WRFDA Overview

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WRFDA is a Data Assimilation system built within the WRF software framework, used for application in both research and operational environments….

Outline

- Basic principal of data assimilation
  - Scalar case
  - Two state variables case
  - General n-dimensional case
- Introduction to WRF Data Assimilation

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What is data assimilation?

- A statistical method to obtain the best estimate of state variables

- In the atmospheric sciences, DA involves combining model forecast (prior) and observations, along with their respective errors characterization, to produce an *analysis (Posterior)* that can initialize a numerical weather prediction model (e.g., WRF)

Scalar Case

- State variable to estimate "x", e.g., consider today's temperature of Boulder at 12 UTC.
- Now we have a "background" (or "prior") information x_b of x, which is from a 6-h GFS or WRF forecast initiated from 06 UTC today.
- We also have an observation y of x at a surface station in Boulder
- What is the best estimate (analysis) x_a of x?

Scalar Case

- We can simply average them: $x_a = \frac{1}{2}(x_b + y)$
  - This means we trust equally the background and observation.
- But if their accuracy is different and we have some estimation of their errors
  - e.g., for background, we have statistics (e.g., mean and variance) of $x_b - y$ from the past
  - For observation, we have instrument error information from manufacturer

Scalar Case

- Then we can do a weighted mean: $x_a = ax_b + by$ in a least square sense, i.e.,
  - Minimize $J(x) = \frac{1}{2}\frac{(x-x_b)^2}{\sigma_b^2} + \frac{1}{2}\frac{(x-y)^2}{\sigma_o^2}$
  - Requires $\frac{dJ(x)}{dx} = \frac{(x-x_b)}{\sigma_b^2} + \frac{(x-y)}{\sigma_o^2} = 0$
  - Then we can easily get

$$x_a = \frac{\sigma_o^2}{\sigma_b^2+\sigma_o^2} x_b + \frac{\sigma_b^2}{\sigma_b^2+\sigma_o^2} y$$

  - We can also write in the form of analysis increment

$$x_a - x_b = \frac{\sigma_b^2}{\sigma_b^2+\sigma_o^2}(y - x_b)$$

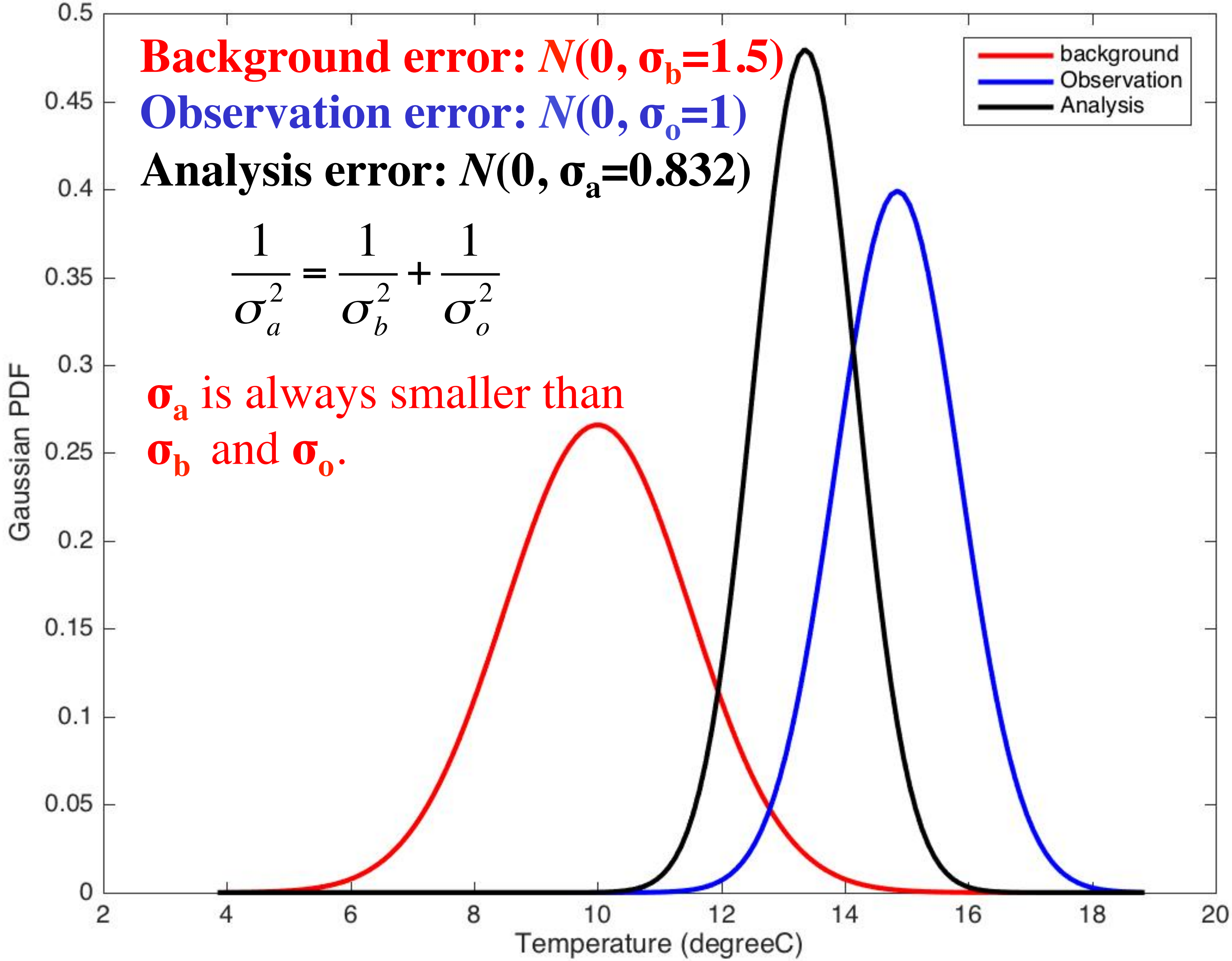
Scalar Case

- Minimize $J(x) = \frac{1}{2}\frac{(x-x_b)^2}{\sigma_b^2} + \frac{1}{2}\frac{(x-y)^2}{\sigma_o^2}$

- Is actually equivalent to maximize a Gaussian PDF

$$ce^{-J(x)}$$

Assume errors of X_b and y are unbiased



Background error: $N(0, \sigma_b=1.5)$
Observation error: $N(0, \sigma_o=1)$
Analysis error: $N(0, \sigma_a=0.832)$
$\frac{1}{\sigma_a^2} = \frac{1}{\sigma_b^2} + \frac{1}{\sigma_o^2}$
σ_a is always smaller than σ_b and σ_o.
background
Observation
Analysis
Gaussian PDF
Temperature (degreeC)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
2
4
6
8
10
12
14
16
18
20

Two state variables case

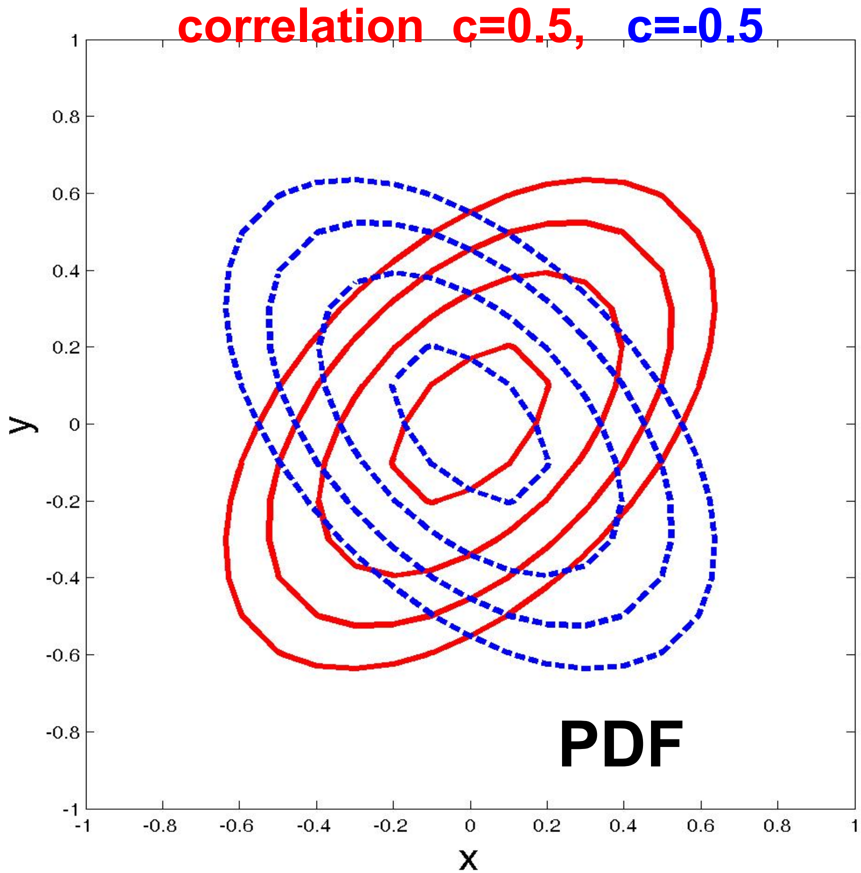
- Consider two state variables to estimate: Boulder and Denver's temperatures x_1 and x_2 at 12 UTC today.
- Background from 6-h forecast: x_1^b and x_2^b
  - and their error covariance with correlation c

$$\mathbf{B} = \begin{bmatrix} \sigma_1^2 & c\sigma_1\sigma_2 \\ c\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

- We only have an observation y_1 at a Boulder station and its error variance σ_o^2

2D PDF

$$PDF(x,y)=\frac{1}{2\pi\sqrt{1-c^2}}\exp\{-\frac{1}{2(1-c^2)}(x^2-2cxy+y^2)\}$$



Analysis increment for two variables

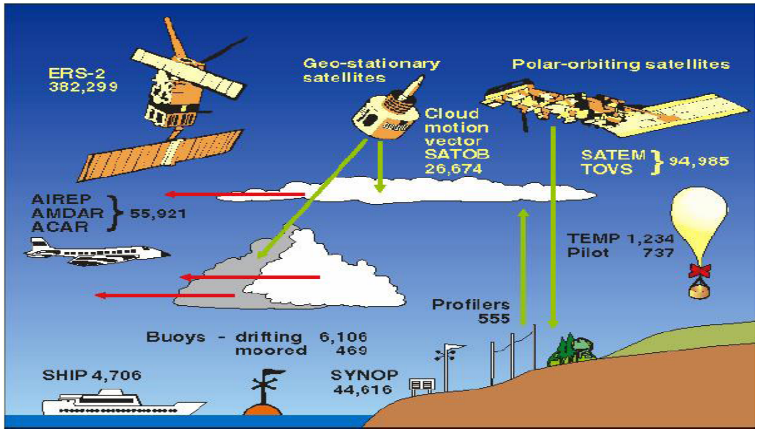
$$x_1^a - x_1^b = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_o^2}(y_1 - x_1^b)$$

$$x_2^a - x_2^b = \frac{c\sigma_1\sigma_2}{\sigma_1^2 + \sigma_o^2}(y_1 - x_1^b)$$

Unobserved variable x_2 gets updated through the error correlation *c* in the background error covariance.

This correlation can be correlation between two locations (spatial), two variables (multivariate), or two times (temporal).

General Case

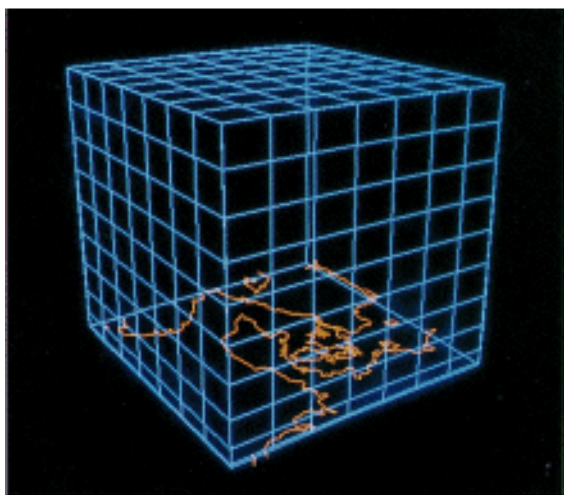


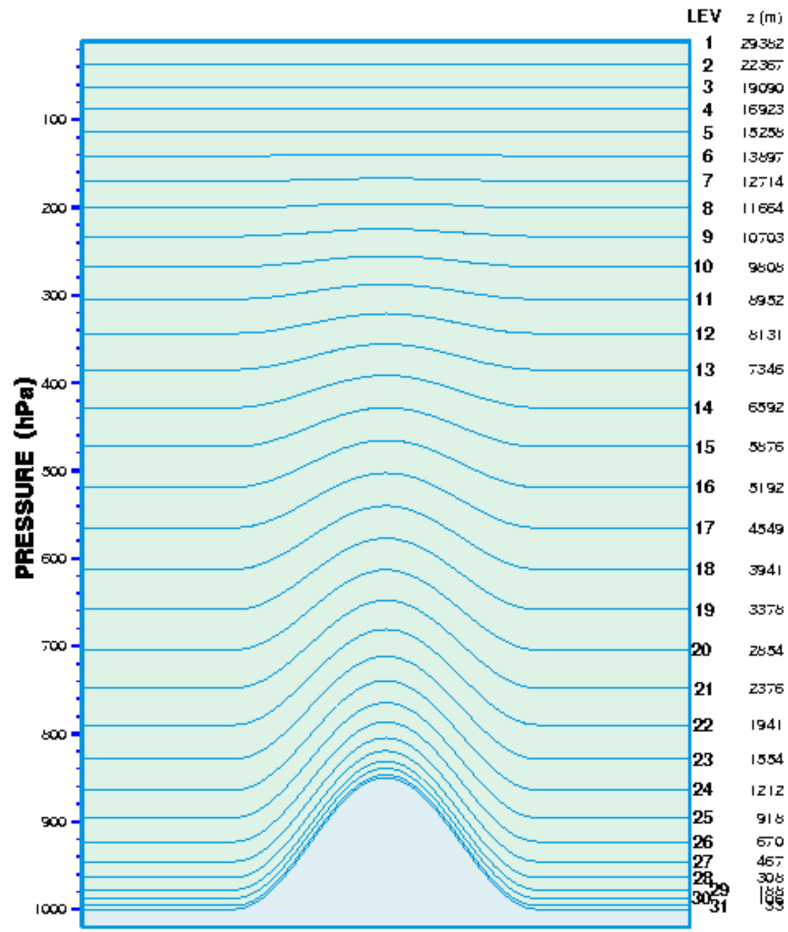
Observations

y^o, ~10^5-10^6

Model state

x, ~10^7





Vertical resolution of the DMI-HIRLAM system

General Case: vector and matrix notation

state vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

observation vector

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

background error covariance

$$\mathbf{B} = \begin{bmatrix} \sigma_1^2 & c_{12}\sigma_1\sigma_2 & \cdots & \cdots \\ c_{12}\sigma_1\sigma_2 & \sigma_2^2 & \cdots & \cdots \\ \cdots & \cdots & \ddots & \cdots \\ \cdots & \cdots & \cdots & \sigma_m^2 \end{bmatrix}$$

Observation error covariance

$$\mathbf{R} = \begin{bmatrix} \sigma_{o1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{o2}^2 & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \sigma_{on}^2 \end{bmatrix}$$

General Case: cost function

$$J(x) = \frac{1}{2}(x - x^b)^T \mathbf{B}^{-1}(x - x^b) + \frac{1}{2}[\mathbf{H}x - y]^T \mathbf{R}^{-1}[\mathbf{H}x - y]$$

H maps x to y space, e. g., interpolation.
Terminology in DA: observation operator

Minimize J(x) is equivalent to maximize a multi-dimensional Gaussian PDF

Constant * $e^{-J(x)}$

General Case: analytical solution

Again, minimize J requires its gradient (a vector) with respect to x equal to zero:

$$\nabla J_{\mathbf{x}}(\mathbf{x}) = \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_{\mathbf{b}}) - \mathbf{H}^{\mathbf{T}}\mathbf{R}^{-1}[\mathbf{y} - \mathbf{H}\mathbf{x}] = 0$$

This leads to analytical solution for the analysis increment:

$$x^a - x^b = \mathbf{B}\mathbf{H}^{\mathbf{T}}(\mathbf{H}\mathbf{B}\mathbf{H}^{\mathbf{T}} + \mathbf{R})^{-1}[y - \mathbf{H}x^b]$$

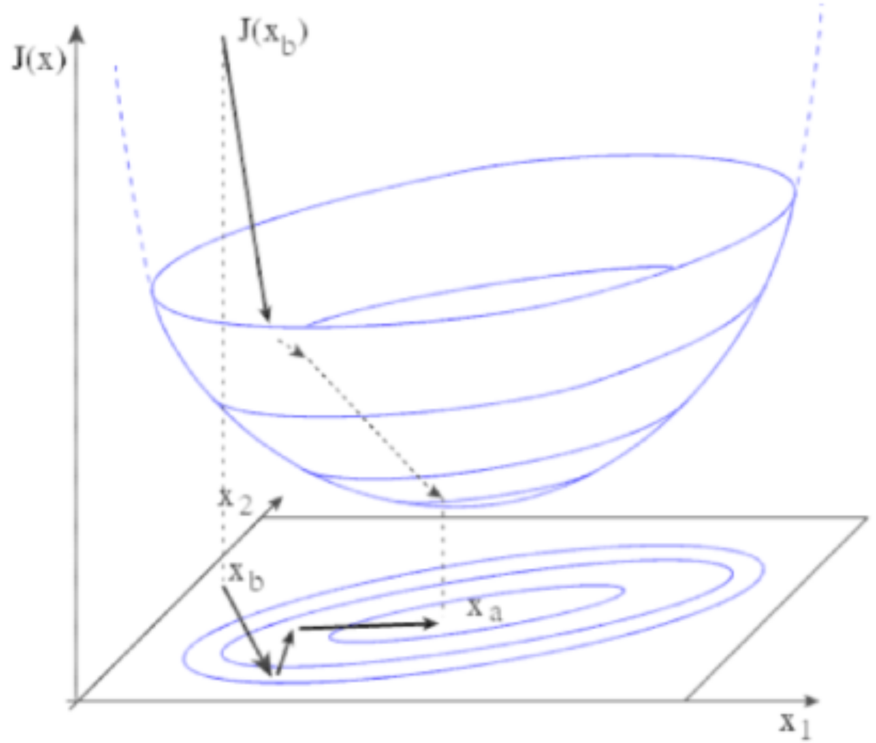
$\mathbf{HBH^T}$: projection of background error covariance in observation space

$\mathbf{BH^T}$: projection of background error covariance in background-observation space

Iterative algorithm to find minimum of cost function

- **Descending algorithms**
  - **Descending direction:** γ_n **(N-dimensional vector)**
  - **Descending step:** μ_n

$$x_{n+1} = x_n + \mu_n \gamma_n$$



from Bouttier and Courtier 1999

Precision of Analysis with optimal B and R

$$\mathbf{A}^{-1} = \mathbf{B}^{-1} + \mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}$$

Generalization of scalar case $\frac{1}{\sigma_a^2} = \frac{1}{\sigma_b^2} + \frac{1}{\sigma_o^2}$

Or in another form: $\mathbf{A} = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{B}$

With

$$\mathbf{K} = \mathbf{B}\mathbf{H}^{T}(\mathbf{H}\mathbf{B}\mathbf{H}^{T} + \mathbf{R})^{-1}$$

called Kalman gain matrix

Precision of analysis: more general formulation

$$\mathbf{A} = (\mathbf{I} - \mathbf{KH})\mathbf{B}_t(\mathbf{I} - \mathbf{KH})^{\mathrm{T}} + \mathbf{KR}_t\mathbf{K}^{\mathrm{T}}$$

where B_t and R_t are "true" background and observation error covariances.

This formulation is valid for any given gain matrix K, which could be suboptimal (e.g., due to incorrect estimation/specification of B and R).

Analysis increment with a single humidity observation

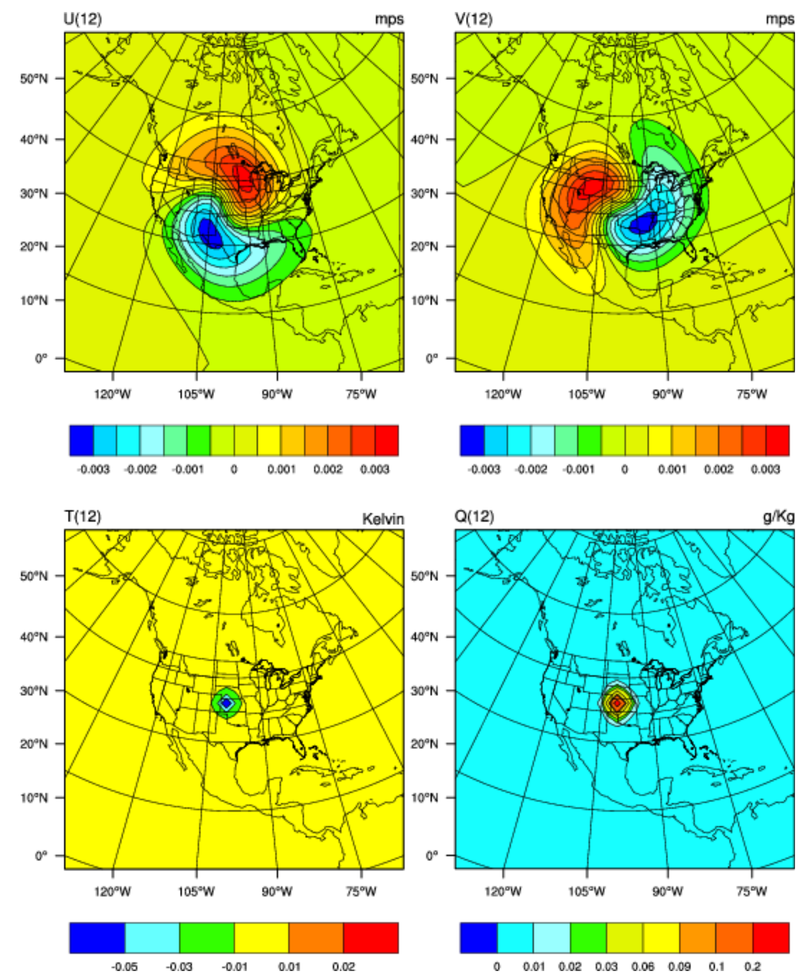
$$x^a - x^b = \mathbf{BH}^{\mathrm{T}}(\mathbf{HBH}^{\mathrm{T}} + \mathbf{R})^{-1}[y - \mathbf{H}x^b]$$

$$x_l^a - x_l^b = \frac{c_{lk}\sigma_l\sigma_k}{\sigma_k^2 + \sigma_{ok}^2}(y_k - x_k^b)$$

It is generalization of previous two variables case:

$$x_1^a - x_1^b = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_o^2}(y_1 - x_1^b)$$

$$x_2^a - x_2^b = \frac{c\sigma_1\sigma_2}{\sigma_1^2 + \sigma_o^2}(y_1 - x_1^b)$$

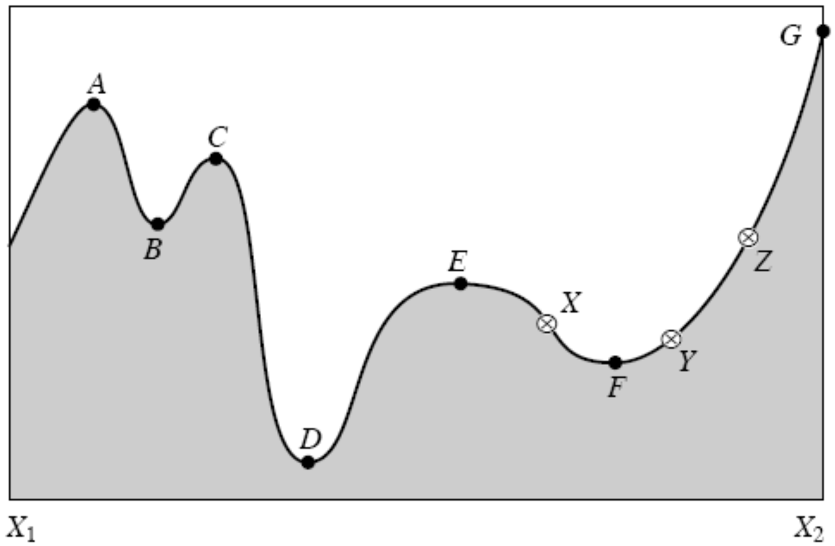


cv_options=6 in WRFDA

Other Remarks

- Observation operator can be non-linear and thus analysis error PDF is not necessarily Gaussian

- J(x) can have multiple local minima. Final solution of least square depends on starting point of iteration, e.g., choose the background x_b as the first guess.



Other Remarks

- **B** matrix is of very large dimension, explicit inverse of **B** is impossible, substantial efforts in data assimilation were given to the estimation and modeling of **B**.

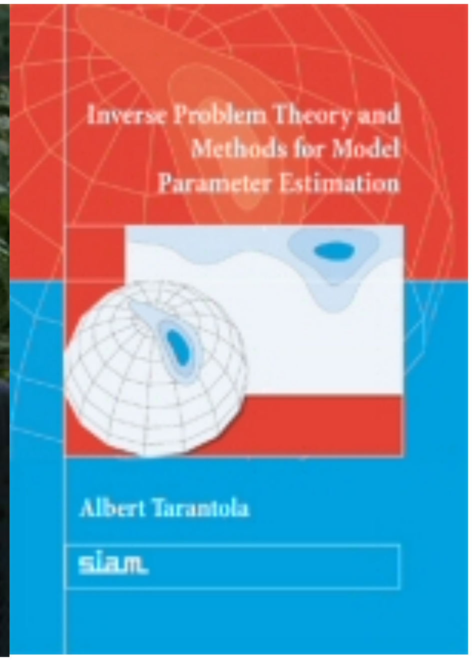
- **B** shall be spatially-varied and time-evolving according to weather regime.

- Analysis can be sub-optimal if using inaccurate estimate of **B** and **R**.

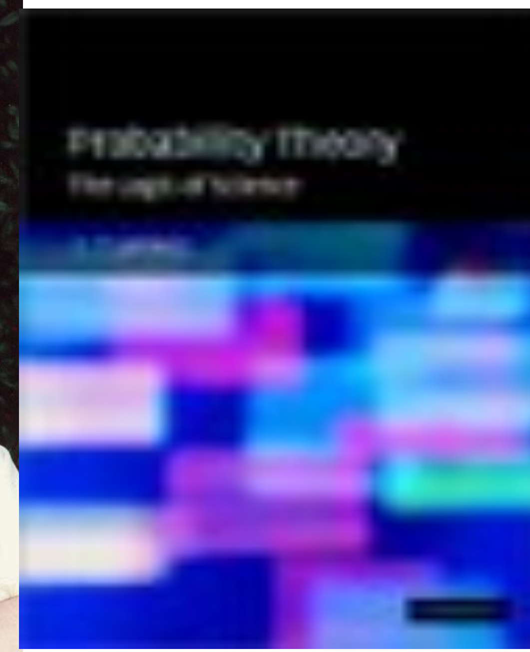
- Could use non-Gaussian PDF
  - Thus not a least square cost function
  - Difficult (usually slow) to solve; could transform into Gaussian problem via variable transform

Two helpful books

Albert Tarantola



Edwin Thompson Jaynes



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Freely available book!

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The Logic of Science

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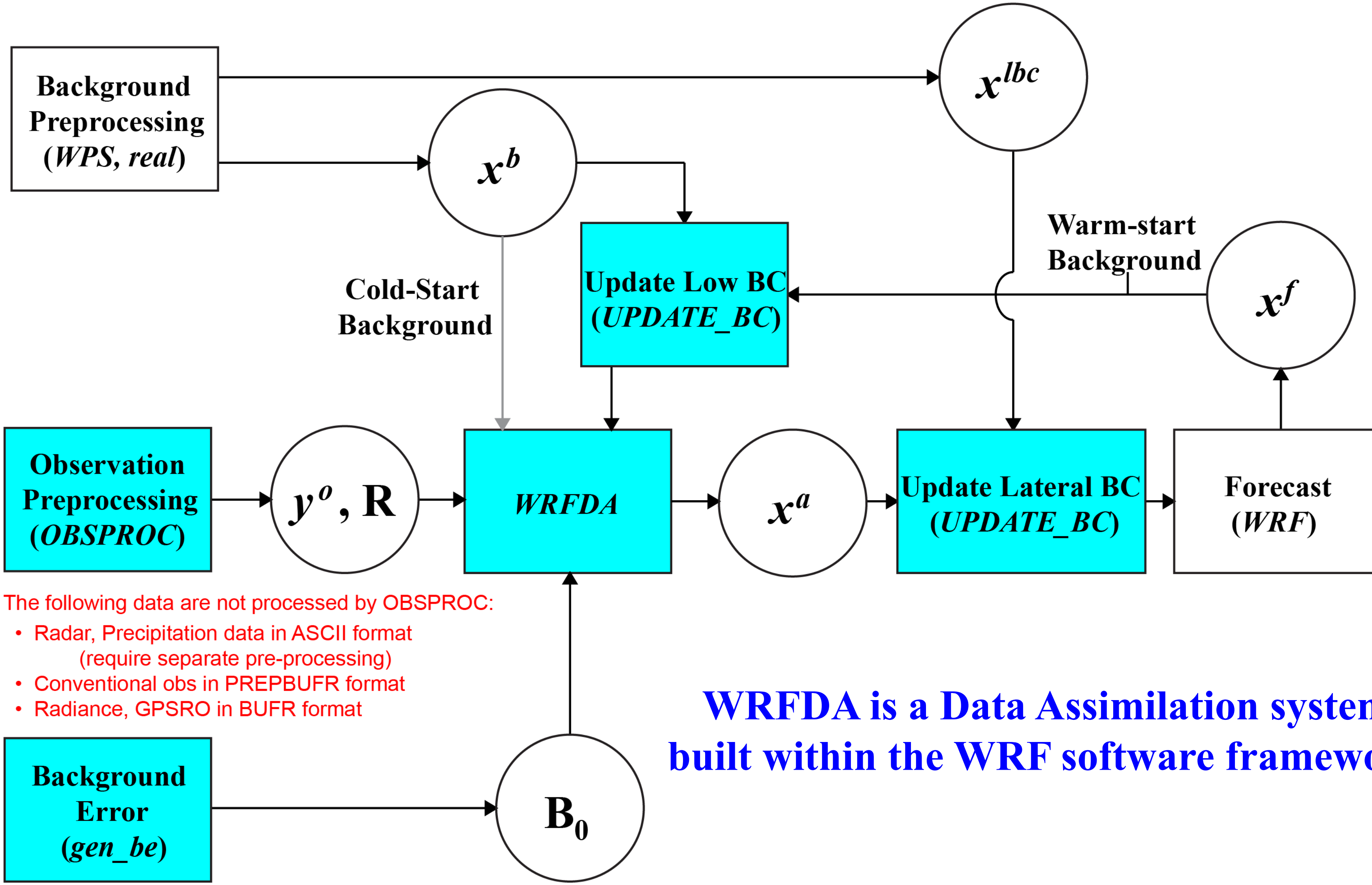
WRFDA in the WRF Modeling System



The following data are not processed by OBSPROC:

- Radar, Precipitation data in ASCII format
  (require separate pre-processing)
- Conventional obs in PREPBUFR format
- Radiance, GPSRO in BUFR format

WRFDA is a Data Assimilation system built within the WRF software framework

What WRFDA can do?

- Provide Initial conditions for the WRF model forecast
- Verification and validation via difference b.w. obs and model
- Observing system design, monitoring and assessment
- Reanalysis
- Better understanding:
  - Data assimilation methods
  - Model errors
  - Data errors
  - …

DA algorithms currently available in WRFDA

- 3DVAR and FGAT
  - Different options for choice of control variables (e.g., Psi/Chi or U/V) and background error covariance modeling (e.g., vertical EOF or vertical recursive filter)
- 4DVAR
  - TL/Adjoint (i.e., WRFPlus code) of WRF up-to-date with WRF
  - Allow LBC control variable and Jc-DFI
- Hybrid-3DEnVar and Hybrid-4DEnVar (since V3.9)
  - Can run in dual-resolution mode
  - Can ingest ensemble from global or regional sources
- ETKF: for generating ensemble analysis

WRFDA Observations

- **In-Situ:**
  - SYNOP
  - METAR
  - SHIP
  - BUOY
  - TEMP
  - PIBAL
  - AIREP, AIREP humidity
  - TAMDAR
- **Bogus:**
  - TC bogus
  - Global bogus
- **Remotely sensed retrievals:**
  - Atmospheric Motion Vectors (geo/polar)
  - SATEM thickness
  - Ground-based GPS **TPW or ZTD**
  - SSM/I oceanic surface wind speed and TPW
  - Scatterometer oceanic surface winds
  - Wind Profiler
  - **Radar data (reflectivity/retrieved rainwater, and radial-wind)**
  - Satellite temperature/humidity/thickness profiles
  - GPS refractivity (e.g. COSMIC)
  - **Stage IV precipitation/rain rate data (4D-Var only)**
- **Radiances:**

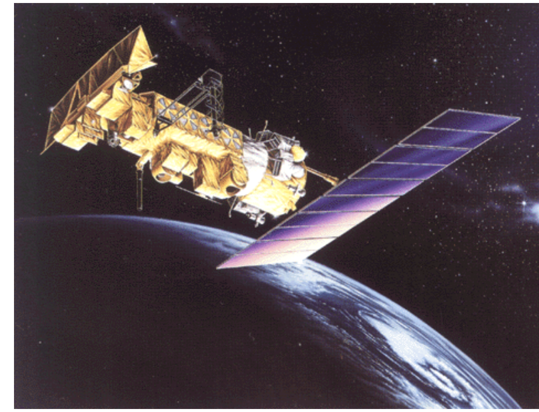
HIRS	NOAA-16, NOAA-17, NOAA-18, NOAA-19, METOP-A
AMSU-A	NOAA-15, NOAA-16, NOAA-18, NOAA-19, EOS-Aqua, METOP-A, METOP-B
AMSU-B	NOAA-15, NOAA-16, NOAA-17
MHS	NOAA-18, NOAA-19, METOP-A, METOP-B
AIRS	EOS-Aqua
SSMIS	DMSP-16, DMSP-17, DMSP-18
IASI	METOP-A, METOP-B
ATMS	Suomi-NPP
MWTS	FY-3
MWHS	FY-3
SEVIRI	METEOSAT
AMSR2	**GCOM-W1 (new in V3.8)**

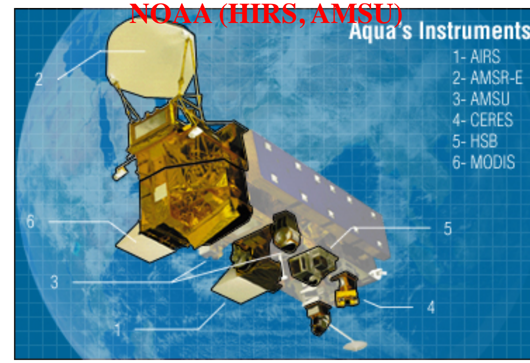
WRFDA is flexible to allow assimilation of different formats of observations:

- **Little_r (ascii), HDF, Binary**
- **NOAA MADIS (netcdf),**
- **NCEP PrepBufr,**
- **NCEP radiance bufr**

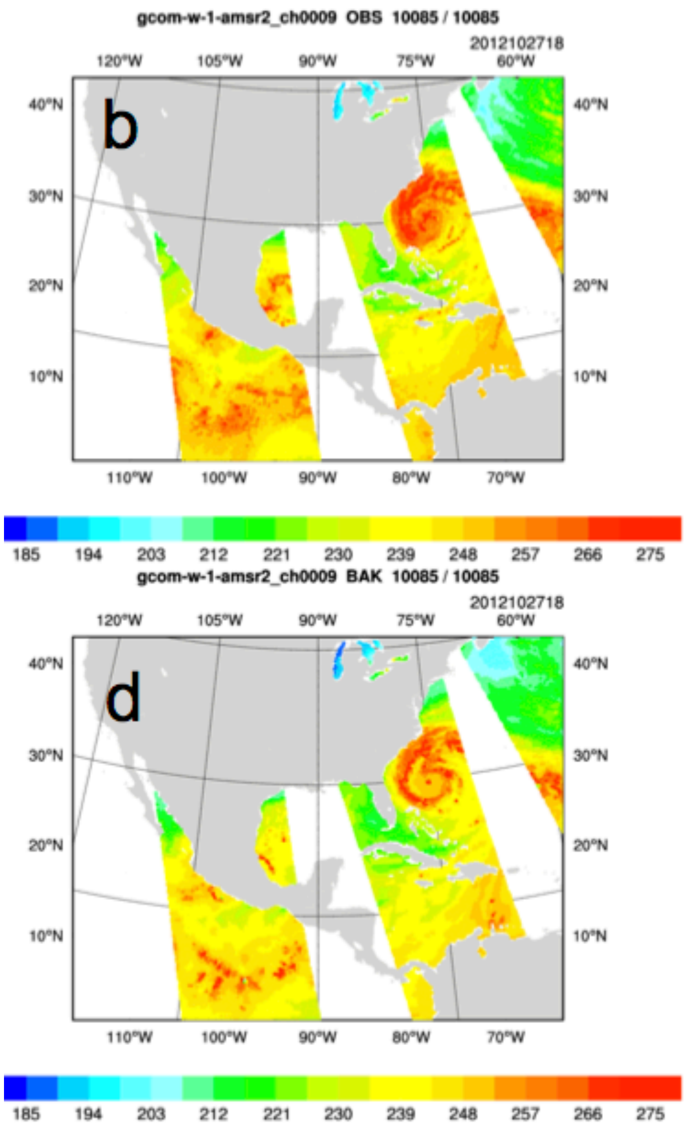
WRFDA
Radiance Assimilation

- Two RTM interfaces
  - RTTOV or CRTM
- Variational Bias Correction
- Modular code design to ease adding new satellite sensors
- Capability for cloudy radiance DA

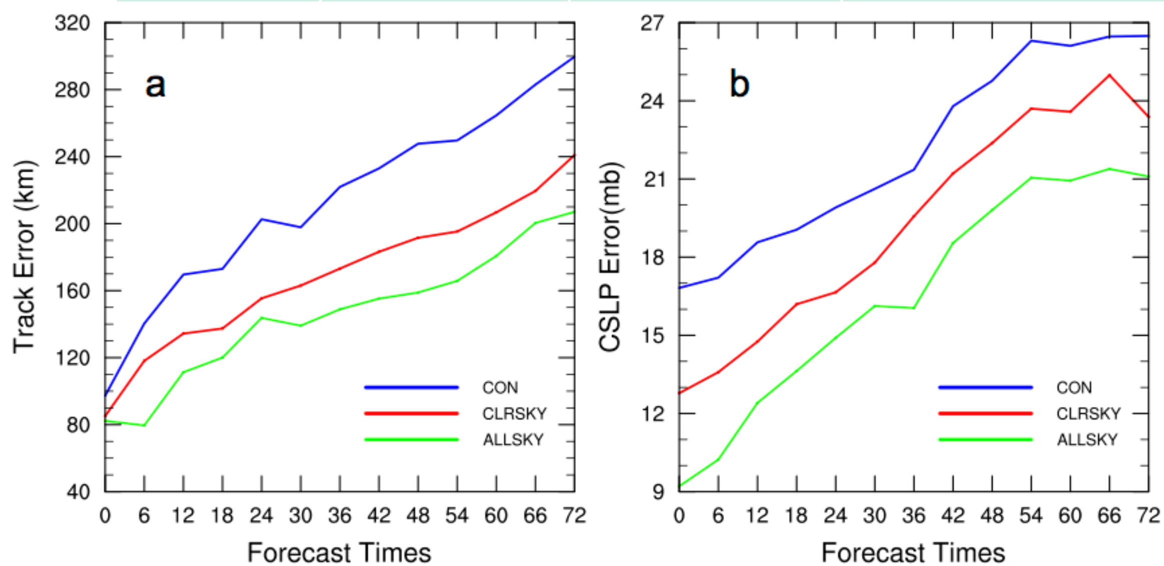




New in V3.9: all-sky radiance DA: AMSR2

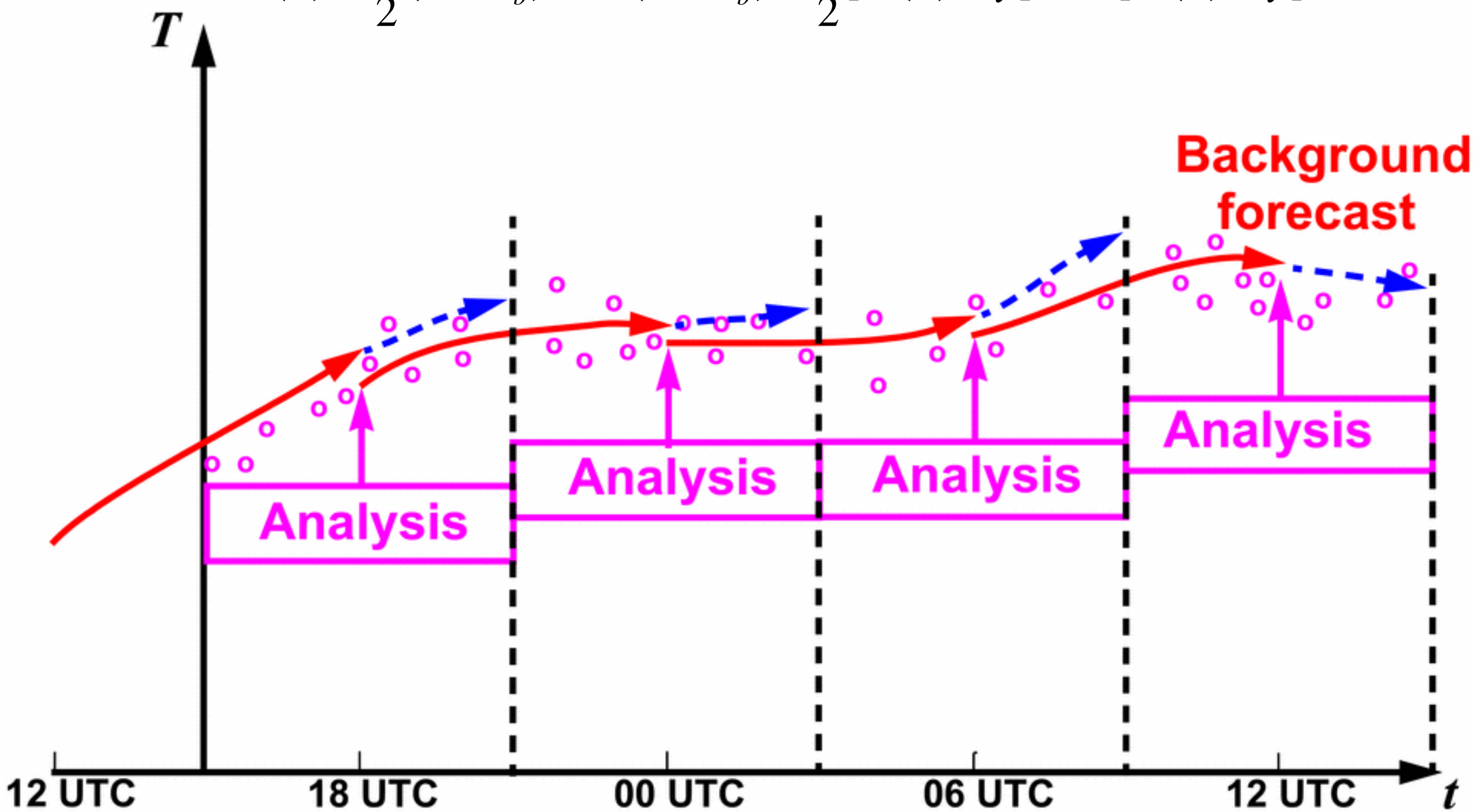


Channel	Frequency (GHz)	Polarization	Footprint (along scan* along track)
1,2	6.925	V,H	35*61 km
3,4	7.3	V,H	35*61 km
5,6	10.65	V,H	24*41 km
7,8	18.7	V,H	13*22 km
9,10	23.8	V,H	15*26 km
11,12	36.5	V,H	7*12 km
13,14	89.0	V,H	3*5 km



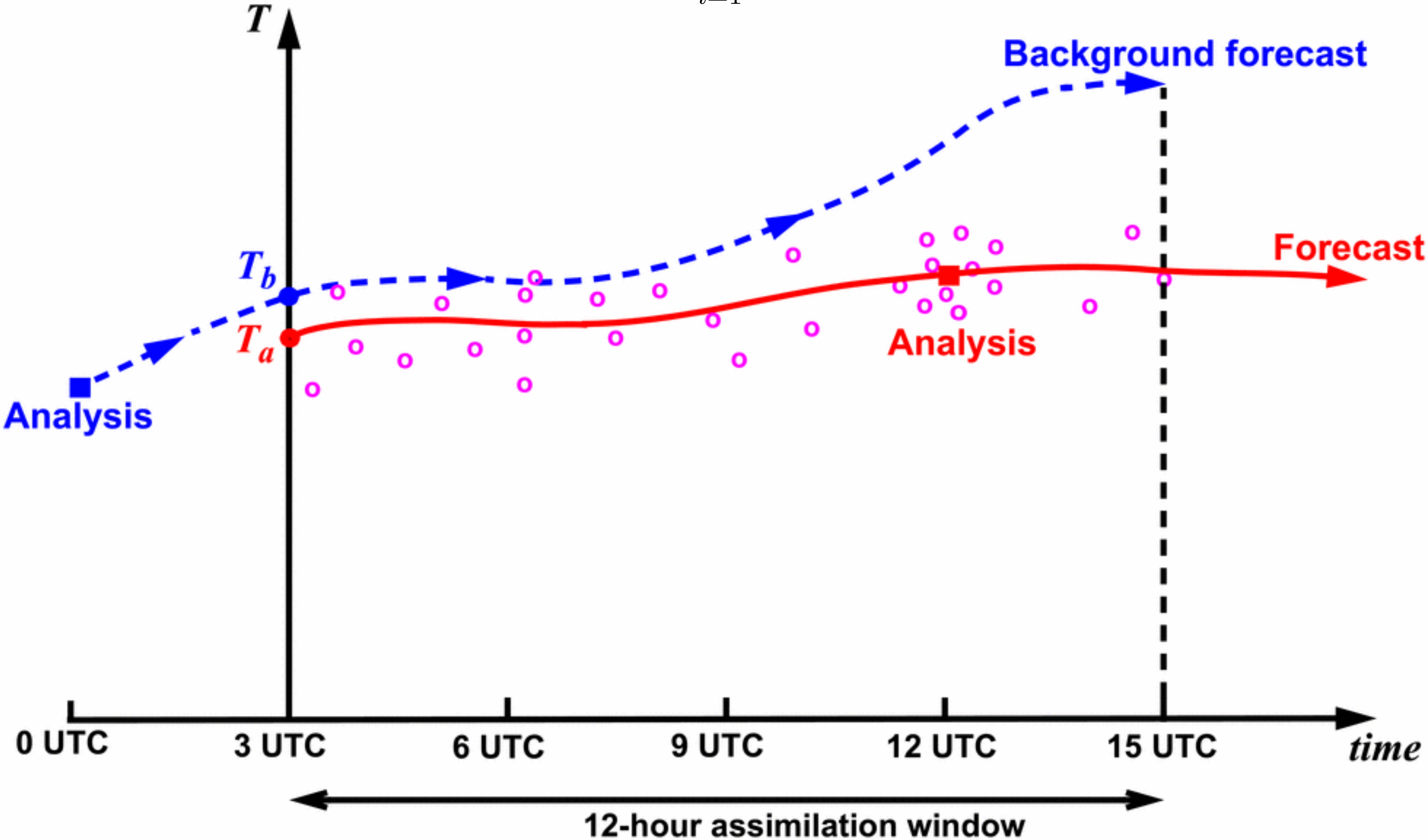
3DVAR (Barker et al. 2004)

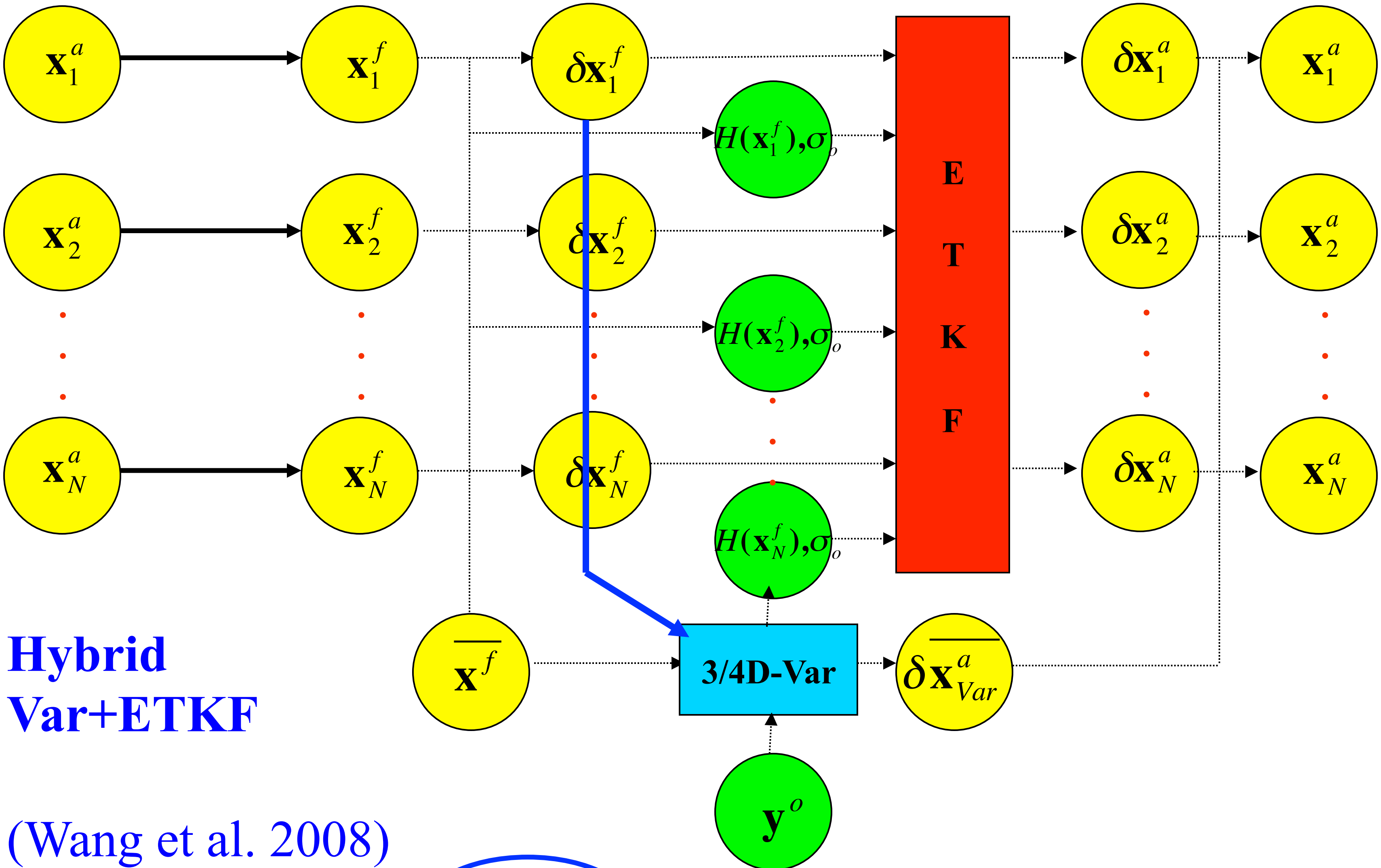
$$J(x) = \frac{1}{2}(x - x_b)^{\mathrm{T}} \mathrm{B}^{-1}(x - x_b) + \frac{1}{2}[H(x) - y]^{\mathrm{T}} \mathrm{R}^{-1}[H(x) - y]$$



4DVAR (Huang et al. 2009)

$$J(\mathbf{x}_0) = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_0^b)^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_0^b) + \frac{1}{2}\sum_{i=1}^{N}[H_i(M_i(\mathbf{x}_0)) - \mathbf{y}_i]^{\mathrm{T}}\mathbf{R}_i^{-1}[H_i(M_i(\mathbf{x}_0)) - \mathbf{y}_i]$$

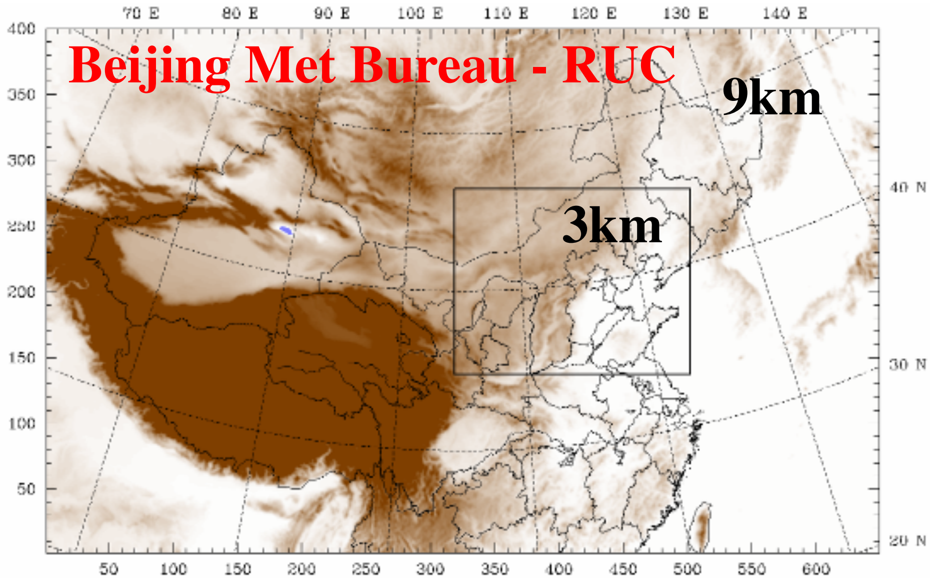


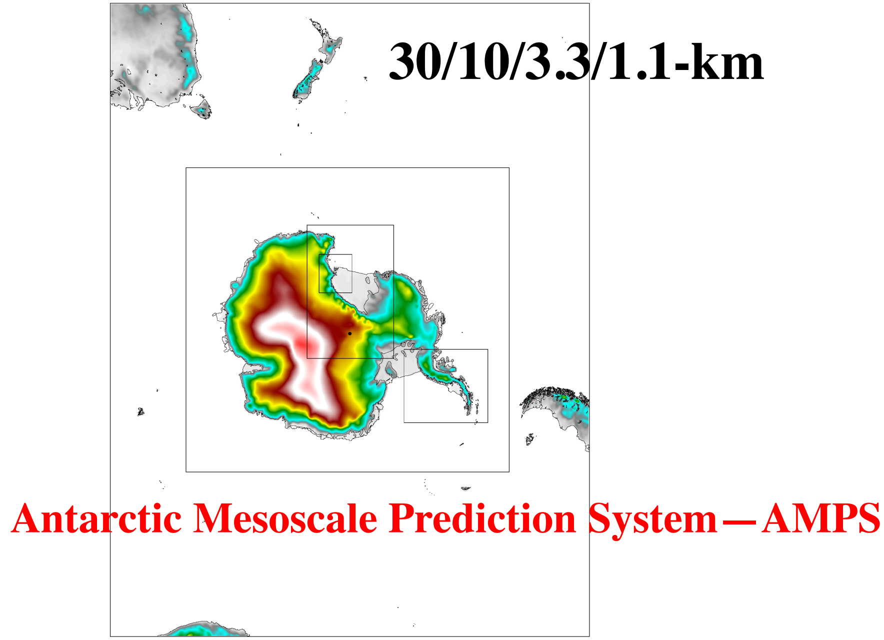


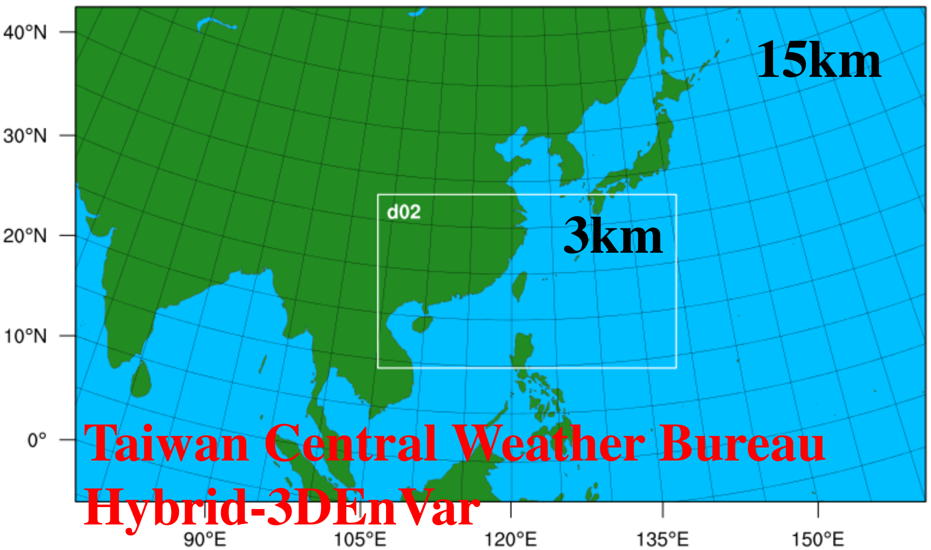
Hybrid
Var+ETKF

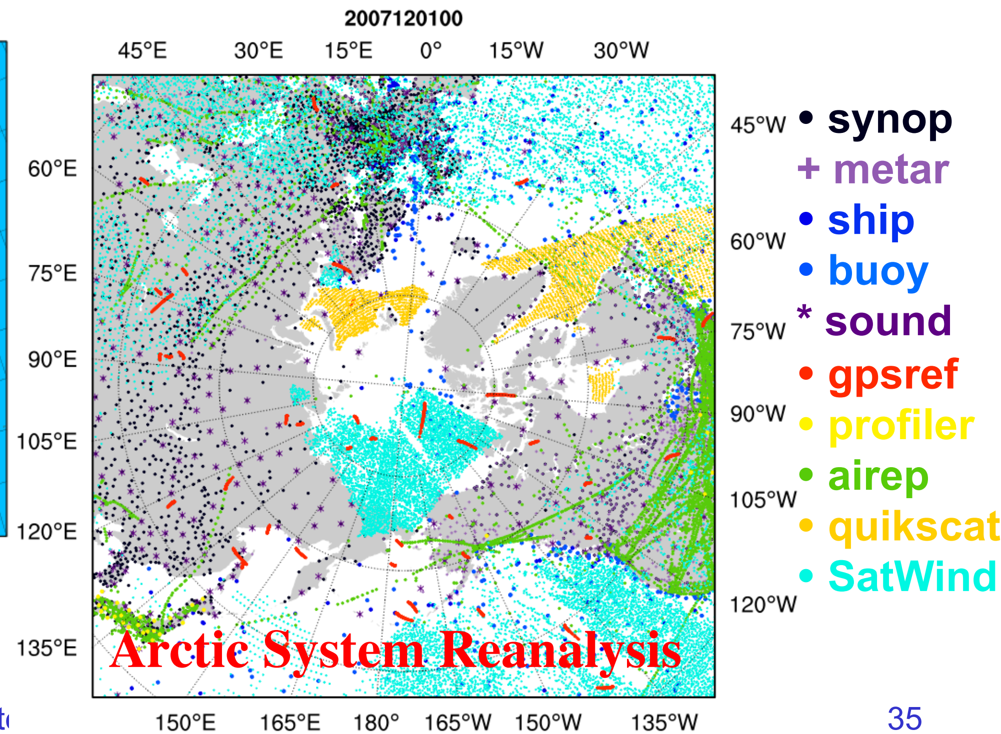
(Wang et al. 2008)

$$J = \frac{W_b}{2}\mathbf{v}^T\mathbf{v} + \frac{W_\alpha}{2}\mathbf{a}^T\mathbf{A}^{-1}\mathbf{a} + \frac{1}{2}\sum_{i=0}^{n}\left[\mathbf{d}_i - \mathbf{H}_i\mathbf{M}_i\mathbf{U}\mathbf{v}\right]^T \mathbf{R}_i^{-1}\left[\mathbf{d}_i - \mathbf{H}_i\mathbf{M}_i\mathbf{U}\mathbf{v}\right]$$


Beijing Met Bureau - RUC
9km
3km
70 E
80 E
90 E
100 E
110 E
120 E
130 E
140 E
40 N
30 N
20 N


30/10/3.3/1.1-km
Antarctic Mesoscale Prediction System—AMPS


15km
d02
3km
Taiwan Central Weather Bureau
Hybrid-3DEnVar
40°N
30°N
20°N
10°N
0°
90°E
105°E
120°E
135°E
150°E


2007120100
Arctic System Reanalysis
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120°E
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180°
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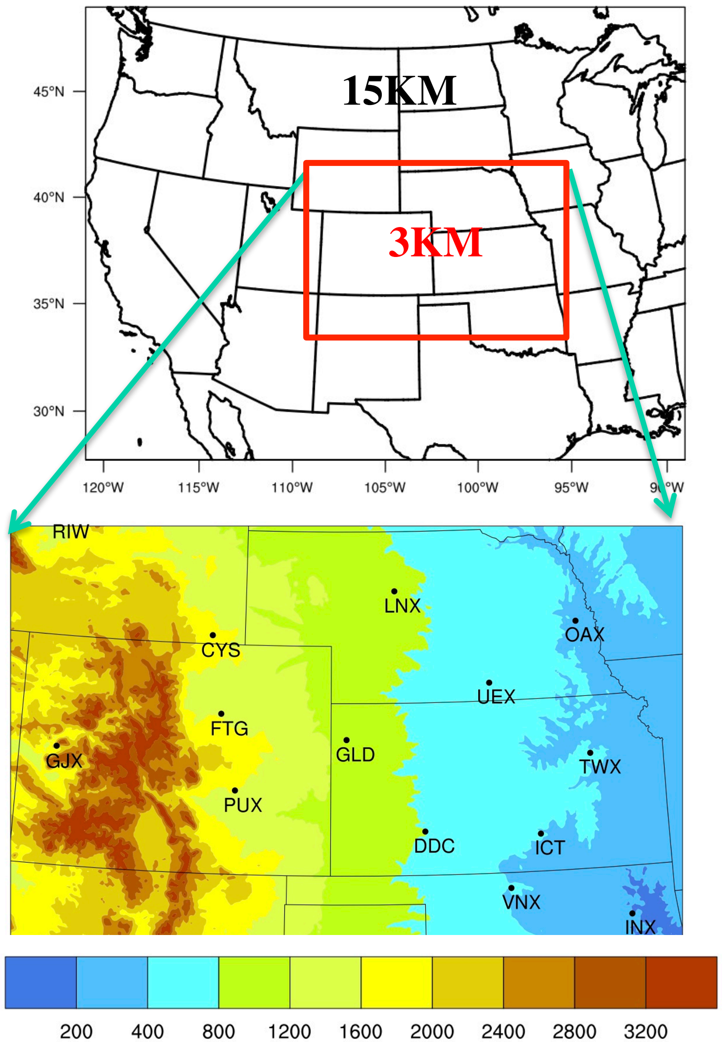
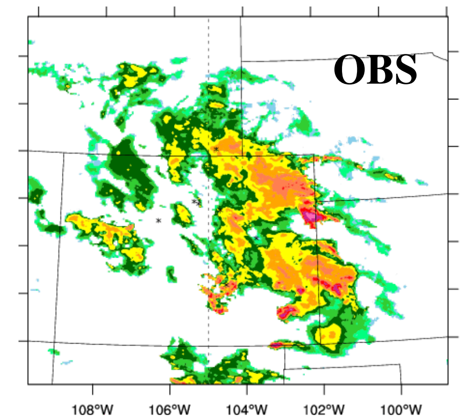
135°W

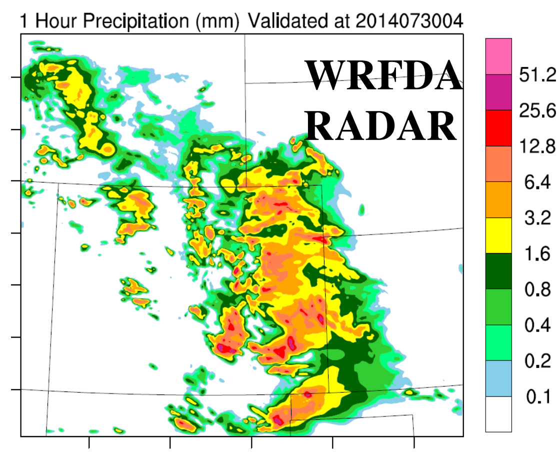
Radar DA for hydrological application

STEP Hydromet Real Time Exp. during spring time

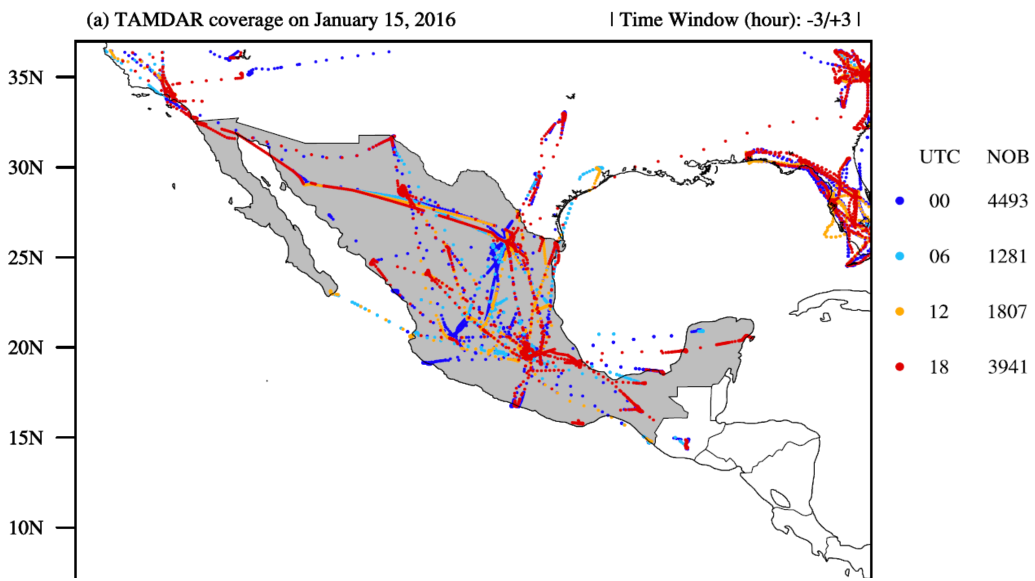


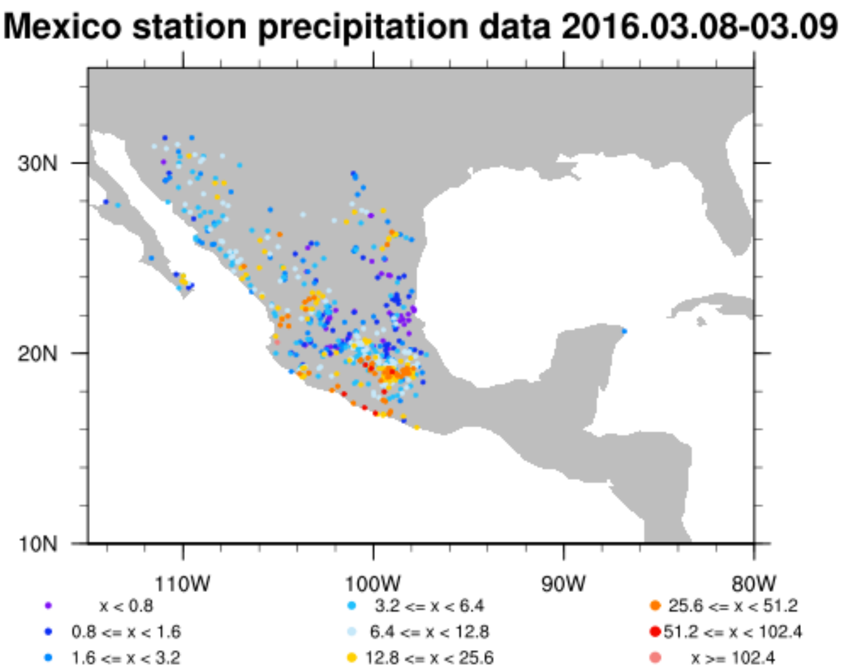
- **The goal is to improve local-scale QPF in coupled hydromet system**
- **< 1 h rapid update**
- **Radar radial velocity and reflectivity assimilation**
- **High resolution vs. ensemble**
- **Impact of terrain**
- **Improved results in capturing localized storms**

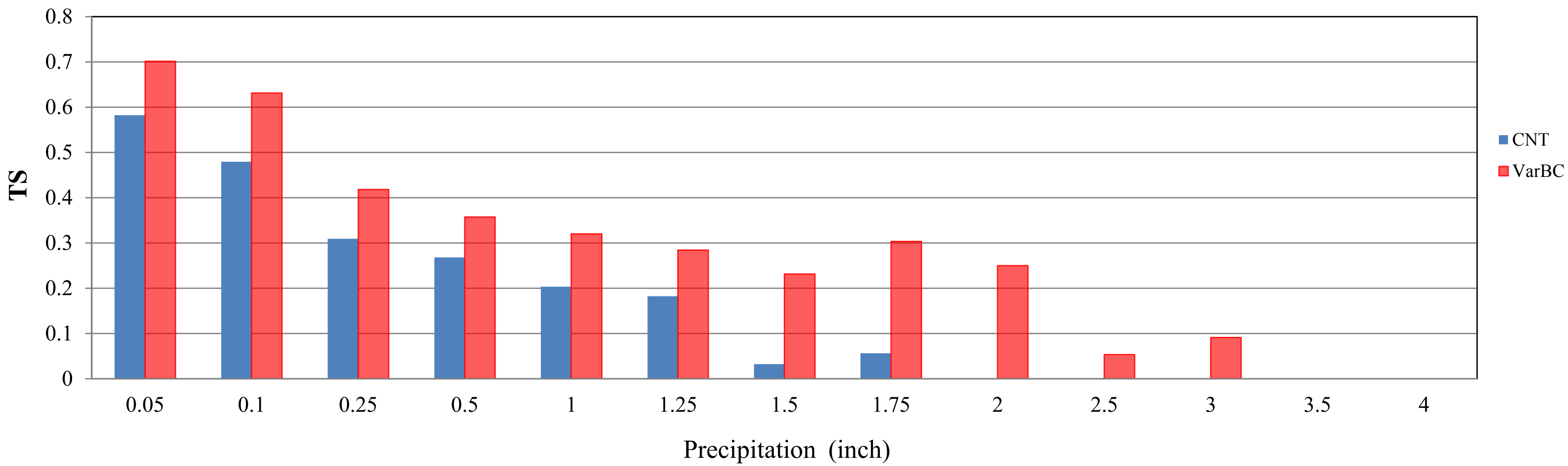




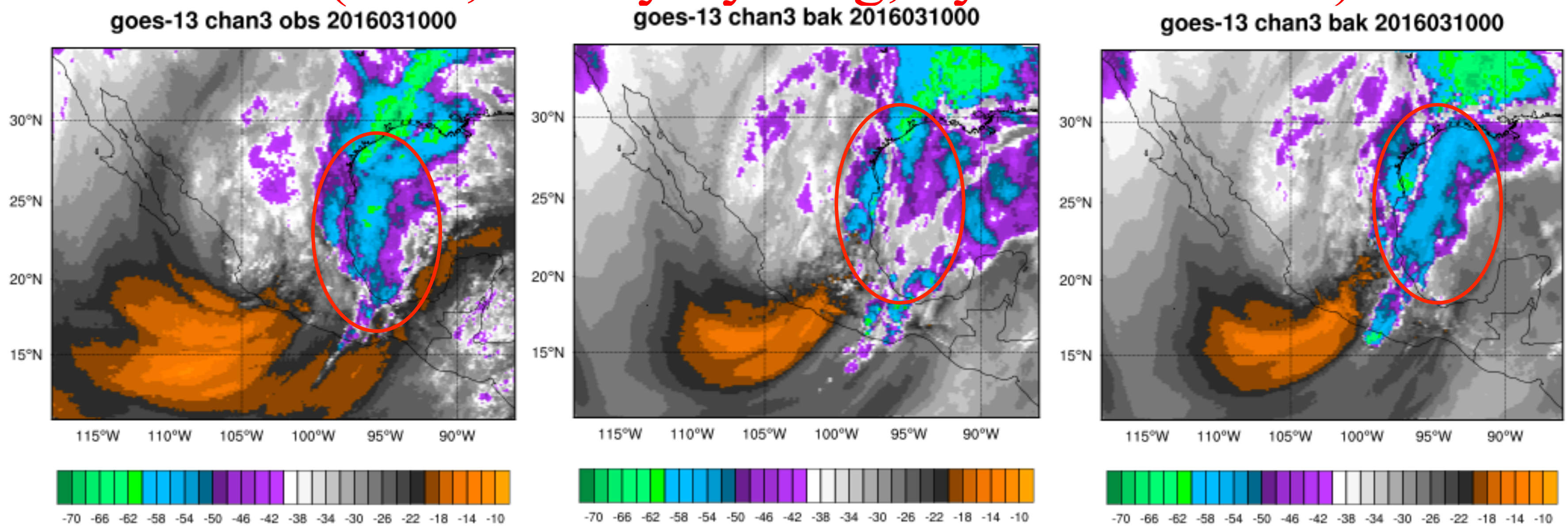
Impact of Aircraft T VarBC on rainfall forecast



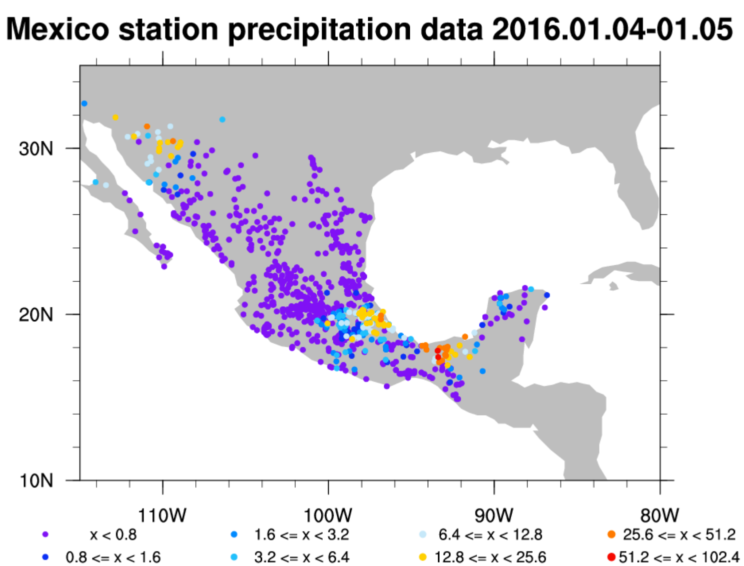


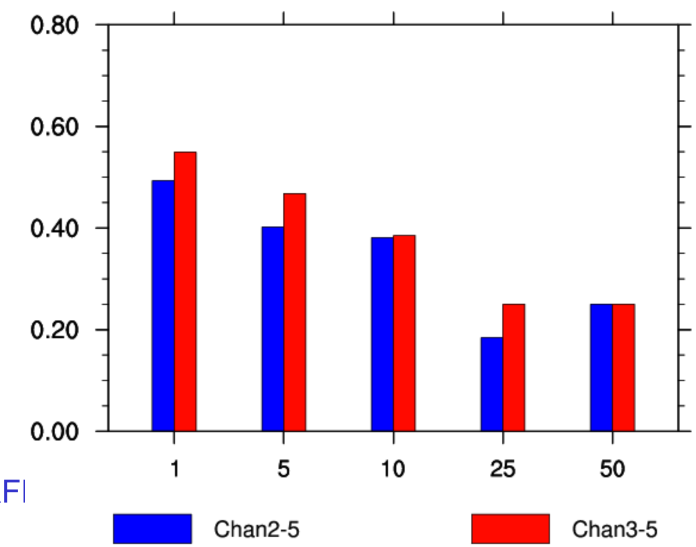


GEOS imager radiance DA at convection-permitting scale (4km, hourly-cycling, hybrid-3DVAR)



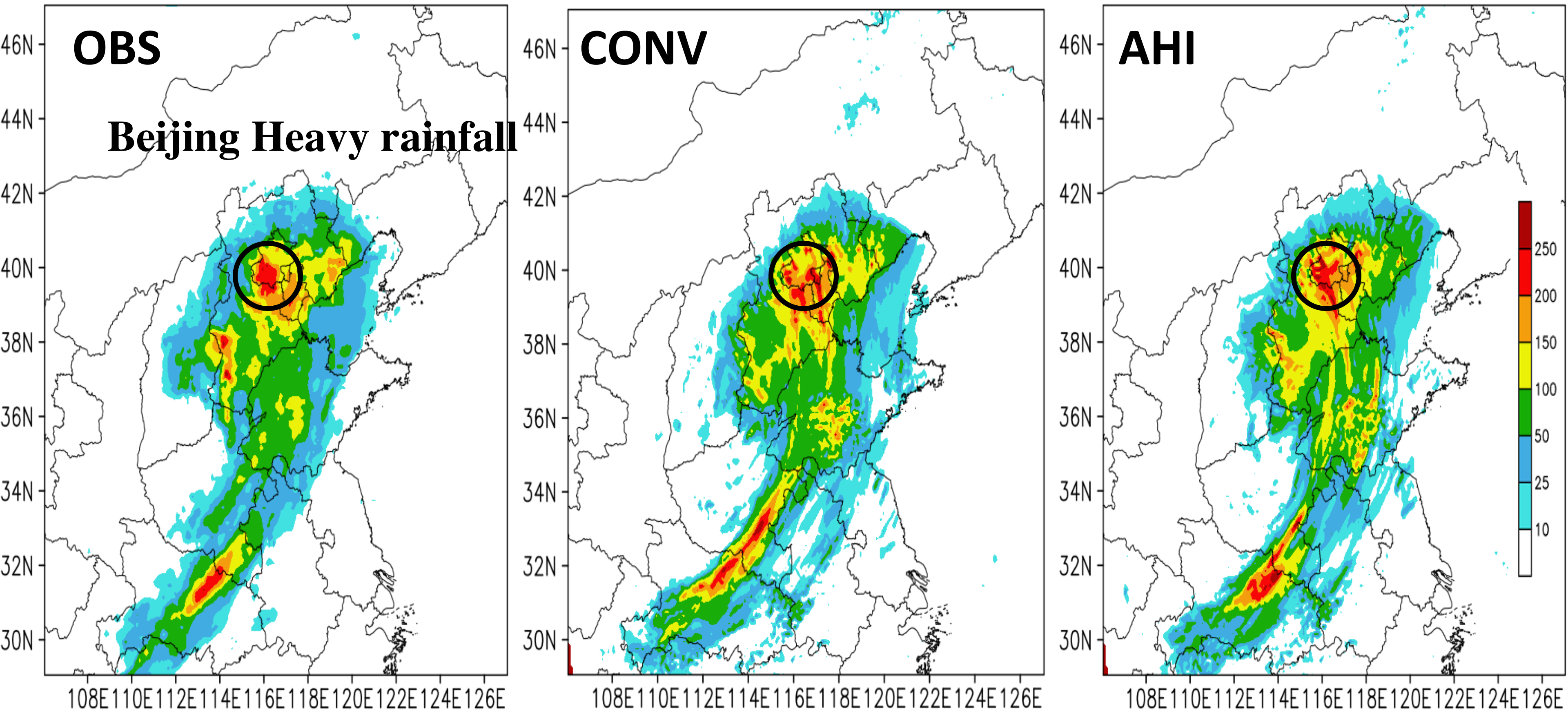
obs **No-GOES DA** **GOES-DA**





Yang et al., 2017, JGR.

24h accumulated rainfall field initialized at 2016071912



Himawari-8 AHI radiance DA impact

Other ongoing work

- Continue developing Multi-Resolution Incremental 4DVAR (MRI-4DVAR)
- Continue developing cloudy radiance/product DA
- High spatial- and temporal-resolution geostationary satellite DA
- Improving surface data assimilation
- Improving radar DA
- WRFPlus-Chem & WRFDA-Chem

Last Remarks

- We welcome contributions from external users/ developers.
  - Contact wrfhelp@ucar.edu or directly email to me liuz@ucar.edu for contributing back your code

- We maintain a WRFDA-related publications list, please inform us your papers to be included
  - http://www2.mmm.ucar.edu/wrf/users/wrfda/publications.html