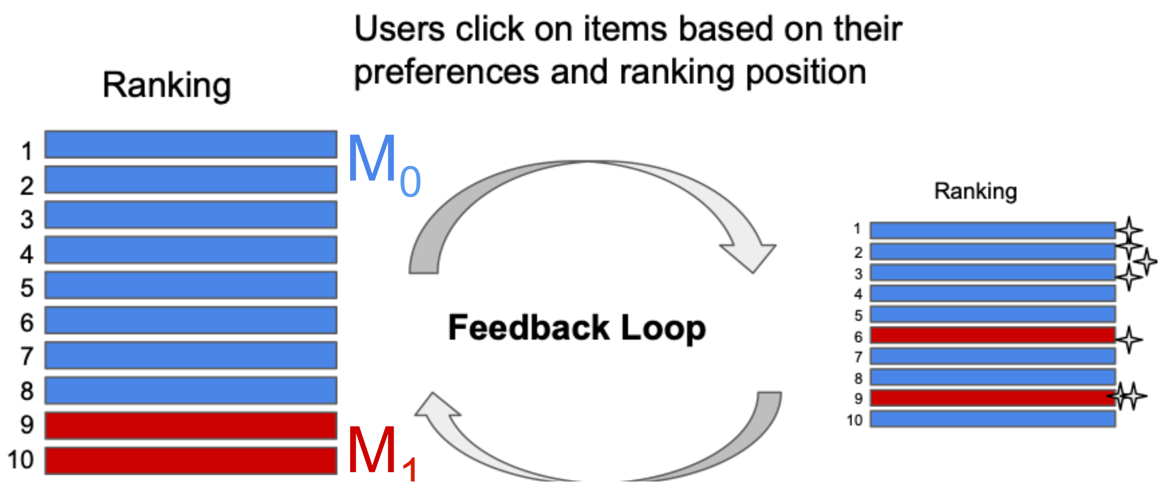
- **Propensities**: user n with $\gamma_n \in \{0,\frac{1}{2},1\}$ has propensity $\varphi_{n,m}$ of clicking on item m:

$$\varphi_{n,m} = \begin{cases} \frac{\gamma_n}{M_0} & \text{if } m \in M_0 \\ \frac{1-\gamma_n}{M_1} & \text{if } m \in M_1. \end{cases} \tag{1}$$

$\gamma_n = 0$ Prefers M_1 (i.e., chooses M_0 with prob. 0)

$\gamma_n = 1/2$ Indifferent between M_0 and M_1 (Agnostic)

$\gamma_n = 1$ Prefers M_0 (i.e., chooses M_0 with prob. 1)



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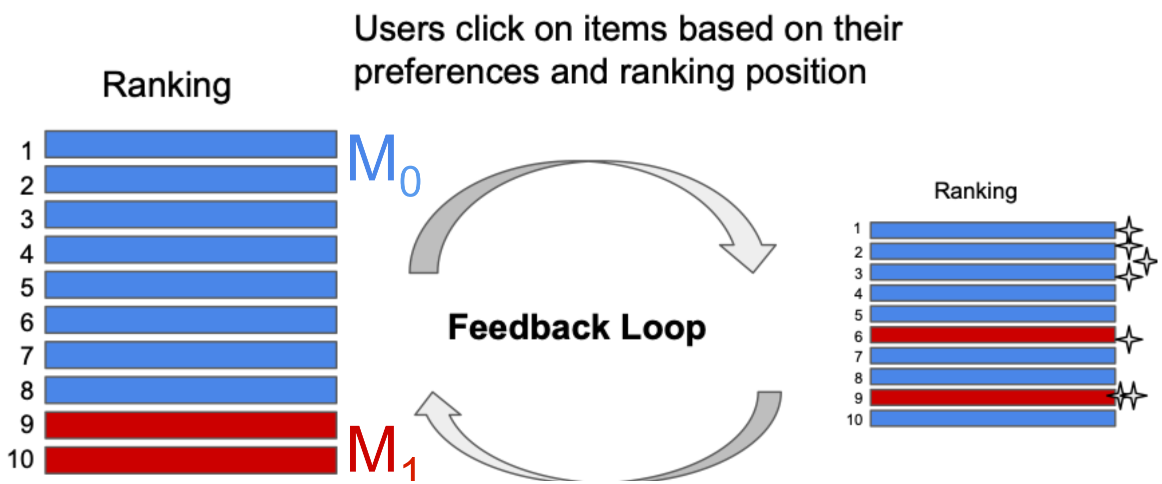
Users enter sequentially with γ_n drawn randomly and independently: $\gamma_n = 0$, $\gamma_n = 1$ each with probability $0 < p < \frac{1}{2}$ and $\gamma_n = \frac{1}{2}$ with (remaining) probability $0 < 1-2p < 1$.



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Agnostic

Prefers M_0

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Prefers M_1
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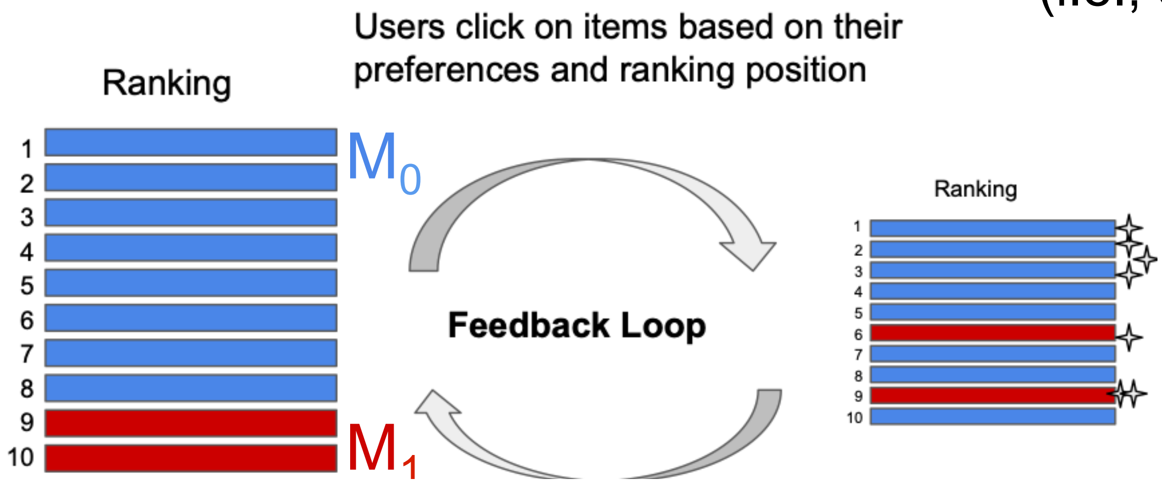

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- They also have an **attention bias** β (> 1), whereby an item ranked exactly one position higher is β times more likely to be clicked.
- **Stochastic choice rule**: user n chooses ranked item m according to

$$\underbrace{\rho_{n,m} = \frac{1}{Z}\underbrace{\beta^{(M-r_{n,m})}}_{\text{attention bias}} \cdot \underbrace{\varphi_{n,m}}_{\text{click propensity}}}_{\text{Prob(user } n \text{ clicks on item } m\text{)}} \qquad \underbrace{Z = \sum_{m' \in M} \beta^{\left(M-r_{n,m'}\right)} \varphi_{n,m'}.}_{\text{normalization constant}} \tag{2}$$

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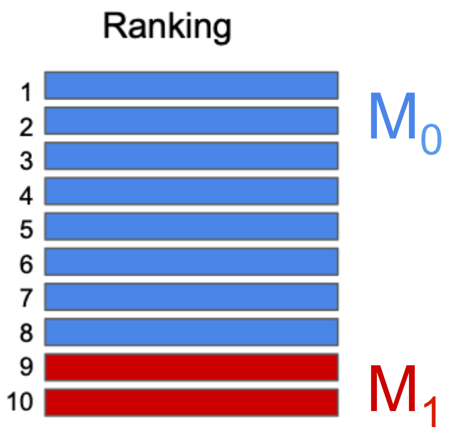
The Few-Get-Richer Effect

Keeping the total number of ranked items M constant, decreasing the number of items in one of the two classes can dramatically increase the total traffic to that class: having **few** *items in the ranking can* **increase** *total number of clicks on those (few) items.*

Simulations I
Trajectories

- $M = 20$ items and $N = 100$ users, and M_1 items are **initially at the bottom**.
- Proportion of users of different types: p_0 and p_1. Agnostic users: $p_2 = 1 - p_0 - p_1$.
- Uniform initialization, with all items having one click.

$\beta = 1.1, \Gamma = \{0.9, 0.1, 0.5\}, p_0 = p_1 = 0.4$



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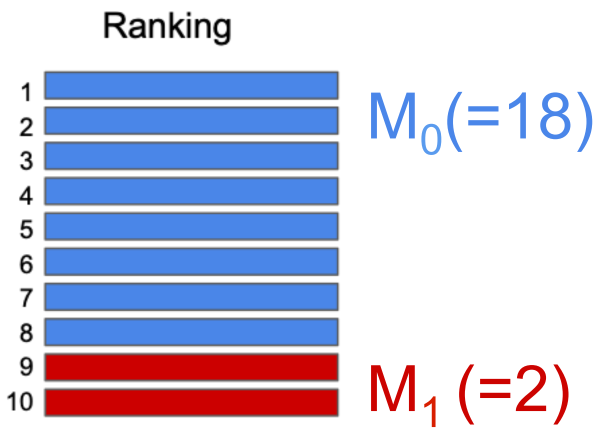

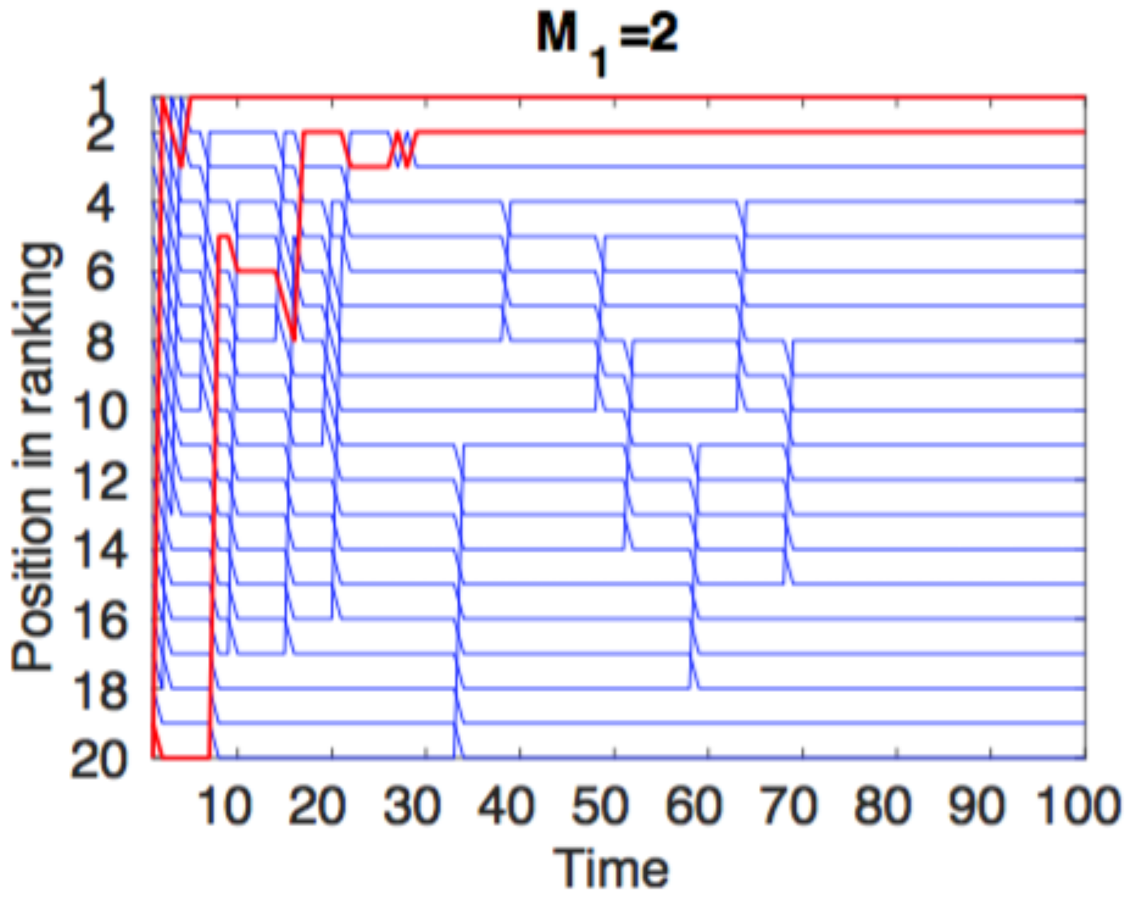

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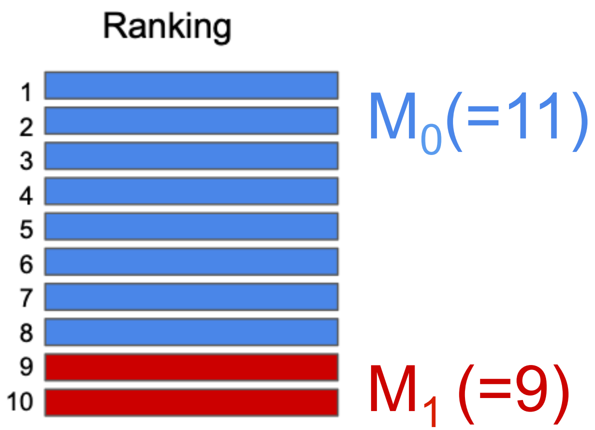

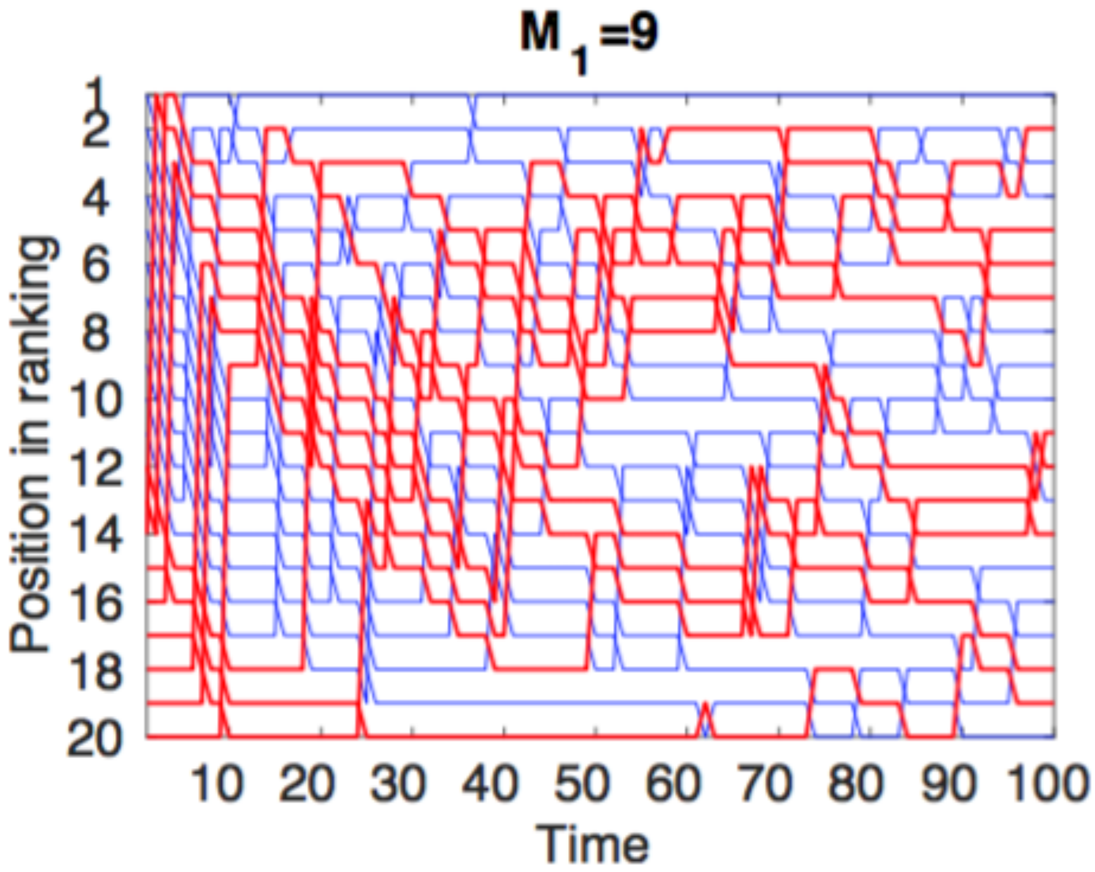

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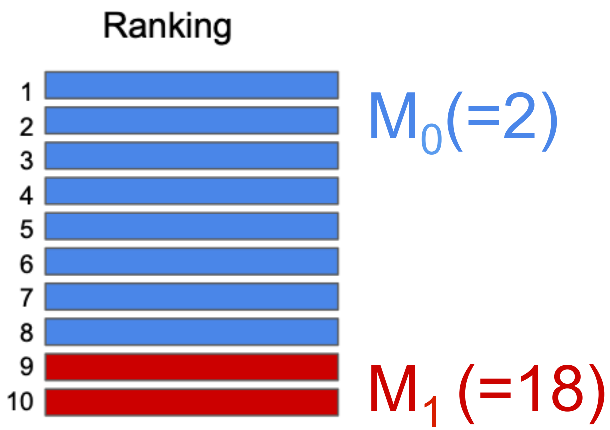

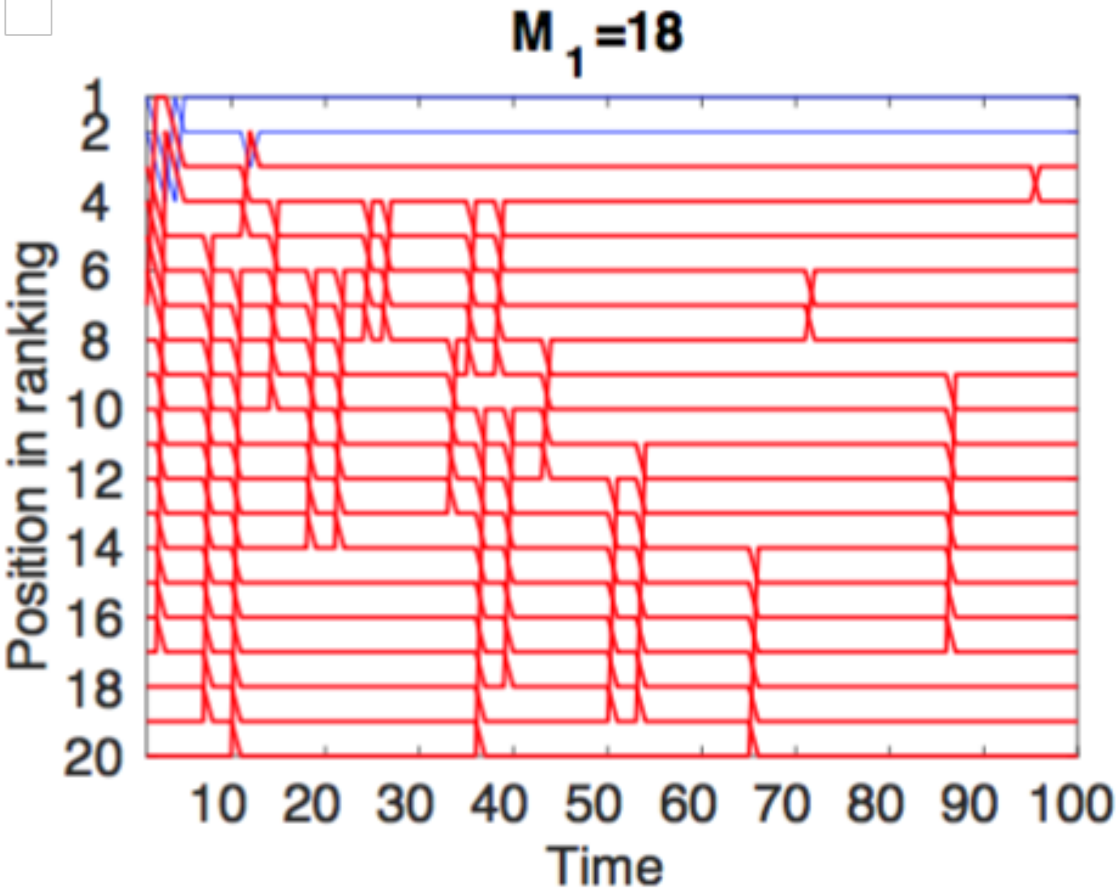

More formally…

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Keeping the total number of ranked items M constant, decreasing the number of items in one of the two classes can dramatically increase the total traffic to that class: having **few** *items in the ranking can* **increase** *total number of clicks on those (few) items.*

More formally…

Fix two popularity-based search environments $\mathcal{E}$ and $\mathcal{E}'$ that differ only in the number of items of class 1 (M_1 and M_1' respectively). Suppose $M_1 < \frac{M}{1+\beta} < \frac{\beta M}{1+\beta} < M_1'$, then there exists $\overline{N}$ such that, for any $N \geq \overline{N}$, the total clicking probability (ρ_{N,M_1}) by individual N on an item in M_1 in environment $\mathcal{E}$ is strictly greater than the total clicking probability ($\rho_{N,M_1'}$) by individual N on an item in M_1' in environment $\mathcal{E}'$, provided $p > 0$ is sufficiently small.

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Not too few 'agnostic' users

The proof is in three steps.

1. we characterize a limit ranking (r_∞) of the process ρ_n (popularities) and show it constitutes a (stable) limit.

2. we show it is the unique such limit ranking.

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2. we show it is the unique such limit ranking.

3. we compute total traffic on all items in M_1 at the limit and show it is over half of total traffic when $M_1 < \frac{M}{1+\beta}$, and hence greater than total traffic on all items in M_1' for $M_1' > \frac{\beta M}{1+\beta}$.

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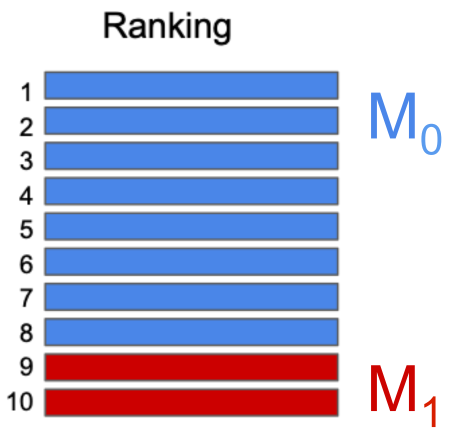
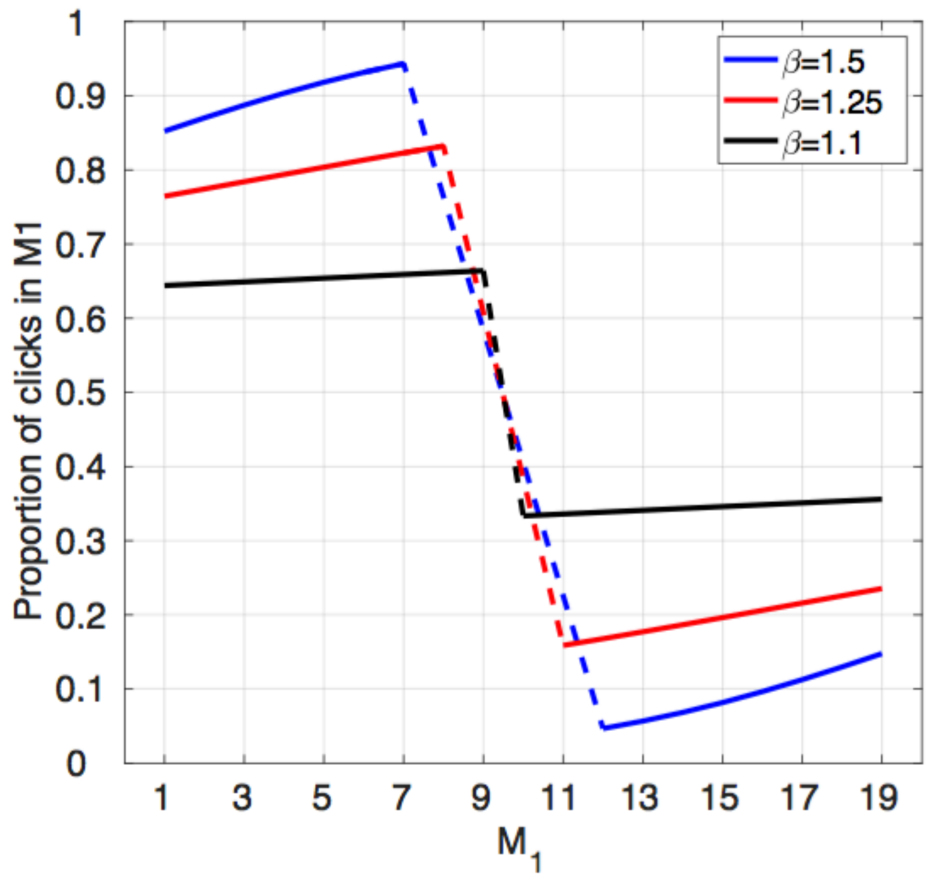
$\Gamma = \{0.8, 0.2, 0.5\}$ $\quad p_0 = p_1 = 0.4$.

▶ Analytical curves for **infinite** users:





Simulations II
Comparative Statics

- $M = 20$ items and $N = 100$ users, and M_1 items are **initially at the bottom**.
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- Uniform initialization, with all items having one click.

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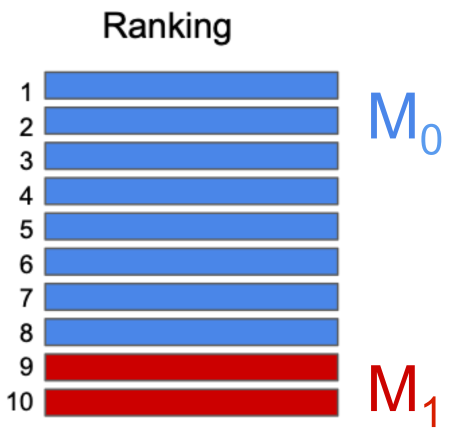
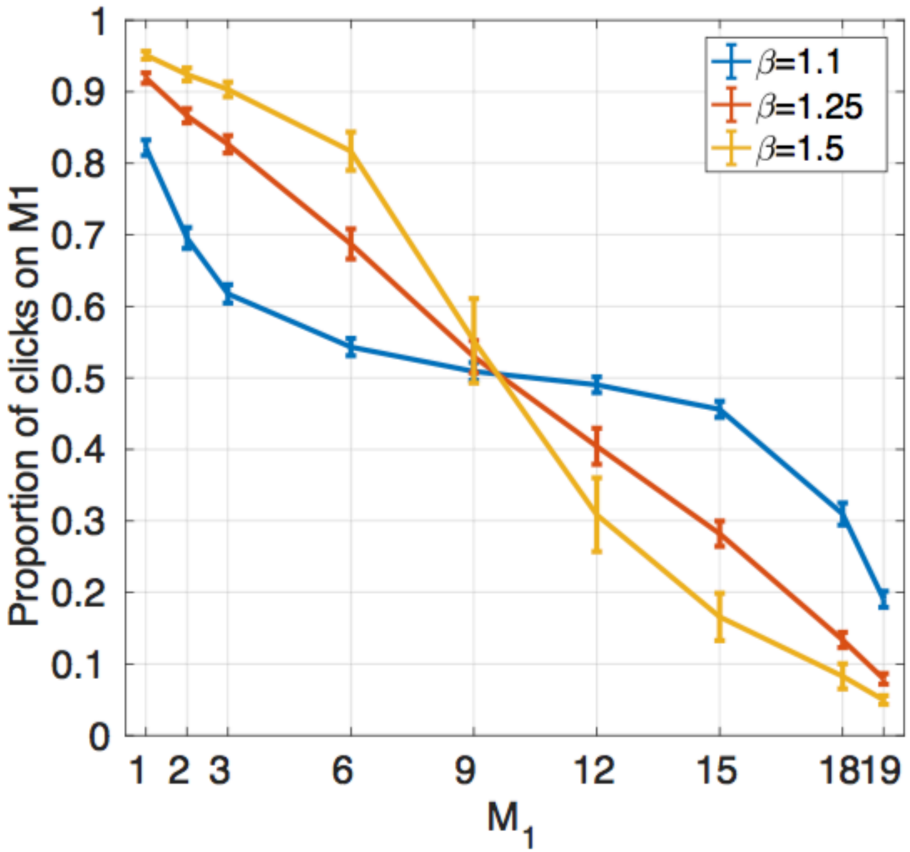
Higher attention bias (effect of ranking on choice)
→
Stronger effect



▶ Dependence on attention bias β:



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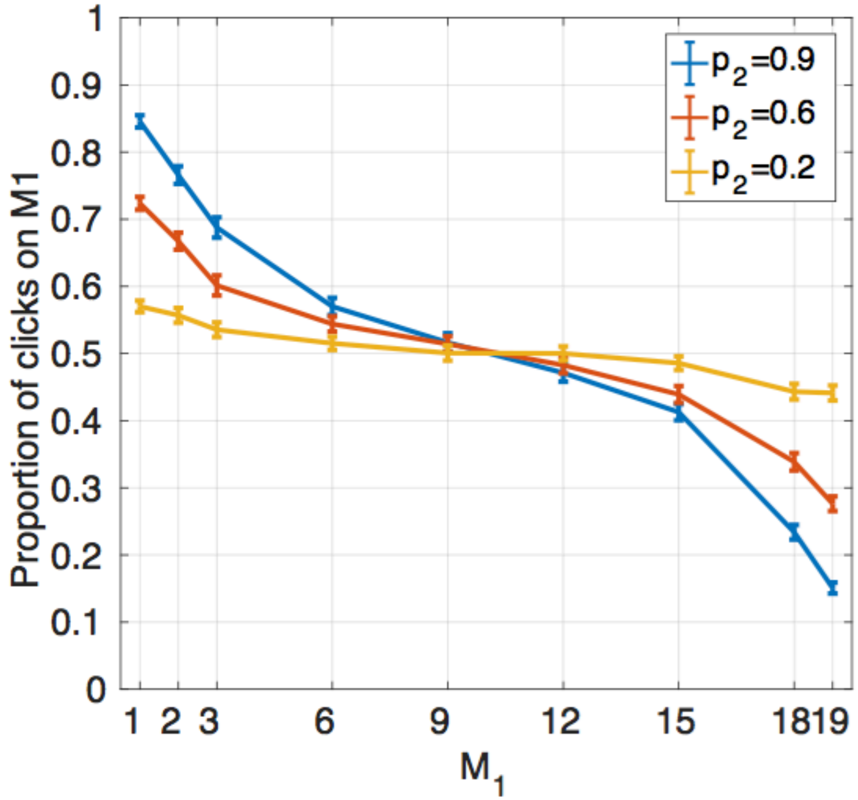
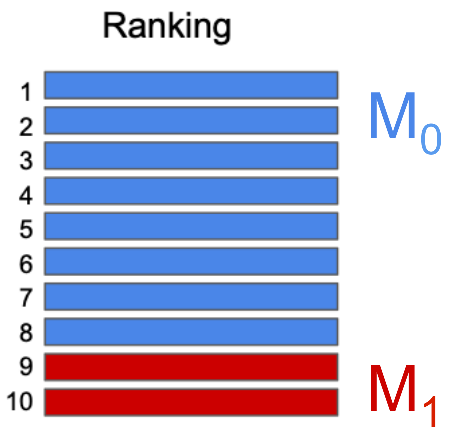
Higher proportion of 'agnostic' users (p_2) → Stronger effect

▶ Dependence on proportion of agnostic users p_2:





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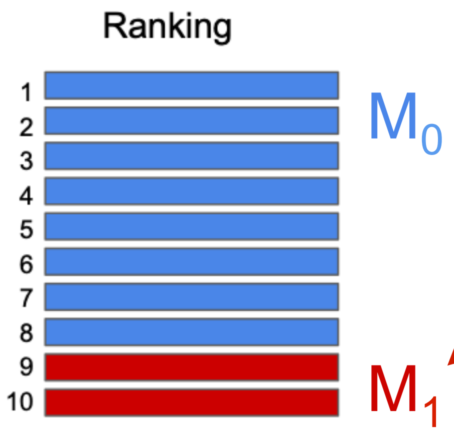
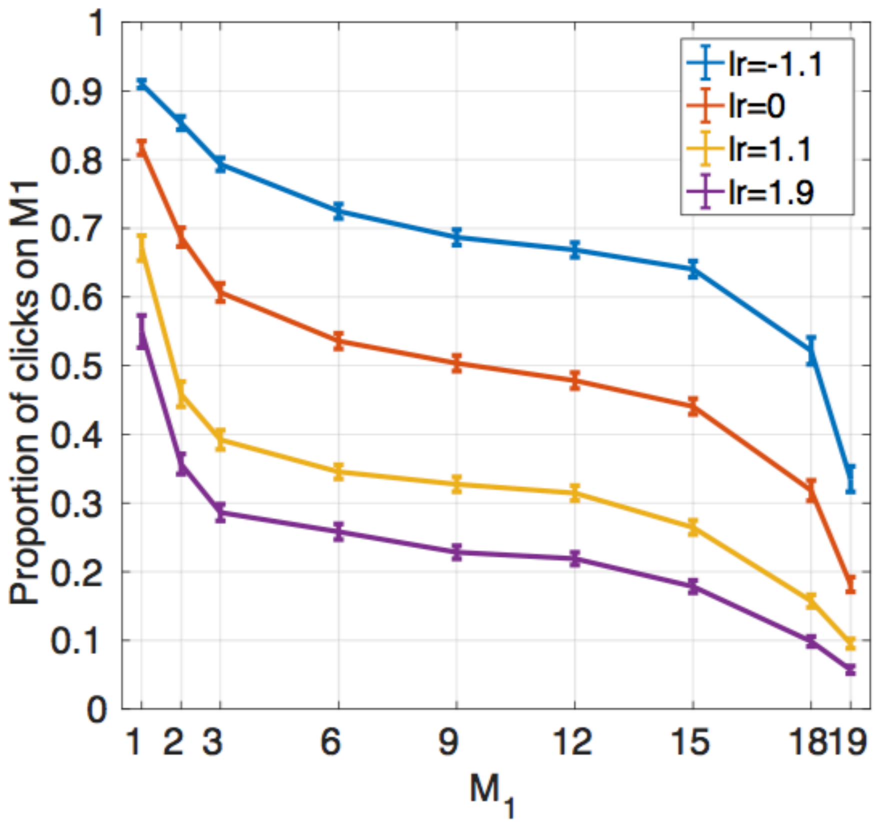
Stronger proportion of users with preference for minority items (p_1/p_0) → Stronger effect



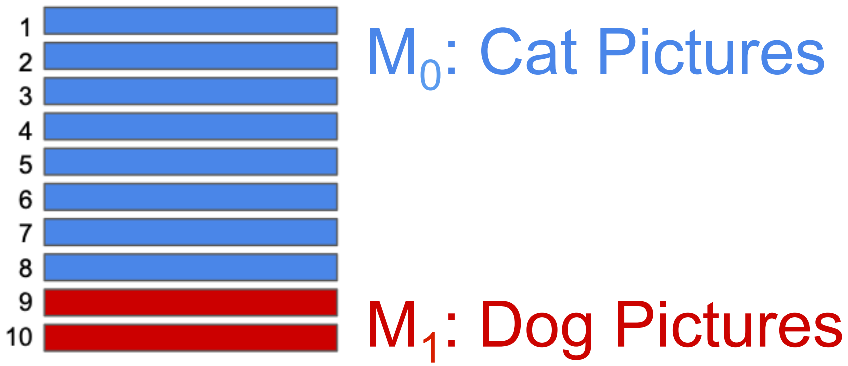
▶ Dependence on the ratio $lr = \log \frac{p_0}{p_1}$:



Experiment

- Amazon Turk : 786 participants.
- $M = 20$ ranked items of 2 types:
  M_0 **Cat Pictures**, M_1 **Dog Pictures**.

- Uniform initialization, with all pictures having one click.



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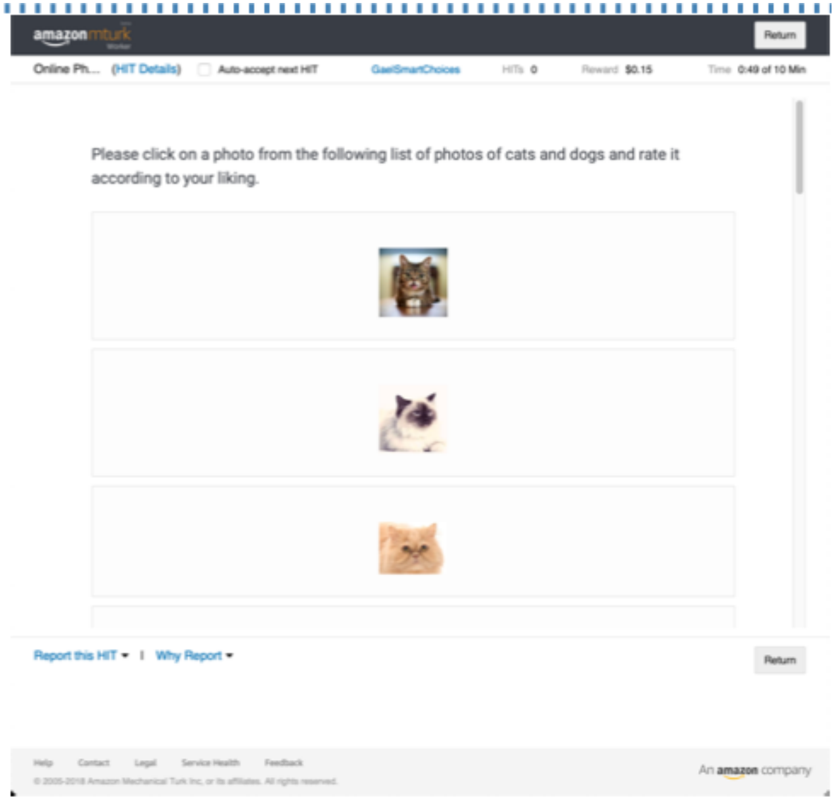

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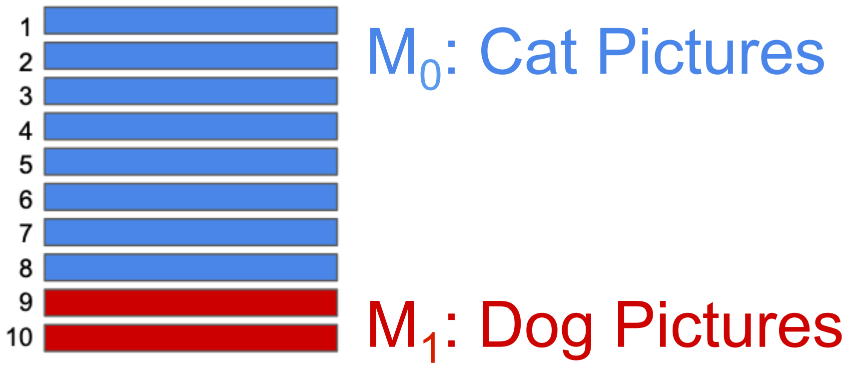

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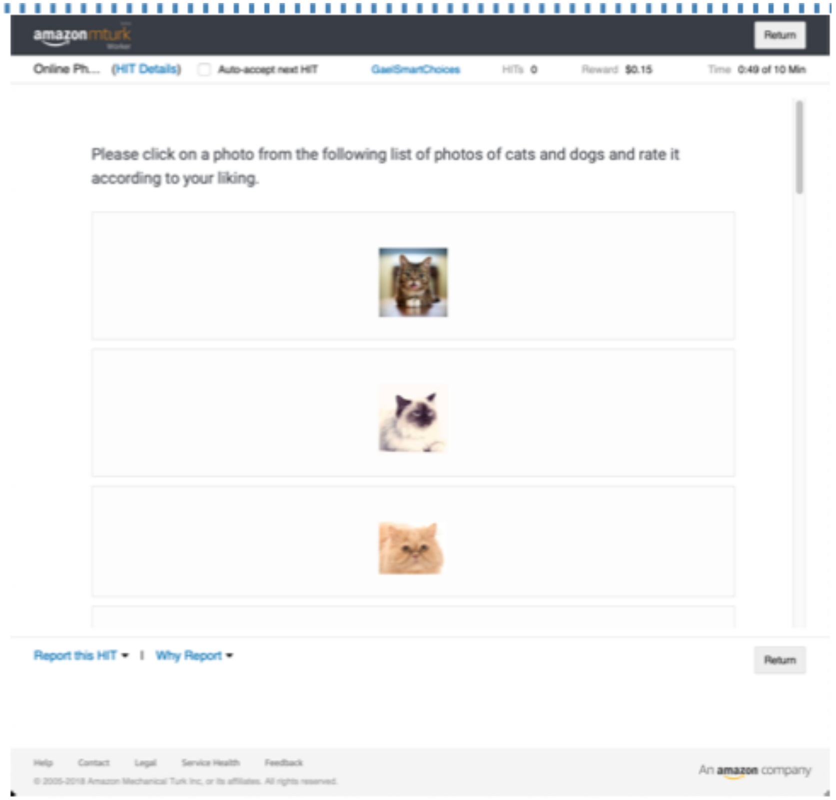
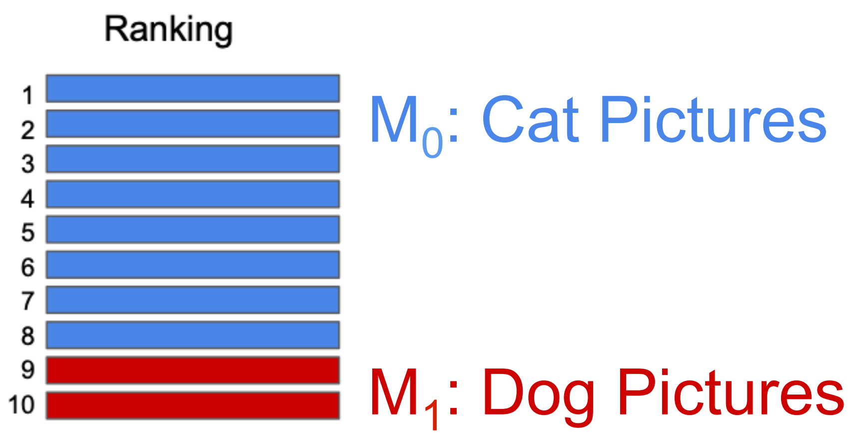


- Uniform initialization, with all pictures having one click.
- User types: ‘**cat person**’, ‘**dog person**’, or ‘**neither a cat nor a dog person**.’

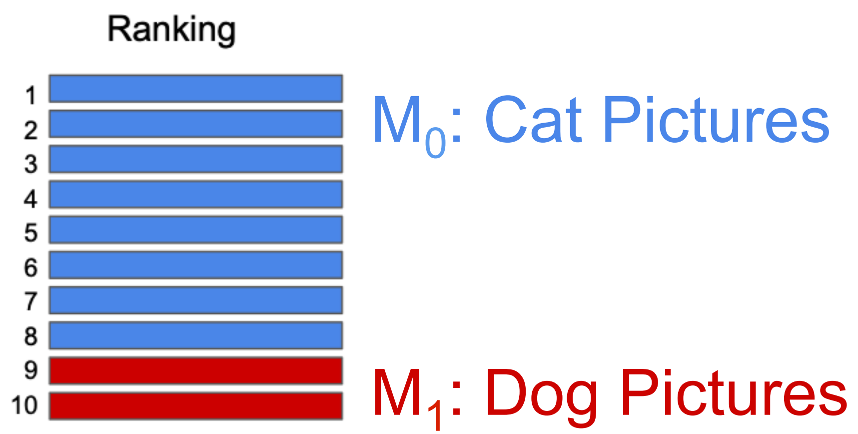


“Are you more of a cat person or a dog person?”

- “I am a cat person”
- “I am neither a cat person nor a dog person”
- “I am a dog person.”

- Amazon Turk : 786 participants.
- $M = 20$ ranked items of 2 types:
  M_0 **Cat Pictures**, M_1 **Dog Pictures**.
- 4 treatments with $M_1 = 3, 8, 12$ or 17 dogs, initially ranked at the bottom.
- 2 sets of ranking conditions
  - ***Static***: dog pictures stay at the bottom. "Control" condition
  - ***Dynamic***: items go up as they are clicked. "Treatment" condition
- Uniform initialization, with all pictures having one click.

- User types: '**cat person**', '**dog person**', or '**neither a cat nor a dog person**.'



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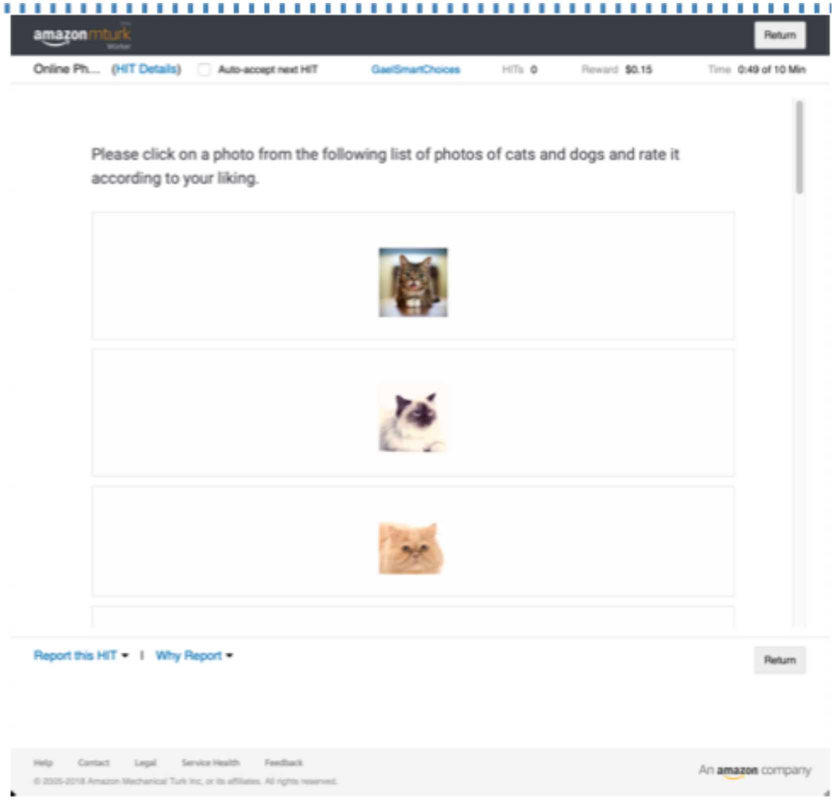
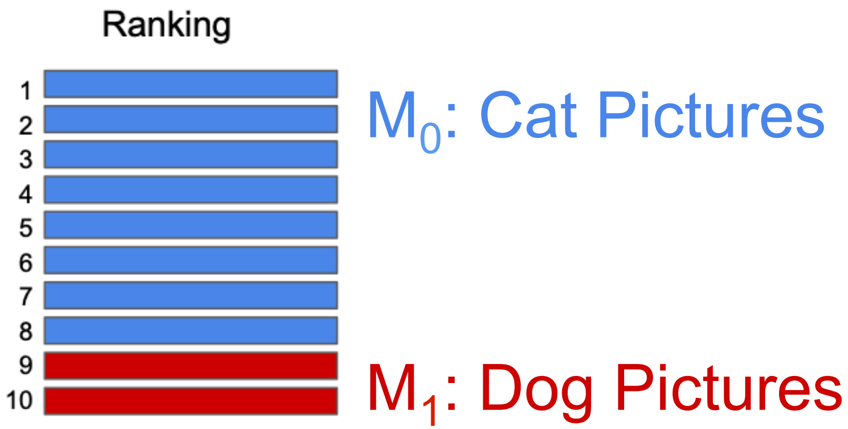


- User types: '**cat person**', '**dog person**', or '**neither a cat nor a dog person**.'



Main finding: Total #clicks attracted by dog pictures is larger when there are few dog pictures (3/20) than when there are many dog pictures (17/20) in the *Dynamic* setting but not in the *Static* setting.

# Cats (M_0)		3	8	12	17
# Dogs (M_1)		17	12	8	3
Dynamic					
Condition		D1	D2	D3	D4
# participants		96	102	99	101
# participants in each type	Cat person				
	Neither				
	Dog person				
Dog traffic share	Experiment				
	Sim1				
	Sim2				
Static					
Condition		S1	S2	S3	S4
# participants		96	101	95	96
# participants in each type	Cat person				
	Neither				
	Dog person				
Dog traffic share	Experiment				
	Sim1				
	Sim2				

# Cats (M_0)		3	8	12	17
# Dogs (M_1)		17	12	8	3
Dynamic					
Condition		D1	D2	D3	D4
# participants		96	102	99	101
# participants in each type	Cat person	34	30	24	29
	Neither	9	21	11	16
	Dog person	53	51	64	56
Dog traffic share	Experiment				
	Sim1				
	Sim2				
Static					
Condition		S1	S2	S3	S4
# participants		96	101	95	96
# participants in each type	Cat person	34	30	25	33
	Neither	13	19	9	15
	Dog person	49	52	61	48
Dog traffic share	Experiment				
	Sim1				
	Sim2				

“Are you more of a cat person or a dog person?”

- “I am a cat person”
- “I am neither a cat person nor a dog person”
- “I am a dog person.”

# Cats (M_0)		3	8	12	17
# Dogs (M_1)		17	12	8	3
Dynamic					
Condition		D1	D2	D3	D4
# participants		96	102	99	101
# participants in each type	Cat person	34	30	24	29
	Neither	9	21	11	16
	Dog person	53	51	64	56
Dog traffic share	Experiment				
	Sim1				
	Sim2				
Static					
Condition		S1	S2	S3	S4
# participants		96	101	95	96
# participants in each type	Cat person	34	30	25	33
	Neither	13	19	9	15
	Dog person	49	52	61	48
Dog traffic share	Experiment				
	Sim1				
	Sim2				

"Are you more of a cat person or a dog person?"

- "I am a cat person"
- "I am neither a cat person nor a dog person"
- "I am a dog person."

Notice: Not exactly constant.

# Cats (M_0)		3			17
# Dogs (M_1)		17			3
Dynamic					
Condition					
# participants					
# participants in each type	Cat person				
	Neither				
	Dog person				
Dog traffic share	Experiment				
	Sim1				
	Sim2				
Static					
Condition					
# participants					
# participants in each type	Cat person				
	Neither				
	Dog person				
Dog traffic share	Experiment	.44	.37	.40	.27
	Sim1				
	Sim2				

Static case:

Dog traffic: <50%

Fewer dog pictures
→ **lower** dog traffic

(no surprise)

# Cats (M_0)		3		17
# Dogs (M_1)		17		3
Dynamic				
Condition				
# participants				
# participants in each type	Cat person			
	Neither			
	Dog person			
Dog traffic share	Experiment	.53		.71
	Sim1			
	Sim2			
Static				
Condition				
# participants				
# participants in each type	Cat person			
	Neither			
	Dog person			
Dog traffic share	Experiment	.44		.27
	Sim1			
	Sim2			

Dynamic case:

Dog traffic: >50%

Fewer dog pictures
→ **greater** dog traffic

Main finding: Total #clicks attracted by dog pictures is larger when there are few dog pictures (3/20) than when there are many dog pictures (17/20) in the *Dynamic* setting but not in the *Static* setting.

# Cats (M_0)		3	8	12	17
# Dogs (M_1)		17	12	8	3
Dynamic					
Condition					
# participants					
# participants in each type	Cat person				
	Neither				
	Dog person				
Dog traffic share	Experiment	.53	.69	.76	.71
	Sim1				
	Sim2				
Static					
Condition					
# participants					
# participants in each type	Cat person				
	Neither				
	Dog person				
Dog traffic share	Experiment	.44	.37	.40	.27
	Sim1				
	Sim2				

Main finding: Total #clicks attracted by dog pictures is larger when there are few dog pictures (3/20) than when there are many dog pictures (17/20) in the *Dynamic* setting but not in the *Static* setting.

# Cats (M_0)		3	8	12	17
# Dogs (M_1)		17	12	8	3
Dynamic					
Condition		D1	D2	D3	D4
# participants		96	102	99	101
# participants in each type	Cat person	34	30	24	29
	Neither	9	21	11	16
	Dog person	53	51	64	56
Dog traffic share	Experiment	.53	.69	.76	.71
	Sim1				
	Sim2				
Static					
Condition		S1	S2	S3	S4
# participants		96	101	95	96
# participants in each type	Cat person	34	30	25	33
	Neither	13	19	9	15
	Dog person	49	52	61	48
Dog traffic share	Experiment	.44	.37	.40	.27
	Sim1				
	Sim2				

More dog lovers in D3 than in D4

Main finding: **Total #clicks attracted by dog pictures is larger when there are few dog pictures (3/20) than when there are many dog pictures (17/20) in the *Dynamic* setting but not in the *Static* setting.**

Simulations III
Using Estimated Model Parameters

Recall:

- $r_{n,m} \in \{1, \ldots, M\}$ is the rank of item m observed by user $n \in \{1, \ldots, N\}$, which depends on the **number of clicks received**.
- **Propensities**: user n with $\gamma_n \in \{0, \frac{1}{2}, 1\}$ has propensity $\varphi_{n,m}$ of clicking on item m:

$$\varphi_{n,m} = \begin{cases} \frac{\gamma_n}{M_0} & \text{if } m \in M_0 \\ \frac{1-\gamma_n}{M_1} & \text{if } m \in M_1. \end{cases} \tag{1}$$

  Users enter randomly and independently with $\gamma_n = 0$ and $\gamma_n = 1$ each with probability $0 < p < \frac{1}{2}$ and with $\gamma_n = \frac{1}{2}$ with (remaining) probability $0 < 1 - 2p < 1$.
- They also have an **attention bias** β (> 1), whereby an item ranked exactly one position higher has β times as much as probability of being clicked.
- **Stochastic choice rule**: user n chooses ranked item m according to

$$\rho_{n,m} = \frac{1}{Z} \underbrace{\beta^{(M - r_{n,m})}}_{\text{attention bias}} \cdot \underbrace{\varphi_{n,m}}_{\text{click propensity}} \qquad Z = \sum_{m' \in M} \beta^{(M - r_{n,m'})} \varphi_{n,m'}. \tag{2}$$

Estimate: $\beta = 1.22$, for 'cat person': $\gamma_n = .74$.

for 'dog person': $\gamma_n = .08$.

# Cats (M_0)		3	8	12	17
# Dogs (M_1)		17	12	8	3
Dynamic					
Condition					
# participants					
# participants in each type	Cat person	34	30	24	29
	Neither	9	21	11	16
	Dog person	53	51	64	56
$\boxed{\text{Dog}}$ traffic share	Experiment	.53	.69	.76	.71
	Sim1	.46	.56	.73	.76
	Sim2				
Static					
Condition					
# participants					
# participants in each type	Cat person	34	30	25	33
	Neither	13	19	9	15
	Dog person	49	52	61	48
$\boxed{\text{Dog}}$ traffic share	Experiment	.44	.37	.40	.27
	Sim1	.41	.37	.39	.28
	Sim2				

<u>Sim1</u>: average traffic attracted by Dog pictures (M_1) over 1000 simulations of the choice model with a setting matching the <u>exact number of participants of each identity type in each condition</u>.

<u>Dynamic setting</u>
Traffic to dog pictures **increases** when there are fewer dog pictures

<u>Static setting</u>
Traffic to dog pictures (sort of) **decreases** when there are fewer dog pictures

# Cats (M_0)		3	8	12	17
# Dogs (M_1)		17	12	8	3
Dynamic					
Condition					
# participants					
# participants in each type	Cat person	34	30	24	29
	Neither	9	21	11	16
	Dog person	53	51	64	56
Dog traffic share	Experiment	.53	.69	.76	.71
	Sim1	.46	.56	.73	.76
	Sim2				
Static					
Condition					
# participants					
# participants in each type	Cat person	34	30	25	33
	Neither	13	19	9	15
	Dog person	49	52	61	48
Dog traffic share	Experiment	.44	.37	.40	.27
	Sim1	.41	.37	.39	.28
	Sim2				

Sim2: average traffic attracted by Dog pictures (M_1) over 1000 simulations of the choice model with 100 users where numbers of users who are a 'dog person', 'neither a dog person nor a cat person' and a 'cat person' are 55, 15 and 30, respectively (same frequencies for all conditions).

# Cats (M_0)		3	8	12	17
# Dogs (M_1)		17	12	8	3
Dynamic					
Condition					
# participants					
# participants in each type	Cat person	34	30	24	29
	Neither	9	21	11	16
	Dog person	53	51	64	56
Dog traffic share	Experiment	.53	.69	.76	.71
	Sim1	.46	.56	.73	.76
	Sim2	.47	.60	.67	.75
Static					
Condition					
# participants					
# participants in each type	Cat person	34	30	25	33
	Neither	13	19	9	15
	Dog person	49	52	61	48
Dog traffic share	Experiment	.44	.37	.40	.27
	Sim1	.41	.37	.39	.28
	Sim2	.44	.39	.35	.30

Sim2: average traffic attracted by Dog pictures (M_1) over 1000 simulations of the choice model with 100 users where numbers of users who are a ‘dog person’, ‘neither a dog person nor a cat person’ and a ‘cat person’ are 55, 15 and 30, respectively (same frequencies for all conditions).

Dynamic setting

Traffic to dog pictures **increases** when there are fewer dog pictures

Static setting

Traffic to dog pictures **decreases** when there are fewer dog pictures

Conclusion

- We used stylized model to **prove** existence of few-get-richer effect.
- Using **simulations**, we showed the few-get-richer effect is robust to some alternative specifications.
- The presence of **attention bias** and of **agnostic users** both play a key role for the size of the effect.
- Results of **online experiment** are consistent with the theory and simulations. It is a proof-of-concept for the few-get-richer effect.

- We used stylized model to **prove** existence of few-get-richer effect.
- Using **simulations**, we showed the few-get-richer effect is robust to some alternative specifications.
- The presence of **attention bias** and of **agnostic users** both play a key role for the size of the effect.
- Results of **online experiment** are consistent with the theory and simulations. It is a proof-of-concept for the few-get-richer effect.

- Implications

  - **Misinformation:** removal of some fake news sources can lead to an *increase* in total traffic attracted by the remaining `alternative' news sources, resulting in *more exposure* to `fake news'.
  - **Recommendation systems:** having less items of one class can actually induce *more exploration* on that class.

What else to do?

- Better experiments; ideally with field data
- Estimate welfare implications for users?
- Devise ‘correction’ mechanism
- Alternative models
- Optimal ranking algorithms/recommendation systems?

Some literature:

- Germano, F., Gómez, V. and Le Mens, G., 2019, May. The few-get-richer: a surprising consequence of popularity-based rankings. In *The World Wide Web Conference* (pp. 2764-2770). ACM.
- Germano, F., and Sobbrio, F., 2019, July. Opinion dynamics via search engines (and other algorithmic gatekeepers), *in prep.*.
  → Welfare implications, asymptotic learning, also looks at personalization.
- Tennenholtz, M., and Kurland, O., 2019, May. Rethinking search engines and recommendation systems: A game-theoretic perspective.
  → Optimal recommendation systems with search engine optimization.

What else to do?

- Better experiments; ideally with field data
- Estimate welfare implications for users?
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Thank you!

Some literature:

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