Neural Networks

Lecture 09

Automatic Image Analysis

June 7, 2021

- The introduced formulation for Logistic Regression has no analytical solution.
- We can search for minima by walking on the error surface in the direction of steepest decent.

$$p(c|x) = y(x) = \sigma(Wx), \quad \sigma_i(x) = \frac{e^{x_i}}{\sum_{\forall j} e^{x_j}}$$

$$\nabla E(\theta) = \sum_i (y_i - t_i) x_i$$

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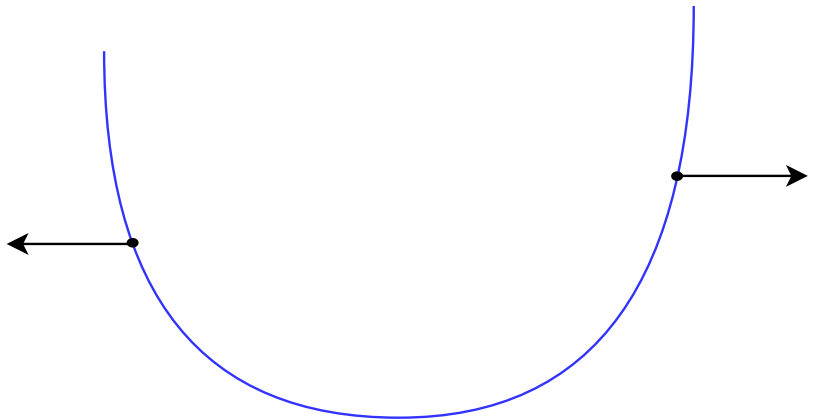
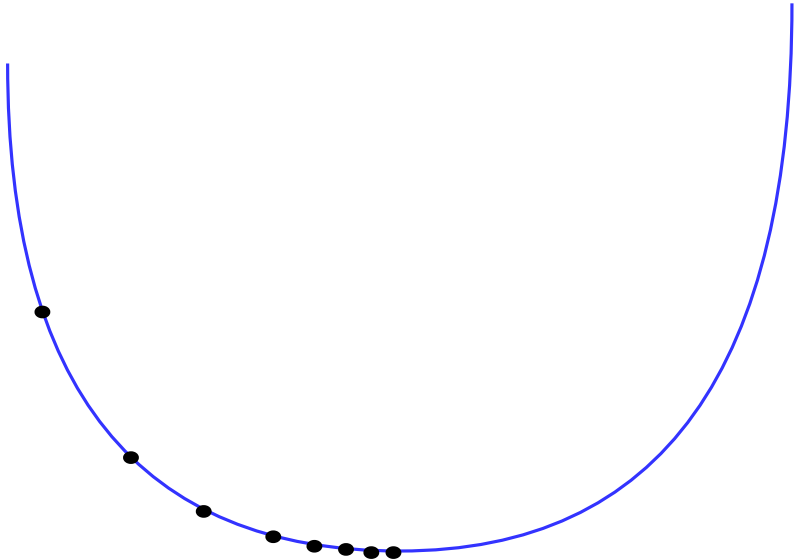
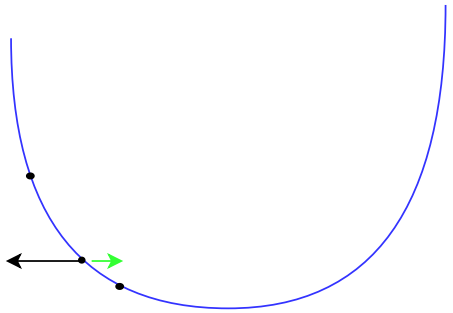
- We start at a random point and search for a minimum by walking on the error surface in the direction of steepest decent.



- For the error $E(\Theta)$ blue, the gradient points into the direction of steepest ascent.



- And end up at a local minimum.



$$\theta_{i+1} = \theta_i - \eta \nabla E(\theta)$$

- We can write the update step formally including the learning rate (step size) η.
- Whereas ∇ is the gradient operator.

$$\theta_{i+1} = \theta_i - \eta \nabla E(\theta)$$

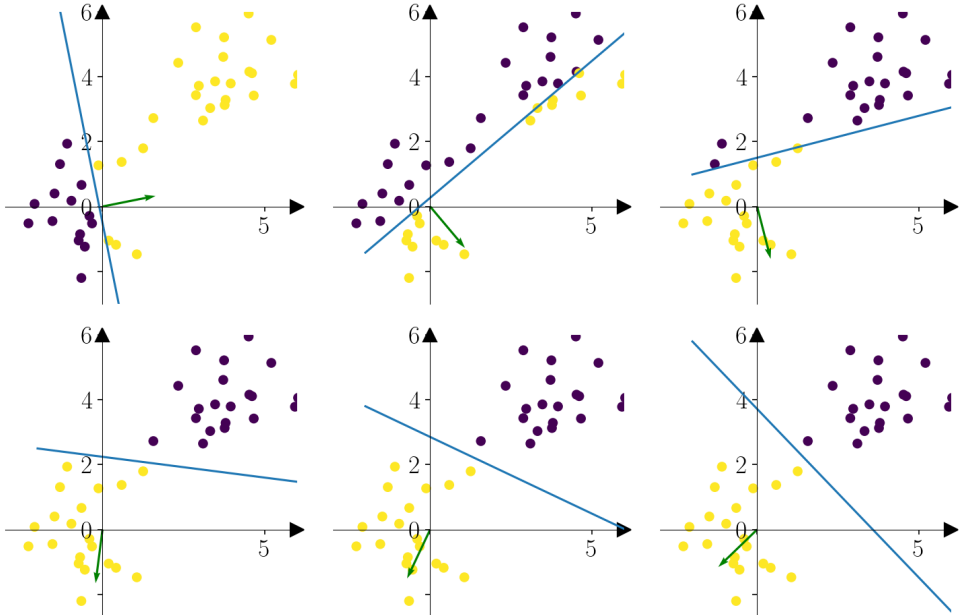
$$z_{i+1} = \beta z_i + \nabla E(\theta)$$
$$\theta_{i+1} = \theta_i - \eta z_{i+1}$$

- Momentum is the heavy backpack for our mountain hiker.
- It helps to overcome small riddles, valleys and local minima.
- Quadratic speedup compared to plain gradient descent on some error functions.
- It is a own field of research, a good starting point for the interested is maybe this article on distill:
  https://distill.pub/2017/momentum/

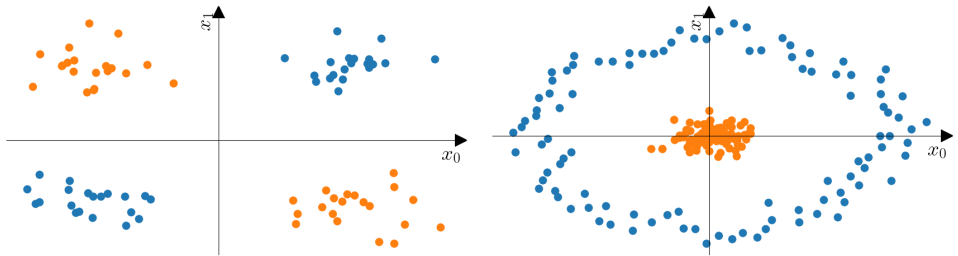
$$E(\theta) = \sum_i E_i(\theta)$$

$$\theta_{i+1} = \theta_i - \eta \nabla \sum_j E_j(\theta)$$

- The error function includes a sum over all data points.
- If we use all data points for the computation of the gradient (batch methods) there would be better ways of doing that than gradient descent.
- Furthermore, the size of the data set often would make it very expensive to use all data points.
- However what we usually do when training neural networks is online learning.
- This means we use only one sample or a subset of samples j (mini-batch) at a time.

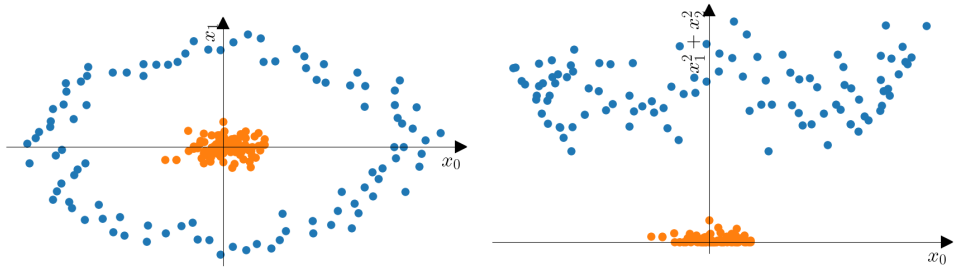


- Progress of gradient decent optimization for linear regression.



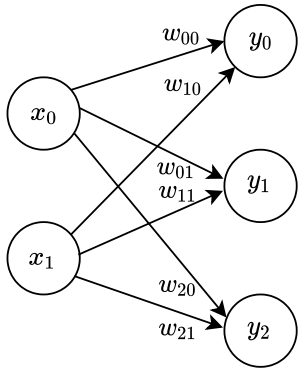
- Not all data distributions can be classified like this.

So far we solved this with feature engineering.



- We mapped the images from the highly complex pixel space using e.g. SIFT or HOG into a feature space.
- And afterwards hoped for a good classification result in feature space using e.g. SVM.

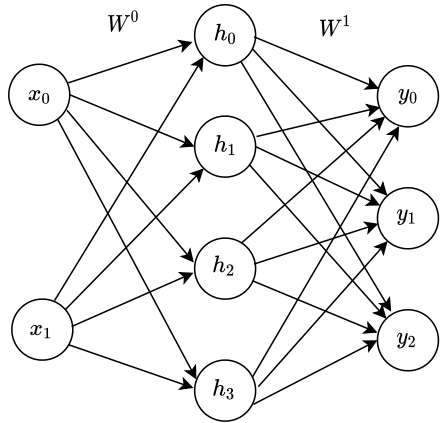
Can we learn these mappings?



$$y = \sigma(Wx)$$
$$\rightarrow y = \sigma(W^1\phi(W^0x))$$

- Let's add another mapping into our Logistic Regression!

Can we learn these mappings?



$$y = \sigma(W^1 \phi(W^0 x))$$
$$h = \phi(W^0 x)$$

- The vector h is representation in learned feature space.
  $\rightarrow$ It can have any dimensionality.
- ϕ is called the activation function.
  $\rightarrow$ It needs to be non-linear. A purely linear mapping into a new feature space wouldn't help.

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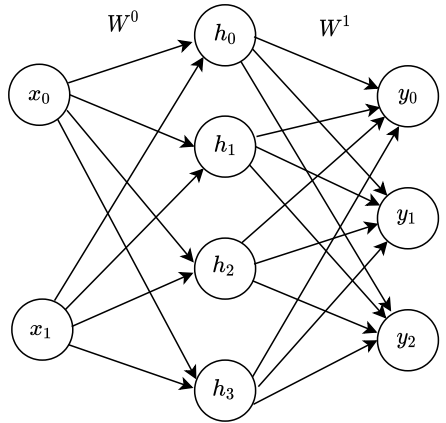

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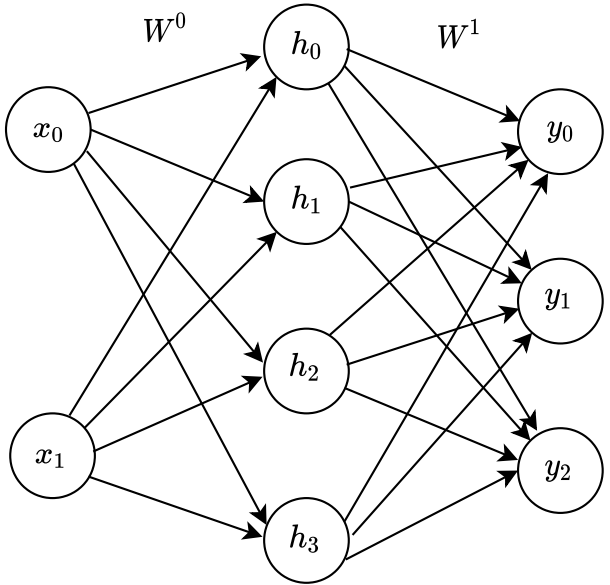
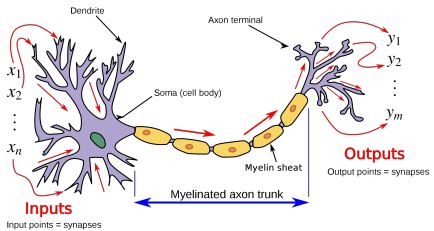
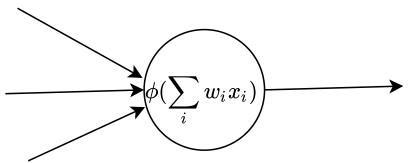
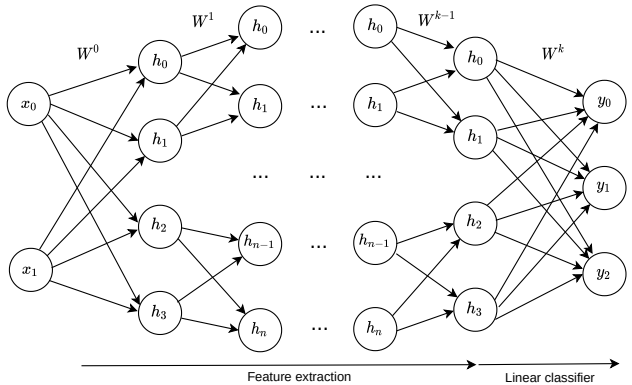


- This is what is called an Artificial Neural Network with one hidden layer.
- It's also sometimes called a Multilayer Perceptron (MLP).
- Universal Approximation Theorem:
  A neural network with one hidden layer can learn to approximate any continuous function.
  The quality of the approximation depends on the number of hidden neurons (size of the hidden layer).
  https://en.wikipedia.org/wiki/Universal_approximation_theorem

Can we learn these mappings? → Artificial Neural Network

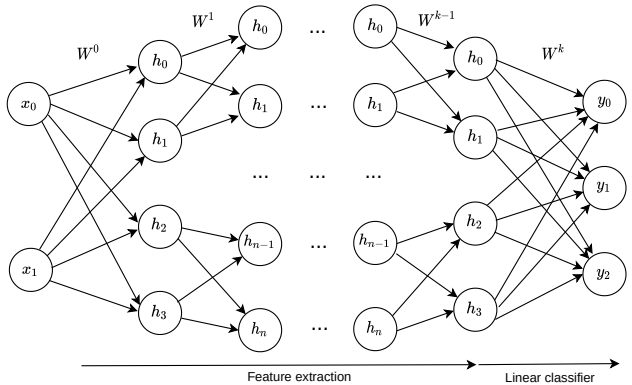






- Adding more hidden layers to this, following the same principle, leads us to what is called deep learning.

But wait. What's the derivative of this?



- Manually deriving the derivative? Puhh ...
- Symbolic derivation leads to expression swell.
- Both are restricted to model definitions with closed-form expressions.

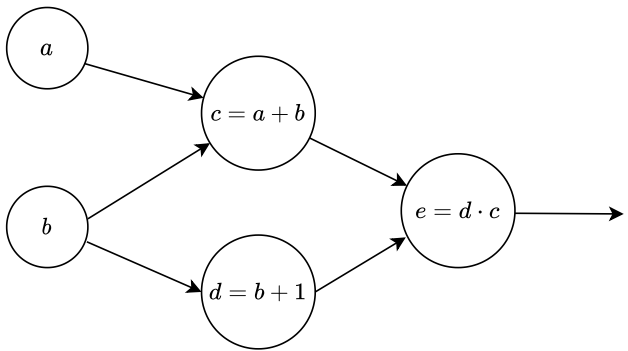
But wait. What's the derivative of this?

$$\frac{\partial E(w)}{\partial w_i} \approx \frac{E(w + he_i) - E(w)}{h}$$

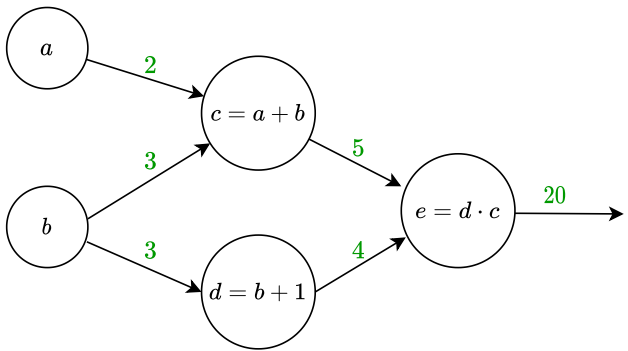
- e_i is the unit vector in the ith direction and h is a small positive number.
- We could numerically evaluate the gradient for every weight.
  $\rightarrow$ Makes a full evaluation of the network for every weight necessary.

- Fortunately, somebody figured, we could do this using the chain rule!
- The approach was developed many times but
  is widely used in Machine Learning mostly because of
  Learning representations by back-propagating errors.
  David E. Rumelhart, Geoffrey E. Hinton, and Ronald J. Williams; Nature; 1986

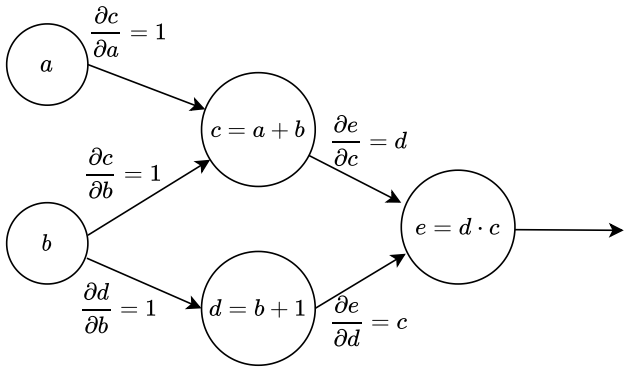
- Let's look at the computational graph for the function $e = (a + b)(b + 1)$
- Example from https://colah.github.io/posts/2015-08-Backprop/

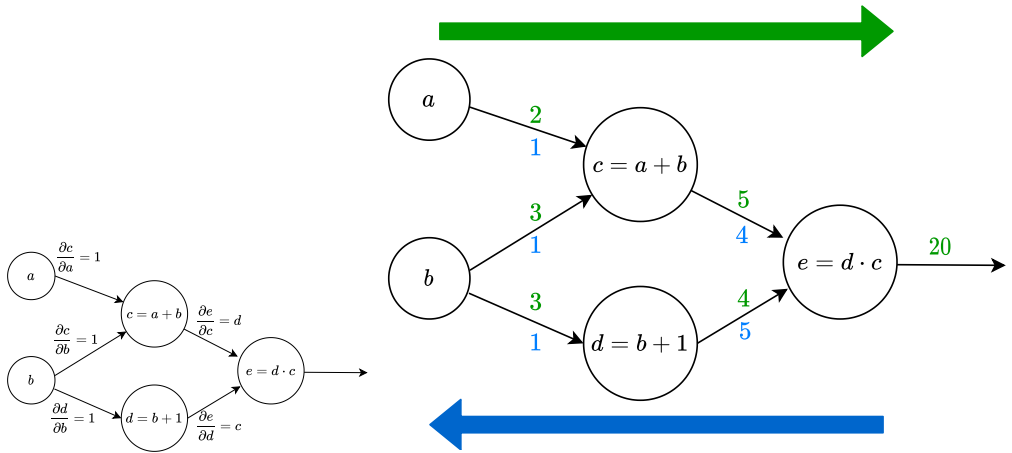


- A forward pass through this graph for $a = 2$ and $b = 3$.

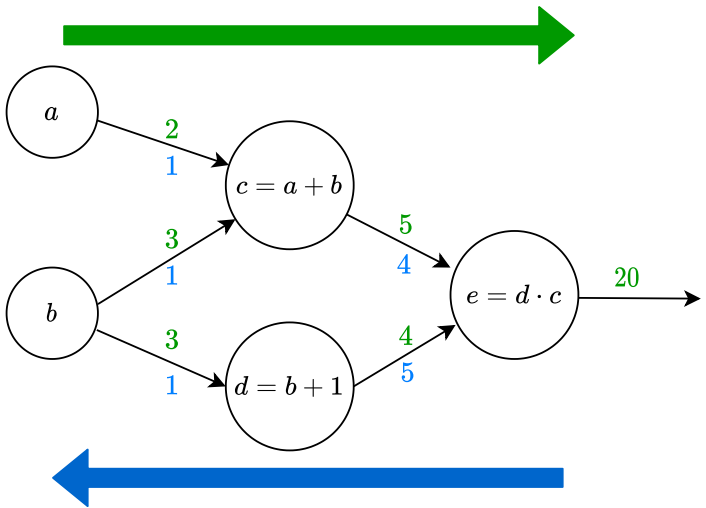


- We can assign the partial derivative to each edge of the graph.



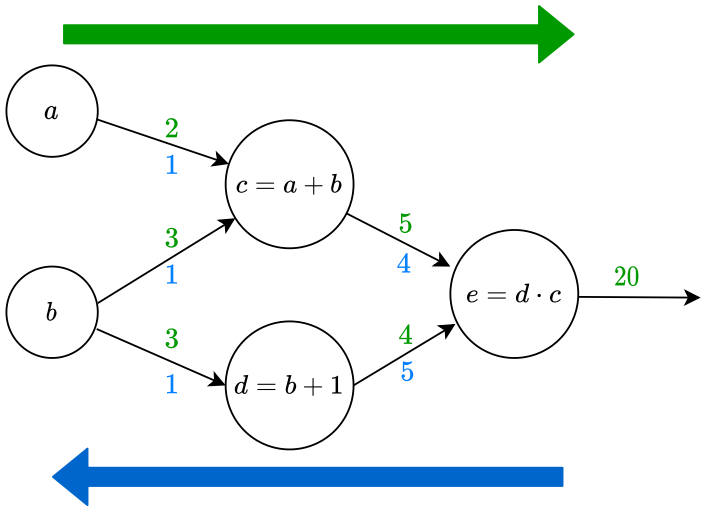


- After a forward pass through the network, we can assign a value to all of the partial derivatives by doing what is called a backward pass.



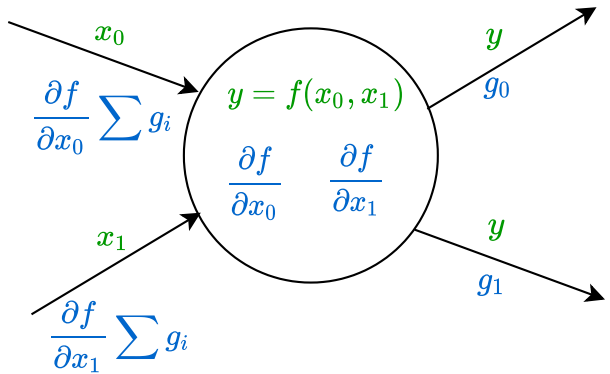
$$\frac{\partial e}{\partial a} = 4 \cdot 1 = \frac{\partial e(c(b))}{\partial c} \frac{\partial c(b)}{\partial b}$$

- If we multiply the values of the partial derivatives of the nodes from output to parameter, we get the partial derivative of the full network with respect to this parameter.
- Which is nothing else than the application of the chain rule.

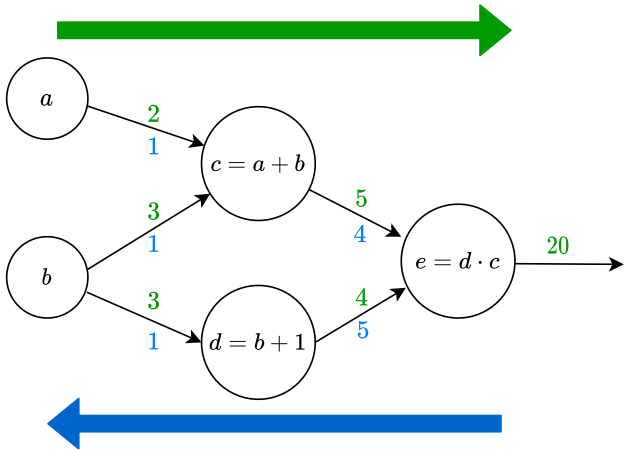


$$\frac{\partial e}{\partial b} = 4 \cdot 1 + 5 \cdot 1$$
$$= \frac{\partial e(c(b))}{\partial c}\frac{\partial c(b)}{\partial b} + \frac{\partial e(d(b))}{\partial d}\frac{\partial d(b)}{\partial b}$$

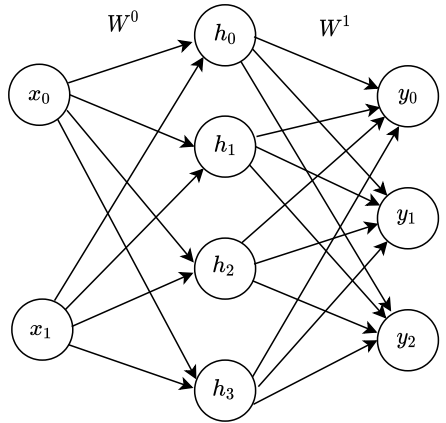
- If there are multiple paths from output to parameter, we have to sum up all the derivatives at every node before propagating the further.
- This also corresponds to the chain rule in the multi variate case.



- Every node needs to implement a function and the partial derivatives of that function with respect to its inputs.
  $\rightarrow$ The values of the partial derivatives of the network with respect to all parameters for a given input, can be computed with one forward and one backward pass.



- Activations (node output) of all layers have to be kept in memory for the backward path!
  → High memory consumption of the network during training.
  → (Yes, Backprop is dynamic programming.)
- Add-nodes distribute gradient equally.
- Multiply-nodes backpropagate their inputs as gradients.
- What does that mean e.g. for the Wx operation?
  → Big input, big gradient on the weights!
  → Preprocessing of input data matters for gradient flow!
  → To understand and monitor gradient flow is crucial for successful training of neural networks.



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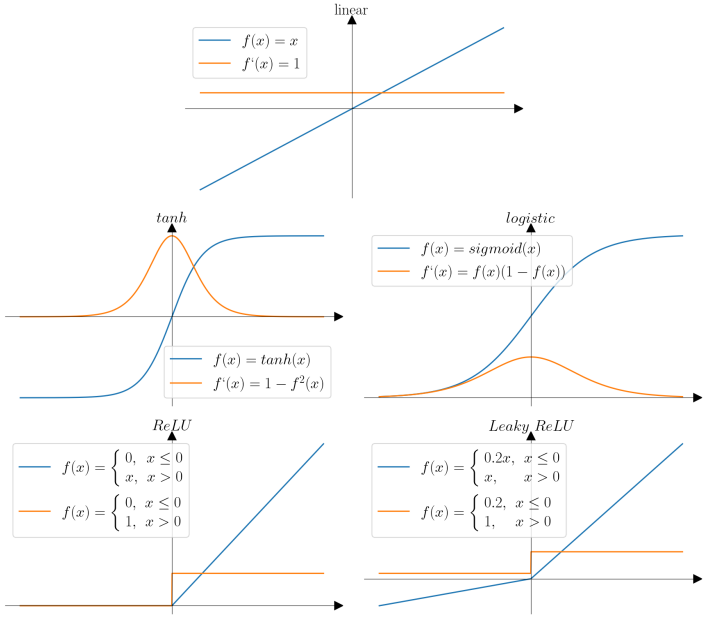
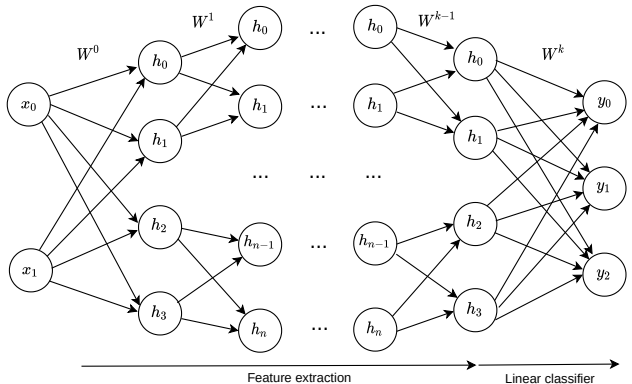
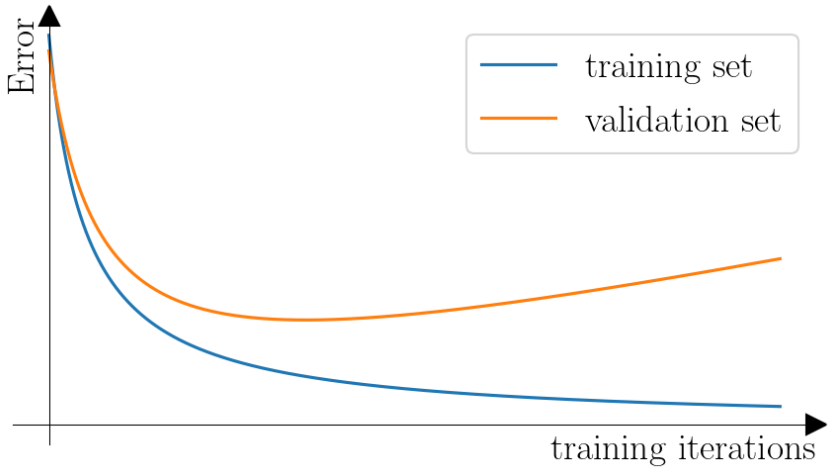


- Let's have a look at possible activation functions.

"With great power comes great overfitting" (Joseph Redmon)



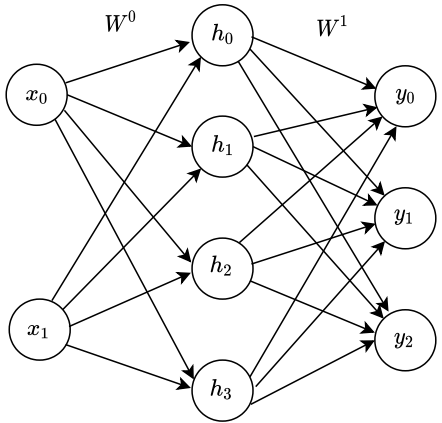
- Deep neural networks have millions of parameters.
- They can learn to fit almost everything. But that's often not what you want.



- When fitting a model with a gradient descent method, we should always look at the error on training and validation sets over training iteration.
- High bias
  → Increase model complexity
- High variance
  → More data and/or regularization
- In deep learning it's less of a trade-off between bias and variance.
  → We can increase model complexity accompanied with more data/regularization until bias is close to zero, without increasing variance much.

- Early stopping
- Weight regularization
- Dropout
- Data augmentation
- Batch normalization

- Regularization is very important. Most approaches use multiple of the regularization methods listed here.
- We will discuss these methods in more detail in the exercises.



$$128 \times 128 \times 3 = 49152$$
$$16 \times 16 \times 36 = 9216$$
$$\rightarrow 49152 \cdot 9216 + 9216 \cdot 10 = 453076992 \approx 450 \cdot 10^6$$

- A small MLP with feature vector comparable to HOG.
- Input: an rgb image relatively low resolution
  $\rightarrow 128 \times 128 \times 3 = 49152$
- Hidden layer: comparable to HOG with 36 dim feature vector computed from 8×8 patches
  $\rightarrow 16 \times 16 \times 36 = 9216$
- Output neurons for e.g. 10 object classes
  $\rightarrow 49152 \cdot 9216 + 9216 \cdot 10 = 453076992 \approx 450$*million* parameters