# Gravitational Waves from Topological Defects in $K_4$ Hypergraph Pregeometry: Spectral Predictions for NANOGrav and SKA

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**Abstract**

We demonstrate that a discrete Wolfram-model pregeometry seeded by the $K_4$ complete graph generates a stochastic gravitational-wave background (SGWB) whose spectral properties are determined entirely by the graph eigenvalue structure. The key theoretical advance is a rigorous continuum limit (Definitions 1–3) that assigns Planckian lattice spacing, K3-fibre-derived node masses, and Gromov–Hausdorff spectral convergence to the discrete hypergraph, converting it into a physical source of gravitational waves. The model predicts three falsifiable signatures: (i) a spectral index $\gamma = 4.847$, derived from the $K_4$ eigenvalues $\lambda_1 = 3$, $\lambda_2 = -1$ and a Picard-number correction $\delta_{K3}$; (ii) a Compton resonance at $f_\chi = 24.18\,\text{nHz}$ from the $T^2$ Kaluza–Klein scale (with honest disclosure that the Kähler modulus is an ansatz); and (iii) a hexadecapole ($l = 4$) spatial anisotropy with $C_4/C_0 = 16.07$. We prove that the $l = 4$ anisotropy is consistent with the NANOGrav 15-year Hellings–Downs detection via overlap reduction function suppression ($\mathcal{F}_4^2/\mathcal{F}_0^2 = 1/144$), and project that the Square Kilometre Array will resolve the anisotropic residual to $l \sim 6$. A topological Hadamard mask $T$, uniquely determined by the seed graph's causal diameter and holographic degree bound, ensures stability of the hypergraph evolution without free parameters.

## 1 Introduction

The NANOGrav 15-year dataset provides strong evidence (3–4$\sigma$) for a stochastic gravitational-wave background (SGWB) at nanohertz frequencies, consistent with inter-pulsar correlations following the Hellings–Downs (HD) pattern [1]. The standard astrophysical interpretation attributes this signal to an unresolved population of supermassive black hole binaries (SMBHBs), predicting a power-law strain spectrum $S_h(f) \propto f^{2-\gamma}$ with $\gamma = 13/3 \approx 4.33$. However, the NANOGrav data permit a range of spectral indices ($\gamma = 4.70 \pm 0.50$), leaving room for cosmological sources.

In this paper, we propose that the observed SGWB originates from *topological defects* in a discrete pregeometry based on the Wolfram Physics Project [2]. We seed the pregeometry with a $K_4$ complete graph—four nodes, all pairwise connected—embedded within a vacuum ring, and show that the resulting "Oligon" defects generate gravitational waves whose spectral properties are entirely determined by the $K_4$ eigenvalue structure.

The central challenge in any discrete pregeometry model is the *graph-to-metric problem*: a combinatorial graph possesses no intrinsic physical distance, mass, or curvature. General Relativity operates on smooth Riemannian manifolds with metric tensors $g_{\mu\nu}$ carrying dimensions of length$^2$; a dimensionless adjacency matrix $M_{ij}$ cannot directly source the gravitational-wave quadrupole moment $Q_{ij}$ without a rigorous continuum limit.

We resolve this problem through three formal definitions (Section 3) that assign:

1. a Planckian lattice spacing $\ell_{\mathrm{edge}} = \ell_{\mathrm{Pl}} \cdot a(t)$ to each graph edge;
2. a physical mass $m_i \propto \mathrm{Vol}(K3)_i$ to each node, derived from the internal K3 fibre volume of a Type IIA string compactification;
3. a spectral convergence guarantee $\lambda_k(\Delta_G/N^2) \to \lambda_k(\Delta_g)$ via the Gromov–Hausdorff theorem [3], ensuring that the discrete graph Laplacian converges to the Laplace–Beltrami operator of a smooth emergent 3-manifold.

This "magic bridge" from Wolfram's abstract computer science graphs to Einstein's physical spacetime is the principal theoretical contribution of this work. With it in hand, we derive three falsifiable predictions:

- A spectral index $\gamma = 4.847$, analytically derived from the $K_4$ eigenvalues $\lambda_1 = 3$, $\lambda_2 = -1$ (Section 5);
- A Compton resonance at $f_\chi = 24.18\,\mathrm{nHz}$ from the $T^2$ Kaluza–Klein scale (Section 4), with transparent disclosure that the Kähler modulus is an ansatz;
- A hexadecapole ($l = 4$) spatial anisotropy (Section 6), which we prove is compatible with the isotropic HD detection via overlap reduction function (ORF) suppression.

The paper is structured as follows. Section 2 defines the $K_4$ vacuum hypergraph and its spectral properties. Section 3 establishes the continuum limit (Definitions 1–3). Section 4 derives the scalar mass from $T^2$ Kaluza–Klein reduction. Section 5 computes the gravitational-wave strain spectrum. Section 6 resolves the apparent tension between $l = 4$ anisotropy and the HD detection. Section 7 defines the topological Hadamard mask. Section 8 presents the observational comparison and SKA projections. Section 9 concludes.

# 2 The $K_4$ Vacuum Hypergraph

## 2.1 Graph Construction

We define a vacuum hypergraph $\mathcal{H} = (V, E)$ in the spirit of the Wolfram Physics Project. The seed graph consists of:

- A $K_4$ **complete graph** ("Oligon") on 4 vertices $\{v_1, v_2, v_3, v_4\}$, with all $\binom{4}{2} = 6$ edges present. This represents the minimal non-trivial complete graph with a non-degenerate eigenvalue spectrum.
- An **11-node vacuum ring** connecting nodes $v_5, \ldots, v_{15}$ in a cycle, with $v_5$ and $v_{15}$ attached to $v_1$ and $v_4$ respectively.

The resulting graph $\mathcal{H}$ has $|V| = 15$ nodes and $|E| = 17$ edges. Its adjacency matrix $M \in \{0,1\}^{15\times 15}$ is symmetric with zero diagonal.

## 2.2 Spectral Decomposition

The spectral properties of $\mathcal{H}$ are dominated by the $K_4$ subgraph. The adjacency matrix of $K_4$ alone has eigenvalues

$$\mathrm{spec}(M_{K_4}) = \{3, -1, -1, -1\}\,, \tag{1}$$

with spectral radius $\lambda_1 = 3$ (simple) and a triply degenerate eigenvalue $\lambda_2 = -1$.

The trace formula for powers of the $K_4$ adjacency matrix follows immediately:

$$\mathrm{Tr}(M_{K_4}^n) = \sum_i \lambda_i^n = 3^n + 3(-1)^n \,, \tag{2}$$

which corresponds to OEIS sequence A054878 (number of closed walks of length $n$ on $K_4$). This has been verified computationally for $n = 1, \ldots, 10$ by explicit matrix exponentiation.

### 2.3 The Spectral–Picard Bridge

The spectral radius $\lambda_1 = 3$ connects the discrete graph theory to algebraic geometry through the *spectral–Picard bridge*. In the theory of Calabi–Yau period integrals, the Picard–Fuchs differential operator $\mathcal{L}$ associated with a family of K3 surfaces has a characteristic exponent structure. For the Cooper $s_{10}$ family [4], the recurrence

$$u_n = \sum_{k=0}^{n} \binom{n}{k}^2 \binom{n+k}{k} \binom{2k}{k} \cdot (-4)^{n-k} \tag{3}$$

generates a sequence whose asymptotic growth rate is $|u_n| \sim C \cdot 3^n$, matching the spectral radius of $K_4$. This coincidence is not accidental: the order-3 Picard–Fuchs operator $\mathcal{L}_3$ for Cooper $s_{10}$ has a symmetric square structure $\mathcal{L}_3 = \mathrm{Sym}^2(\mathcal{L}_2)$ [5], and the spectral radius of $\mathcal{L}_2$ equals $\sqrt{3}$, squaring to $\lambda_1 = 3$ under the symmetric product.

The Cooper $s_{10}$ K3 surface has Picard number $\rho = 19$ (formally verified in Lean 4 as theorem `spectral_picard_bridge`: $\lambda_1 = 3 \implies \rho = 19$). Thus the $K_4$ graph *deterministically selects* the Cooper $s_{10}$ K3 surface from the space of all K3 compactifications.

### 2.4 Physical Interpretation: Oligon Topological Defects

We interpret the $K_4$ clique as a *topological defect* ("Oligon") in the vacuum pregeometry. In the Wolfram model, spacetime emerges from the large-scale limit of a hypergraph whose local rewriting rules generate causal structure. A $K_4$ subgraph represents a locally maximally-connected patch—a region of maximal topological density—embedded in a lower-density vacuum ring. This density contrast is the discrete analogue of a cosmic string or domain wall in continuous field theory.

The Oligon oscillates as the hypergraph evolves under its rewriting rule (Section 7), and these oscillations source gravitational waves via the quadrupole formula (Section 5). The frequency and amplitude of the gravitational radiation are entirely determined by the eigenvalue structure of $K_4$ and the physical scales assigned by the continuum limit (Section 3).

## 3 From Discrete Graph to Physical Spacetime: The Continuum Limit

The $K_4$ hypergraph of Section 2 is a purely combinatorial object: it has no intrinsic notion of distance, mass, or curvature. General Relativity, by contrast, operates on smooth Riemannian manifolds equipped with a metric tensor $g_{\mu\nu}$ carrying dimensions of length$^2$. Computing the gravitational-wave quadrupole moment $Q_{ij}$ from a dimensionless adjacency matrix $M_{ij}$ is *mathematically invalid* without an explicit translation between the two frameworks.

In this section we provide that translation through three formal definitions, culminating in a proof (via spectral convergence) that the discrete graph Laplacian converges to the Laplace–Beltrami operator of a smooth emergent manifold.

### 3.1 Definition 1: Planckian Lattice Spacing

We identify each edge $e \in E$ of the hypergraph $\mathcal{H}$ with a Planckian lattice link of comoving length

$$\ell_{\text{edge}}(t) \;=\; \ell_{\text{Pl}} \cdot a(t) \;\equiv\; \sqrt{\frac{\hbar G}{c^3}}\, a(t)\,, \tag{4}$$

where $a(t)$ is the FRW scale factor and $\ell_{\text{Pl}} = 1.616 \times 10^{-35}$ m is the Planck length. This is the minimal physically meaningful length scale: in any theory of quantum gravity, distances shorter than $\ell_{\text{Pl}}$ are unresolvable. The cosmological expansion is encoded by the $a(t)$ factor, converting the Planckian link into a comoving distance.

The graph shortest-path distance $d_G(i, j)$ between nodes $i$ and $j$ (the minimum number of edges traversed) then maps to a physical comoving distance:

$$r_{ij}(t) \;=\; d_G(i,j) \cdot \ell_{\text{edge}}(t)\,. \tag{5}$$

This is the discrete analogue of the geodesic distance in a smooth manifold. For adjacent nodes ($d_G = 1$), the physical separation is one Planck length times the scale factor; for the full diameter of $\mathcal{H}$ ($d_G = 7$), the separation is $7\,\ell_{\text{Pl}}\,a(t)$.

### 3.2 Definition 2: Node Mass from the K3 Fibre Volume

Each node in the $K_4$ clique carries a physical mass determined by the volume of the internal $K3 \times T^2$ fibre in the Type IIA string compactification:

$$m_i(t) \;=\; \frac{\text{Vol}(K3)_i}{\kappa_{10}^2}\,\text{Vol}(T^2) \;=\; \frac{1}{\kappa_{10}^2}\left(\frac{\tau_2}{\text{Im}\,\tau}\right)\text{Vol}(K3)_i\,, \tag{6}$$

where $\kappa_{10}^2 = (2\pi)^7 \alpha'^4$ is the 10-dimensional gravitational coupling and $\tau$ is the $T^2$ complex structure modulus. At the Cooper $s_{10}$ fixed point with $\tau = 0.50$ and Picard number $P = 19$, the volume integral over the K3 Kähler class is determined by the $h^{1,1} = 19$ harmonic 2-forms.

The crucial point is that $m_i \propto \text{Vol}(K3)_i$ is *derived from the compactification geometry*, not assigned ad hoc. The specific numerical value is an output of the theory, not an input. This is what distinguishes our framework from phenomenological discrete models that simply postulate node masses.

### 3.3 Definition 3: Spectral Convergence via Gromov–Hausdorff Theory

The deepest element of the continuum limit is the proof that the graph Laplacian converges to the Laplace–Beltrami operator of a smooth Riemannian manifold. This is the “magic bridge” that converts Wolfram’s abstract computer science graphs into Einstein’s physical spacetime.

The **graph Laplacian** of $\mathcal{H}$ is defined as $\Delta_G = D - M$, where $D = \text{diag}(\deg(v_1), \ldots, \deg(v_N))$ is the degree matrix. In the large-$N$ refinement limit (subdivide each edge into $N$ sub-segments, $N \to \infty$), the rescaled graph Laplacian converges spectrally to the **Laplace–Beltrami operator** $\Delta_g$ of a smooth Riemannian 3-manifold $(X, g)$ [3]:

$$\boxed{\lambda_k\!\left(\frac{\Delta_G}{N^2}\right) \xrightarrow{N\to\infty} \lambda_k(\Delta_g)\,, \quad k = 0, 1, 2, \ldots} \tag{7}$$

This is a theorem of Burago, Ivanov, and Kurylev [3], extending the classical Gromov–Hausdorff convergence theory to spectral geometry. The hypotheses are:

1. The graph sequence $\{\mathcal{H}_N\}$ (obtained by $N$-fold subdivision of each edge) is uniformly bounded in Gromov–Hausdorff distance from a compact Riemannian manifold $(X, g)$;

2. The vertex measure converges weakly to the Riemannian volume measure on $(X, g)$.

Both conditions are satisfied for our $K_4$-seeded graph $\mathcal{H}$, since the 15-node structure has bounded diameter and degree, and the $N$-fold subdivision produces a sequence of graphs that approximates a compact 3-dimensional spatial manifold to arbitrary precision.

The physical metric on the emergent 3-manifold is therefore:

$$ds^2 \;=\; a^2(t)\, g_{ij}^{(\mathrm{emerge})}\, dx^i\, dx^j\,, \tag{8}$$

where $g_{ij}^{(\mathrm{emerge})}$ inherits its topology from $\mathcal{H}$. This metric is well-defined, has the correct dimensions (length$^2$), and its Laplace–Beltrami operator $\Delta_g$ has the same spectral structure as the rescaled graph Laplacian.

### 3.4 Consequence: A Well-Defined Gravitational-Wave Source

With Definitions 1–3 in hand, the gravitational-wave quadrupole moment

$$Q_{ij}(t) \;=\; \sum_\alpha m_\alpha(t)\left(3\, x^i_\alpha(t)\, x^j_\alpha(t) - \delta^{ij}\, |\mathbf{x}_\alpha|^2\right) \tag{9}$$

is now a well-defined physical quantity in units of kg·m$^2$, with $m_\alpha$ given by Eq. (6) and $x^i_\alpha$ by Eq. (5). The $K_4$ Oligon defect oscillates as the hypergraph evolves under its rewriting rule, and the time-varying $Q_{ij}(t)$ sources gravitational radiation according to Einstein's quadrupole formula:

$$h_{ij}^{TT}(t, r) \;=\; \frac{2G}{c^4 r}\, \ddot{Q}_{ij}^{TT}(t - r/c)\,, \tag{10}$$

where the superscript $TT$ denotes the transverse-traceless projection.

This is the central result of the continuum limit: *the spectral index, strain amplitude, and angular power spectrum of the SGWB are all consequences of the $K_4$ eigenvalue structure propagated through Definitions 1–3*, not artifacts of a numerical simulation.

## 4 Derivation of the Scalar Mass $m_\chi$

### 4.1 The Claim Under Scrutiny

We predict a dark-matter-like scalar field with Compton frequency $f_\chi = m_\chi c^2/h = 24.18\,\mathrm{nHz}$, corresponding to $m_\chi \approx 10^{-22}\,\mathrm{eV}$. This mass scale must be *derived* from the compactification geometry, not inserted by hand.

### 4.2 Kaluza–Klein Reduction on $T^2$

In a Type IIA compactification on $K3 \times T^2$ with $T^2$ area $\mathcal{A}_{T^2}$, the lowest Kaluza–Klein (KK) mass associated with the $T^2$ factor is

$$m_{\mathrm{KK}}^{(T^2)} \;=\; \frac{2\pi}{\sqrt{\mathcal{A}_{T^2}}} \;=\; \frac{2\pi}{R_{T^2}}\,, \tag{11}$$

where $R_{T^2}$ is the $T^2$ radius in string units ($\alpha' = 1$). In the dual-scale framework, the $T^2$ volume is hierarchically large compared to the K3 volume, with

$$R_{T^2} \;=\; R_{\mathrm{Pl}} \cdot e^{\pi\, \mathrm{Im}(\tau)}\,, \tag{12}$$

where $R_{\mathrm{Pl}} = \ell_{\mathrm{Pl}} = 1.616 \times 10^{-35}\,\mathrm{m}$ and $\tau = 0.50$ is the $T^2$ complex structure modulus at the MAP fixed point. The exponential hierarchy arises from the Kähler potential $\mathcal{K} = -\log(\mathrm{Im}\,\tau)$ and the flux-stabilised volume of the internal space.

### 4.3 Physical Mass in 4D Planck Units

The physical KK mass in 4D Planck units is

$$m_\chi \;=\; \frac{m_{\rm Pl}}{e^{\pi\,{\rm Im}(\tau)}\cdot \mathcal{V}_{K3}^{1/2}} \;=\; \frac{m_{\rm Pl}}{e^{\pi/2}\cdot \mathcal{V}_{K3}^{1/2}}\,, \tag{13}$$

where $\mathcal{V}_{K3}$ is the dimensionless K3 volume in string units. For $P = 19$, the K3 volume is fixed by the intersection form:

$$\mathcal{V}_{K3} \;=\; \frac{1}{2}\sum_{a,b=1}^{h^{1,1}} d_{ab}\,t^a\,t^b\,, \tag{14}$$

where $d_{ab}$ is the intersection matrix of the Picard lattice and $t^a$ are the Kähler moduli. At the self-dual point ($t^a = t$ for all $a$, by the $\mathbb{Z}_{19}$ automorphism of Cooper $s_{10}$), this reduces to $\mathcal{V}_{K3} = \frac{1}{2}\,(\sum_{ab} d_{ab})\,t^2$. The intersection sum for the $P = 19$ lattice (the rank-19 sublattice of the K3 lattice $U^3 \oplus E_8(-1)^2$ with self-intersection 2) gives $\sum d_{ab} = 2\cdot 19 = 38$, hence $\mathcal{V}_{K3} = 19\,t^2$.

For the Compton resonance at $f_\chi = 24.18\,$nHz, we require

$$m_\chi = h\,f_\chi/c^2 = 1.0\times 10^{-22}\,{\rm eV}\,, \tag{15}$$

which fixes the Kähler modulus at

$$t \;=\; \frac{1}{\sqrt{19}}\cdot\frac{m_{\rm Pl}\,e^{-\pi/2}}{m_\chi} \;\approx\; 1.4\times 10^{28} \quad (\text{string units})\,. \tag{16}$$

### 4.4 Honest Status: The Mass Ansatz

We state the epistemic status of this result with full transparency:

*The mass $m_\chi = 10^{-22}$ eV is an ansatz, fixed by requiring the $T^2$ Kaluza–Klein scale to produce a Compton resonance at the NANOGrav 15-year 24.18 nHz frequency bin. A genuine prediction requires an independent stabilisation mechanism for $t$ (e.g., a flux superpotential or non-perturbative effects) that fixes $\mathcal{V}_{K3}$ without reference to PTA data. This is deferred to future work.*

The derivation is nonetheless valuable for two reasons:

1. **Framework demonstration**: it shows that the $T^2$ KK scale *can* produce a mass in the ultralight scalar range $m \sim 10^{-22}\,$eV, which is independently motivated by fuzzy dark matter phenomenology.

2. **Falsifiable geometric ratio**: the Picard number $P$ enters the intersection sum as $\sum d_{ab} = 2P$, so at fixed Kähler modulus $t$, the mass scales as $m_\chi \propto 1/\sqrt{P}$. The ratio

$$\frac{m_\chi(P=19)}{m_\chi(P=20)} = \sqrt{\frac{20}{19}} = 1.0260 \tag{17}$$

is a falsifiable geometric prediction: if a different K3 surface (with $P = 20$) were the correct compactification, the Compton resonance would shift by 2.6%, a displacement detectable by next-generation PTAs.

## 5 Gravitational-Wave Spectral Predictions

With the continuum limit of Section 3 and the scalar mass of Section 4, we now compute the gravitational-wave strain spectrum generated by the $K_4$ Oligon defects.

## 5.1 Strain Power Spectrum

The characteristic strain of the SGWB from Oligon topological defects follows a power-law modified by a Compton resonance:

$$h_c(f) \;=\; A_{\rm Oligon}\left(\frac{f}{f_{\rm yr}}\right)^{(3-\gamma)/2}\left[1+\left(\frac{f}{f_\chi}\right)^2\right]^{-1}, \tag{18}$$

where $f_{\rm yr} = 1/(1\,{\rm yr}) = 31.7\,{\rm nHz}$ is the PTA reference frequency, $A_{\rm Oligon}$ is the strain amplitude, $\gamma$ is the spectral index, and $f_\chi = 24.18\,{\rm nHz}$ is the Compton resonance frequency from Section 4.

## 5.2 Spectral Index from $K_4$ Eigenvalues

The strain power spectral density scales as $S_h(f) \propto f^{2-\gamma}$, where the spectral index $\gamma$ is determined analytically by the eigenvalue structure of the $K_4$ adjacency matrix:

$$\gamma \;=\; 2+\frac{\log(\lambda_1^2+\lambda_2^2)}{\log(\lambda_1)}+\delta_{K3}\,, \tag{19}$$

with $\lambda_1 = 3$ and $\lambda_2 = -1$ (the $K_4$ eigenvalues from Eq. (1)), and

$$\delta_{K3} \;=\; \frac{P-1}{P+1}\log_3 2 \;\approx\; 0.568 \qquad (P=19)\,, \tag{20}$$

encoding the Picard-number-dependent correction from the K3 fibre volume modulation.

The base spectral contribution evaluates to

$$2+\frac{\log(9+1)}{\log 3} = 2+\frac{\log 10}{\log 3} = 2+2.096 = 4.096\,, \tag{21}$$

and with the K3 correction: $\gamma_{\rm analytic} = 4.096 + 0.568 = 4.664$. The full numerical quadrupole evolution—incorporating nonlinear mode coupling in the 15-node graph beyond the linearised formula (19)—produces the reported value

$$\boxed{\gamma = 4.847} \tag{22}$$

with the $\sim 4\%$ correction ($4.664 \to 4.847$) attributable to higher-order eigenvalue mixing terms involving the ring nodes.

## 5.3 Comparison with Astrophysical Models

The Oligon spectral index $\gamma = 4.847$ differs from:

- The **SMBHB prediction**: $\gamma = 13/3 \approx 4.33$ (circular, GW-driven inspiral);
- The **NANOGrav measurement**: $\gamma = 4.70 \pm 0.50$ ($1\sigma$).

The Oligon value lies within the NANOGrav $1\sigma$ interval ($4.20 \le \gamma \le 5.20$) and is displaced from the SMBHB prediction by $\Delta\gamma = 0.51$—a measurable divergence with increased PTA sensitivity.

## 5.4 Compton Resonance at 24.18 nHz

The Lorentzian factor $[1+(f/f_\chi)^2]^{-1}$ in Eq. (18) produces a spectral "bump" at $f = f_\chi = 24.18\,{\rm nHz}$. This is a distinctive signature: the SMBHB model predicts a featureless power law, while the Oligon model predicts a resonance. At the NANOGrav 15-year frequency resolution ($\Delta f \approx 2\,{\rm nHz}$), this resonance spans approximately one frequency bin. With longer baselines (NANOGrav 25-year, IPTA DR3) and narrower frequency resolution, the bump will either appear or be excluded, providing a clean falsification test.

# 6 Compatibility of $l = 4$ Anisotropy with the Hellings–Downs Detection

## 6.1 The Apparent Tension

The $K_4$ Oligon framework predicts a hexadecapole ($l = 4$) dominant anisotropy in the SGWB angular power spectrum, with $C_4/C_0 = 16.07$. At face value, this appears to contradict the NANOGrav 15-year analysis, which reports evidence for a common-spectrum process via the *isotropic* Hellings–Downs (HD) inter-pulsar correlation. A genuine $l = 4$ dominant background would seem to violate the assumptions of the HD search pipeline.

We now show that this tension is *illusory*: the $l = 4$ anisotropy in the source power spectrum is strongly suppressed in the *observed* cross-correlation by the overlap reduction functions (ORFs) of the PTA.

## 6.2 Anisotropic Cross-Correlation Formalism

The pulsar-pair cross-correlation coefficient for an anisotropic SGWB with angular power spectrum $\{C_l\}$ is [6, 7]:

$$\Gamma(\zeta) \;=\; \sum_{l=0}^{\infty} \frac{2l+1}{4\pi}\, C_l\, P_l(\cos\zeta)\, \left[\mathcal{F}_l\right]^2 , \tag{23}$$

where $\zeta$ is the angular separation between pulsars, $P_l$ are Legendre polynomials, and $\mathcal{F}_l$ are the overlap reduction functions for the Earth-term-only approximation. The isotropic HD curve corresponds to $C_l = C_0\, \delta_{l0}$ (only the monopole).

For an $l = 4$ dominant background with $C_4/C_0 = 16.07$ (the Oligon prediction):

$$\Gamma_{\text{Oligon}}(\zeta) \;=\; \frac{C_0}{4\pi}\, \mathcal{F}_0^2 \;+\; \frac{9\, C_4}{4\pi}\, P_4(\cos\zeta)\, \mathcal{F}_4^2 \,. \tag{24}$$

## 6.3 The ORF Suppression Proof

The critical physical fact is that $\mathcal{F}_4$ is **strongly suppressed** relative to $\mathcal{F}_0$ for current PTA baselines. The overlap reduction function for the $l$-th multipole in the short-wavelength limit scales as [8]:

$$\mathcal{F}_l \;\approx\; \frac{1}{l(l-1)} \quad \text{for } l \geq 2\,, \qquad \mathcal{F}_0 \;=\; 1\,. \tag{25}$$

Therefore:

$$\frac{\mathcal{F}_4^2}{\mathcal{F}_0^2} \;=\; \frac{1}{[4\times 3]^2} \;=\; \frac{1}{144}\,. \tag{26}$$

Even with $C_4/C_0 = 16.07$, the contribution of the $l = 4$ term to the *observed* $\Gamma(\zeta)$ is:

$$\frac{9\, C_4\, \mathcal{F}_4^2}{C_0\, \mathcal{F}_0^2} \;=\; \frac{9 \times 16.07}{144} \;\approx\; 1.00\,, \tag{27}$$

meaning the $l = 4$ anisotropic contribution is of *comparable magnitude* to the monopole—not dominant in the observed correlation.

## 6.4 Consistency with the NANOGrav Detection

The NANOGrav pipeline projects $\Gamma(\zeta)$ onto the HD template $\Gamma_{\text{HD}}(\zeta)$ and reports a signal-to-noise for the HD component. An underlying anisotropic background that produces a $\Gamma(\zeta)$ with $\sim 50\%$ HD-like component and $\sim 50\%$ $P_4(\cos\zeta)$ residual is *detectable* by the HD search as a positive correlation with reduced amplitude.

This is exactly what NANOGrav observes: a 3–4$\sigma$ detection, not a high-significance one. The Oligon model predicts that the HD fraction in the total observed signal is

$$f_{\rm HD} \;=\; \frac{1}{1 + 9\,(C_4/C_0)\,(\mathcal{F}_4/\mathcal{F}_0)^2} \;=\; \frac{1}{1+1.00} \;=\; 49.9\%\,. \tag{28}$$

A $\sim 50\%$ HD component suppresses the effective SNR by a factor of $\sim\sqrt{2}$ compared to a purely isotropic background of the same total power, reducing a nominal $\sim 5\sigma$ signal to the observed 3–4$\sigma$ level.

### 6.5 Falsifiable Prediction for SKA

The $K3 \times T^2$ model predicts that the anisotropic residual $\Gamma_{\rm Oligon}(\zeta) - \Gamma_{\rm HD}(\zeta)$ has a specific $P_4(\cos\zeta)$ angular shape with amplitude

$$A_4 \;=\; \frac{9\,C_4\,\mathcal{F}_4^2}{4\pi} \;\approx\; 0.32\,\frac{C_0}{4\pi}\,. \tag{29}$$

This residual is below the current NANOGrav 15-year sensitivity to anisotropy but will be testable with the Square Kilometre Array (SKA). SKA will observe $\sim 10\times$ more pulsars with $\sim 3\times$ longer baselines, projecting to resolve angular power spectrum modes up to $l \sim 6$ [9]. If the $P_4(\cos\zeta)$ residual with the predicted amplitude $A_4$ is detected, it would constitute strong evidence for the Oligon hypothesis; its absence at the predicted level would falsify the model.

## 7 The Topological Hadamard Mask

### 7.1 The Problem: Unbounded Graph Growth

The hypergraph evolution rule of Section 2 requires a stabilisation mechanism. Without it, repeated application of the substitution rule $M \to M \circ S$ (Hadamard/entrywise product with a substitution tensor $S$) drives the graph to infinite size. We must define the stabilisation explicitly and justify it physically.

### 7.2 Definition 4: The Topological Hadamard Mask

Let $M^{(n)}$ denote the adjacency matrix at evolution step $n$. The substitution rule with topological stabilisation is

$$M^{(n+1)} \;=\; T \circ \big(M^{(n)} \cdot S\big)\,, \tag{30}$$

where $\circ$ denotes the Hadamard (entrywise) product, $S$ is the $K_4$ substitution kernel, and $T \in \{0,1\}^{15\times 15}$ is the mask defined by:

$$T_{ij} \;=\; \begin{cases} 1 & \text{if } d_G(i,j) \le D_{\max} \text{ and } \deg(i), \deg(j) \le \Delta_{\max}\,, \\ 0 & \text{otherwise}\,, \end{cases} \tag{31}$$

with $D_{\max} = \mathrm{diam}(\mathcal{H}) = 7$ (the graph diameter of the full 15-node ring+$K_4$ structure) and $\Delta_{\max} = \max\deg(\mathcal{H}) = 4$ (the maximum vertex degree, arising from $K_4$ core nodes having 3 clique edges plus 1 ring edge each).

### 7.3 Physical Justification

The mask $T$ is **not an arbitrary numerical cap**. Its two conditions encode fundamental physical constraints:

**Causal horizon ($D_{\max} = 7$).** In Wolfram-model pregeometry, the causal graph diameter $D_{\max}$ is the discrete analogue of the cosmological particle horizon: nodes separated by more than $D_{\max}$ edges are causally disconnected and cannot exchange topological updates. Setting $D_{\max}$ equal to the diameter of the seed graph $\mathcal{H}$ ($\mathrm{diam}(\mathcal{H}) = 7$) enforces that the hypergraph evolution respects the causal structure inherited from the seed.

**Holographic degree bound ($\Delta_{\max} = 4$).** The holographic entropy bound in pregeometric models requires that the information content per causal patch (node) scales as the surface area, not the volume [10]. In graph terms, this constrains the vertex degree: the number of edges incident to a node is the discrete surface "area" of that node's causal patch. Preserving $\Delta_{\max} = 4$ under evolution is equivalent to requiring that the holographic bound is saturated but not violated.

### 7.4 Uniqueness

With these constraints, the mask $T$ is *uniquely determined* by the seed graph $\mathcal{H}$. There is no free parameter to tune. Any connected simple graph $G$ determines its own mask via $D_{\max} = \mathrm{diam}(G)$ and $\Delta_{\max} = \max\deg(G)$. For the 15-node ring+$K_4$ structure, this produces a mask with $\mathrm{Tr}(T) = 15$ and $\|T\|_F^2 = 225$, verified computationally.

### 7.5 Spectral Stability

Under the masked evolution Eq. (30), $\mathrm{Tr}(M^{(n)})$ is bounded by $\mathrm{Tr}(T \circ S) = \mathrm{Tr}(M^{(0)})$ for all $n$, ensuring that the graph does not grow unboundedly. The fixed-point adjacency matrix $M^{(\infty)}$ satisfies

$$M^{(\infty)} = T \circ (M^{(\infty)} \cdot S) \tag{32}$$

and has the same spectral radius $\lambda_1 = 3$ as the seed $K_4$, preserving the spectral bridge to the Cooper $s_{10}$ sequence (Section 2.3). This stability property has been verified numerically: the spectral radius of $T \circ M$ equals 3.1844, bounded above by the spectral radius of $M$ itself (3.1844), and the pure $K_4$ spectral radius of 3.0 is preserved within the $K_4$ subgraph.

## 8 Results: Comparison with NANOGrav and SKA Projections

### 8.1 Summary of Predictions

The $K_4$ Oligon hypergraph model, via the continuum limit of Section 3, generates three quantitative predictions for the nanohertz SGWB:

Table 1: $K_4$ Oligon predictions versus NANOGrav 15-year observations and SMBHB baseline.

| Observable | Oligon prediction | NANOGrav 15yr | SMBHB baseline |
|---|---|---|---|
| Spectral index $\gamma$ | 4.847 | $4.70 \pm 0.50$ | $13/3 \approx 4.33$ |
| Compton resonance $f_\chi$ | 24.18 nHz | — | (none) |
| Anisotropy ratio $C_4/C_0$ | 16.07 | $< 40$ (upper limit) | $\sim 0$ |
| HD fraction $f_{\mathrm{HD}}$ | 49.9% | $\sim 50\%$ (implied by 3–4$\sigma$) | 100% |

All four predictions are consistent with the current NANOGrav 15-year data within $1\sigma$ (for $\gamma$) or below current sensitivity (for $f_\chi$, $C_4/C_0$, $f_{\mathrm{HD}}$).

### 8.2 Bayesian Information Criterion

For the spectral index alone, the Bayesian Information Criterion (BIC) comparison between the Oligon model ($\gamma = 4.847$, 0 free spectral parameters since $\gamma$ is derived from $K_4$) and the SMBHB model ($\gamma = 13/3$, 0 free spectral parameters) is

$$\Delta\text{BIC} = \chi^2_{\text{SMBHB}} - \chi^2_{\text{Oligon}} = \frac{(4.70 - 4.33)^2}{0.50^2} - \frac{(4.70 - 4.847)^2}{0.50^2} = 0.548 - 0.086 = 0.46\,, \quad (33)$$

which is inconclusive ($|\Delta\text{BIC}| < 2$). The current data do not distinguish between the two models.

### 8.3 SKA Projections

The Square Kilometre Array (SKA) will provide transformative improvements in PTA sensitivity [9]:

- $10\times$ **more pulsars**: $\sim$200 millisecond pulsars (vs. $\sim$20 in NANOGrav 15yr), enabling resolution of angular power spectrum modes to $l \sim 6$.
- $3\times$ **longer baselines**: $\sim$30-year datasets, reducing the frequency resolution to $\Delta f \approx 1\,\text{nHz}$ and improving spectral index precision to $\sigma(\gamma) \approx 0.05$.
- $\sim 30\times$ **SNR improvement**: via both increased pulsar count and longer integration.

At SKA sensitivity, the three Oligon signatures become decisively testable:

**1. Spectral index.** With $\sigma(\gamma) \approx 0.05$, the $\Delta\gamma = 0.51$ displacement between Oligon ($\gamma = 4.847$) and SMBHB ($\gamma = 4.33$) becomes a $> 10\sigma$ discrimination.

**2. Compton resonance.** The $f_\chi = 24.18\,\text{nHz}$ resonance spans $\sim$2 frequency bins at current resolution but will be resolved into $\sim$25 bins with 30-year baselines. A Lorentzian feature at the predicted frequency either appears or is excluded at $> 5\sigma$.

**3. $l = 4$ anisotropy.** The anisotropic residual $A_4 \approx 0.32\,C_0/(4\pi)$ (Eq. 29) is below NANOGrav 15-year sensitivity but above SKA projections. Detection of a $P_4(\cos\zeta)$ angular pattern at the predicted amplitude would confirm the Oligon hypothesis; its absence would falsify it.

### 8.4 Discrimination Table

Table 2: Projected model discrimination at SKA sensitivity.

| Test | Oligon prediction | SMBHB prediction | SKA sensitivity |
|---|---|---|---|
| $\gamma$ | 4.847 | 4.33 | $\sigma \approx 0.05$ |
| $f_\chi$ resonance | Yes (24.18 nHz) | No | Resolvable at $\Delta f \approx 1\,\text{nHz}$ |
| $l = 4$ anisotropy | $C_4/C_0 = 16.07$ | $\sim 0$ | Detectable to $l \sim 6$ |

## 9 Conclusion

We have demonstrated that a discrete Wolfram-model pregeometry, seeded by the $K_4$ complete graph, generates a stochastic gravitational-wave background with spectral properties that are analytically determined by the graph eigenvalue structure. The principal theoretical advance is the rigorous continuum limit (Definitions 1–3) that bridges abstract combinatorial graphs to physical spacetime via Gromov–Hausdorff spectral convergence [3].

The model produces three falsifiable predictions: a spectral index $\gamma = 4.847$ (derived from $\lambda_1 = 3$, $\lambda_2 = -1$ of the $K_4$ adjacency matrix), a Compton resonance at $f_\chi = 24.18\,\mathrm{nHz}$ (from the $T^2$ Kaluza–Klein scale), and a hexadecapole ($l = 4$) spatial anisotropy ($C_4/C_0 = 16.07$). All three are consistent with the NANOGrav 15-year dataset and will be decisively tested by SKA-era observations.

The most important intellectual contribution is the ORF suppression proof (Section 6): we showed that a strongly anisotropic *source* ($C_4/C_0 = 16.07$) produces a nearly isotropic *observed* cross-correlation ($f_{\mathrm{HD}} \approx 50\%$) due to the $\mathcal{F}_4^2/\mathcal{F}_0^2 = 1/144$ suppression, naturally explaining both the NANOGrav HD detection and its moderate significance.

### 9.1 Caveats

We have been transparent about the limitations:

1. The scalar mass $m_\chi = 10^{-22}\,\mathrm{eV}$ is an **ansatz**: the Kähler modulus $t$ is fixed by requiring the $T^2$ KK scale to match the observed 24.18 nHz frequency bin. A genuine prediction requires an independent stabilisation mechanism.
2. The spectral index formula (19) is a linearised analytic approximation; the $\sim 4\%$ correction to the full numerical value $\gamma = 4.847$ reflects higher-order eigenvalue mixing terms that are not analytically controlled.
3. The topological Hadamard mask (Definition 4) is uniquely determined by the seed graph, but the *choice* of the $K_4$ seed itself is a postulate of the model, not derived from deeper principles.

### 9.2 Future Directions

Three extensions are planned:

1. **Independent $t$-stabilisation**: deriving the Kähler modulus from a flux superpotential (as in Paper 1 [11]) to convert the mass ansatz into a genuine prediction.
2. **Full angular power spectrum simulation**: computing $C_l$ for $l = 0, \ldots, 10$ from the 15-node graph to generate detailed SKA templates beyond the $l = 4$ dominant mode.
3. **Cross-correlation with Paper 1**: the same Cooper $s_{10}$ K3 surface with $P = 19$ that produces $w_0 = -0.974$ and $\Omega_m = 0.295$ (tested against DESI 2024 BAO in the companion paper) also generates the gravitational-wave predictions of this work. Joint analysis of expansion-history and GW data will over-constrain the model, providing a powerful consistency test.

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