WRFDA
Background Error Estimation

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Talk overview

- What is Background Error (BE) ?
- Some properties of BE
- Role of BE in WRFDA
- Various components of BE
- Impact of BE on minimization and forecasts
- How to compute ("gen_be" utility)?
- Single Observation Test
- Upcoming new features
- Introduction to Practice Session

What is BE?

- The BE covariance matrix describes the probability distribution function (PDF) of forecast errors, assumed Gaussian
- BE is the covariance of (forecast - truth) in analysis control variable space

$$BE = <(x-x^t), (x-x^t)^T>$$

- Since truth (x^t) is not known, it needs to be estimated
- Common methods for estimating BE
  - Innovation Method
  - NMC Method: $(x-x^t) \approx (x^{t1} - x^{t2})$
    (Forecast differences valid for the same time)
  - Ensemble Method: $(x-x^t) \approx (x^{ens} - <x^{ens}>)$
    = (Ensemble - Ensemble mean)

Some properties of BE

- **B** matrix is square and symmetric. Thus, its eigenvalues are all real and eigenvectors are mutually orthogonal
- It is positive semi-definite. Thus, its eigenvalues are all non-negative. It is very important property because without this minimum of the cost function may not exist
- It consists of correlation (C)and variance (∑) parts, $B = \Sigma C \Sigma$
- If V is an orthogonal matrix ($V^T V = I$) transforming vector X to U ($U = VX$), then the background error for X (B) and of U (B^u) will be related as $B^u = V^T B V$
- A special representation of B is the eigen-representation, where B^u is diagonalized. Eigenvectors of B forms the columns of V and the eigenvalues of B are the diagonal elements of B^u

Role of BE

- **B** spreads information, both vertically & horizontally with proper weights to observations and FG. This effect may be understood by introducing a single observation of one (**k**th) element of **x** in the analysis equation

$$\mathbf{x}^a = \mathbf{x}^b + \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}[\mathbf{y}^o - H(\mathbf{x}^b)]$$

In this case *H* or **H** is a row vector with all elements zero except the **k**th, which is = 1 and $y^o = y$; $R = \sigma^2$. Thus analysis equation gives,

$$x_l^a = x_l^b + B_{lk}\frac{y - x_k^b}{B_{kk} + \sigma^2} = x_l^b + \frac{B_{lk}}{B_{kk} + \sigma^2}y - \frac{B_{lk}}{B_{kk} + \sigma^2}x_k^b$$

Thus non-zero off-diagonal terms for **B** leads to analysis increment for **l**th element

- In data assimilation, this is not the only mechanism of spreading the information. Observation operators (*H* & **H**) also does this job
- If $\sigma^2 << B_{kk}$; $x_k^a \approx y$ and if $\sigma^2 >> B_{kk}$; $x_k^a \approx x_k^b$

  Thus if BE is very large compared to observation error, analysis is closer to observation otherwise it is closer to FG

Role of BE Contd.

- B matrix spreads information between variables and imposes balance
- Since B is the last operator in the analysis equation, the analysis increments lies in the subspace of B.
- B provides a means by which observations can act in synergy. B allows observations to reinforce each other in a way that improves the analysis to a degree that is greater than their individual contributions.
- B is used for preconditioning the analysis equation.

How BE is represented in WRFDA?

- It is represented with a suitable choice of U as follows

$$B = U^T U \qquad \text{with} \quad U = U_p U_v U_h$$

U_h Horizontal Transform

U_v Vertical Transform

U_p Physical Transform

- Horizontal transformation (U_h) is via

  Regional ----- Recursive filters

  Global ----- Power spectrum

- Vertical transformation (U_v) is via EOF's
- Physical transformation (U_p) depends upon the choice of the analysis control variable

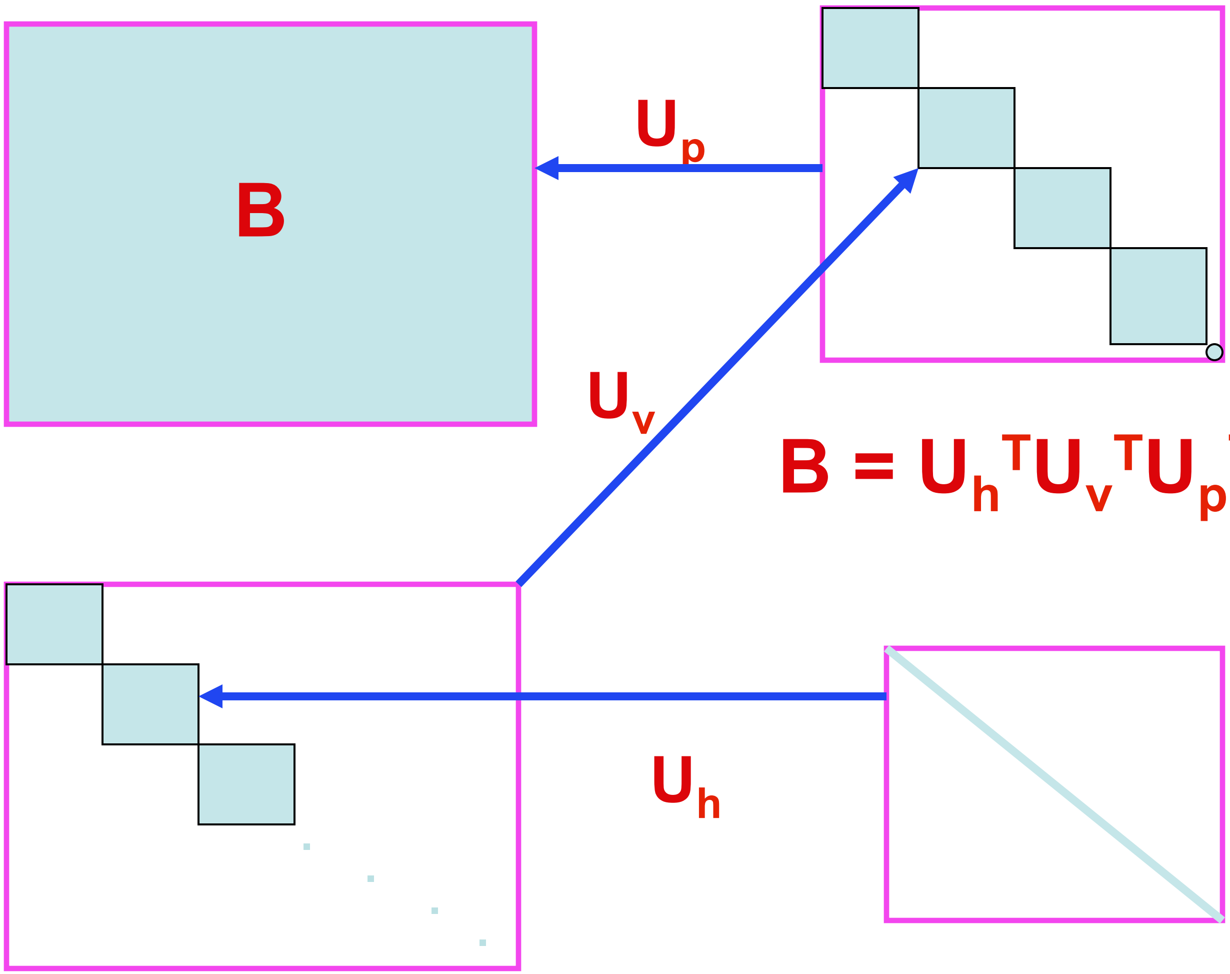
How BE is represented? *Contd.*

- Size of B is typically of the order of $10^7 \text{x} 10^7$
- It is reduced by designing the analysis control variables in such a way that cross covariance between these variables are minimum
- Currently, analysis control variables for WRFDA are the amplitudes of EOF's of

  stream function (ψ)
  Unbalanced part of velocity potential (χ_u)
  Unbalanced part of temperature (T_u)
  Relative Humidity (q)
  Unbalanced part of surface pressure (p_{s_u})

- With this choice of analysis control variables off-diagonal elements of BE is very small and thus its size typically reduces to the order of 10^7

How BE is represented? *Contd.*



$$B = U_h^T U_v^T U_p^T U_p U_v U_h$$

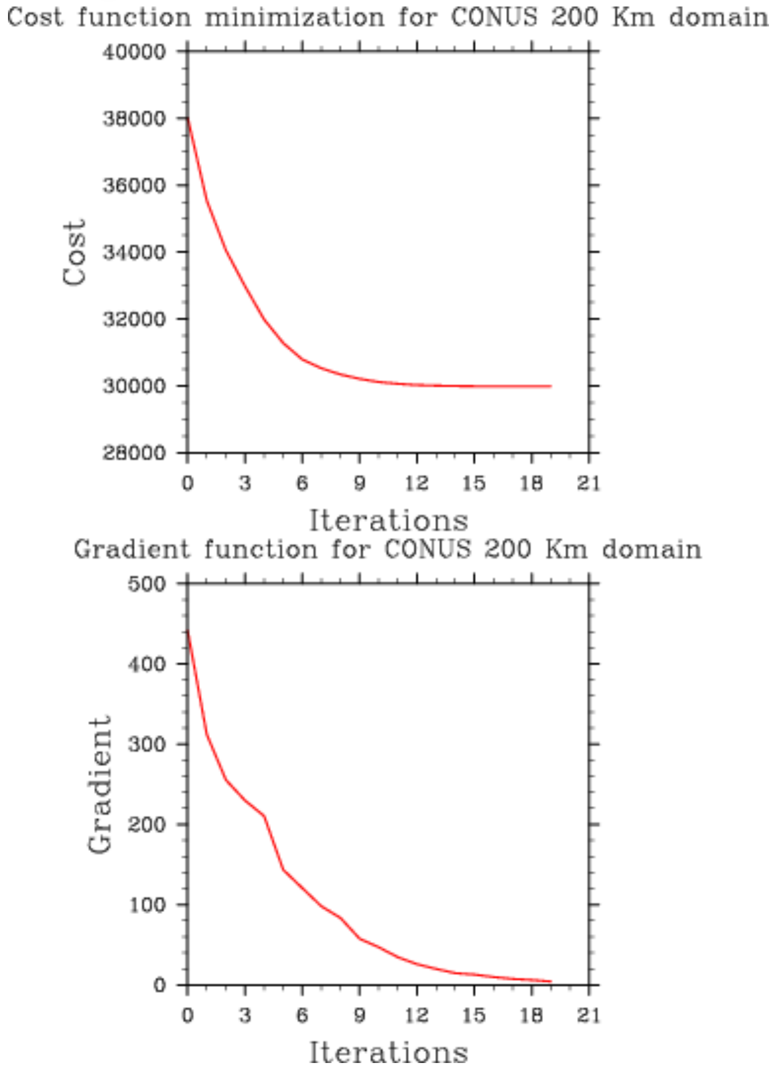
Components of BE

- Regression Coefficient for balanced part of Velocity potential, Temperature and Surface pressure

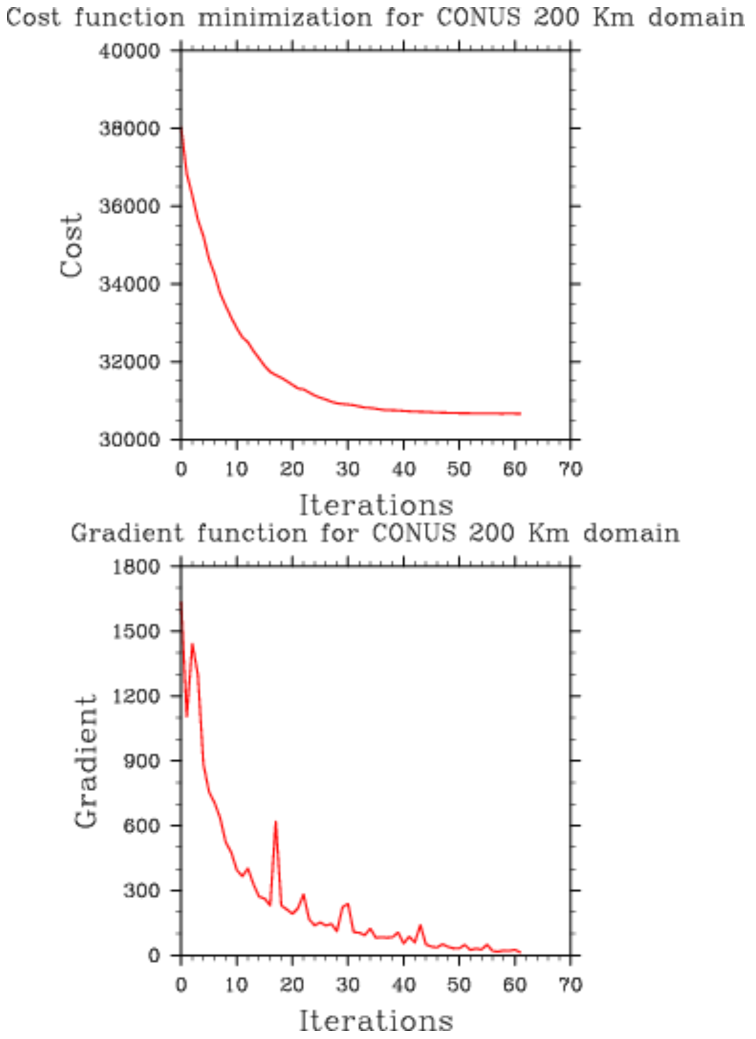
- Eigen vectors and Eigen values for stream function, unbalanced velocity potential, unbalanced temperature and moisture field

- Horizontal length-scales of control variables for regional option
- Power spectrum of control variables for global option

Impact of BE on Minimization

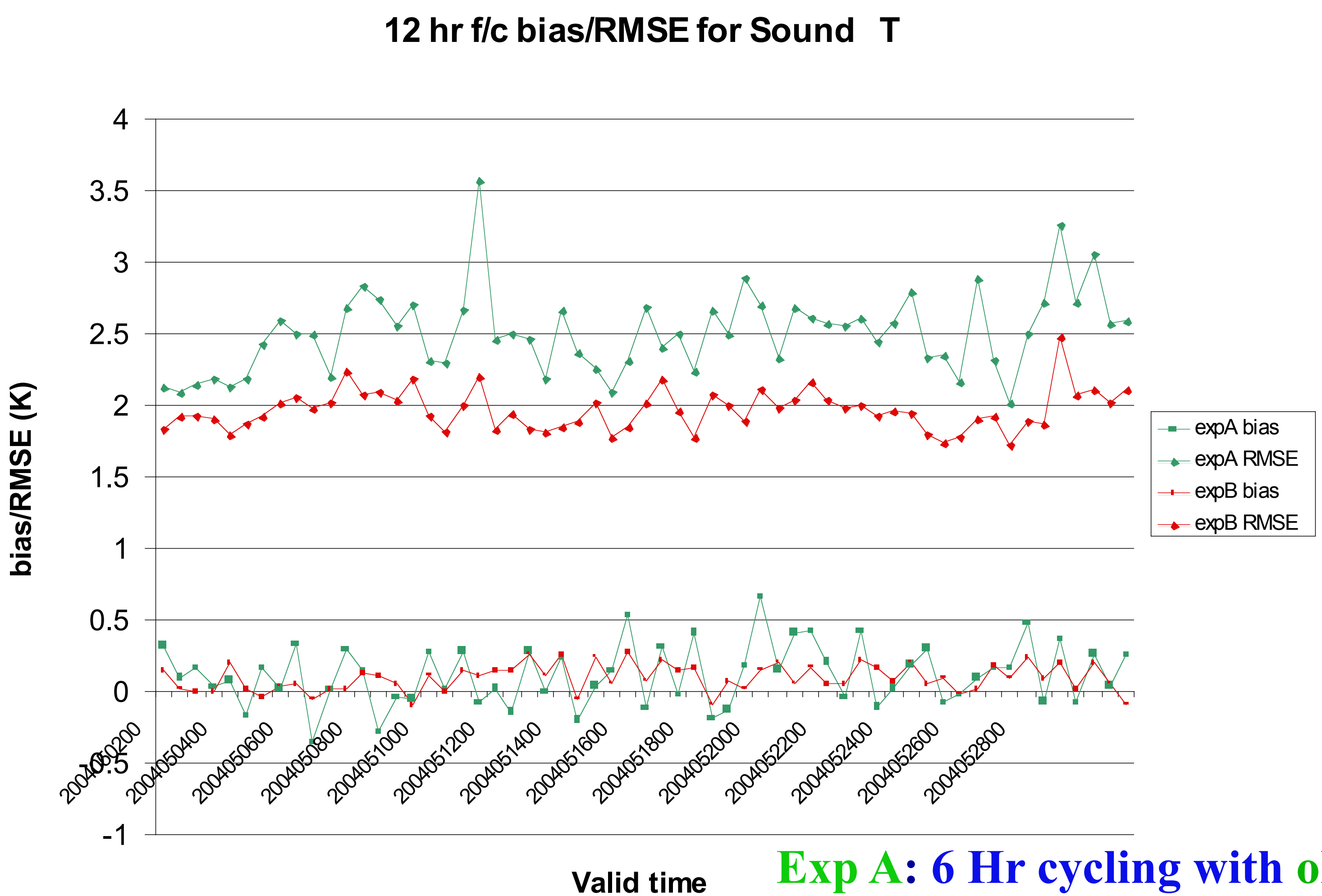


Good BE



Bad BE

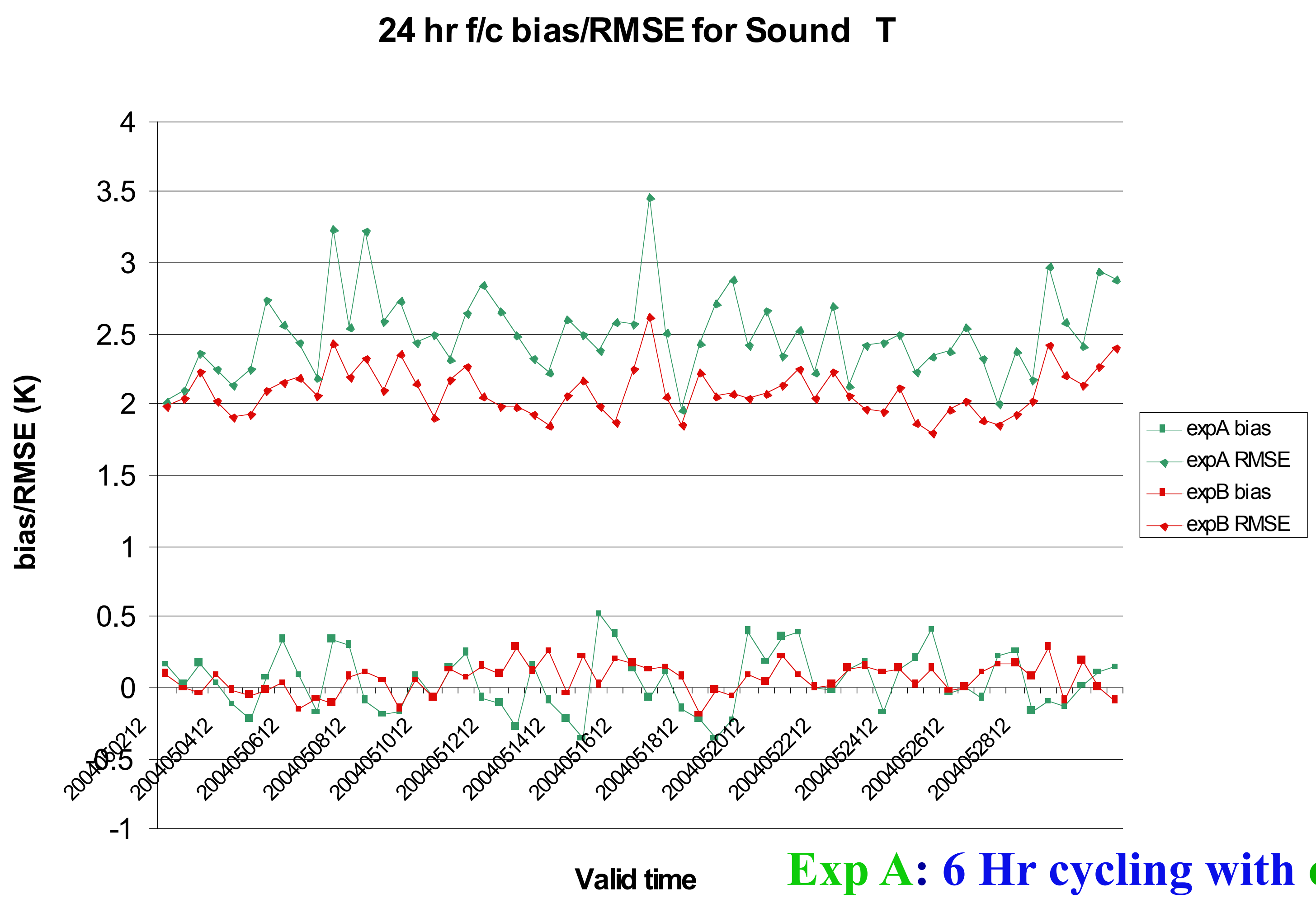
Impact of BE on Temperature forecast



Exp A: 6 Hr cycling with old BE

Exp B: 6 Hr cycling with new BE

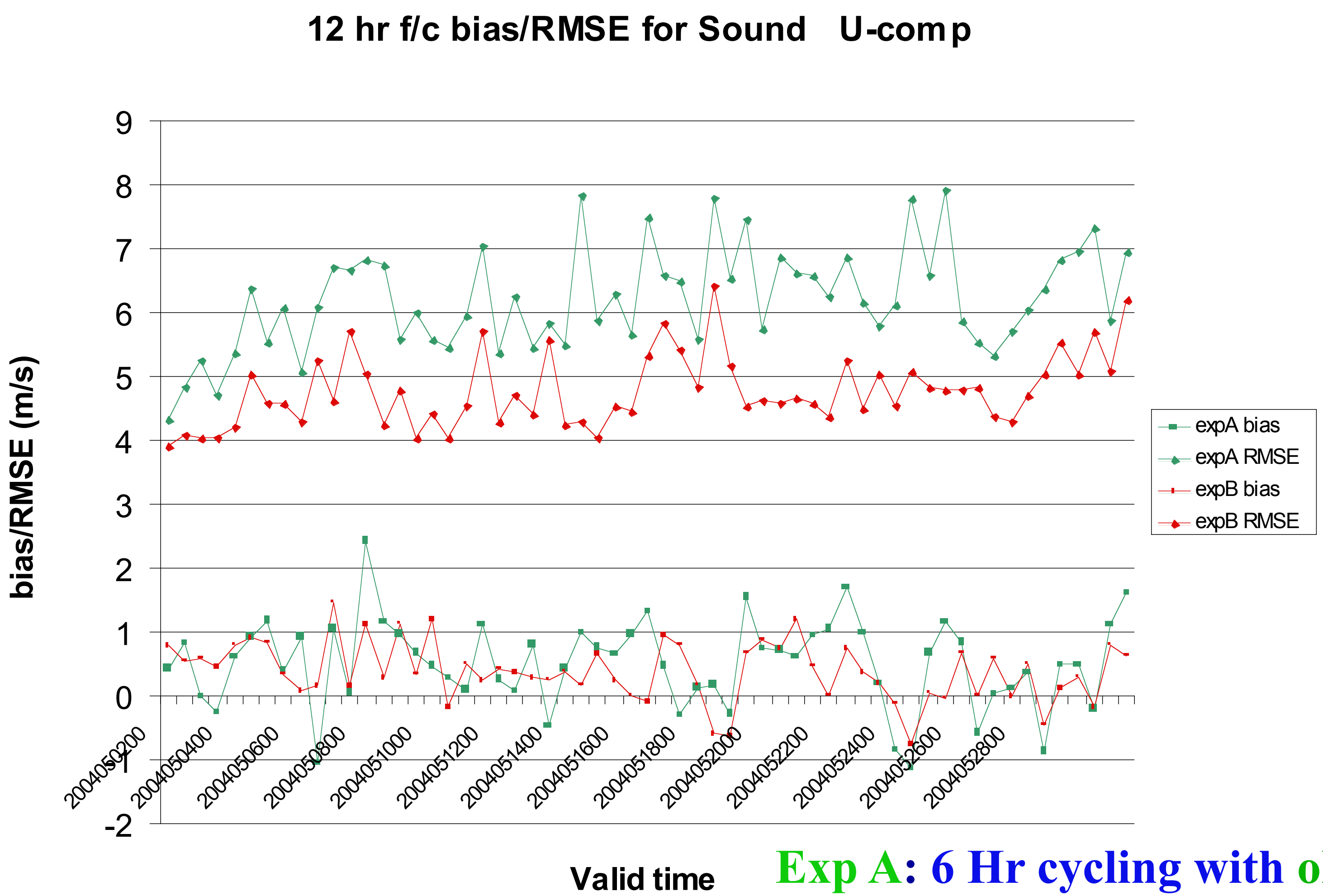
Impact of BE on Temperature forecast



Exp A: 6 Hr cycling with old BE

Exp B: 6 Hr cycling with new BE

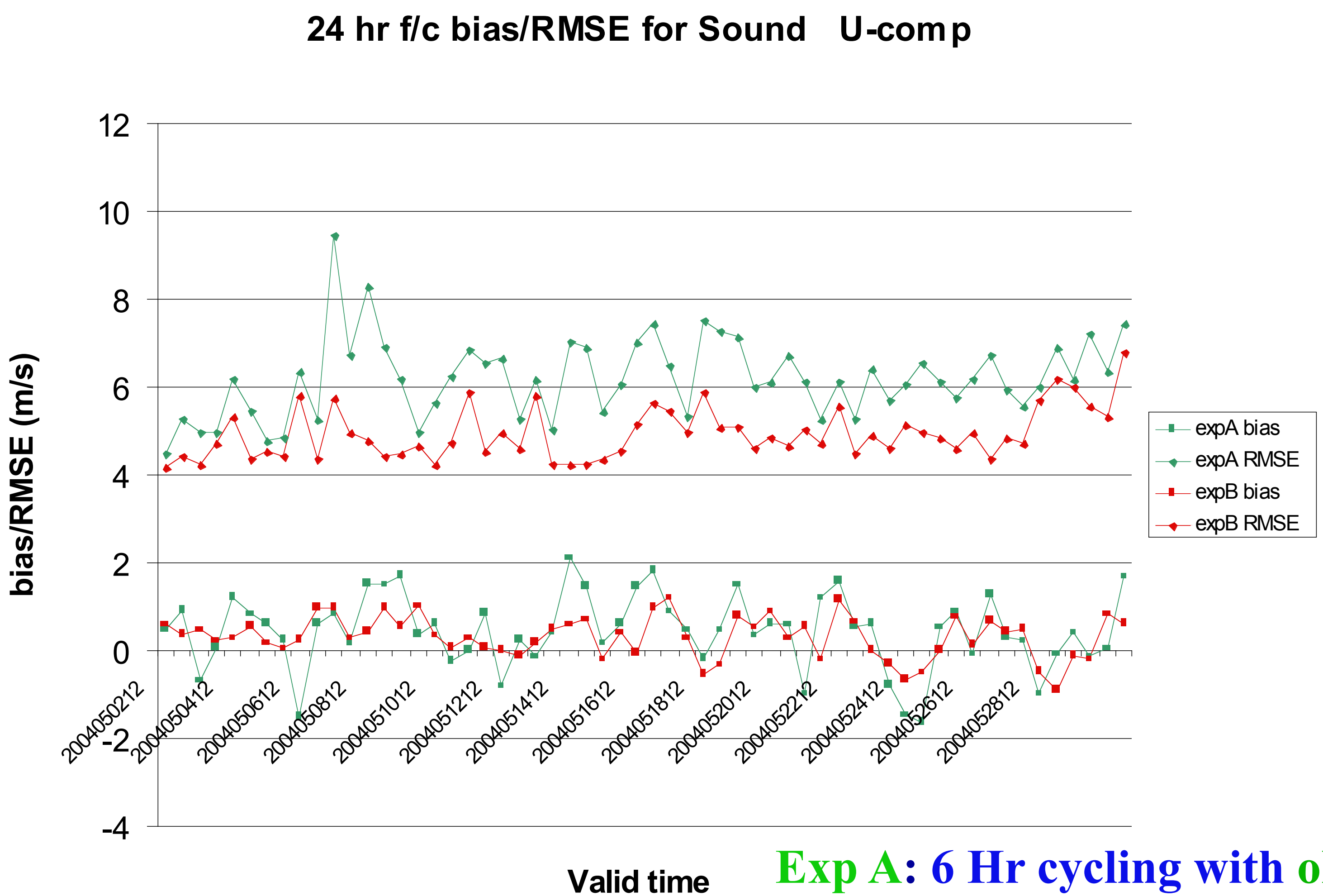
Impact of BE on Wind (U Comp.) forecast



Exp A: 6 Hr cycling with old BE

Exp B: 6 Hr cycling with new BE

Impact of BE on Wind (U Comp.) forecast



Exp A: 6 Hr cycling with old BE

Exp B: 6 Hr cycling with new BE

WRFDA “gen_be” utility:

- It resides in WRFDA under “var” directory
- Computes various components of BE statistics
- Designed both for NMC and Ensemble methods (“BE_METHOD”)
- It consists of five stages
- Basic goal is to estimate the error covariance in analysis control variable space (Coefficients of the EOF’s for ψ, χ_u, T_u, rh and p_{s_u}) with input from model space (U, V, T, q & P_s)

“gen_be” - Stage0

- Computes (ψ , χ) from (u,v)
- Forms desired differences for the following fields

ψ - Stream Function

χ - Velocity potential

T - Temperature

q - Relative Humidity

p_s - Surface Pressure

“gen_be” - Stage1

- Reads “gen_be_stage1” namelist
- Fixes “bins” for computing BE statistics
- Computes “mean” of the differences formed in stage0
- Removes respective “mean” and forms perturbations for

Stream Function	(ψ')
Velocity potential	(χ')
Temperature	(T')
Relative Humidity	(q')
Surface Pressure	(p_s')

“gen_be” - Stage2 & 2a

- Reads “gen_be_stage2” namelist
- Reads field written in stage1 and computes covariance of the respective fields
- Computes regression coefficient & balanced part of χ, T & p_s

$$\chi_b = C\,\psi'$$

$$T_b(k) = \sum_l G(k,l)\,\psi'(l)$$

$$p_{s\_b} = \sum_l W(k)\,\psi'(k)$$

- Computes unbalanced part

$$\chi_u' = \chi' - \chi_b$$

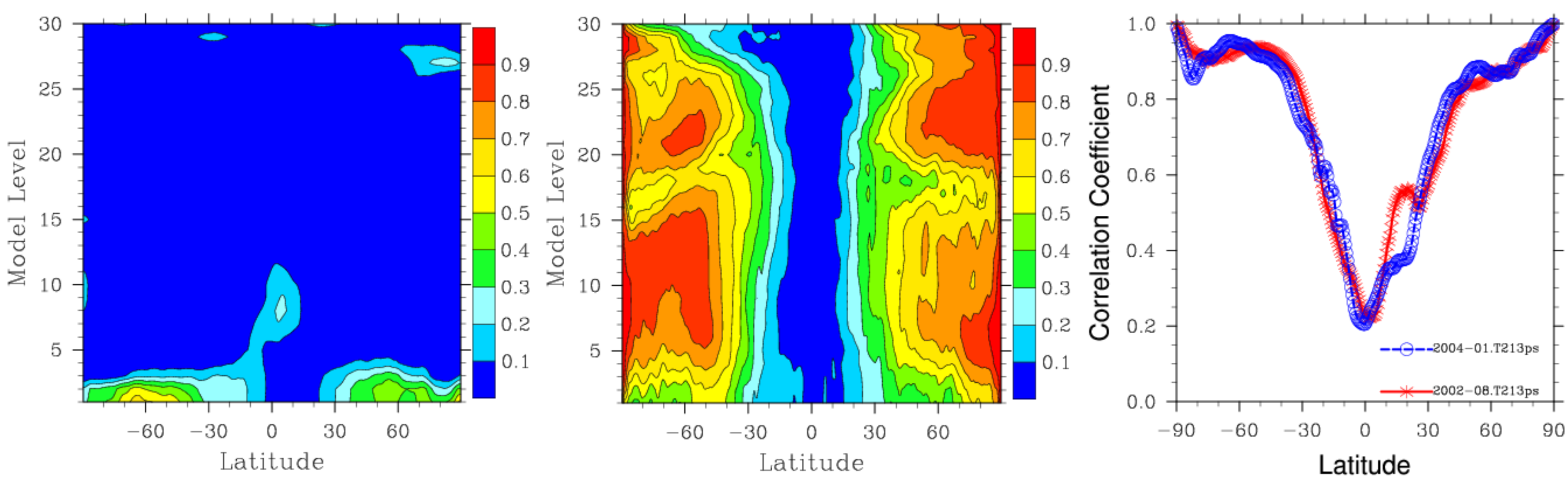
$$T_u' = T' - T_b$$

$$p_{s\_u}' = p_s' - p_{s\_b}$$

WRFDA Balance constraints

- WRFDA imposes statistical balanced constraints between
  - Stream Function & Velocity potential
  - Stream Function & Temperature
  - Stream Function & Surface Pressure
- How good are these balanced constraints?

$\langle \chi_b \bullet \chi \rangle / \langle \chi \bullet \chi \rangle$ $\langle T_b \bullet T \rangle / \langle T \bullet T \rangle$ $\langle p_{sb} \bullet p_s \rangle / \langle p_s \bullet p_s \rangle$



Computed based on KMA global model

“gen_be” - Stage3

- Reads “gen_be_stage3” namelist
- Removes mean for χ_u', T_u' & p_{s_u}'
- Computes eigenvectors and eigen values for vertical error covariance matrix of ψ' , χ_u', T_u' & q
- Computes variance of p_{s_u}'
- Computes eigen decomposition of ψ' , χ_u', T_u' & q

“gen_be” - Stage4

- Reads “gen_be_stage4” namelist
- For each variable & each eigen mode, for regional option computes “lengthscale (s)”

$$\boxed{B(r) = B(0)\exp\{-r^2/8s^2\}}$$

$$\boxed{y(r) = 2\sqrt{2}[\ln(B(0)/B(r)]^{1/2} = r/s}$$

- For global option, computes “power spectrum (D_n)”

$$\boxed{D_n = \sum_{m=-n}^{n}\left(F_n^m\right)^2 = \left(F_n^0\right)^2 + 2\sum_{m=1}^{n}\left[\left(\mathrm{Re}(F_n^m)\right)^2 + \left(\mathrm{Im}(F_n^m)\right)^2\right]}$$

Single observation test

- Through single observation test, one can understand
  - structure of BE
  - It identifies the "shortfalls" of BE
  - It gives a broad guidelines for tuning BE

Basic concept:

Analysis equation: $x^a = x^b + BH^T(HBH^T + R)^{-1}[y^o - H(x^b)]$

Set single observation (U,V,T etc.) as follows:

$[y^o - H(x^b)] = 1.0$; $R = I$

Thus,

$x^a - x^b = B$ * constant delta vector

How to activate Single obs test (PSOT)?

"single obs utility" or "psot" may be activated by setting the following namelist parameters

num_pseudo = 1

pseudo_var =" Variable name" like "U", "T", "P", etc.

pseudo_x = "X-coordinate of the observation"

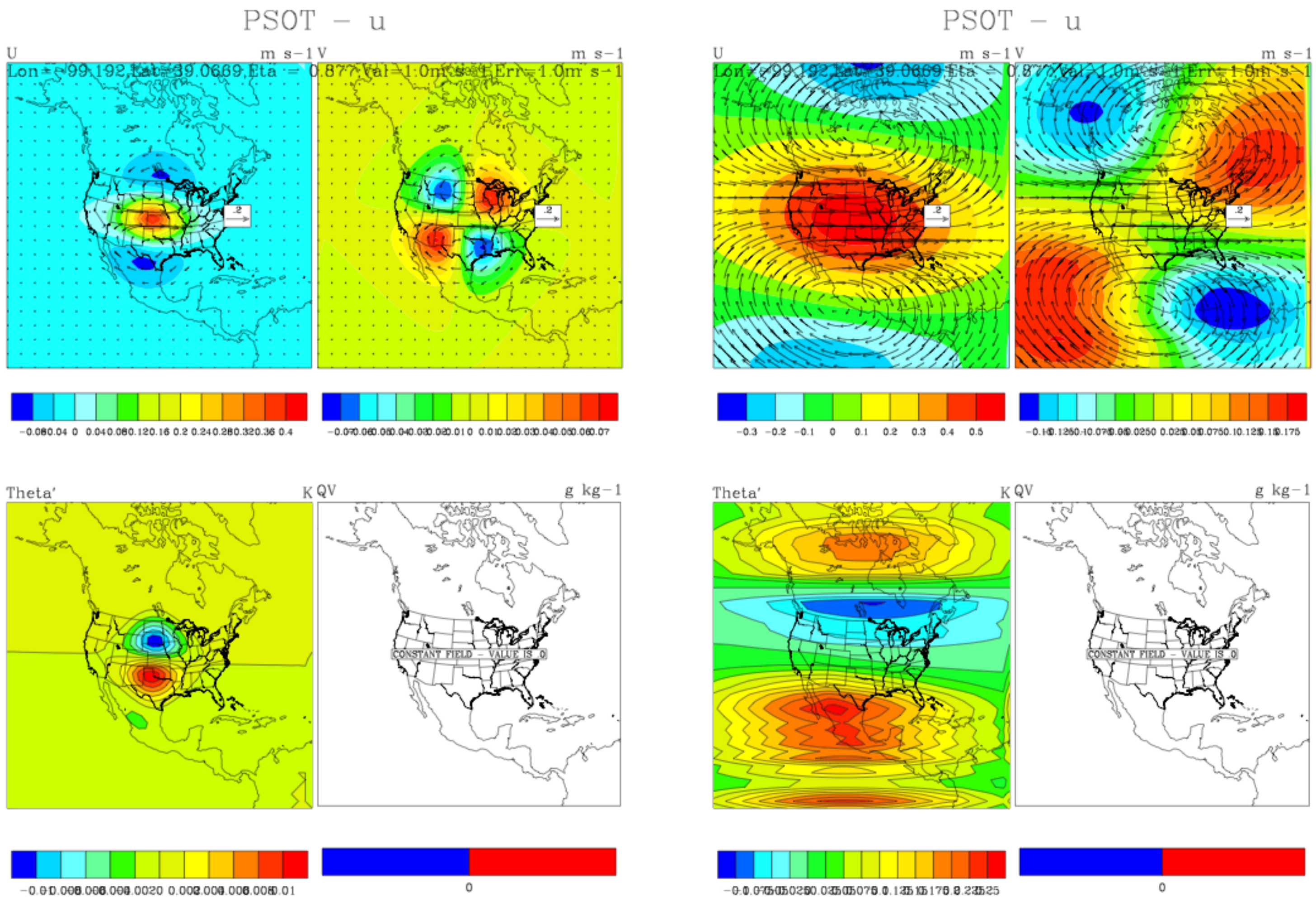
pseudo_y = "Y-coordinate of the observation"

pseudo_z = "Z-coordinate of the observation"

pseudo_val = "Observation innovation", departure from FG"

pseudo_err = "Observation error"

Single Obs (U) test with different BE



How to perform tuning of BE?

- Horizontal component of BE can be tuned with following namelist parameters

  LEN_SCALING1 - 5 (Length scaling parameters)

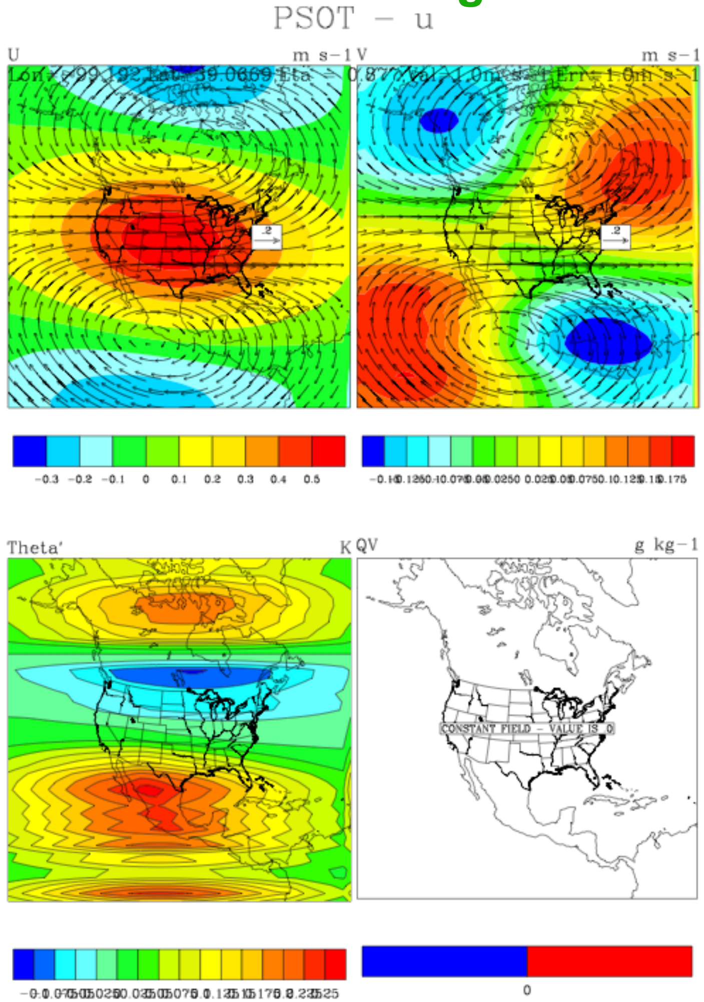
  VAR_SCALING1 - 5 (Variance scaling parameters)

- Vertical component of BE can be tuned with following namelist parameter

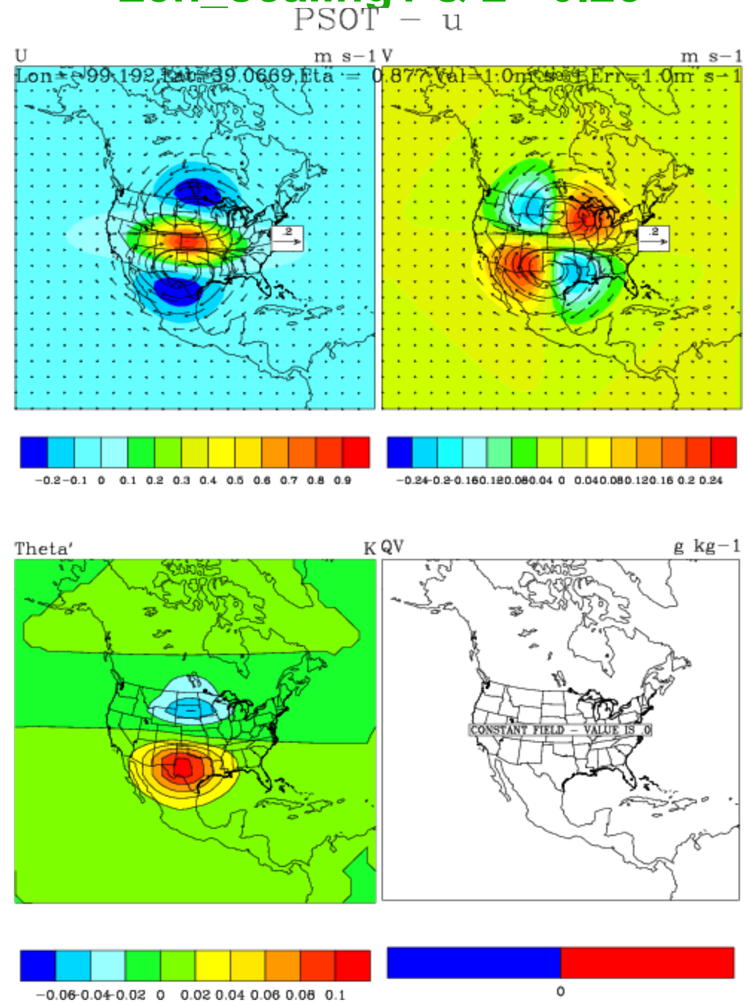
  MAX_VERT_VAR1 - 5 (Vertical variance parameters)

Results with BE Tuning

No tuning



Len_scaling1 & 2 =0.25



Multivariate formulation of BE

- New set of analysis control variables (cv_options=6) have been designed

$$\chi_b(i,j,k) = \alpha_{\chi\psi} * \psi(i,j,k)$$

$$T_b(i,j,k) = \sum_{l=1}^{N_k} \alpha_{T\Psi}(i,j,k,l) * \psi(i,j,k,l) + \sum_{l=1}^{N_k} \alpha_{T\chi_u}(i,j,k,l) * \chi_u(i,j,l)$$

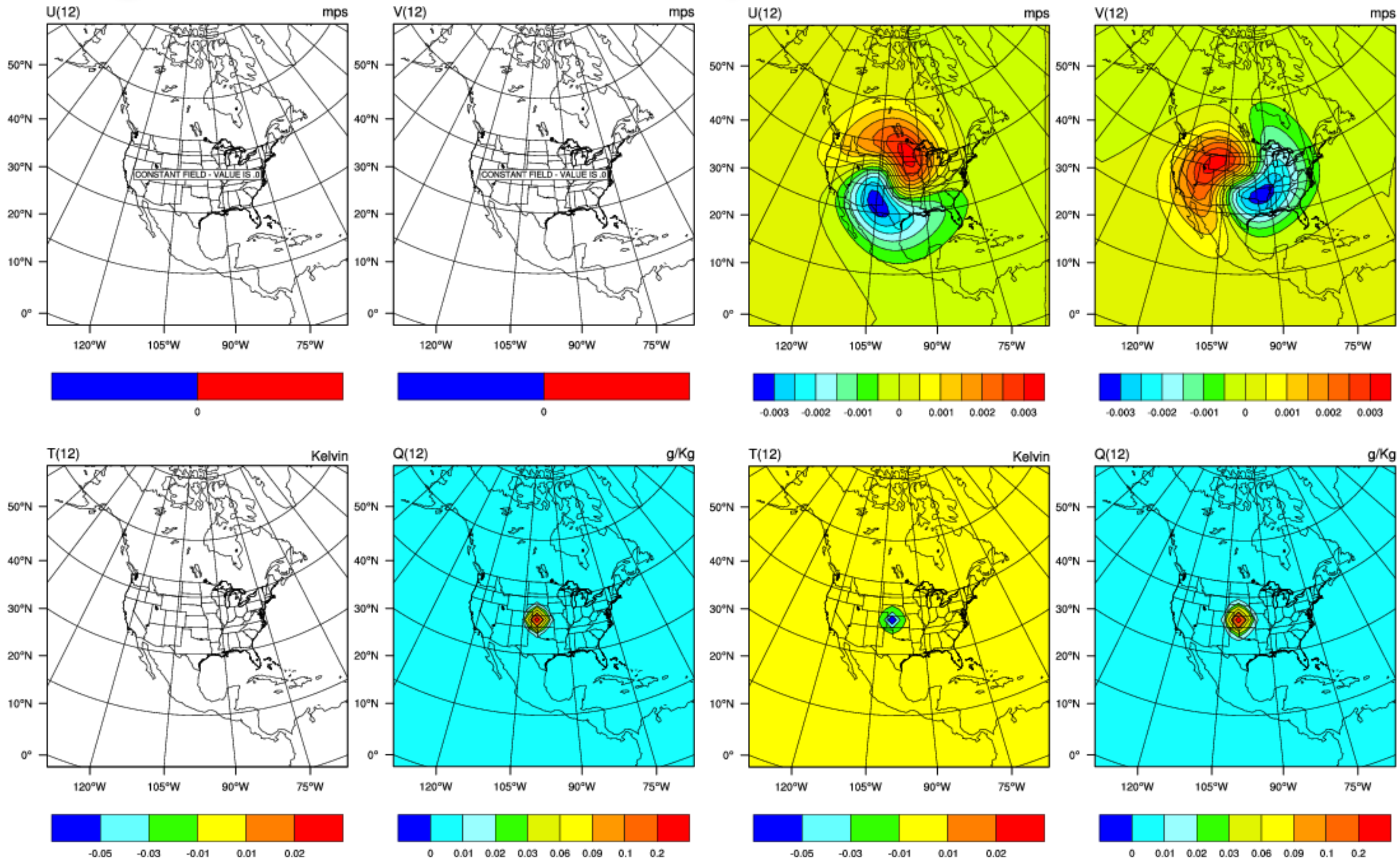
$$Q_b(i,j,k) = \sum_{l=1}^{N_k} \alpha_{Q\psi}(i,j,k,l) * \psi(i,j,l) + \sum_{l=1}^{N_k} \alpha_{Q\chi_u}(i,j,k,l) * \chi_u(i,j,l) +$$

$$+ \sum_{l=1}^{N_k} \alpha_{QT_u}(i,j,k,l) * T_u(i,j,l) + \sum_{l=1}^{N_k} \alpha_{ps_u Q}(i,j,l) * ps_u(i,j)$$

$$P_s(i,j) = \sum_{l=1}^{N_k} \alpha_{P_s\Psi}(i,j,l) * \psi(i,j,l) + \sum_{l=1}^{N_k} \alpha_{P_s\chi_u}(i,j,l) * \chi_u(i,j,l)$$

Indexes i,j and k are corresponding to West-East, North-South and vertical sigma levels respectively, N_k is the number of sigma levels and α is the regression coefficient between the variables indicated in its subscript.

Single Obs (Moisture) test



cv_options=5, BE

Cv_optins=6, BE

Upcoming new features

- Some filtering options at various stages
- Background error for cloud hydrometeors like cloud water vapor, ice, snow and rain
- Introduction of new “bin_type=7” for four types of “rain” categories
- Additional diagnostics to study the frequency distribution of background error statistics
- Implementation of Holm (2002) type background error
- Stand alone branch of “gen_be”

Advanced Practice Session – "gen_be"

- Compilation of "gen_be" utility
- Generation of BE statistics
- Familiarization with various graphical utilities to display "gen_be" diagnostics
- Running single observation tests to understand the structure of BE
- BE error tuning

Generation of BE

- "gen_be_wrapper.ksh" script for generating BE for "CONUS" at 200 Km domain with:

  Grid Size : 45 x 45 x 28
  BE Method : NMC Method
  Data Input : January, 2007 forecasts, both from 00 & 12 UTC IC

Basic environment variables that needs to be set are:

- Gen_be executables location (WRFVAR_DIR)
- Forecast input data (FC_DIR)
- Run directory (BE_DIR)
- Data Range (START_DATE, END_DATE)

"gen_be" wrapper script basically executes "var/scripts/gen_be/gen_be.ksh" script

Gen_be diagnostics

- "gen_be" creates various diagnostic files which may be used to display various components of BE statistics.
- Important files are:

Eigen vectors: fort.174, fort.178, fort.182, fort.186

Eigen values: fort.175, fort.179, fort.183, fort.187

scalelength: fort.194, fort.179, fort.183, fort.187

Correlation between χ_u & χ_b (chi_u.chi.dat)

Correlation between T_u & T_b (T_u.T.dat)

Correlation between p_{s_u} & (ps_u.ps.dat)

Important Strings that needs to be defined in the wrapper script "var/script/gen_be/gen_be_plot_wrapper.ksh"

BE_DIR --- gen_be Run directory

How to run Single Observation Test ?

- Familiarization with single observation “wrapper” script (“da_run_suite_wrapper_con200.ksh”) to run Single Observation test
- Key parameters are

  Type of observation (pseudo_var)

  Obs co-ordinates (pseudo_x, pseudo_y & pseudo_z)

  Observation value (pseudo_val)

  Observation error (pseudo_err)

- Display analysis increments to understand BE structure

BE tuning

- Understand the role of BE-tuning parameters through namelist options

LEN_SCALING1 - 5 (Length scaling parameters)
VAR_SCALING1 - 5 (Variance scaling parameters)
MAX_VERT_VAR1 - 5 (Vertical variance parameters)

Note: If BE is available for the same domain configuration then it's tuning is not required