WRFDA Background Error (Modeling and Estimation)

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Talk Overview

- Background Error (BE) and its role in DA?
- Modeling of BE
- Estimation of BE ("gen_be" utility)
- Single observation test and tuning of BE
- Impact of BE on analysis and NWP forecast
- Hands on practice session

Background Error (BE)

- If $\mathbf{x}$ is the forecast of the analysis variable and $\mathbf{x}^{\mathrm{t}}$ is the corresponding true state, the BE is defined as the covariance of forecast minus truth $(\mathbf{x} - \mathbf{x}^{\mathrm{t}})$.

$$\mathbf{BE} = \langle(\mathbf{x} - \mathbf{x}^{\mathrm{t}}), (\mathbf{x} - \mathbf{x}^{\mathrm{t}})^{\mathrm{T}}\rangle$$

- Thus, the BE covariance matrix ($\mathbf{B}$) describes the probability distribution function (PDF) of the forecast errors $(\mathbf{x} - \mathbf{x}^{\mathrm{t}})$

Role of BE in DA

- **B** appears in the cost function and the analysis equation as,

$$J(x) = \frac{1}{2}(x - x^b)^T B^{-1}(x - x^b) + \frac{1}{2}[y - H(x)]^T R^{-1}[y - H(x)]$$

$$x^a - x^b = \mathrm{BH}^T(\mathrm{HBH}^T + \mathrm{R})^{-1}[y^o - H(x^b)]$$

- Thus, **B** gives proper weight to the background term ($\boldsymbol{x} - \boldsymbol{x}^{\mathrm{b}}$) in defining the analysis cost function (J)
- Since **B** is the last operator in the analysis equation, the analysis increment ($\boldsymbol{x}^{\mathrm{a}} - \boldsymbol{x}^{\mathrm{b}}$) lies in the subspace of **B**
- **B** spreads information, both vertically and horizontally with proper weights to observation ($\boldsymbol{y}^{\mathbf{o}}$) and the background ($\boldsymbol{x}^{\mathrm{b}}$)

Role of BE in DA

- **B** spreads information between variables and imposes balance across different analysis variables. Thus, a pressure or the temperature observation has the ability to modify the wind analysis and vise-versa
- **B** provides a means by which observations can act in synergy, meaning **B** allows observations to reinforce each other in a way that improves the analysis to a degree that is greater than their individual contributions
- **B** is used for preconditioning the analysis equation

Modeling of BE

Why ?

- **B** is a square, symmetric and positive definite matrix ($x^T\mathbf{B}x > 0$ for all non-zero vectors x) with dimension equal to the number of the analysis variables
- Thus, typically the size of **B** is of the order of $10^7 \times 10^7$ and so, it is not possible to either store or compute its inverse

How ?

- The size of **B** is reduced by designing the actual analysis control variables in such a way that the cross covariance between these variables are minimum (zero)
- Thus assuming all the off-diagonal elements as zero, the size of **B** is typically reduced to the order of 10^7

Modeling of BE

- Let us define a control variable transform (CVT),

$$\delta x = B^{1/2} v$$

or,

$$\delta x = Uv$$

Where,

$$\delta x = x - x^b$$

$$U = B^{1/2} \quad \text{and} \quad vv^T = I$$

- Thus $\mathbf{B} = \boldsymbol{U}\boldsymbol{U}^{\mathrm{T}}$, and so the modeling of back ground error amounts to approximating the control variable transform ($\boldsymbol{U}$)
- $\boldsymbol{U}$ is approximated with a sequence of three linear transforms

$$U = U_p U_v U_h$$

- Thus,

$$B = U_p U_v U_h U_h^T U_v^T U_p^T$$

Control variable transform (CVT)

$$\boldsymbol{U} = \boldsymbol{U}_p \boldsymbol{U}_v \boldsymbol{U}_h$$

- $\boldsymbol{U}_h \longrightarrow$ Horizontal transform. It is applied via recursive filter (Hayden and Purser(1995)
- $\boldsymbol{U}_v \longrightarrow$ Vertical transform. It is applied through empirical orthogonal functions (EOFs). The EOFs are the eigenvectors of the vertical error covariance matrix (**E**). Thus,

$$\boldsymbol{U}_v = \mathbf{E}\, \Lambda^{1/2}$$

Where, $\Lambda^{1/2}$ is a diagonal matrix holding square root of the eigenvalues of vertical error covariance matrix (**E)**

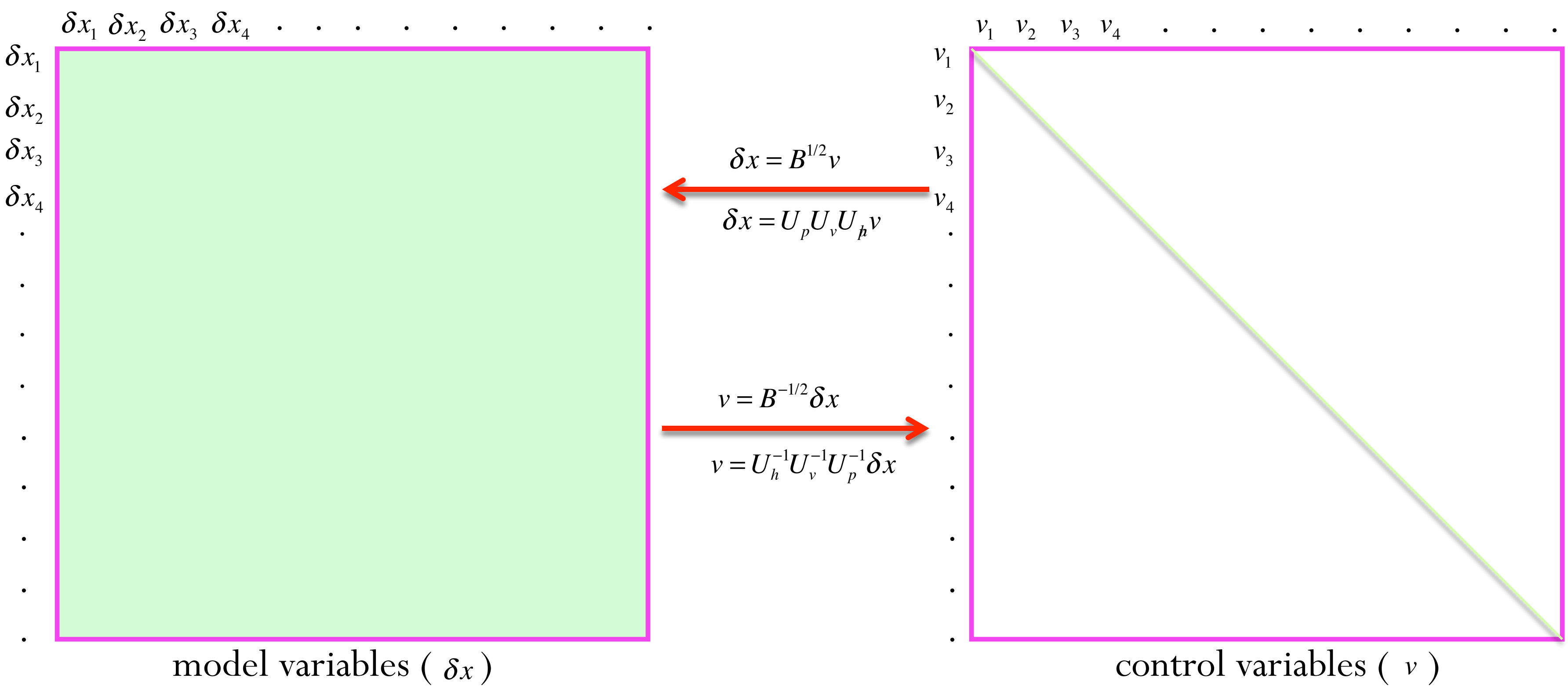
- $\boldsymbol{U}_p \longrightarrow$ Physical transform. It is applied via statistical balance

Modeling of BE

Thus, for modeling of background error, following is estimated

- Horizontal length-scale for $\boldsymbol{U}_{\mathrm{h}}$ transform
- Eigenvectors and eigenvalues for $\boldsymbol{U}_{\mathrm{v}}$ transform
- Regression coefficients for $\boldsymbol{U}_{\mathrm{p}}$ transform

How CVT ($U=U_pU_vU_h$) works ?



cost function (J): $\mathrm{J}(v) = \frac{1}{2} v^T v + \frac{1}{2} (d - \mathrm{H}Uv)^T R^{-1} (d - \mathrm{H}Uv)$

gradient of J: $\nabla_v J = v + U^T R^{-1} (d - \mathrm{H}Uv)$

analysis: $x^a = x^b + Uv$

Choice of analysis variables

cv_options	Analysis variables
3	Ψ, unbalance X, unbalance t, pseudo rh and unbalance log (P_s)
5	Ψ, unbalance X, unbalance t, pseudo rh and unbalance P_s
6	Ψ and unbalance X, unbalance t, unbalance pseudo rh and unbalance P_s
7	u, v, t, Ps and pseudo rh

Estimation of Background Error

- For simplicity, the background error distribution is assumed Gaussian
- Since the truth ($\mathbf{x}^{t}$) is not known, the forecast error ($\mathbf{x} - \mathbf{x}^{\mathbf{t}}$) needs to be estimated
- There are two common methods for estimating ($\mathbf{x} - \mathbf{x}^{\mathbf{t}}$)

  a) NMC method: $(\mathbf{x} - \mathbf{x}^{\mathbf{t}}) \approx (\mathbf{x}^{\mathbf{t1}} - \mathbf{x}^{\mathbf{t2}})$

  The difference of forecasts (with t1 and t2 ICs) valid for the same time

  b) Ensemble method: $(\mathbf{x} - \mathbf{x}^{\mathbf{t}}) \approx (\mathbf{x}^{\mathbf{ens}} - <\mathbf{x}^{\mathbf{ens}}>)$

  Ensemble minus Ensemble mean

“gen_be” utility

“gen_be” utility estimates the different components of the BE

- It is designed to work both for NMC and Ensemble methods with “namelist” option:

  BE_METHOD= “NMC” for NMC-method

  BE_METHOD =“ENS” for Ensemble method
- It consists of five stages (0 – 4)

Stage0: (forecast error samples)

- Step 1 - (u,v) to horizontal divergence (D) and vorticity (ζ)

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \qquad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

- Step 2 – Convert D and ζ) to Ψ and χ

$$\nabla^2 \psi = \zeta \qquad \nabla^2 \chi = D$$

- Finally, the forecast errors ($\mathbf{x}$-$\mathbf{x}^t$) are generated for

Ψ - Stream function

χ - Velocity potential

T - Temperature

q - Relative humidity

p_s - Surface pressure

Stage1: (removes temporal mean)

- Computes temporal mean of the forecast error samples generated in stage0
- Removes temporal mean to form the perturbations for

  Stream function (ψ')

  Velocity potential (χ')

  Temperature (T')

  Relative humidity (q')

  Surface pressure (p_s')

Stage2: (Regression coefficients)

- Regression coefficient (α_{xy}) between two variables ***x*** and ***y*** is estimated as

$$\alpha_{xy} = \frac{\langle x.y \rangle}{\langle x.x \rangle}$$

Where,

$\langle x,y \rangle$ is the covariance between ***x*** and ***y***

$\langle x,x \rangle$ is the variance of ***x***

Stage2a: (Input for U_p-transform)

The U_p transform is defined as,

$$\begin{pmatrix} \Psi \\ \chi \\ t \\ Ps \\ rh \end{pmatrix} = \begin{pmatrix} I & 0 & 0 & 0 & 0 \\ M & I & 0 & 0 & 0 \\ N & 0 & I & 0 & 0 \\ Q & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{pmatrix} \begin{pmatrix} \psi \\ \chi_u \\ t_u \\ Ps_u \\ rh \end{pmatrix}$$

Where,

I - identity matrix, **0** – zero matrix and **M, N, Q** are respectively the regression coefficient matrices for (χ , ψ), (t, ψ), and (Ps_u, ψ)

Stage2a: (cv_options=6, the MBE)

The U_p transform is defined as,

$$\begin{bmatrix} \Psi \\ \chi \\ \mathbf{t} \\ \mathbf{Ps} \\ \mathbf{rh} \end{bmatrix} = \begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{M} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{N} & \mathbf{P} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{Q} & \mathbf{R} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{S}_1 & \mathbf{S}_2 & \mathbf{S}_3 & \mathbf{S}_4 & \mathbf{I} \end{pmatrix} \begin{bmatrix} \psi \\ \chi_{\mathbf{u}} \\ \mathbf{t}_{\mathbf{u}} \\ \mathbf{Ps}_{\mathbf{u}} \\ \mathbf{rh}_{\mathbf{u}} \end{bmatrix}$$

Where,

$\mathbf{P}$, $\mathbf{R}$, $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$ and $\mathbf{S}_4$ are respectively the regression coefficient matrices for (t, χ_u), (Ps_u, χ_u), (rh, ψ), (rh, χ_u), (rh,t_u) and (rh,Ps_u)

Stage3: (Input for U_v-transform)

For all 3-D analysis variables,

- Compute vertical error correlation matrix
- Compute eigenvectors (**E**) and eigenvalues (Λ) of the vertical error covariance matrix
- Perform $\Lambda^{-1/2}\,\mathbf{E}^T$ to compute the amplitude of the corresponding EOFs

Stage4 (Input for U_h-transform)

a) Computes covariance (**z**) of the coefficients of the EOF's in distance-wise bins

b) Assuming the horizontal covariance has exponential decay (Gaussian function) as,

$$\boxed{z(r) = z(0)\exp\{-r^2 / 8s^2\}}$$

c) Estimate the horizontal length-scale (**s**) of the covariance using linear curve fitting method as,

$$\boxed{y(r) = 2\sqrt{2}[\ln(z(0) / z(r))]^{1/2} = r / s}$$

"gen_be" Bin's Choice

bin_type	Total number of bins (num_bins) and bin's description
0	num_bins= total number of grid points (no binning)
1	num_bins=nj * nk (each latitude is a bin)
2	num_bins= bin_width_lat * bin_width_hgt
3	num_bins=bin_width_lat * n_k (bin_width_lat is defined with lats.)
4	num_bins=bin_width_lat * n_k (bin_width_lat is defined with the number of points in south-north direction)
5	num_bins=n_k (bins with all horizontal points)
6	num_bins=1 (average over all the grid (3D) points)

- n_j – number of points in south-north direction
- n_k – number of points in vertical
- **Remarks:** Default option is "bin_type=5"

Single observation test (PSOT)

Why?

Assimilation of a single observation helps in understanding the following aspects of the background error

- Its role and the structure
- Identify the "shortfalls"
- Broad guidelines for tuning

PSOT – Basic concept

Let y be the single observation for the k[th] element of $\mathbf{x}^b$ with standard observation error σ. Then the analysis equation

$$x^a = x^b + \mathrm{BH}^T(\mathrm{HBH}^T + \mathrm{R})^{-1}[y^o - H(x^b)]$$

leads to

$$x_l^a - x_l^b = \frac{B_{lk}}{B_{kk} + \sigma^2}(y - x_k^b)$$

Thus,

- If $\sigma^2 << B_{kk} \quad \Rightarrow \quad x_k^a = y$
- If $\sigma^2 >> B_{kk} \quad \Rightarrow \quad x_k^a = x_k^b$
- Thus, if BE is very large compared to observation error, analysis is closer to observation, otherwise it is closer to the first guess (FG) or the background
- A non-zero off-diagonal term B_{lk} of **B** leads to non-zero analysis increment for the l^{th} element of $\mathbf{x}^a$

PSOT – Basic concept

- Set single observation (u, v, t, ps etc.) as,

  unit innovation, $[y^o - H(x^b)] = 1.0$

  unit observation error, $R = 1.0$

The analysis equation

$$x^a = x^b + \mathrm{BH}^T(\mathrm{HBH}^T + \mathrm{R})^{-1}[y^o - H(x^b)]$$

gives,

$$x^a - x^b = B\delta$$

Where, δ is a constant delta vector

- Thus, analysis increments with single observation, displays the structure of the background error

How to activate PSOT?

PSOT utility may be activated by setting the following namelist parameters

num_pseudo = 1

pseudo_var = Variable name like u, t, p, etc.

pseudo_x = X-coordinate of the observation

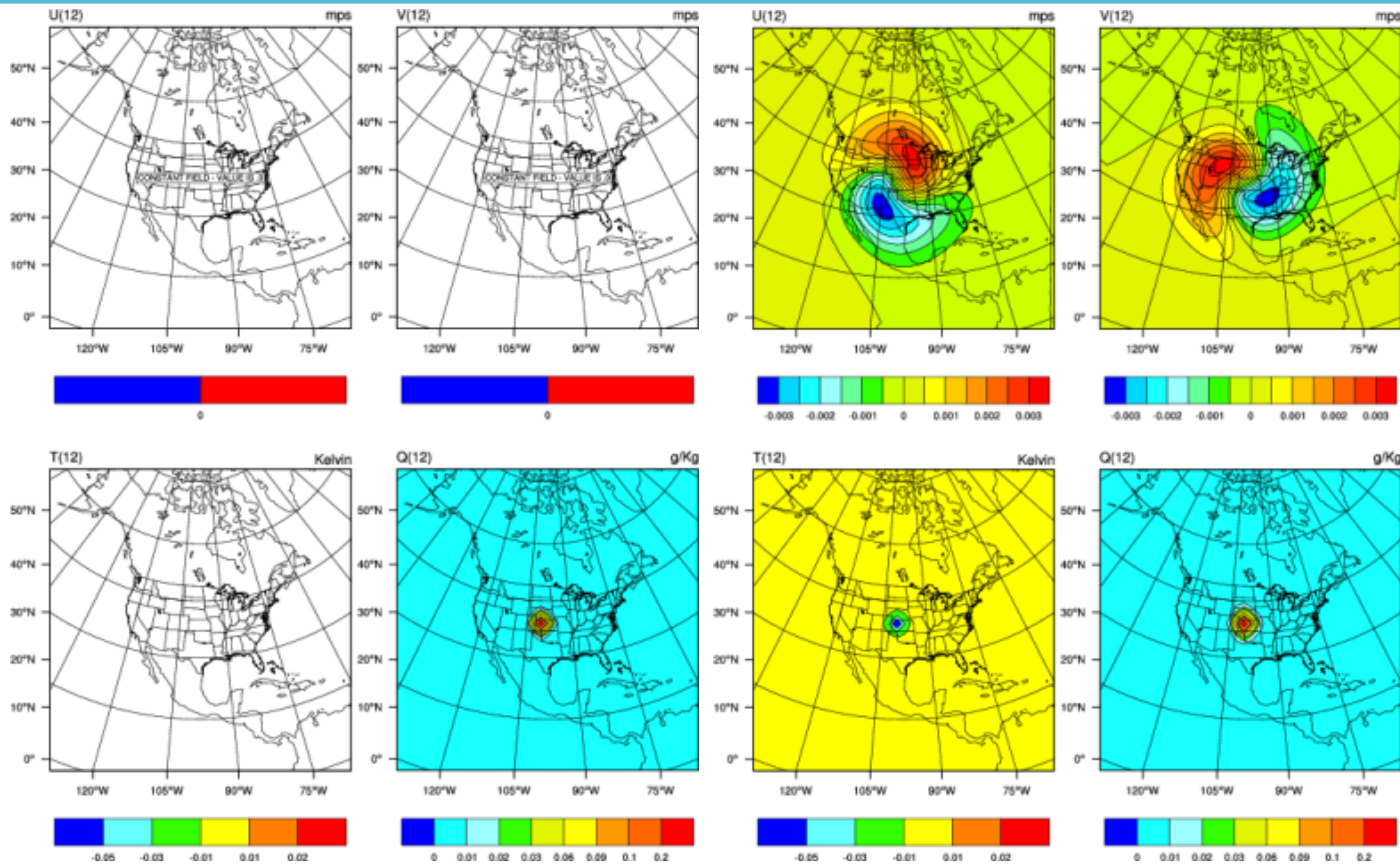
pseudo_y = Y-coordinate of the observation

pseudo_z = Z-coordinate of the observation

pseudo_val = Observation innovation, departure from FG

pseudo_err = Observation error

Analysis increments with PSOT-q



cv_options=5

cv_options=6

Tuning of BE

- Horizontal component of BE can be tuned with following ten namelist parameters

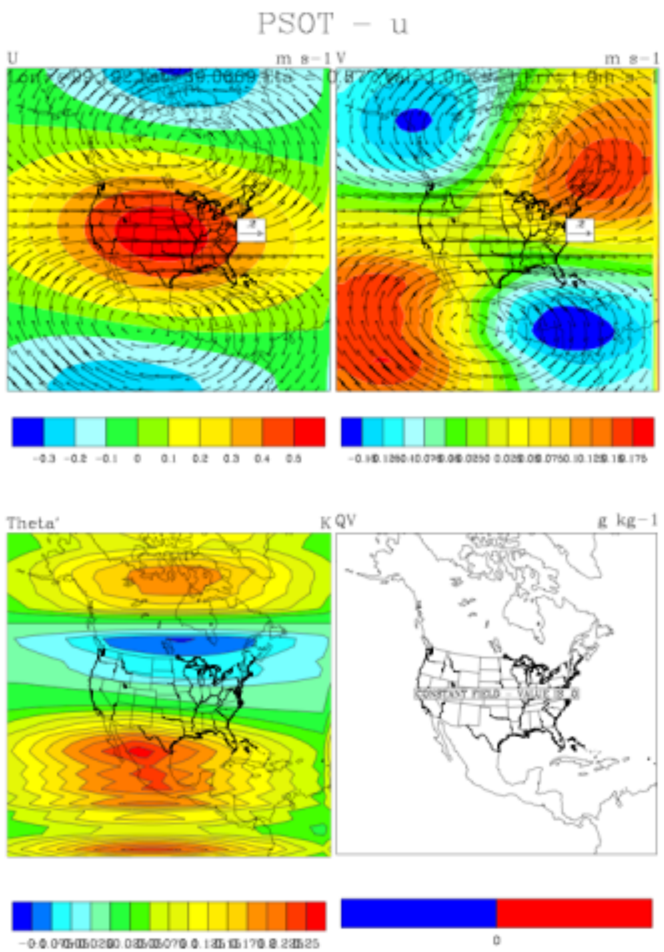
  LEN_SCALING1 - 5 (Length scaling parameters)

  VAR_SCALING1 - 5 (Variance scaling parameters)

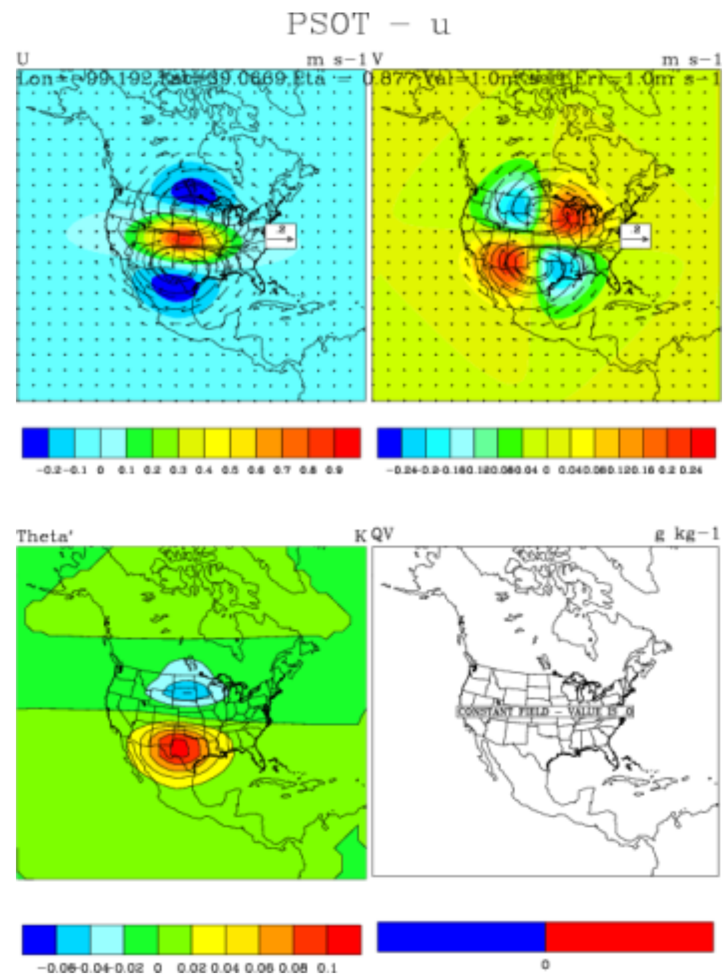
- Vertical component of BE can be tuned with the following five namelist parameters

  MAX_VERT_VAR1 - 5 (Vertical variance parameters)

BE tuning (length-scale)

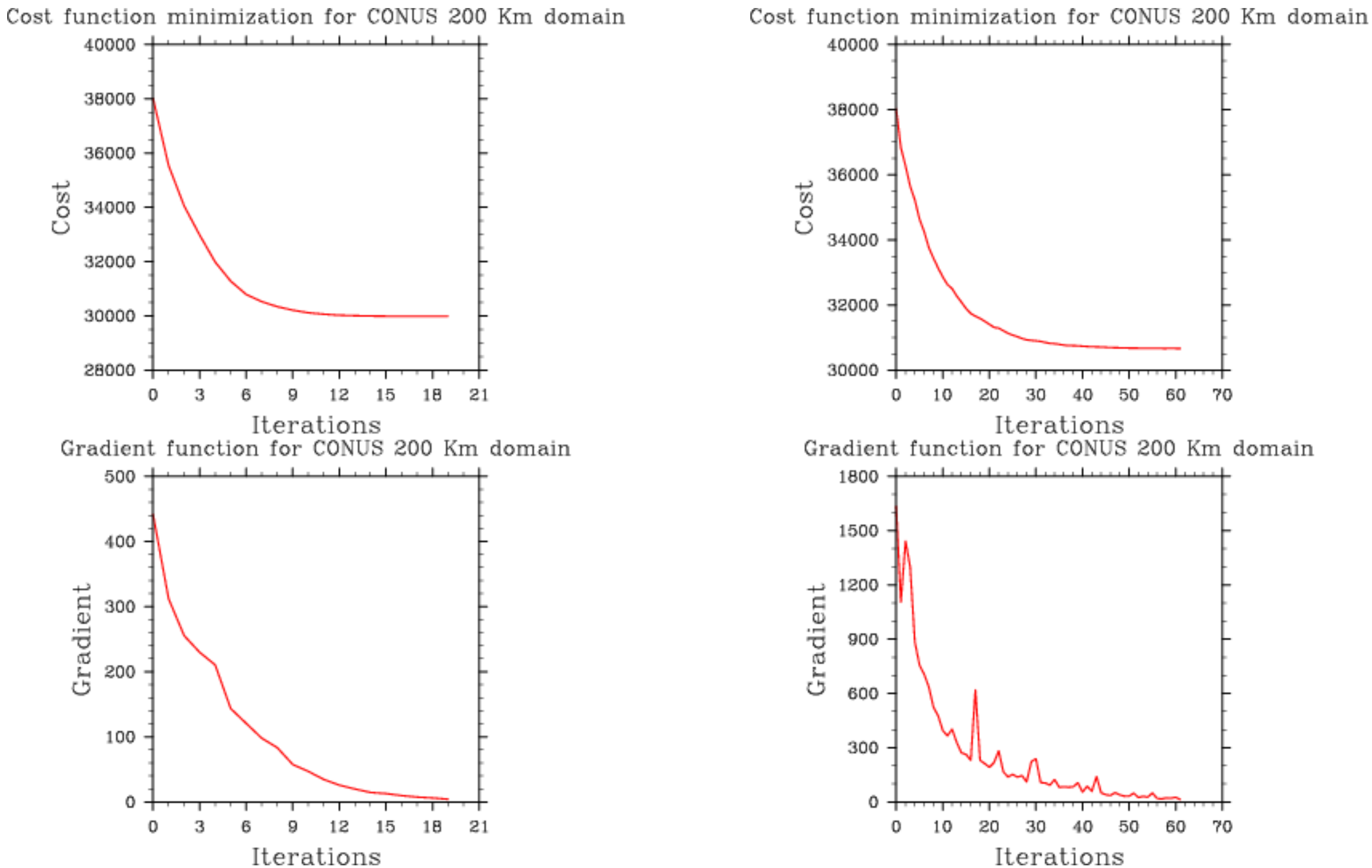


no tuning



tuning (len_scaling1 & 2 =0.25)

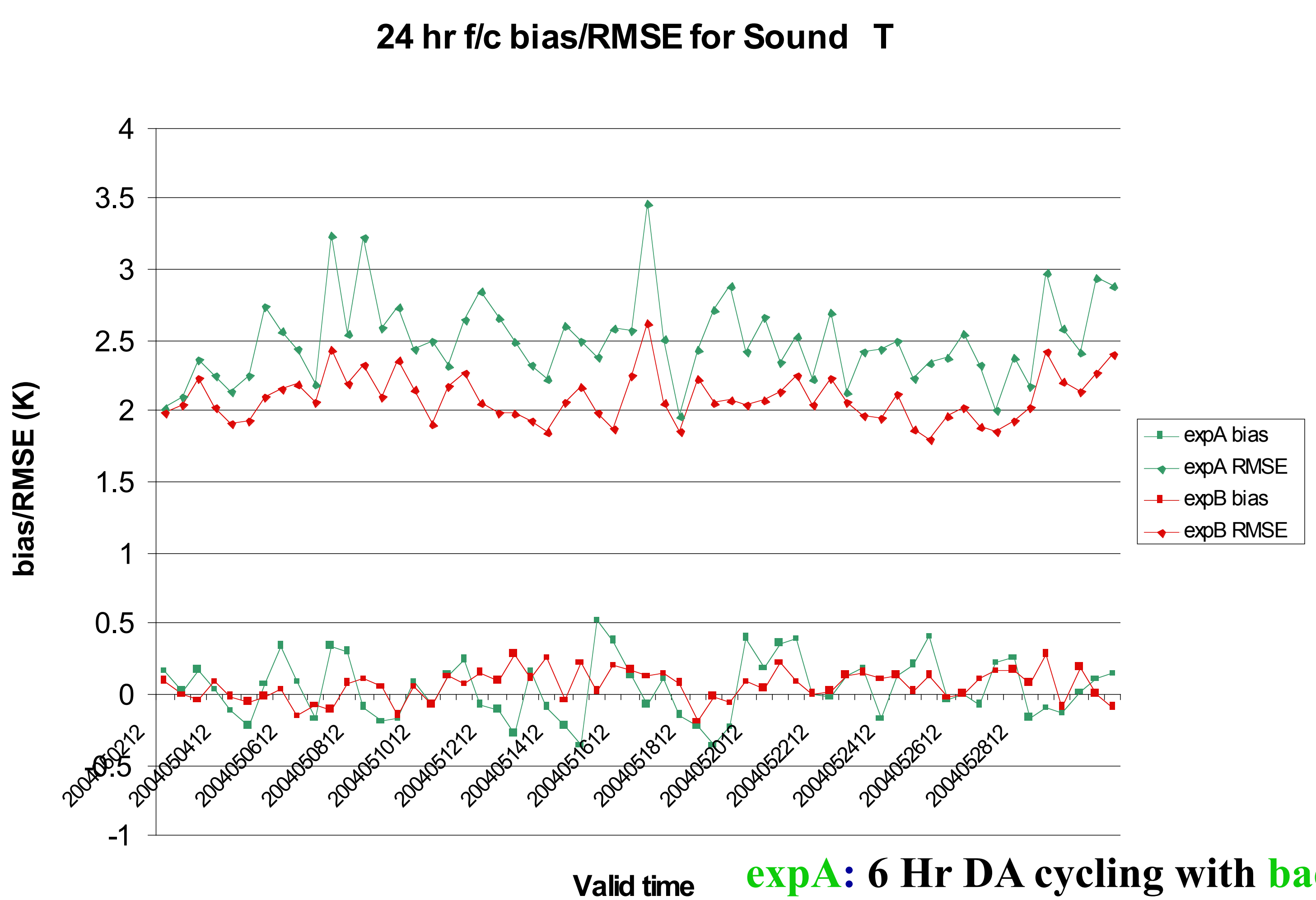
Impact of BE on Minimization



good BE

bad BE

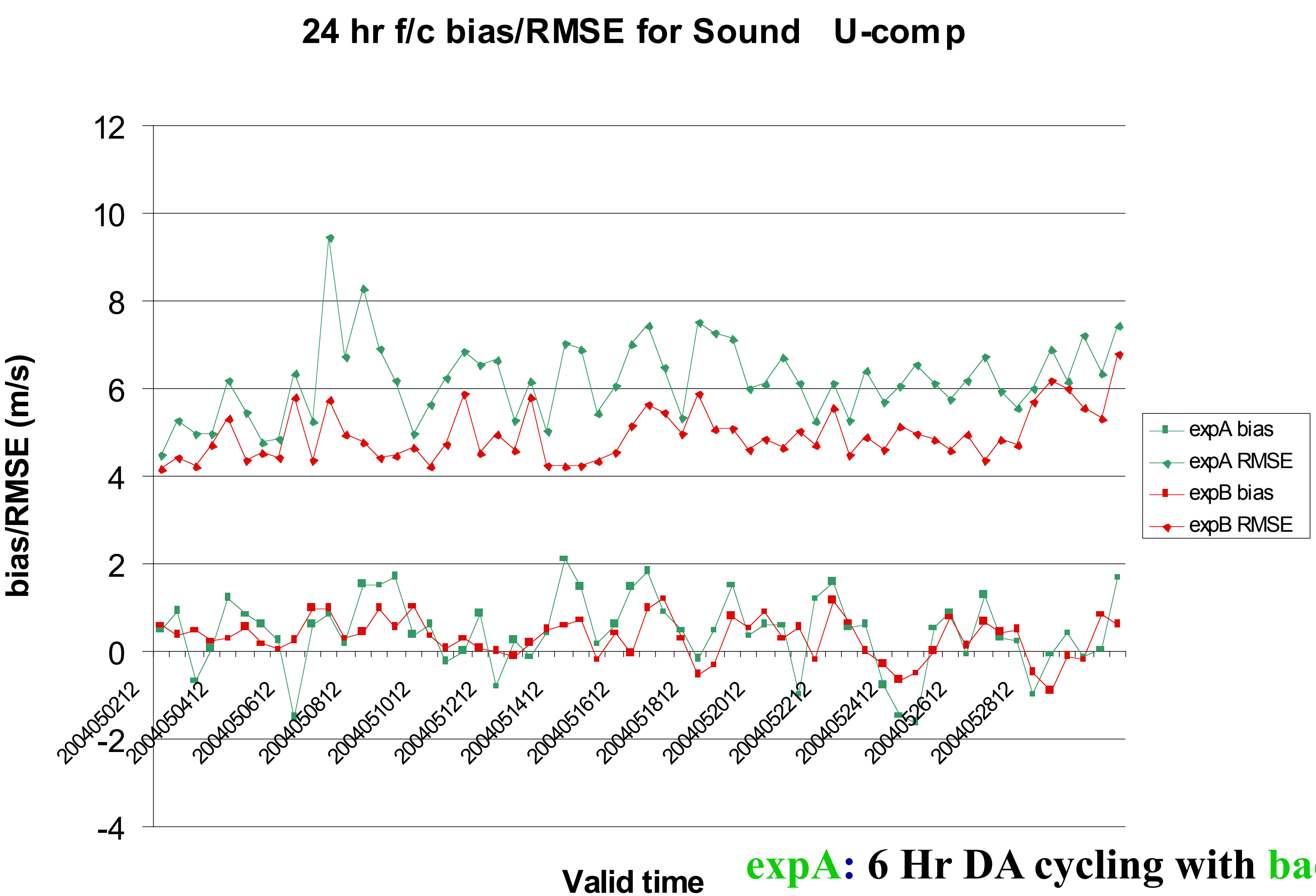
Impact of BE on Temp. Forecast



expA: 6 Hr DA cycling with bad BE

expB: 6 Hr DA cycling with good BE

Impact of BE on U-comp Forecast



expA: 6 Hr DA cycling with bad BE

expB: 6 Hr DA cycling with good BE

Advanced practice "gen_be"

- Compilation of "gen_be" utility
- Generation of BE statistics
- Familiarization with "gen_be" diagnostics
- Running PSOT to understand the structure of BE
- BE tuning

Generation of BE

"gen_be_wrapper.ksh" script for generating BE at 60 km "CONUS" domain with:

Grid Size	: 90 X 60 X 41 (staggered grid points)
BE Method	: NMC Method
Data Input	: 12 and 24 hour forecasts (already run)

Basic environment variables to be set in the wrapper script:

WRFVAR_DIR (code location) ; FC_DIR (forecast location)

START_DATE (1st pert time) ; END_DATE (last pert time)

NUM_LEVELS (half sigma levels) ; RUN_DIR (run directory)

gen_be diagnostics

- "gen_be" creates various diagnostic files which may be used to display different components of BE
- Important diagnostics files are:

Eigen vectors: fort.174, fort.178, fort.182, fort.186

Eigen values: fort.175, fort.179, fort.183, fort.187

scale-length: fort.194, fort.195, fort.196, fort.197

Correlation between χ_u & χ_b (chi_u.chi.dat)

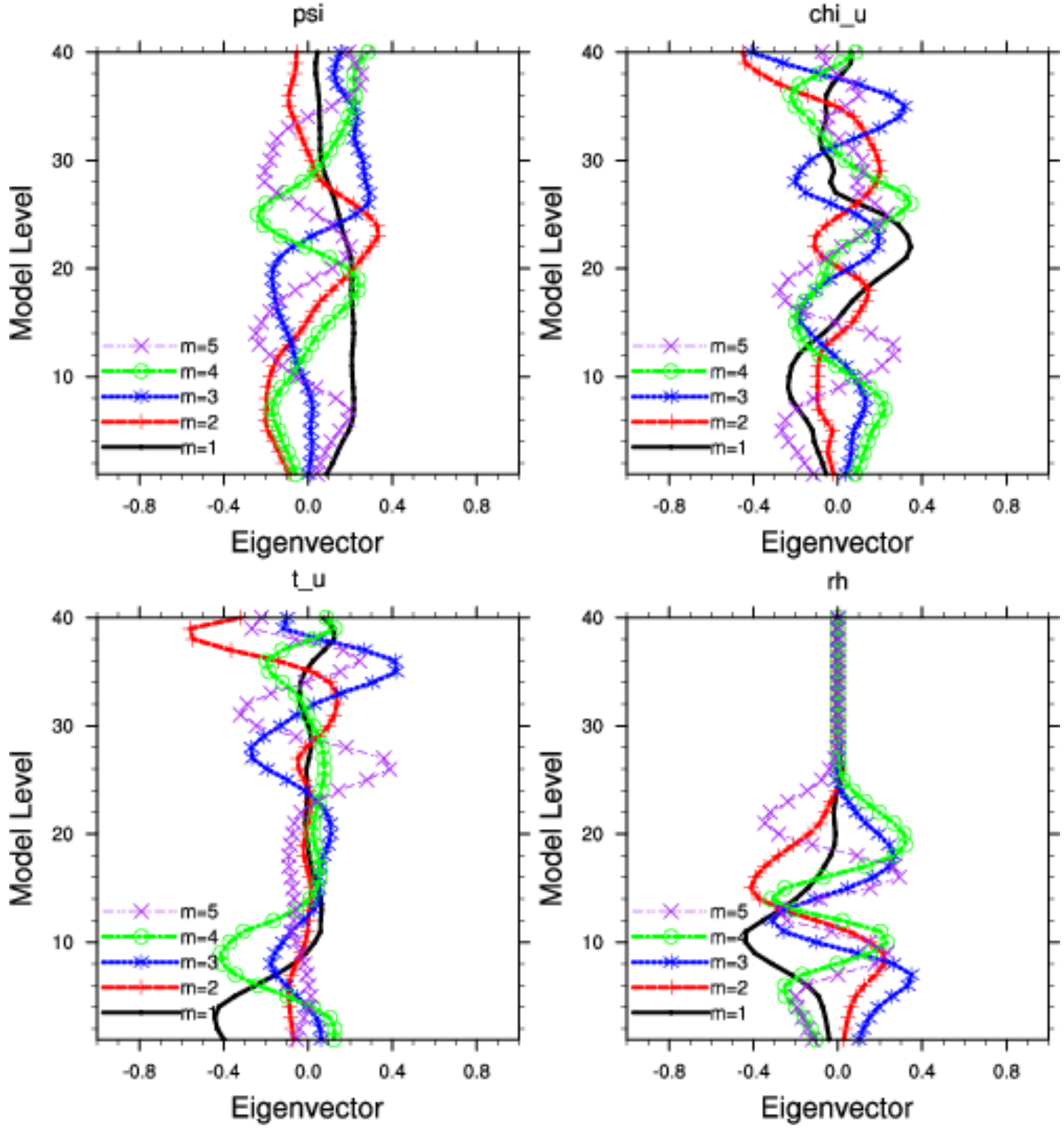
Correlation between T_u & T_b (T_u.T.dat)

Correlation between p_{s_u} & (ps_u.ps.dat)

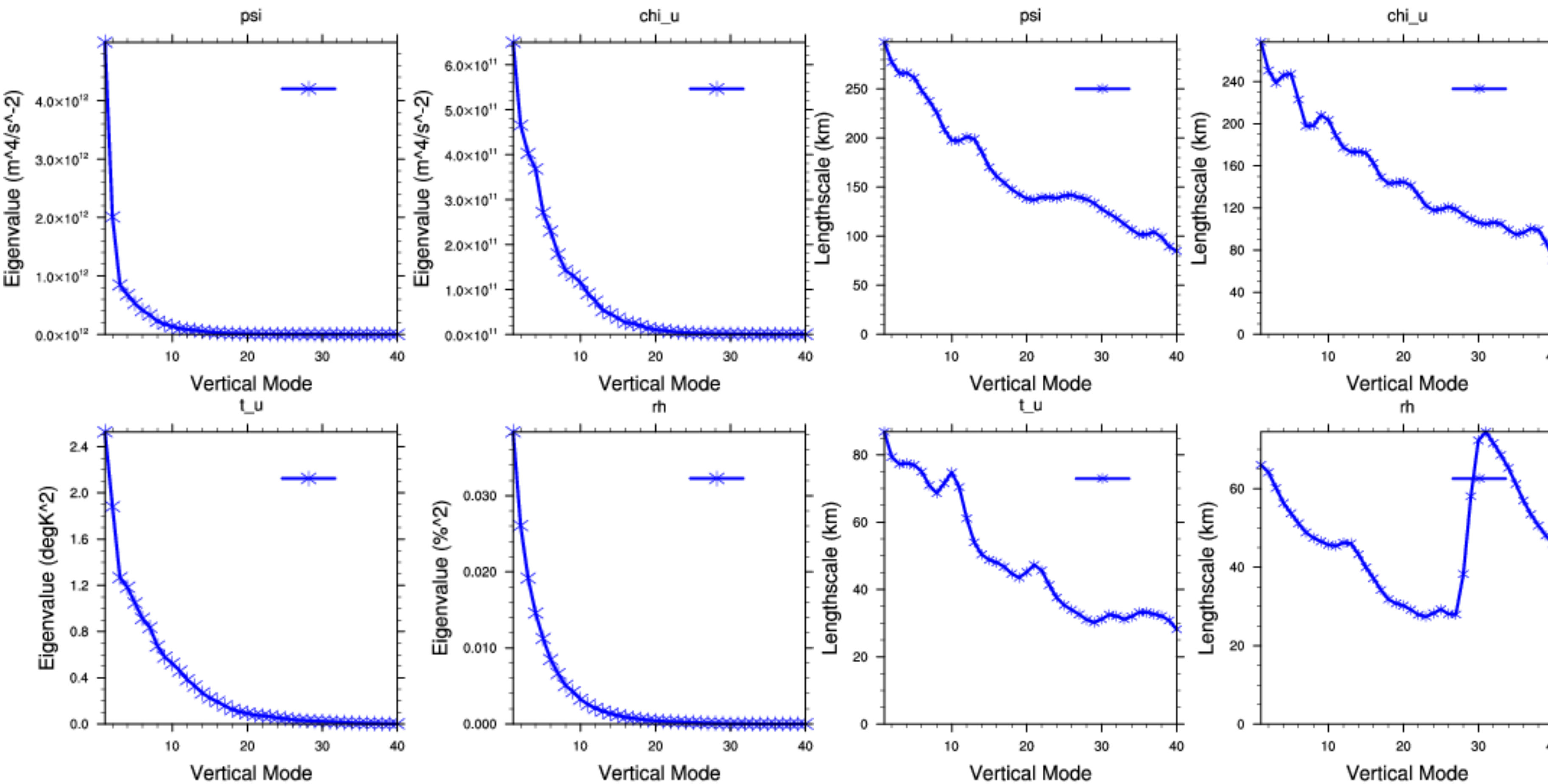
- Sample wrapper script for the display of BE diagnostics "var/script/gen_be/gen_be_plot_wrapper.ksh"

Note: BE_DIR is set to "gen_be" RUN_DIR directory

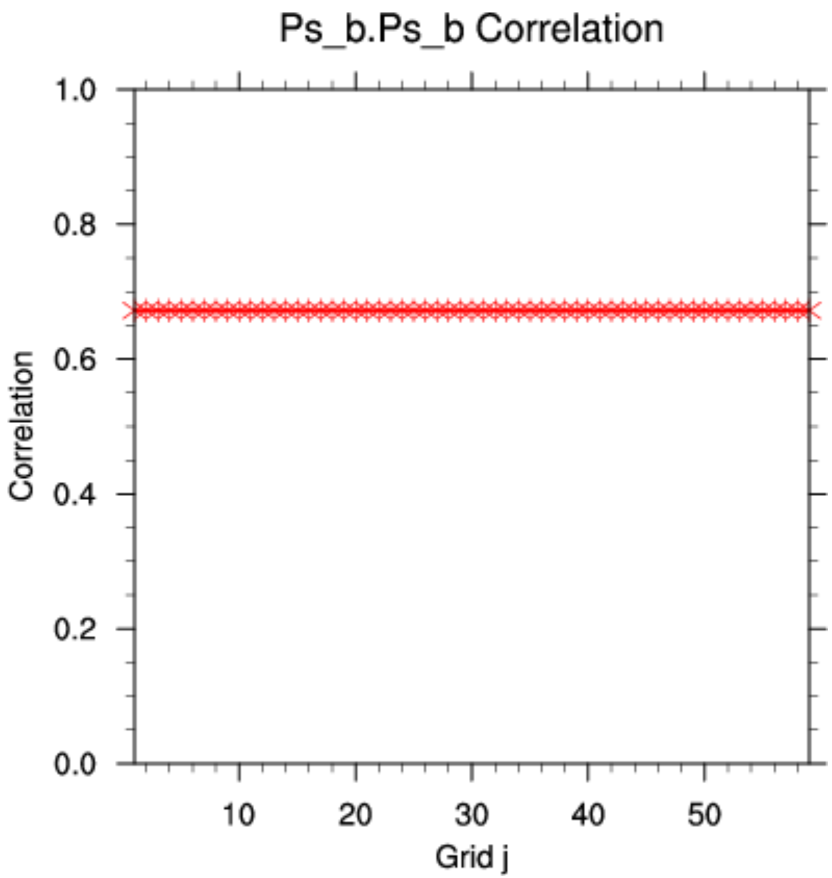
Leading (first 5) Eigenvectors

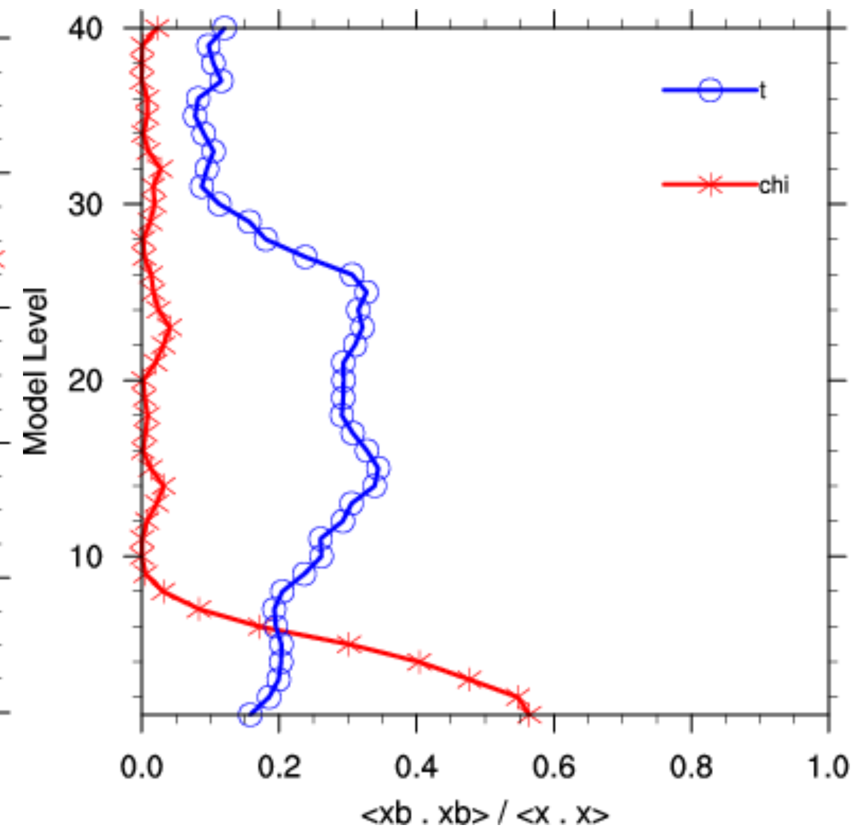


Eigenvalues and length-scales



Balance Correlations





How to run PSOT?

- Use following script from the WRFDA TOOLS package to build the PSOT wrapper script "var/scripts/wrappers/da_run_suite_wrapper_con200.ksh"
- Key parameters to set are:

  Type of observation (pseudo_var)

  Obs co-ordinates (pseudo_x, pseudo_y & pseudo_z)

  Observation value (pseudo_val)

  Observation error (pseudo_err)
- Display analysis increments to understand BE structure

Tuning of BE

- Understand the role of BE-tuning parameters through namelist options

  LEN_SCALING1 - 5 (Length scale)

  VAR_SCALING1 - 5 (Horizontal variance)

  MAX_VERT_VAR1 – 5 (Vertical variance)