Hybrid Variational/Ensemble Data Assimilation

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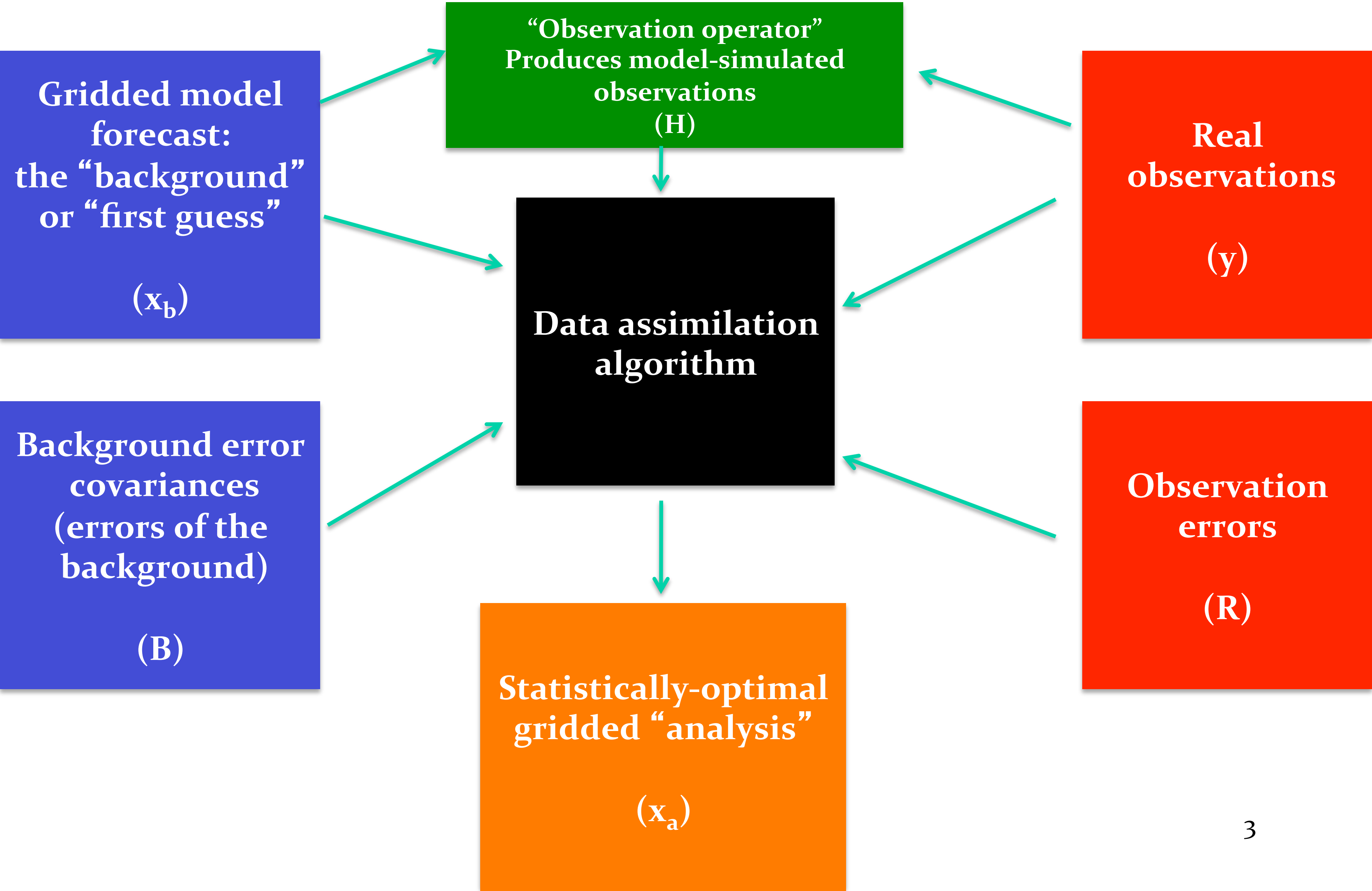
Outline

- Background
- Some results
- Introduction to hybrid practice

What is data assimilation?



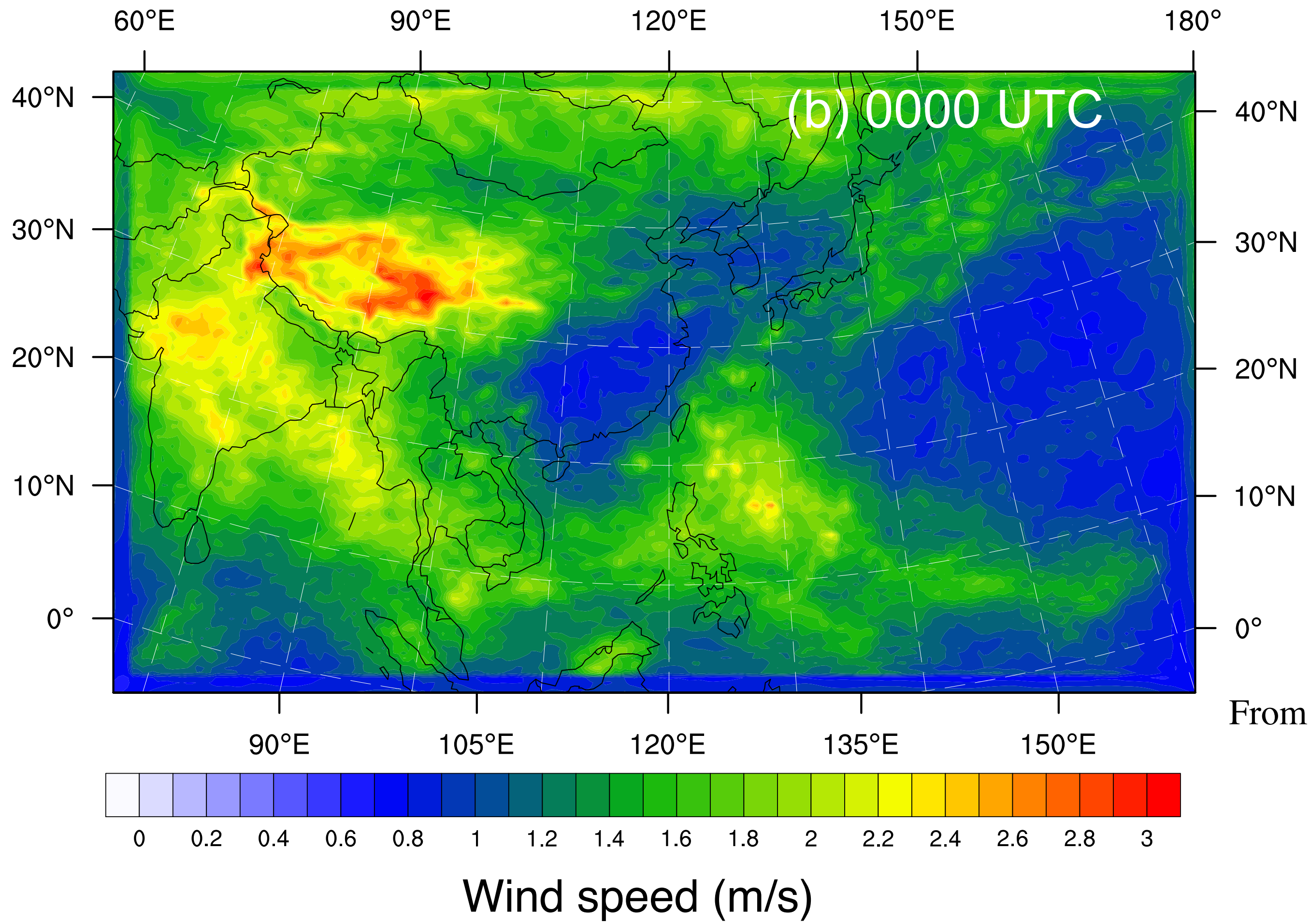
Some data assimilation methods

- Three-dimensional variational (3DVAR)
  - Background error covariances (BECs) typically fixed/ time-invariant
  - May yield poor results when actual flow differs from that encapsulated within the fixed "climatology"

- Ensemble Kalman filter (EnKF)
  - Time-evolving, "flow-dependent" BECs estimated from a short-term ensemble forecast
  - Many different flavors (e.g., ETKF, EAKF)

Ensemble BECs (i.e., spread)

- Average ensemble spread of wind speed over ~3 weeks at 0000 UTC



From Schwartz et al. (2013)

Ensemble BECs (i.e., spread)

- General definition of covariance:

$$\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})$$

- In vector matrix form (here, *n* is ensemble size):

$$=\frac{1}{n-1}\sum_{i=1}^{n}(\mathbf{x}_i-\overline{\mathbf{x}})(\mathbf{x}_i-\overline{\mathbf{x}})^{\mathbf{T}}$$

$$=\frac{1}{n-1}\sum_{i=1}^{n}(\delta\mathbf{x}_i)(\delta\mathbf{x}_i)^{\mathbf{T}}$$

“Hybrid” variational/ensemble DA

- “Hybrid” variational/ensemble
  - Incorporates ensemble background errors within a variational (e.g., 3DVAR) framework
  - Combination of fixed and time-evolving background errors
  - Main additional expense compared to 3DVAR is running an ensemble of forecasts

What is Hybrid DA?

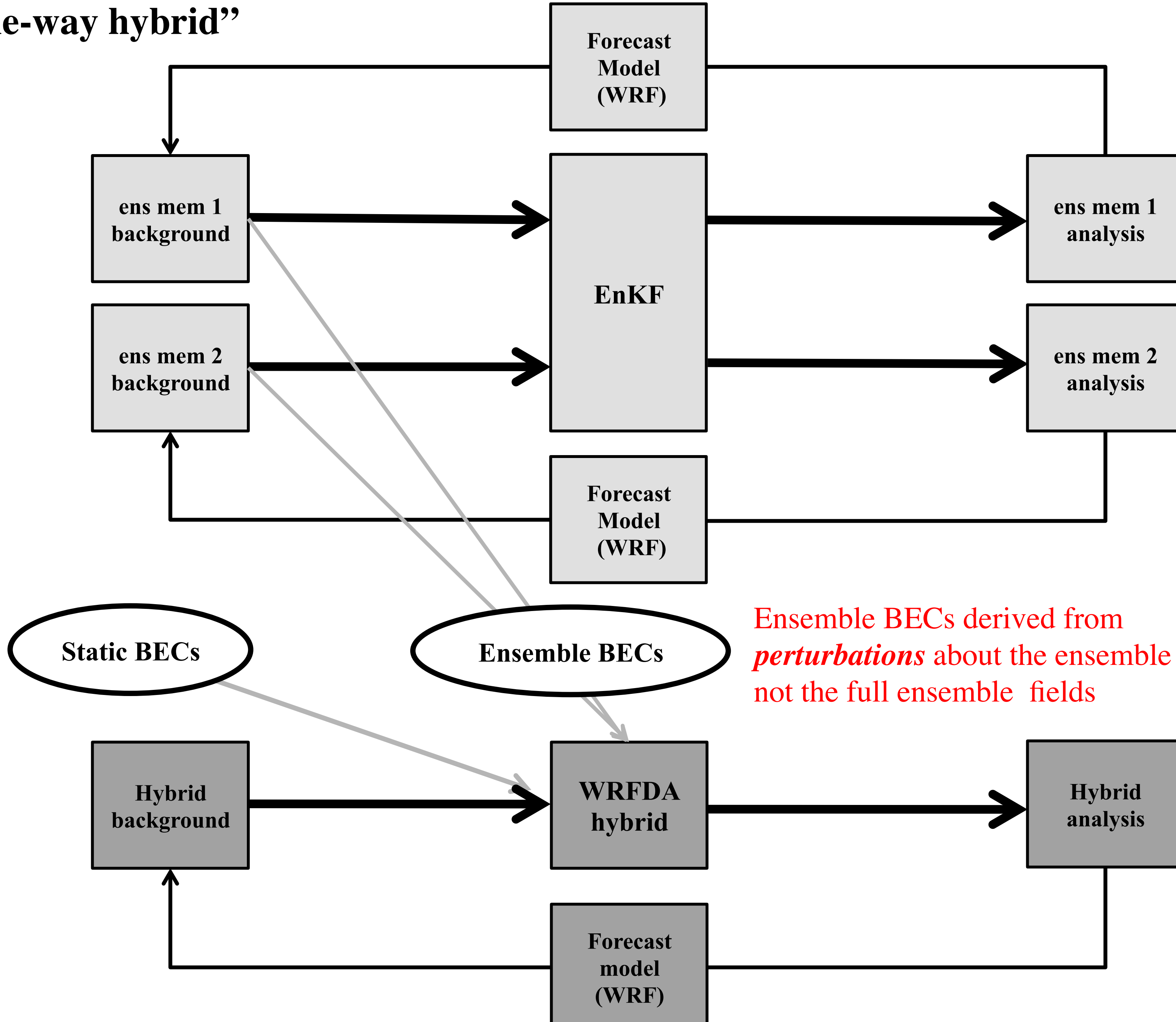
- Deterministic background is analyzed by a variational algorithm (i.e., minimize a cost function)
  - It combines the 3DVAR "climatological" BECs and "errors of the day" from ensemble perturbations

- Traditionally generates a deterministic analysis (like 3DVAR)

- Need a separate system to update ensemble
  - Could be ensemble forecasts already available from operational centers
  - Could be an EnKF-based DA system
  - Could be a multiple model/physics ensemble

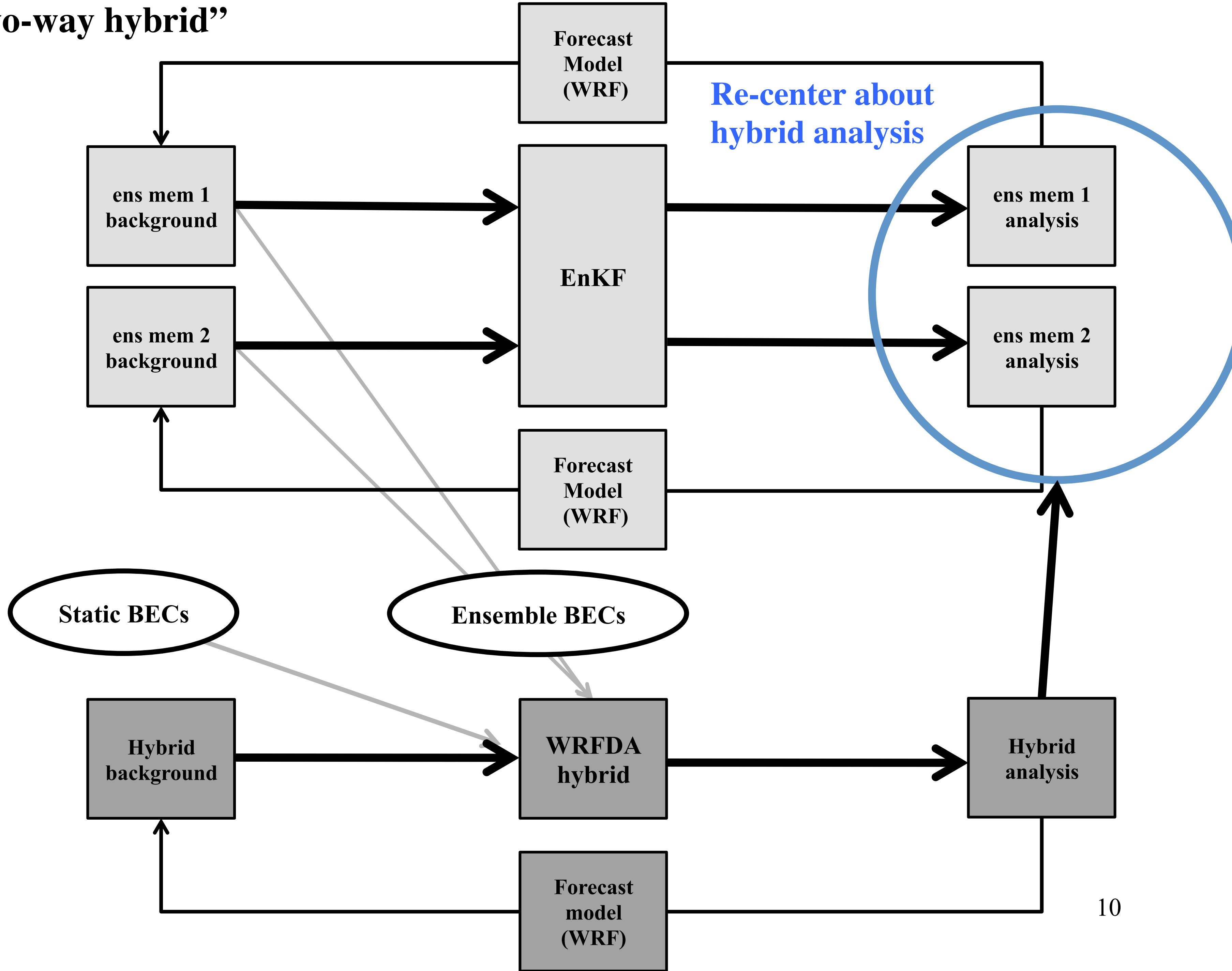
- Ensemble needs to be good to well-represent "errors of the day"

“One-way hybrid”



Ensemble BECs derived from ***perturbations*** about the ensemble mean, not the full ensemble fields

"Two-way hybrid"



Hybrid formulation

(Hamill and Snyder, 2000)

- 3DVAR cost function

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \mathbf{x}_b)^{\mathrm{T}} \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) + \frac{1}{2}[H(\mathbf{x}) - \mathbf{y}]^{\mathrm{T}} \mathbf{R}^{-1} [H(\mathbf{x}) - \mathbf{y}]$$

- Idea: replace **B** by a weighted sum of static $\mathbf{B}_{\mathrm{s}}$ and the ensemble $\mathbf{B}_{\mathrm{e}}$

$$\mathbf{B} = a_s \mathbf{B}_{\mathrm{s}} + a_e \mathbf{B}_{\mathrm{e}} \circ \mathbf{C}, \qquad a_s = 1 - a_{\mathrm{e}}$$

  - Term **C** is localization for the ensemble
  - Terms a_s and a_e can be tuned to determine how much $\mathbf{B}_{\mathrm{s}}$ and $\mathbf{B}_{\mathrm{e}}$ are weighted

- This form is difficult to implement for a large NWP model
  - Most systems use "extended control variables"

Hybrid formulation used in WRFDA

(Lorenc, 2003)

• Ensemble covariance is included in the 3DVAR cost function through augmentation of control variables

$$J(\mathbf{x}_1,\alpha)=\beta_s\frac{1}{2}(\mathbf{x}_1-\mathbf{x}_b)^{\mathbf{T}}\mathbf{B}^{-1}(\mathbf{x}_1-\mathbf{x}_b)+\overbrace{\beta_e\frac{1}{2}\sum_{i=1}^{\mathrm{N}}\alpha_i^{\mathbf{T}}\mathbf{C}^{-1}\alpha_i}^{\text{ensemble control variable } \alpha_i \ (M\times 1)}$$

$$+\frac{1}{2}[\mathbf{y}-H(\mathbf{x}_1+\mathbf{x}'_e)]^{\mathbf{T}}\mathbf{R}^{-1}[\mathbf{y}-H(\mathbf{x}_1+\mathbf{x}'_e)]$$

$\mathbf{x}'_e=\sum_{\mathrm{i}=1}^{\mathrm{N}}\alpha_i\circ\mathbf{x}_i^{'}$, where $\mathbf{x}_i^{'}$ is the ensemble perturbation for the ensemble member i.

$\circ$ denotes element-wise product. α_{i} is in effect the ensemble weight.

C: correlation matrix (effectively loclization of ensemble perturbations)

• More simply: $J(\mathbf{x}_1,\alpha)=J_b+J_e+J_o$

• β_s and β_e ($1/\beta_s$ + $1/\beta_e$ =1) can be tuned to have different weight between static and ensemble part

3DEnVar and 4DEnVar

- In "3DEnVar", ensembles valid at only one time are used:

$$J(\mathbf{x}_1,\alpha) = \beta_s \frac{1}{2}(\mathbf{x}_1 - \mathbf{x}_b)^{\mathbf{T}} \mathbf{B}^{-1}(\mathbf{x}_1 - \mathbf{x}_b) + \overbrace{\beta_e \frac{1}{2}\sum_{i=1}^{\mathrm{N}} \alpha_i^{\mathbf{T}} \mathbf{C}^{-1} \alpha_i}^{\text{ensemble control variable } \alpha_i \ (M\times 1)}$$

$$\boxed{+\frac{1}{2}[\mathbf{y} - H(\mathbf{x}_1 + \mathbf{x}'_e)]^{\mathbf{T}} \mathbf{R}^{-1} [\mathbf{y} - H(\mathbf{x}_1 + \mathbf{x}'_e)]}$$

Ensemble ($\mathbf{x}'_e$) only needed at the analysis time

- In "4DEnVar", ensembles at *multiple times are used*, and observations are binned as in FGAT:

$$J(\mathbf{x}_1,\alpha) = \beta_s \frac{1}{2}(\mathbf{x}_1 - \mathbf{x}_b)^{\mathbf{T}} \mathbf{B}^{-1}(\mathbf{x}_1 - \mathbf{x}_b) + \overbrace{\beta_e \frac{1}{2}\sum_{i=1}^{\mathrm{N}} \alpha_i^{\mathbf{T}} \mathbf{C}^{-1} \alpha_i}^{\text{ensemble control variable } \alpha_i \ (M\times 1)}$$

$$\boxed{+\frac{1}{2}\sum_{k=1}^{K}[\mathbf{y}_k - H_k(\mathbf{x}_1 + \mathbf{x}'_{e,k})]^{\mathbf{T}} \mathbf{R}_k^{-1} [\mathbf{y}_k - H_k(\mathbf{x}_1 + \mathbf{x}'_{e,k})]}$$

Ensemble needed at K times

More on 4DEnVar

• In 4DEnVar, the static contribution is the same as in 3DVAR/3DEnVar

• The ensemble perturbation weights (α) are time-invariant

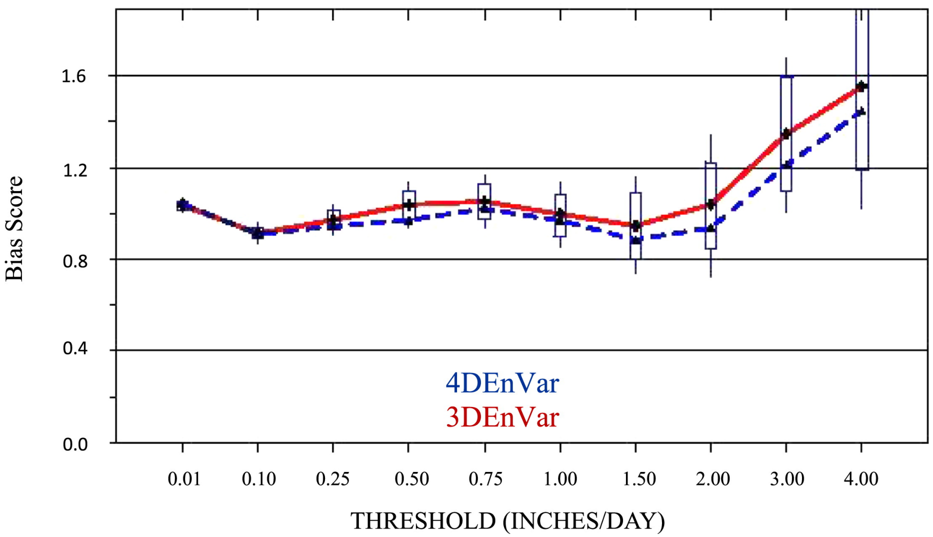
• Only difference compared to 3DEnVar is use of ensembles at multiple forecast times and binning of observations

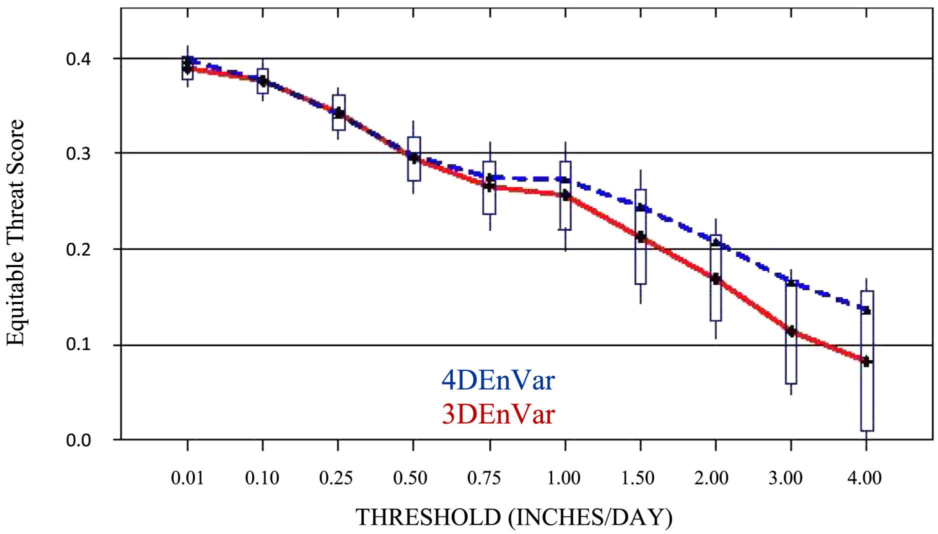
$$J(\mathbf{x}_1,\alpha) = \beta_s \frac{1}{2}(\mathbf{x}_1 - \mathbf{x}_b)^{\mathbf{T}} \mathbf{B}^{-1}(\mathbf{x}_1 - \mathbf{x}_b) + \overbrace{\beta_e \frac{1}{2}\sum_{i=1}^{\mathrm{N}} \alpha_i^{\mathbf{T}} \mathbf{C}^{-1} \alpha_i}^{\text{ensemble control variable } \alpha_i \ (M \times 1)}$$

$$\boxed{+\frac{1}{2}\sum_{k=1}^{K}[\mathbf{y}_k - H_k(\mathbf{x}_1 + \mathbf{x}'_{e,k})]^{\mathbf{T}} \mathbf{R}_k^{-1}[\mathbf{y}_k - H_k(\mathbf{x}_1 + \mathbf{x}'_{e,k})]}$$

More on 4DEnVar

- 4DEnVar is now operational for the GFS and NAM models and can yield forecast improvements compared to 3DEnVar:

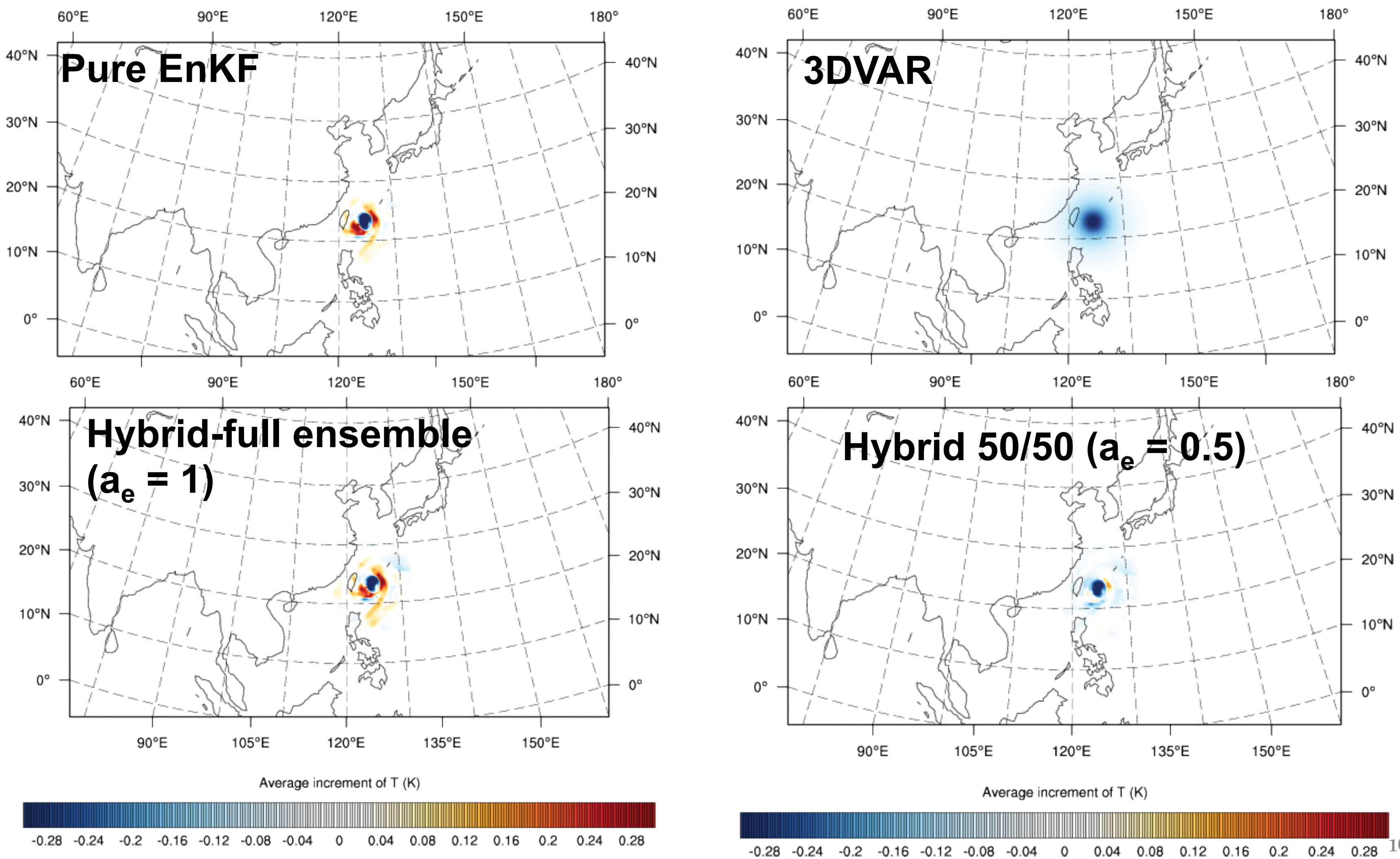




From Wu et al. (2017)

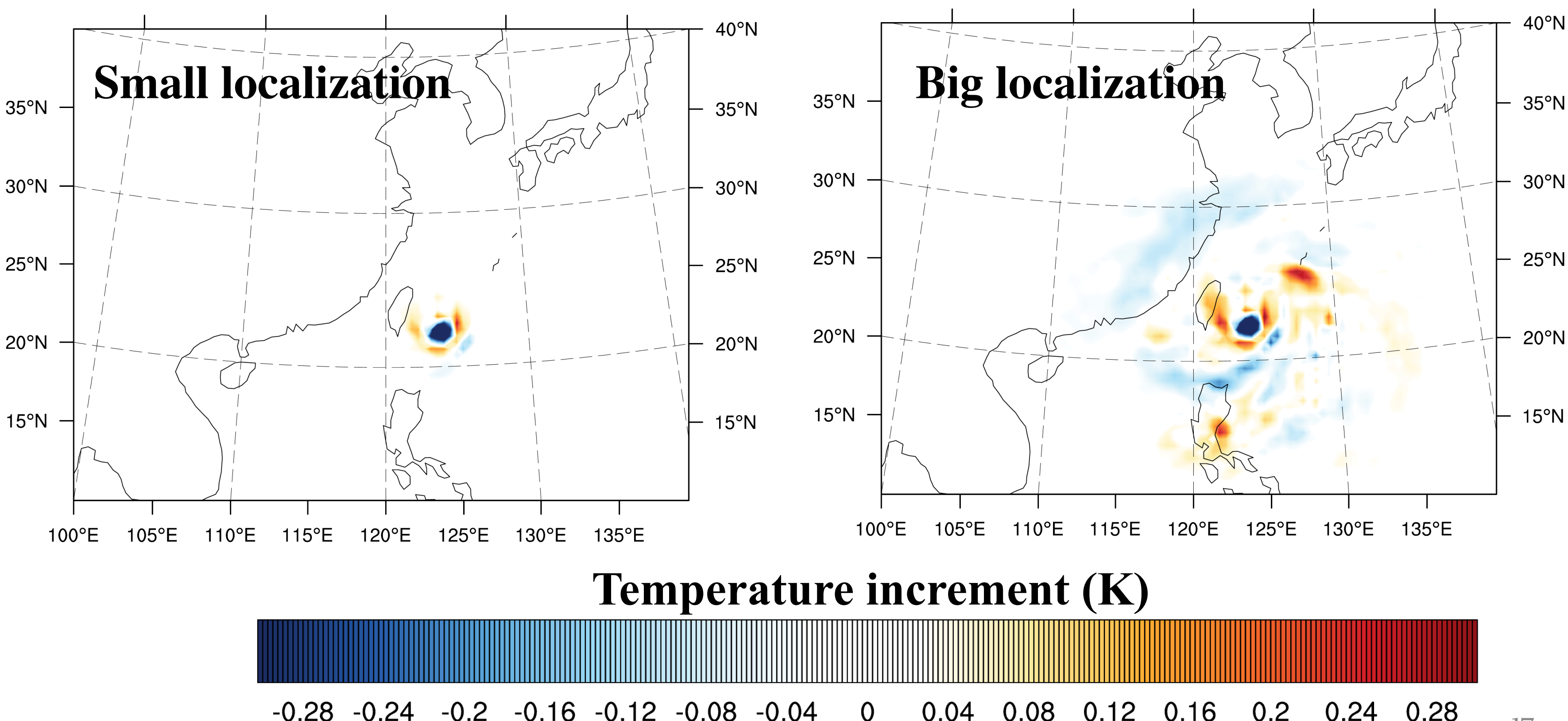
Single observation tests

- **Potential temperature increment, 21st model level**



Meaning of localization

- Localization defines the extent to which an observation can produce an analysis increment
- In this example, 100% of the BECs are from ensemble

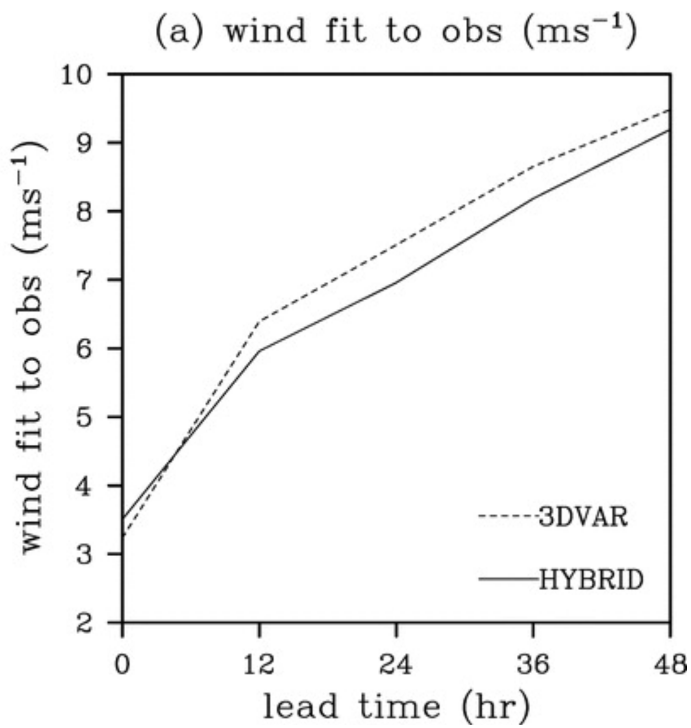


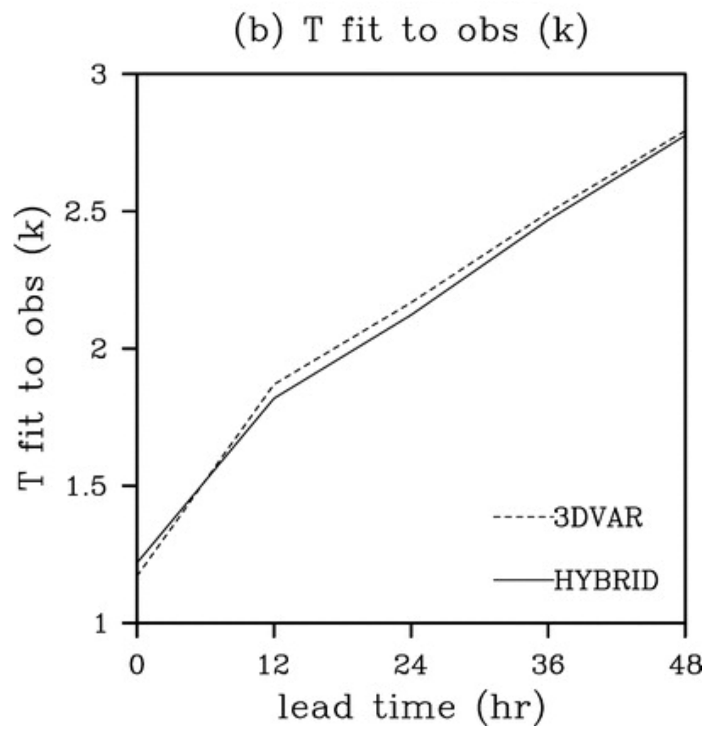
Advantages of Hybrid DA

- Hybrid localization is in model space while EnKF localization is usually in observation space
- For some observation types (e.g., radiances), localization is not well defined in observation space
- Easier to make use of existing radiance VarBC in hybrid
- For small ensembles, use of static **B** could be beneficial to have a higher-rank covariance.

Sample results

- Example over North America at coarse grid spacing
- Similar results have been obtained by many studies

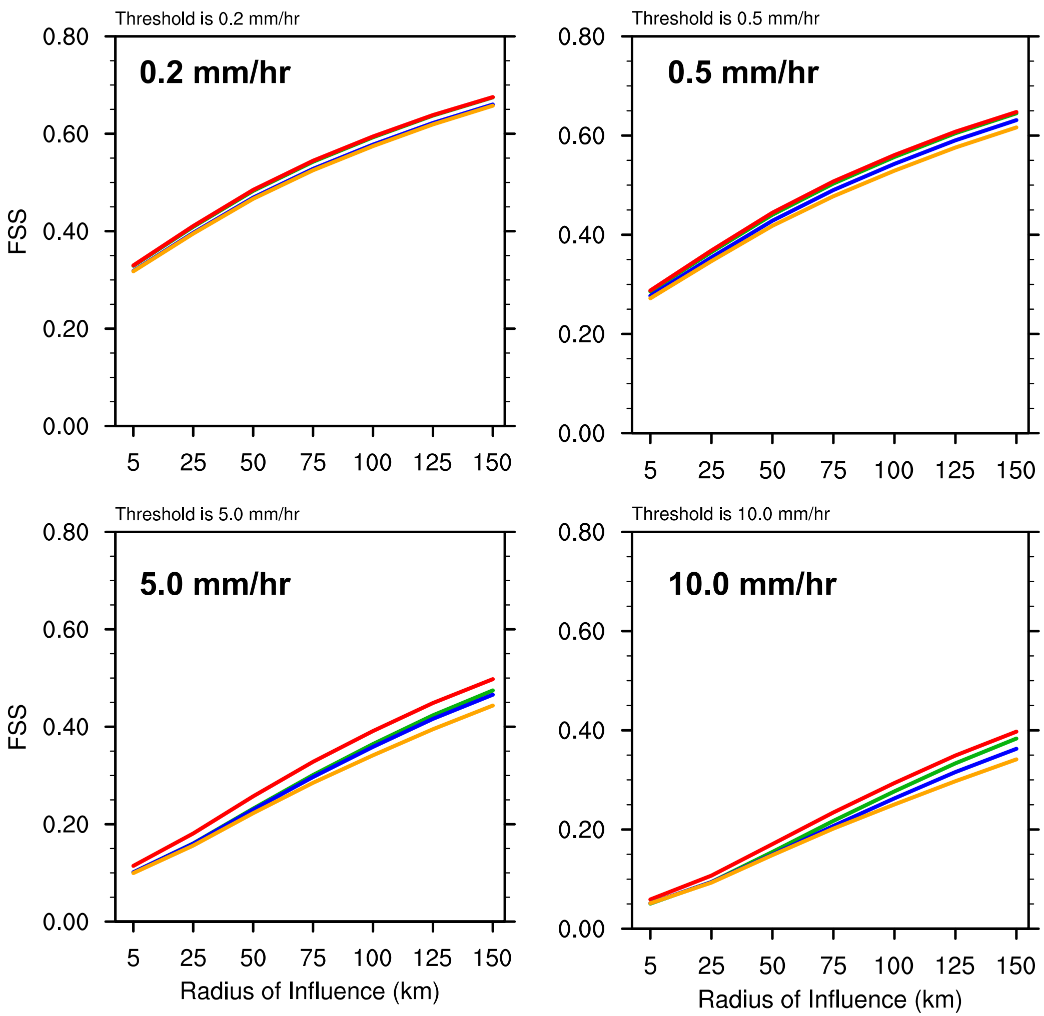




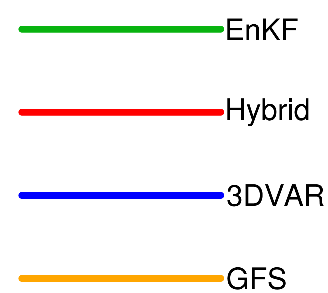
From Wang et al. (2008)

Hybrid vs. 3DVAR and EnKF

- Fractions skill scores for rainfall (higher is better)



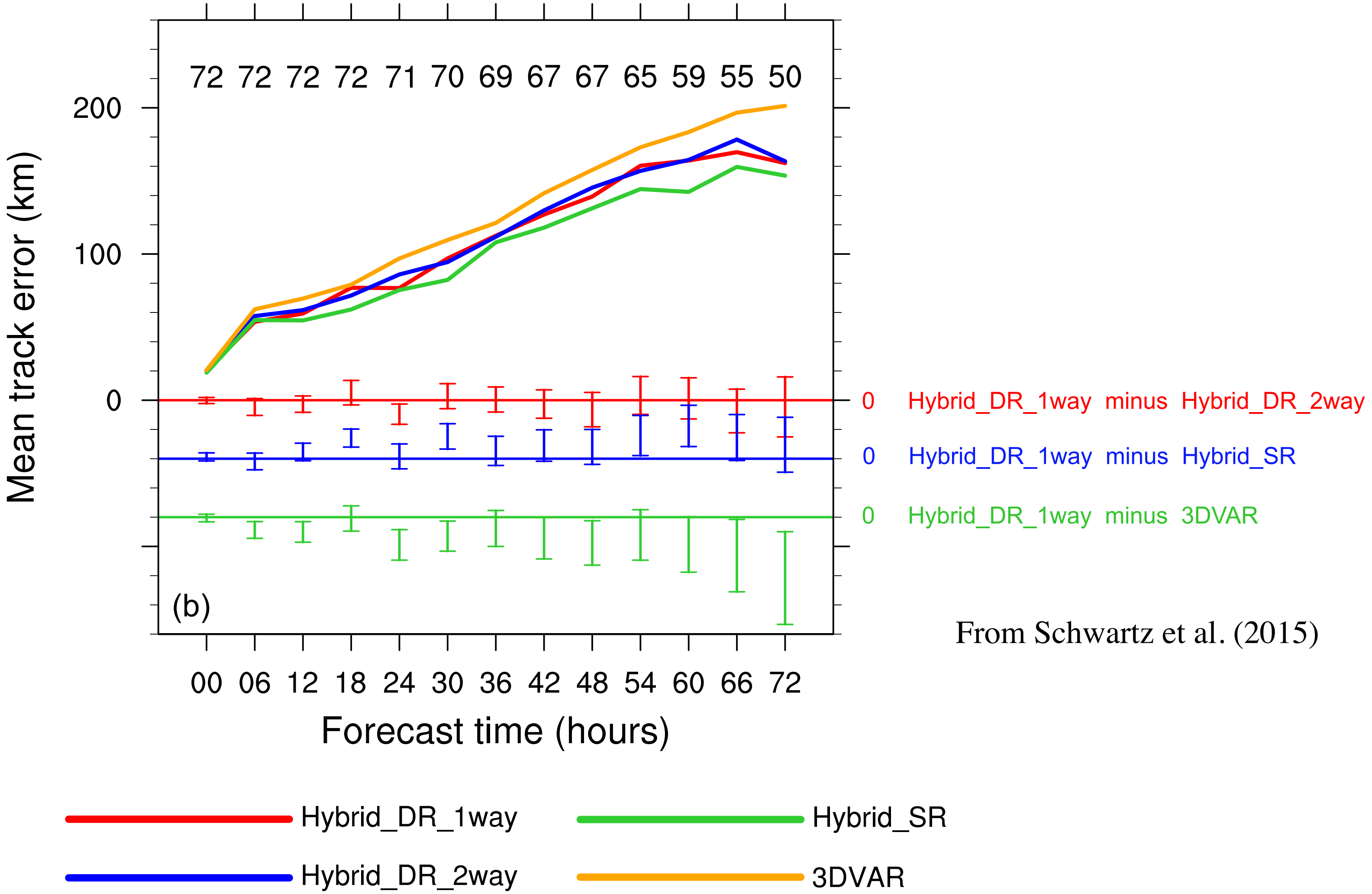
Aggregated over hourly 18-36-hr forecasts of precipitation



Modified from Schwartz and Liu (2014)

Typhoon example

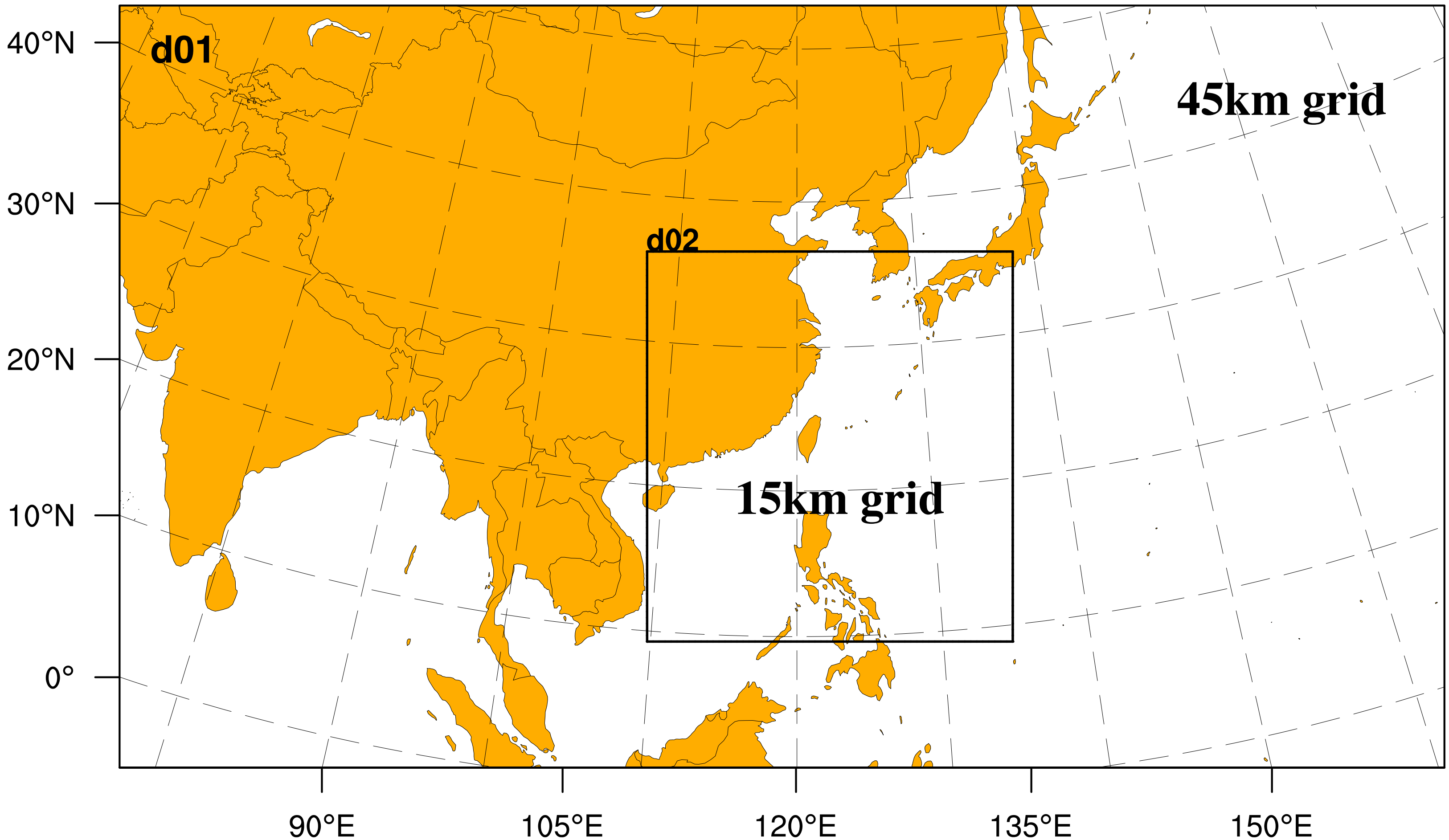
- Mean tropical cyclone track errors



From Schwartz et al. (2015)

Dual-Resolution hybrid (V3.6)

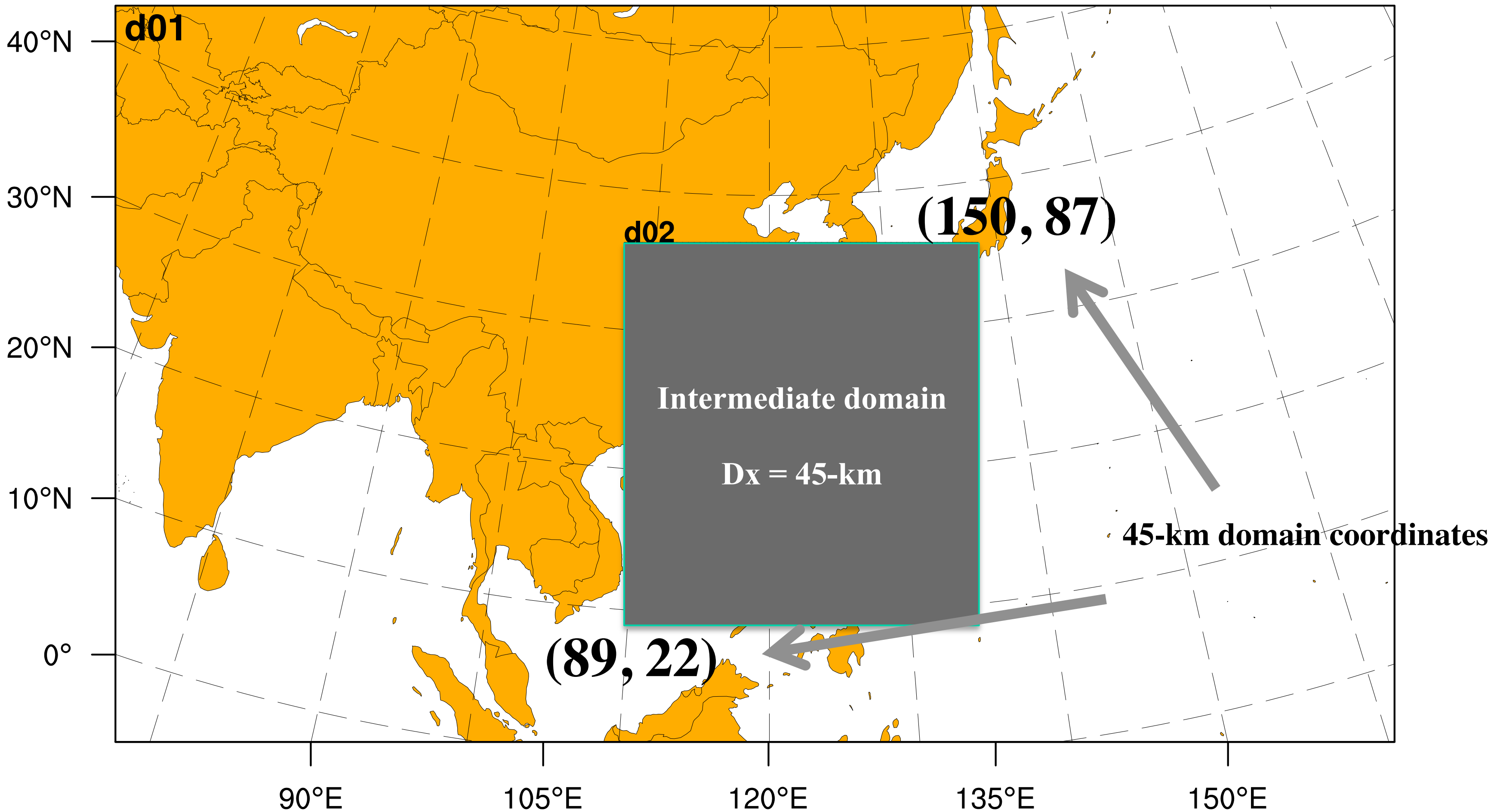
Schwartz et al. (2015; MWR)



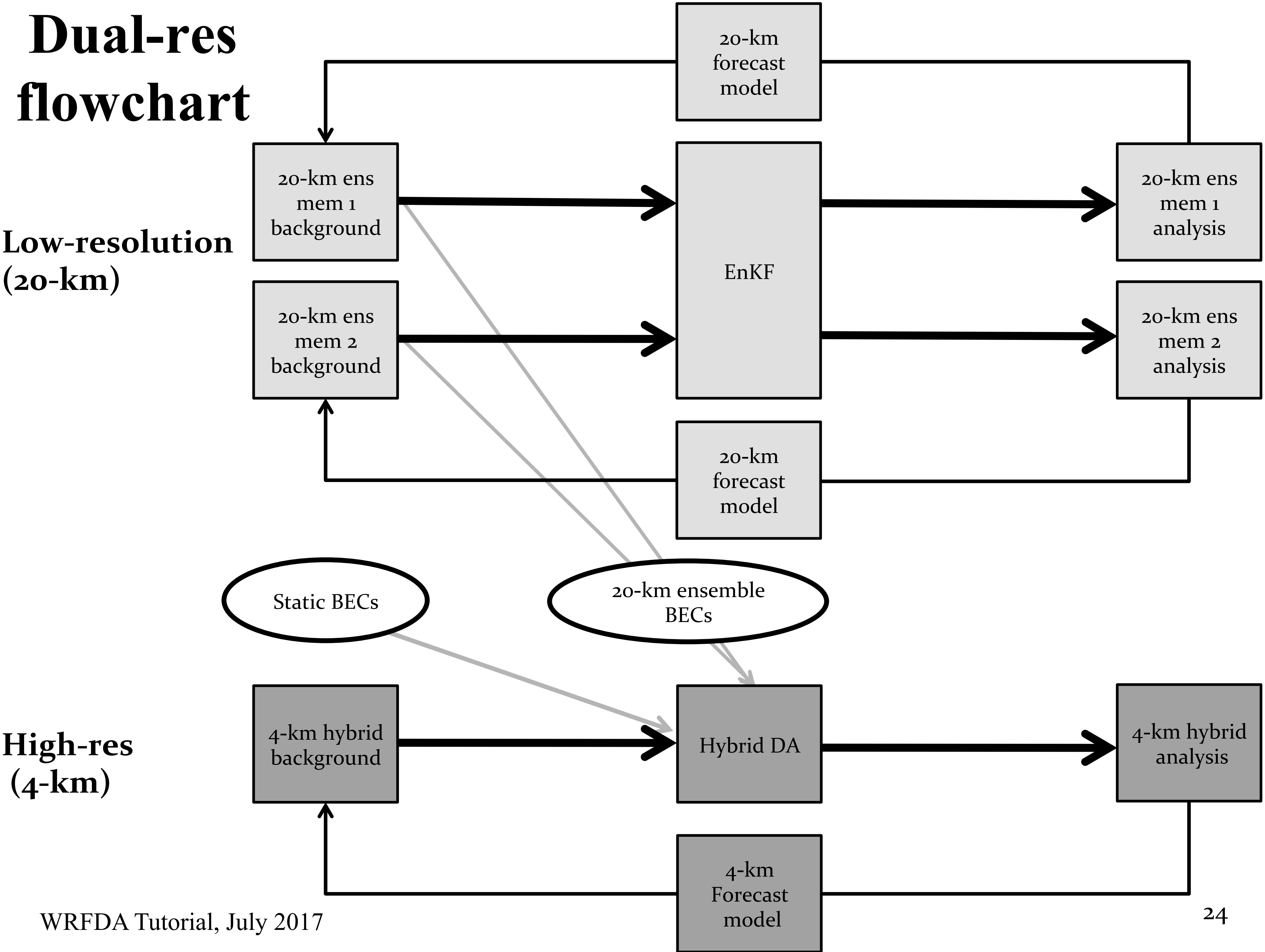
Hybrid analysis on 15-km grid but with ensemble perturbation input from 45-km grid

Intermediate domain

- WRFDA directly reads in d01 ensembles, then cuts to d02 size (making use of WRF model nest namelist setting)

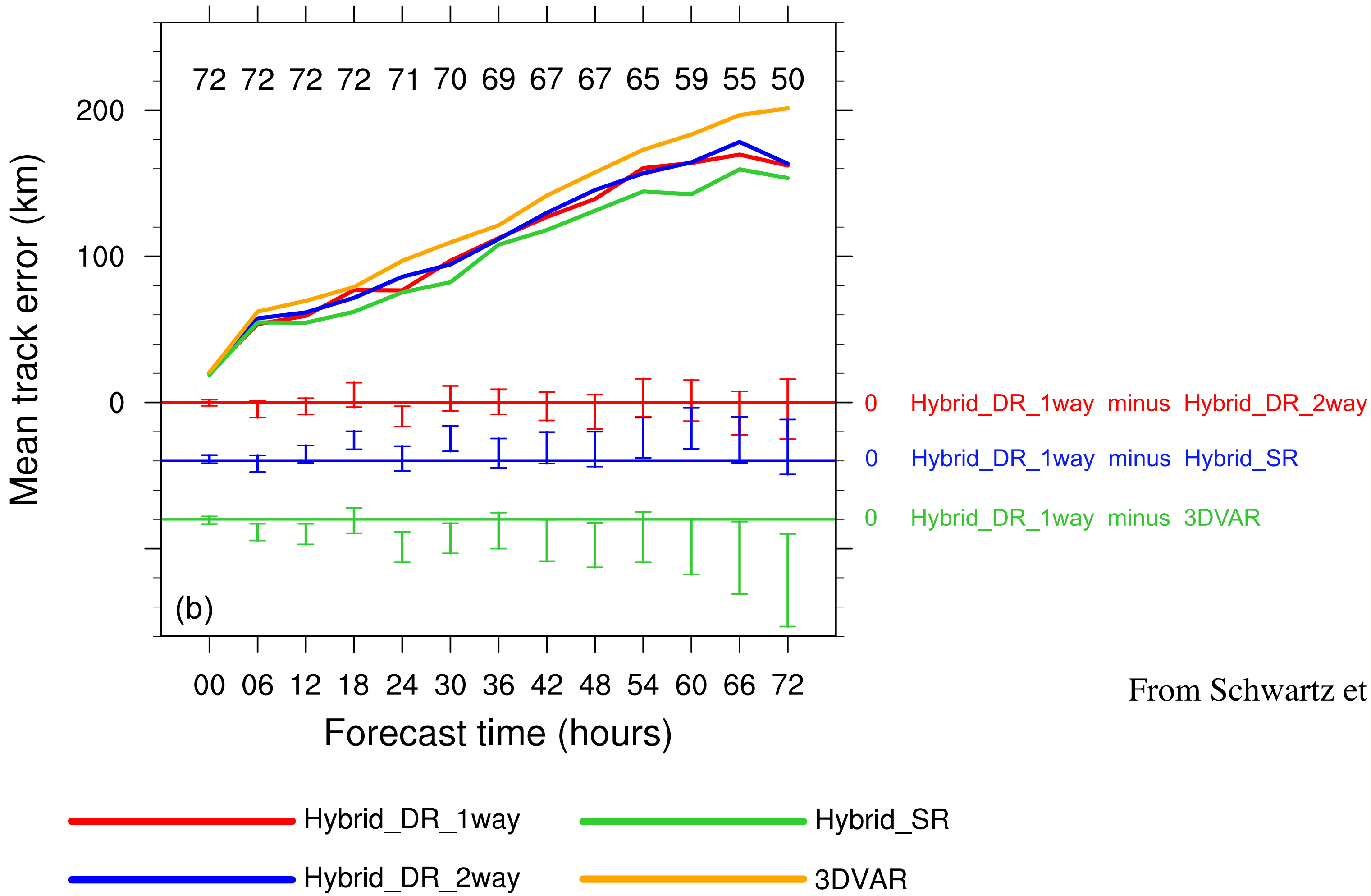


Dual-res flowchart



Impact of dual-resolution

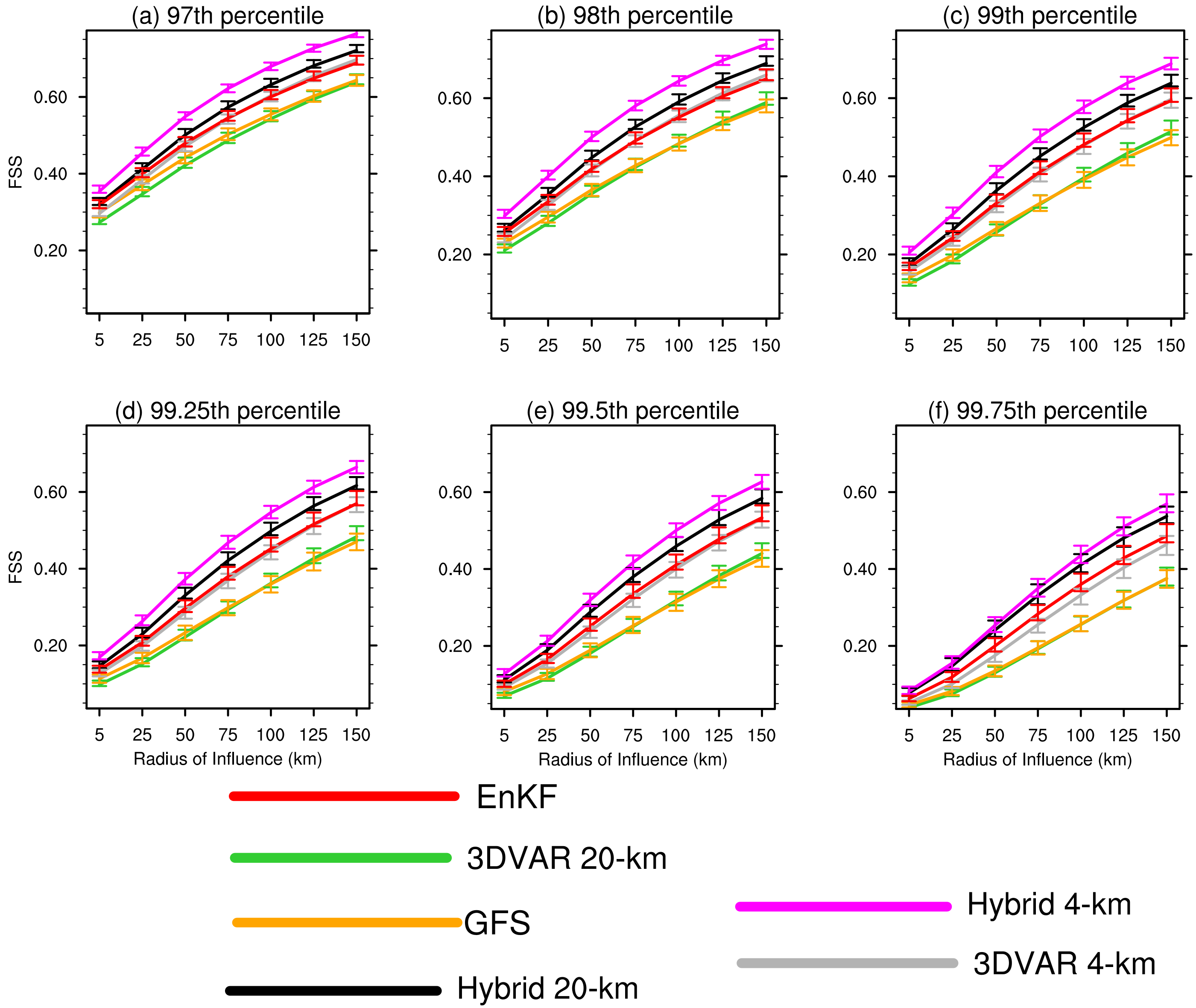
- Mean tropical cyclone track errors



From Schwartz et al. (2015)

Impact of dual-resolution

- Fractions skill score (FSS) aggregated over the first 12 forecast hours and 55 4-km forecasts



Hybrid practice

- **Computation steps:**
  - Compute ensemble mean (**gen_be_ensmean.exe**)
  - Extract ensemble perturbations (**gen_be_ep2.exe**)
  - Run WRFDA in “hybrid” mode (**da_wrfvar.exe**)
  - Display results for: ens_mean, std_dev, ensemble perturbations, hybrid increments, cost function
  - If time permits, play with different namelist settings: “je_factor” and “alpha_corr_scale”
- **Scripts to use:**
  - Some NCL scripts to display results

- **Ensemble generation part not included in current practice**

Namelist for WRFDA in hybrid mode

&wrfvar7
je_factor=2,   # half/half for ensemble and static B weightings (tunable parameter)

&wrfvar16
use_4denvar = .false.   # .true. will activate 4DEnVar

hybrid_dual_res = .false.   # If true, hybrid is in "dual-resolution" mode

alphacv_method=2,   # ensemble part is in model space (u,v,t,q,ps)

ensdim_alpha=10,   # ensemble size. Hybrid mode activated when ensdim_alpha > 0

alpha_corr_type=3,   # 1=Exponential; 2=SOAR; 3=Gaussian

alpha_corr_scale=750.,   # correlation scale in km (tunable parameter)

alpha_std_dev=1.,

alpha_vertloc=true, [use program "**gen_be_vertloc.exe**" to generate file
(output is be.vertloc.dat)]

Namelist for dual-resolution hybrid

- Dual-resolution hybrid uses WRF nesting to define grids, so also need to specify **nested domain** geometry in the namelist

- Analysis on the nested domain (i.e., "d02"), but using the ensemble from the parent domain (i.e., "d01")

- When running in dual-resolution mode, also need to link "d01" file to run directory as "./fg_ens":

ln –sf ${dir}/wrfinput_d01 ./fg_ens (ensemble grid)

ln –sf ${dir}/wrfinput_d02 ./fg (high-res background)

```
&wrfvar16
hybrid_dual_res = .true.

&domains
e_we              = 222, 316
e_sn              = 128, 274
s_vert            = 1,1
e_vert            = 45, 45
dx                = 45000, 15000,
dy                = 45000, 15000,
hypsometric_opt   = 2
max_dom           = 2
grid_id           = 1, 2,
parent_id         = 0, 1
i_parent_start    = 0, 74,
j_parent_start    = 0, 17,
parent_grid_ratio = 1, 3
```

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