Validation of the OpenAI Yang–Mills construction

Based on Lean code submitted October 28, 2026

Summary of results and independent validation

Yang–Mills group Nov 10, 2026 release

Initial assessment. This validation covers the first OpenAI Lean submission which formalizes the continuum pure Yang–Mills theory. In our judgment the submission proves that a relativistic quantum field theory satisfying the Wightman axioms can be constructed as a scaling limit of four-dimensional Wilson lattice gauge theory for any simple compact gauge group. It does not prove that the infinite-volume limit is unique or that there is a mass gap.

The proof constructs continuum correlation functions as a scaling limit of four-dimensional Wilson lattice gauge theory correlation functions at a fixed physical volume. Subsequential convergence to an infinite-volume limit is then proven using compactness, and this limit is not proven to be unique. Any chosen subsequential limit admits a decomposition into component theories (pure phases), each proven to satisfy the Osterwalder–Schrader axioms. Then, through the OS reconstruction theorem (also proven in the submission) this defines a QFT satisfying the Wightman axioms. Its lattice origin, local fields, normalized stress tensor, leading short-distance laws and operator product expansion provide substantial evidence that the construction describes Yang–Mills theory. We believe that the failure to prove uniqueness of the large volume limit and of the vacuum is not inconsistent with the general expectation that the infinite-volume limit and the true Yang–Mills vacuum are unique.

Part I of this assessment compares the theorem statements of OpenAI's Yang–Mills theory construction to the desired properties of the construction including those discussed in Jaffe–Witten's statement of the Yang–Mills Millennium Prize Problem, while Part II directly examines and checks the Lean code. OpenAI's construction does not certify the full Jaffe–Witten problem: no mass gap is claimed. Uniqueness across volume subsequences and agreement of the component theories are also unresolved. Under the existence criterion adopted here, we are not requiring a precise characterization of the vacuum state.

Validation. A clean rebuild, axiom audit, and Comparator check passed. Our additional Lean checks establish that the submitted construction satisfies the independently formulated requirements described below, using the submission's proved results. Human review of the definitions is also documented below. The repository `yang-mills-group/YM` contains this assessment, a copy of the submission, the additional Lean checks, verification records, and instructions for reproducing the checks.

Sources and attribution. This assessment uses the Lean sources submitted by OpenAI on Oct 28, 2026 and available in the repository `yang-mills-group/OpenAI-YM-20261028`. It does not draw on the separate informal proof manuscript. We used OpenAI's Astra to assist with the digestion of the supplied Lean code. We do not claim to fully understand the proof yet. We also drew on previous formalizations of the OS axioms and of lattice gauge theory, cited in [DMY, LGT].

The OpenAI work rests on prior work by many physicists and mathematicians. A full review with careful attribution will be provided in later releases by the Yang–Mills group.

Our group's only connection to OpenAI (besides accepting their submissions) is the use of their Astra model to help with analysis. Our group has no present connection to the Clay Mathematics Institute.

Contents

I The construction and its results **3**
1 The problem requirements . . . 3
2 What the construction provides . . . 3
3 Continuum Yang–Mills theory and conventions . . . 4
4 Wilson lattice gauge theory and the ultraviolet limit . . . 5
4.1 Labels and lattice representatives . . . 5
4.2 Renormalized observables . . . 5
4.3 The continuum limit at fixed volume . . . 6
5 The infinite-volume limit . . . 7
6 Reconstruction and the component theories . . . 7
6.1 Phase decomposition . . . 8
6.2 Relation between component and mixed theories . . . 9
6.3 Euclidean invariance . . . 9
7 Evidence that the theory is Yang–Mills . . . 9
7.1 The stress tensor and its Ward identity . . . 9
7.2 A nonzero trace and its short-distance law . . . 10
7.3 What is excluded about free fields . . . 11
7.4 Asymptotic freedom and perturbative agreement . . . 11
7.5 The operator product expansion and contact extensions . . . 12
8 Open Questions . . . 13

II Independent validation and review status **14**
9 How the formal proof was checked . . . 14
9.1 Clean rebuild, inventory, and kernel check . . . 14
9.2 Comparator results . . . 15
9.3 Additional review after the Lean kernel check . . . 16
9.4 Identifying the constructed theory as Yang–Mills . . . 16
10 Independent verification of the OS axioms . . . 17
10.1 Independence and scope of the check . . . 17
11 Other independent checks and their implications . . . 18
12 Verification records and reproduction . . . 19
12.1 Versions and repository layout . . . 19
12.2 Checks and their records . . . 19
12.3 Reproducing the checks . . . 21
Acknowledgements . . . 22
References . . . 22
A Lean statement glossary . . . 23

Part I
The construction and its results

1 The problem requirements

Four features of a constructed Yang–Mills theory are required in the problem statement by Jaffe–Witten. [JW]

1. **A nontrivial theory satisfying QFT axioms.** Positivity, spacetime symmetry, locality, regularity and a vacuum representation must hold at the strength of the Wightman or OS frameworks with genuine local observables.
2. **Yang–Mills local fields.** The space of operators must correspond to gauge-invariant polynomials in the curvature F and its covariant derivatives.
3. **The predicted short-distance behavior.** This includes asymptotic freedom and perturbative renormalization theory, a stress tensor, and an operator product expansion with the predicted singularities.
4. **A positive mass gap.** For the nonnegative Hamiltonian H, one requires $\sigma(H) \subset \{0\} \cup [\Delta, \infty)$ for some $\Delta > 0$, with a finite supremum of such gaps.

2 What the construction provides

1. The construction starts with the Wilson definition of lattice gauge theory, associates renormalized lattice observables to gauge-invariant polynomials in the curvature F and its covariant derivatives, and shows that their correlations converge to limiting correlations defining a Euclidean QFT. OS Reconstruction is then used to build a relativistic QFT that satisfies their own version of the Wightman axioms `YangMills.Hierarchy.Reconstructed.axioms`. They show that the theory is non-trivial in `YangMills.Hierarchy.Properties.nonfree`.
2. Renormalized local operators corresponding to gauge-invariant polynomials in F and its covariant derivatives are defined, including a conserved stress-tensor. Some short-distance checks are made; see, for example, Theorem 7.1.
3. The construction provides an operator product expansion `YangMills.Hierarchy.Properties.ope` with bounds on the error but does not show that it matches the predictions of a perturbative expansion. The construction also provides the scaling for β in `YangMills.WilsonRegularity.running` required in asymptotic freedom. The construction defines the trace of the stress tensor `YangMills.Hierarchy.centeredTrace` and proves a number of properties about it in `YangMills.Hierarchy.Properties.nonfree`.
4. The construction provides no evidence for or against a mass gap in the theory.

3 Continuum Yang–Mills theory and conventions

We briefly review the definitions used in continuum Yang–Mills theory. These will be used to define the local fields associated with gauge-invariant polynomials in the curvature and covariant derivatives.

The submitted theorem takes as input a closed matrix subgroup $G \subset U(m)$ with nonabelian simple real Lie algebra $\mathfrak{g}$. For a smooth $\mathfrak{g}$-valued connection A on $\mathbb{R}^4$, its curvature and covariant derivative are defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu], \qquad D_\mu X = \partial_\mu X + [A_\mu, X]. \tag{1}$$

Use the invariant inner product $\langle X, Y\rangle_{\mathfrak{g}} = -m^{-1} \operatorname{Re} \operatorname{Tr}(XY)$. In these conventions Euclidean Yang–Mills theory has the formal expression

$$Z^{-1} \exp\left[-\frac{1}{2g^2} \int_{\mathbb{R}^4} \sum_{\mu<\nu} \|F_{\mu\nu}(x)\|_{\mathfrak{g}}^2 \, d^4x\right] DA. \tag{2}$$

There are many reasons such an expression is ill-defined, but one may still hope to rigorously define a probability measure corresponding to (2) on a suitable state space. This is not what is asserted by the submission. Instead, the continuum objects constructed in the submission are correlation functions of local observables and their reconstructed quantum fields. The constructed correlation functions of the submission relate to (2) as they are defined through scaling limits of correlation functions of Wilson lattice gauge theory.

The local observables considered in the submission are gauge-invariant polynomials in the curvature and covariant derivatives at one point. Instead of directly defining gauge-invariant polynomials as functions or distributions on $\mathbb{R}^4$, the submission defines them as abstract field labels belonging to a vector space and provides correlation functions associated to collections of labels. Each label is assigned a weight (degree): curvature has weight two, a derivative promotes the weight by one, and the weight of a product of labels is the sum of the weights of the labels. The labels of weight $\leq d$ form a finite-dimensional complex vector space V_d. For example,

$$-\operatorname{Tr}(F_{\mu\nu}F_{\rho\sigma}) \in V_4, \qquad -\operatorname{Tr}\big((D_\lambda F_{\mu\nu})(D_\kappa F_{\rho\sigma})\big) \in V_6.$$

See `YangMills.WilsonFamily.Token` and `YangMills.Hierarchy.Field`.

The correlations of local observables constructed in the proof will be defined by testing against functions $f \in C_c^\infty(\{(x_1, \ldots, x_n) \in (\mathbb{R}^4)^n : x_i \neq x_j \text{ for } i \neq j\})$ avoiding collisions.

The set of all finite degree labels is linearly dependent, for instance satisfying relations following from the Bianchi identities. Equalities of labels in these relations are formulated rigorously in terms of correlation functions. The quantum correlation functions obey similar linear relations, which imply that certain observables vanish. The main theorem statement concerns labels in the union over all degrees of the quotient spaces $F_d = V_d/N_d$ where N_d is the space spanned by all the linear relations among the labels.

The submission also proves that, for each degree d, the quotient space F_d is linearly isomorphic to a space of gauge-invariant polynomials coming from classical Maxwell theory. These polynomials have weighted degree at most d in a free curvature and its derivatives, with polynomial expressions identified whenever they agree on formal curvature data satisfying the source-free Maxwell equations and Bianchi identities. Every phase in the decomposition introduced in Section 6.1 has the same correlation-null spaces, so the same comparison holds in each phase.

4 Wilson lattice gauge theory and the ultraviolet limit

The continuum Yang–Mills theory is constructed as a scaling limit of Wilson lattice gauge theory on the discrete torus of side length L_p and lattice spacing a_N, indexed by $N, p \in \mathbb{N}$. The construction chooses both sequences, and thus the UV convergence theorem does not assert convergence over all possible sequences of spacings.

Recall that Wilson action lattice gauge theory is a probability measure on configurations assigning $U_e \in G$ to each positively oriented edge and U_e^{-1} to its reverse. With $U_\square$ the ordered plaquette product, the law of the theory is

$$d\mu_{N,p}(U) = Z_{N,p}^{-1} \exp\left[-\beta_{N,p} \sum_{\square} \left(1 - \frac{1}{m} \operatorname{Re} \operatorname{Tr} U_\square\right)\right] \prod_e dh(U_e), \tag{3}$$

where h is normalized Haar measure, $Z_{N,p}$ normalizes the law, and each positively oriented plaquette is counted once. The coefficient $\beta_{N,p}$ is also adjusted with the volume and lattice spacing. With the convention in equation (2), β should be interpreted as $\beta = 1/g_0^2$, where g_0 is the cutoff coupling. See `YangMills.WilsonFamily`.

4.1 Labels and lattice representatives

The UV convergence theorem asserts the convergence of correlation functions of certain sequences of gauge-invariant lattice observables. More precisely, for any list of labels, there is a sequence of lists of lattice observables such that their lattice correlations converge to a continuum correlation function indexed by the list of labels as the lattice spacing shrinks to 0. See `YangMills.WilsonFamily.Token`, `YangMills.Hierarchy.Field`, and `YangMills.Hierarchy.labels`.

For $v \in V_d$ the construction chooses a gauge-invariant local lattice polynomial L_v, complex linear in v, depending on a fixed finite pattern of edges through their matrix entries and conjugates. The lattice polynomial is constructed by first supposing that U arises as the parallel transport of a smooth connection A. Then the requirement of the map is that the Taylor expansion (up to order d) in the lattice spacing a of L_v matches the continuum observable labeled by v after rescaling by the appropriate power of a.

For example, on smooth links, $U_{\mu\nu} = I_m + a^2 F_{\mu\nu}(0) + O(a^3)$, so

$$a^{-4}\left(1 - \frac{1}{m} \operatorname{Re} \operatorname{Tr} U_{\mu\nu}\right) \longrightarrow -\frac{1}{2m} \operatorname{Tr}(F_{\mu\nu}(0)^2), \tag{4}$$

or with the label notation, if $v = -\frac{1}{2m} \operatorname{Tr}(F_{\mu\nu}^2)$, then $1 - \frac{1}{m} \operatorname{Re} \operatorname{Tr} U_{\mu\nu}$ has the correct expansion to be a candidate for $L_v(0; U)$. The precise construction of the proof requires additional symmetry and does not assign $L_v(0; U) = 1 - \frac{1}{m} \operatorname{Re} \operatorname{Tr} U_{\mu\nu}$ in this example.

4.2 Renormalized observables

The true assignment of labels to lattice observables involves renormalization. Suppose a label $v \in V_d$ has weight d. Then with respect to a basis of labels $v_1, \ldots, v_M \in V_d$, the UV convergence statement associates to v a renormalized lattice observable

$$I_{N,p}(v, x; U) = \sum_{j=1}^{M} c_j(N, p; v) L_{v_j}(x; U), \tag{5}$$

where coefficients depend linearly on v. The coefficients include powers of the spacing and thus may diverge in the limit. The UV convergence theorem asserts the existence of renormalization coefficients $c_j(N, p; v)$ that lead to convergence of correlation functions. The submission provides a formula for them in terms of the lattice RG transformations used in the proof, but they are not explicitly expressed in terms of sums or integrals. See `YangMills.WilsonFamily.insertion`.

4.3 The continuum limit at fixed volume

For a label list $\boldsymbol{v} = (v_1, \dots, v_n)$ and a smooth compactly supported joint test function f on $(\mathbb{R}^4)^n$, the convergence statement involves smeared correlation functions defined by

$$C_{N,p}(\boldsymbol{v}; f) = a_N^{4n} \sum_{x_1,\dots,x_n \in \Lambda_{N,p}} f(a_N x_1, \dots, a_N x_n) \mathbb{E}_{\mu_{N,p}} \left[\prod_{j=1}^{n} I_{N,p}(v_j, x_j; U) \right]. \tag{6}$$

Here $\Lambda_{N,p} := (\mathbb{Z}/a_N^{-1} L_p \mathbb{Z})^4$. Allowed test functions have support away from all coincidences (i.e. $\mathrm{supp}(f) \subset \{(x_1, \dots, x_n) : x_i \neq x_j \text{ for } i \neq j\}$) and strictly inside $((-L_p/2, L_p/2)^4)^n$. The joint test function f is also not restricted to be a product of one-point test functions.

Theorem 4.1 (Fixed-volume ultraviolet limit)**.** *For every closed matrix subgroup $G \subset U(m)$ with corresponding simple real Lie algebra $\mathfrak{g}$ with Casimir constant $C_{\mathfrak{g}}$, the construction supplies a sequence of lattice spacings $(a_N)_{N \geq 1}$, physical side-lengths $(L_p)_{p \geq 1}$, and inverse squared couplings $(\beta_{N,p})_{N,p \geq 1}$ such that $a_N \to 0$ as $N \to \infty$, and*

$$\lim_{N \to \infty} \sup_{p} \left| \frac{\beta_{N,p}}{\log(a_N^{-1})} - \frac{11 C_{\mathfrak{g}}}{24\pi^2} \right| = 0, \tag{7}$$

and for any volume index p, there exist coefficients $c_j(N, p, v_i)$ defining the operators appearing in $C_{N,p}(\boldsymbol{v}; f)$ such that, for every fixed label list $\boldsymbol{v}$ and allowed test function f in that volume,

$$S_p(\boldsymbol{v}; f) = \lim_{N \to \infty} C_{N,p}(\boldsymbol{v}; f) \tag{8}$$

exists along the entire chosen spacing sequence. Moreover, for every volume index p and finite label list $\boldsymbol{v}$, there exists a correlation kernel $S_p(\boldsymbol{v}; \boldsymbol{x})$ that is C^∞ away from coincidences modulo L_p, meaning wherever $x_i - x_j \notin L_p \mathbb{Z}^4$ for all $i \neq j$. For every allowed test function f on $((-L_p/2, L_p/2)^4)^n$,

$$S_p(\boldsymbol{v}; f) = \int_{(\mathbb{R}^4)^n} f(\boldsymbol{x})\, S_p(\boldsymbol{v}; \boldsymbol{x})\, d\boldsymbol{x}. \tag{9}$$

See `YangMills.WilsonRegularity.running` and `YangMills.Hierarchy.Properties.wilson`.

The proof compares $C_{N,p}(\boldsymbol{v}; f)$ at sufficiently fine cutoffs using the proof's RG normalization and shows that the sequence $C_{N,p}(\boldsymbol{v}; f)$ is Cauchy in N. The proof defines an explicit RG map for the leading (marginal) terms in the action, following the general approach of Balaban but with many improvements such as a gauge fixing invariant under discrete lattice symmetries. This agrees with the standard one-loop RG, and bounds on the irrelevant terms are shown to propagate through this map. This leads to parametric bounds on the irrelevant terms in the UV and control of the RG trajectory.

Convergence holds along the entire sequence of lattice spacings a_N used in the proof `YangMills.WilsonFamily.denominator`, but there is no universality result for the types of sequences of spacings $a_N \to 0$. Also, the convergence is only proven for test functions in $C_c^\infty(\{(x_1, \dots, x_n) \in (\mathbb{R}^4)^n : x_i \neq x_j \text{ for } i \neq j\})$.

5 The infinite-volume limit

After removing the UV cutoff at fixed volume, the construction sends the physical side length L_p to infinity. This infinite-volume limit is only taken subsequentially. However, it is proven that one subsequence works for all correlation functions simultaneously. We write its limit as S^{mix}, anticipating the phase decomposition explained in Section 6.1.

Theorem 5.1 (Subsequential infinite-volume limit)**.** *With the notation of Theorem 4.1, for every strictly increasing sequence $p : \mathbb{N} \to \mathbb{N}$ of volume indices, there exists a strictly increasing $q : \mathbb{N} \to \mathbb{N}$ and kernels $S^{\mathrm{mix}}(\boldsymbol{v}; \boldsymbol{x})$, smooth on $\{(x_1, \dots, x_n) \in (\mathbb{R}^4)^n : x_i \neq x_j \text{ for } i \neq j\}$, such that*

$$\lim_{k\to\infty} S_{p(q(k))}(\boldsymbol{v}; f) = \int_{(\mathbb{R}^4)^n} f(\boldsymbol{x}) S^{\mathrm{mix}}(\boldsymbol{v}; \boldsymbol{x})\, d\boldsymbol{x} \tag{10}$$

for every finite label list $\boldsymbol{v}$ and every $f \in C_c^\infty(\{(x_1, \dots, x_n) \in (\mathbb{R}^4)^n : x_i \neq x_j \text{ for } i \neq j\})$.

See `YangMills.Hierarchy.Properties.wilson`, `YangMills.Hierarchy.correlation`, and `YangMills.Hierarchy.Reconstructed.smooth`.

The subsequential infinite-volume limit is obtained via compactness through an Arzelà–Ascoli argument. Specifically, for fixed local labels and every fixed compact set K of distinct-point configurations in $(\mathbb{R}^4)^n$, the construction proves that there is $p_0(K)$ such that

$$\sup_{p \geq p_0(K)} \sup_{\boldsymbol{x} \in K} |S_p(\boldsymbol{v}; \boldsymbol{x})| \leq M_K, \qquad |S_p(\boldsymbol{v}; \boldsymbol{x}) - S_p(\boldsymbol{v}; \boldsymbol{y})| \leq \omega_K(|\boldsymbol{x} - \boldsymbol{y}|), \tag{11}$$

where the second bound holds for $p \geq p_0(K)$ and $\boldsymbol{x}, \boldsymbol{y} \in K$, and $\omega_K(r) \to 0$ as $r \downarrow 0$. Both bounds hold in fixed boxes uniformly in the volume. Taking as input prior estimates obtained through proving the UV convergence, the proof derives local boundedness and equicontinuity through general arguments based on reflection positivity, chessboard estimates, transfer-operator representation, and Cauchy estimates. These general mechanisms are not specific to Yang–Mills; the construction-specific input is control of the renormalized lattice observables, uniformly in the spacing.

The conclusion is *subsequential infinite-volume existence, and it remains unresolved whether the limiting correlations depend on the chosen subsequence.* Jaffe–Witten's formulation of the Millennium Prize Problem regards a subsequential limit obtained via compactness as insufficient, unless additional arguments establish the requisite properties of the limit. The OpenAI reconstruction and local-field theorems supply substantial information beyond compactness. Section 7 explains this further evidence supporting the identification of the theory as Yang–Mills. [JW]

6 Reconstruction and the component theories

The classical reconstruction framework is described in [OS]. Throughout this section, fix one infinite-volume theory extracted from Theorem 5.1. Reflection positivity defines a positive semidefinite form on products of fields at positive Euclidean times. Quotienting its null space and taking the completion with respect to this inner product gives the OS Hilbert space. The OS reconstruction theorem is not initially applied to this Euclidean theory as it is not proven to satisfy clustering. The workaround involves decomposing the Euclidean theory into components which satisfy clustering, and applying OS reconstruction to the component theories.

6.1 Phase decomposition

The submission proves that the correlation function arising from a subsequential infinite-volume limit has a phase decomposition into component correlations $S^\xi(\boldsymbol{v}, f)$,

$$S^{\mathrm{mix}}(\boldsymbol{v}; f) = \int_{\mathcal{X}} S^\xi(\boldsymbol{v}; f)\, d\nu(\xi), \tag{12}$$

for ν a probability measure on a compact metric space $\mathcal{X}$, and any list of labels $\boldsymbol{v}$ and allowed test function f. This decomposition is proven by applying the Gelfand representation theorem for commutative C^*-algebras, together with the Riesz–Markov representation theorem. The C^*-algebra of operators fed into the theorem is generated by time translated field words restricted to the subspace of the OS Hilbert space spanned by vacuum states with output projected back to this subspace. The key point is that these component theories are proven to obey the variants of the OS axioms including clustering. For more details see `YangMills.Hierarchy.Character`.

The submission states and proves its own version of the Osterwalder–Schrader reconstruction theorem and applies it to each component S^ξ. The S^ξ are analytically continued to complex times, and the limits as the imaginary part of time tends to zero (taken after integration against Schwartz test functions) give the real-time Wightman distributions W^ξ. Let $\mathcal{P}$ be the vector space of finite complex linear combinations of formal products $(v_1, f_1)\cdots(v_n, f_n)$, where each v_i is a field label and each $f_i \in \mathcal{S}(\mathbb{R}^4; \mathbb{C})$ is a Schwartz test function. The empty product is also included. On $\mathcal{P}$, define

$$\langle u, v\rangle_\xi = W^\xi(u^* v), \qquad \mathcal{N}_\xi = \{u : \langle u, u\rangle_\xi = 0\}, \qquad \mathcal{H}_\xi = \overline{\mathcal{P}/\mathcal{N}_\xi}.$$

Here $*$ reverses words, adjoints labels, and conjugates test functions and coefficients. The quotient $\mathcal{P}/\mathcal{N}_\xi$, viewed as a dense subspace $\mathcal{D}_\xi$ of its completion $\mathcal{H}_\xi$, is the common domain of the smeared field operators. The empty word defines Ω_ξ and $\Phi^\xi_v(f)[w] \coloneqq [(v, f)w]$. Consequently,

$$W^\xi_n(v_1, f_1; \ldots; v_n, f_n) = \langle \Omega_\xi, \Phi^\xi_{v_1}(f_1)\cdots\Phi^\xi_{v_n}(f_n)\Omega_\xi\rangle.$$

See `Hierarchy.H` and `Hierarchy.theory`. The reconstruction establishes the following Wightman axioms and correlation correspondence for the relativistic QFT.

Theorem 6.1 (Reconstruction of the component theories)**.** *For every character $\xi \in \mathcal{X}$, the component correlations S^ξ reconstruct to a Wightman theory on $\mathcal{H}_\xi$, with vacuum Ω_ξ and fields Φ^ξ on $\mathcal{D}_\xi$. The reconstructed theory satisfies the following properties, with their precise formulations given in the linked glossary entries:*

1. ***Translation spectrum, positive energy, and vacuum uniqueness.***
2. ***Common domain, regularity, adjoints, and cyclicity.***
3. ***Poincaré covariance.***
4. ***Local commutativity.***
5. ***Analytic continuation and recovery of correlations.***
6. ***Faithfulness of the fields.***

See `YangMills.Hierarchy.Reconstructed.axioms`, `YangMills.Hierarchy.Reconstructed.analytic`, `YangMills.Hierarchy.Reconstructed.root`, `YangMills.Hierarchy.Reconstructed.boundary`, and `YangMills.Hierarchy.Reconstructed.faithful` for the definitions of these properties as they appear in the formalization.

6.2 Relation between component and mixed theories

The source proves that an OS reconstruction theorem can also be applied to the original infinite-volume theory corresponding to any particular subsequential limit; however, the reconstructed Wightman theory is not proven to have a unique vacuum state. Clustering of the Euclidean theory would imply uniqueness of the vacuum state.

On the Euclidean side, each component theory correlation function S^{ξ} clusters, but this does not imply clustering of S^{mix}. If the original correlation function S^{mix} satisfied clustering, then a short calculation shows that $\int |S^{\xi}(A) - S^{\mathrm{mix}}(A)|^2 d\nu = 0$. Continuity and full support would then identify every component. Even though clustering would show all component theories of a particular subsequential limit are equal, it does not provide sufficient infrared control to imply uniqueness of the infinite-volume limit.

6.3 Euclidean invariance

Proving Euclidean invariance in the continuum infinite-volume limit is difficult because the lattice regularization breaks Euclidean symmetry, leaving a finite group of symmetries of the lattice. The OpenAI proof has three main parts. The first is that the gauge fixing preserves the lattice symmetries, which would not have been the case for (say) axial gauge fixing. This is done by averaging over symmetry breaking choices. The second part is to show that there exists a divergence-free stress tensor field, and to use it to establish local Ward identities. Finally, one must show that the Euclidean invariance is not spontaneously broken in the limiting state. This follows if the flux of stress-energy falls off sufficiently rapidly at infinity. This is shown by an intricate argument using reflection positivity in multiple orthogonal planes.

7 Evidence that the theory is Yang–Mills

We now turn from the general field-theoretic structure to the properties that identify this construction as Yang–Mills. This includes the existence of a normalized stress tensor and corresponding Ward identity, specific short-distance laws, non-triviality, asymptotic freedom, and an OPE. Throughout the section we use $S^s_{\boldsymbol{v}}(\cdot)$ and $S^s(\boldsymbol{v};\cdot)$ interchangeably, where s refers either to a particular phase or the mixed theory.

7.1 The stress tensor and its Ward identity

In the classical setting, the Yang–Mills stress tensor is defined through variation of the Yang–Mills action with respect to the metric. In the quantum setting of the submission, the stress tensor is uniquely identified as follows. The symmetric tensors with components in the quotient space of labels F_4 that are fixed by field conjugation, centered with respect to S^{mix}, and transform as rank-two tensors under coordinate permutations and sign changes, whose divergence vanishes in F_5, form a one-dimensional real vector space. The stress tensor is a vector in this one-dimensional space with normalization fixed by the Ward identity described in the next paragraph.

For local field labels $v_0, v_1, \ldots, v_n$ and distinct points $x_0, x_1, \ldots, x_n$, let χ be a real smooth, compactly supported function, equal to one in a neighborhood of x_0 and supported away from the other points.

The Ward identity reads

$$-\sum_{\mu=0}^{3}\int d^4y\,\partial_\mu\chi(y)\,S^{\xi}_{T_{\mu\nu},v_0,\dots,v_n}(y,x_0,\dots,x_n)=\partial^{\nu}_{x_0}S^{\xi}_{v_0,\dots,v_n}(x_0,\dots,x_n)\,. \tag{13}$$

In plain English, the stress flux around a point generates coordinate translation. Since $\partial\chi$ vanishes in neighborhoods of $x_0, x_1, \dots, x_n$, the integrand in (13) avoids coincidences of points.

7.2 A nonzero trace and its short-distance law

In the selected component theory indexed by a phase ξ_*, define the label (local field)

$$\Theta \coloneqq \sum_{\mu=0}^{3} T_{\mu\mu} - S^{\xi_*}\left(\sum_{\mu=0}^{3} T_{\mu\mu}\right)\mathbf{1}. \tag{14}$$

More precisely, the above formula asserts that for any list of labels $v_1, ..., v_n$ and insertion points $x_0, x_1, \dots, x_n$, writing $v_0 \coloneqq \sum_{\mu=0}^{3} T_{\mu\mu}$,

$$S^{\xi_*}_{\Theta,v_1,\dots,v_n}(x_0,x_1,...,x_n)=S^{\xi_*}_{v_0,v_1,\dots,v_n}(x_0,x_1,...,x_n)-S^{\xi_*}_{v_0}(x_0)\,S^{\xi_*}_{v_1,\dots,v_n}(x_1,...,x_n)\,. \tag{15}$$

The following theorem asserts non-triviality of the Yang–Mills two-point correlation function associated to the trace of the stress tensor as defined in the preceding paragraph.

Theorem 7.1 (Original trace asymptotic)**.** *With the centered trace Θ defined above,*

$$\lim_{r\downarrow 0} r^8\log^2(1/r)\,S^{\xi_*}_{\Theta,\Theta}(-re_0/2,re_0/2)=3\dim_{\mathbb{R}}\mathfrak{g}/\pi^4. \tag{16}$$

Lengths are measured in the fixed units of the construction.

See `YangMills.Hierarchy.Properties.nonfree`.

In particular, this two-point function is nonzero for sufficiently small r, and some smeared $\Phi_\Theta(f)$ is nonzero. This result may seem absurd since for a smooth connection, the stress tensor has zero trace; however, Θ is in general nonzero as a result of renormalization. This fact is sometimes known as the trace anomaly.

Euclidean covariance gives the equivalent all-directions leading law for $K_\Theta(x) = \mathrm{Re}\,S^{\xi_*}_{\Theta,\Theta}(x,0)$:

$$|x|^8\log^2(1/|x|)K_\Theta(x)\longrightarrow 3\dim_{\mathbb{R}}\mathfrak{g}/\pi^4 \qquad (x\to 0,\ x\neq 0). \tag{17}$$

The coefficient in (17) agrees with the perturbative prediction. The normalization calculation and its supporting references are given in Section 5 of the worked example `LeadingTrace-20261028.lean`. The numerical one-loop beta-function coefficient cancels from this leading trace amplitude; the separate running-coupling result in Section 7.4 specifies its Yang–Mills value. The Lean adapter checks that the constructed trace satisfies the law with this explicit coefficient. [A4]

7.3 What is excluded about free fields

The submission excludes a precise class of free-field descriptions. For $m \geq 0$ and an integer $q \geq 0$, consider

$$K_{m,q,h}(r) = \int_{\mathbb{R}^3} \frac{E_m(\boldsymbol{p})^q h(\boldsymbol{p})}{2E_m(\boldsymbol{p})} e^{-rE_m(\boldsymbol{p})}\, d\boldsymbol{p}, \qquad E_m(\boldsymbol{p}) = \sqrt{m^2 + |\boldsymbol{p}|^2}, \tag{18}$$

where h is continuous, real-valued, and homogeneous with nonnegative integer degree. No finite complex linear combination of finite products of these kernels equals the trace correlation on any interval $0 < r < \delta$. This includes suitable polynomial observables and derivatives in free fields with finitely many masses. The inverse-square logarithm in Theorem 7.1 obstructs those descriptions. See `YangMills.Hierarchy.TraceTest.excludes`.

The theorem excludes the specified finite free-field descriptions but does not rule out every description in terms of composites of Gaussian fields.

7.4 Asymptotic freedom and perturbative agreement

Integrating out fluctuations below a length r changes the coefficient of the curvature-squared term in the effective action. Its inverse square root, with fixed normalization, is a running coupling. Asymptotic freedom means that it tends to zero as $r \downarrow 0$.

More precisely, the submission starts with the family of lattice Wilson theories whose bare couplings are supplied by Theorem 4.1 and performs RG blocking to the first lattice spacing below r. It defines $g_W(r)$ as the inverse square root of the resulting normalized action coefficient, after taking the initial lattice cutoff to zero and a subsequential infinite-volume limit. The submission does not prove $g_W(r)$ is independent of the subsequential infinite-volume limit. Set $b_0 = 11C_{\mathfrak{g}}/(48\pi^2)$ where $C_{\mathfrak{g}}$ is the Casimir constant associated to the Lie algebra $\mathfrak{g}$.

Theorem 7.2 (Effective coupling at short distances)**.** *For small r and any selected subsequential infinite-volume limit, $g_W(r) > 0$, and*

$$\log(1/r) g_W(r)^2 \longrightarrow \frac{1}{2b_0} \qquad (r \downarrow 0). \tag{19}$$

A second coupling is defined through separated continuum correlations. The construction normalizes a scalar two-point derivative and an integrated tensor–scalar–scalar correlation to their free values, then uses a derivative of a tensor–scalar correlation to measure the coupling. The submission then proves that the ratio of this coupling to g_W tends to one as the scale $r \downarrow 0$. Thus the running law is visible in continuum observables as well as in the effective action.

There is further evidence for free-field agreement at small scales. The submission proves that along one common sequence $r_k \downarrow 0$, averaged n-point stress tensor correlations multiplied by r_k^{4n} and evaluated at $r_k x_1, \ldots, r_k x_n$ converge to those of $\dim \mathfrak{g}$ independent free Maxwell fields. Convergence holds with every fixed number of derivatives, uniformly on compact sets avoiding collisions. Maxwell fields here mean the curvature is a Gaussian two-form obtained as the exterior derivative of a $\mathfrak{g}$-valued 1-form Gaussian free field. The submission includes additional comparisons with fields that are not the stress tensor.

The free field comparison just described holds for the mixed theory along a particular scaling sequence approaching zero, while Theorems 7.1 and 7.2 hold as r tends to zero arbitrarily and Theorem 7.1 applies to a selected phase. Together they provide substantial leading short-distance evidence.

7.5 The operator product expansion and contact extensions

Let v_i and u_j be local field labels, each v_i having degree d_i, and define the total degree $w = \sum_i d_i$. The OPE supplies labels $o_\alpha \in F_{d_\alpha}$, finitely many per degree, such that for every $D \geq 0$ and $\varepsilon > 0$,

$$S^s_{\boldsymbol{v},\boldsymbol{u}}(r\boldsymbol{x}, \boldsymbol{y}) = \sum_{d_\alpha \leq D} c_\alpha(\boldsymbol{v}; r\boldsymbol{x})\, S^s_{o_\alpha,\boldsymbol{u}}(0, \boldsymbol{y}) + R^s_D, \qquad |R^s_D| \leq r^{D+1-w-\varepsilon}, \tag{20}$$

where s denotes the mixed theory or a component theory. For fixed labels, D and ε, the bound holds for sufficiently small $r > 0$. The estimate holds uniformly in s and on any fixed compact set of configurations where the x_i are distinct and the y_i are distinct and nonzero.

The coefficients are independent of u_j, y_j, s and D. Their independence of choice of subsequential infinite-volume limit is not proved. Common coefficients of two expansions do not by themselves imply equal correlations as can be seen when the collection of u_j is empty.

The cutoff D counts local observable label degrees, not loop orders. Perturbative agreement to all orders remains unproved.

The submission also proves nested OPE expansions hold under additional separation and scale conditions. Using the OPE, the submission is also able to extend the definition of correlations to allow for test functions with support overlapping with the collision set $\bigcup_{i<j}\{x_i = x_j\}$. However, uniqueness, the full OS axioms for these extensions, and lattice convergence are not proven for these extensions.

8 Open Questions

Here we reiterate some of the major aspects of the construction of Yang–Mills that are not addressed in the current OpenAI work.

1. Establish convergence (rather than finding convergent subsequences) and uniqueness of the infinite-volume limit.
2. Define a probability measure for the continuum theory on an appropriate space, leading to the Schwinger functions of the OpenAI construction.
3. Establish a uniform mass gap, giving exponential decay of corresponding correlations and a spectral gap in the Osterwalder–Schrader Hamiltonian.
4. Construct Wilson loop observables and relate them to the local observables in the OpenAI construction. Establish the area law for Wilson loops.
5. Construct single-particle states and analyze their energy-momentum spectrum. Such an analysis has been carried out for many constructive models in 2 and 3 space-time dimensions.

Part II
Independent validation and review status

9 How the formal proof was checked

We start by explaining the process: verification, validation, and additional checks that the tests are sufficient and not being subverted in some way.

Formal verification checks that a proof establishes a stated mathematical claim from its definitions and permitted axioms. In Lean, proofs are represented by terms that a small kernel checks against the inference rules. Automated proof tactics must produce terms that pass this check.

Validation asks the further question of whether the formal claim correctly expresses the intended Yang–Mills construction described in Part I. We address this through review of the definitions and through independent adapters—Lean proofs showing that the constructed objects satisfy requirements from previous formalizations. We also traced the fields and correlations from the specified Wilson lattice theory through the UV and large-volume limits, the phase decomposition, and the relativistic reconstruction. Section 9.4 explains how we checked these connections.

Lean is highly flexible: users define the mathematical objects and theorem statements they work with. This flexibility also makes unwanted changes possible. A submitted development can, in the process of verification, change the problem statement or one of its underlying definitions and get an invalid proof to appear to verify. This is why a separate comparison with a fixed reference specification is needed, and this is done by the tool Comparator.

Comparator checks a submitted solution against a separately fixed reference statement. It compares the target statements and their underlying definitions, audits the proof's axiom dependencies, and rechecks the exported proof with a Lean kernel. This checks that the solution proves the prescribed formal claim using only the permitted assumptions. The precise targets and verification configurations used in our review are recorded below.

After their original submission, OpenAI subsequently informed us that its original public Comparator target `YangMills.jointExistence` omitted some of the connections to the lattice construction established by its proof. This weakness of the original target does not undermine our review: our combined target, `YMReview.Acceptance.combined_acceptance`, requires those connections by retaining the native construction certificates for the lattice observables, limits, phases and reconstructed fields. Our assessment of these connections therefore rests on that stronger target and the checks described below, rather than on the public existence statement alone.

9.1 Clean rebuild, inventory, and kernel check

The recorded clean build used Lean 4.32.0 on Apple Silicon, with the pinned official Mathlib dependency cache and no pre-existing project build artifacts. It rebuilt 9,470 project modules, completed in about 184 minutes, and reported zero errors. A subsequent incremental confirmation also passed. All 9,477 supplied Lean source files matched the submission manifest. [M1]

The inventory inspected all 250,319 project declarations imported by the construction, including generated declarations. It found no project axiom declarations, no direct references to Lean's placeholder axiom `sorryAx`, and no declarations marked `unsafe`. The 302 generated `._unsafe_rec`

declarations are partial recursion helpers; none occurs among the dependencies of the audited principal results. The sole source-level `sorry` belongs to the reference statement in `Challenge.lean`, which is loaded separately from the solution. The solution supplies a proof without importing that placeholder.

The submission's verifier `VerifyJoint.lean` compared the target statement and its reachable shared definitions in the reference and solution environments. It then rechecked the 100,168 project declarations on which 17 principal results depend, using a fresh kernel environment initialized with the pinned external libraries. It also audited 14,364 external declarations referenced at the boundary of those project dependencies. The principal proofs use only the permitted foundational axioms `propext`, `Classical.choice`, and `Quot.sound`. The inventory covers all imported project declarations, and the kernel check covers the dependencies of the 17 principal results. This submitted verifier is separate from the upstream Comparator used for the three runs reported next.

The clean build emitted 2,453 warnings in 1,063 files:

Warning group	Count
Unused variables, section variables, or simp arguments	1,955
Unused/unreachable tactics, unnecessary focus or `simpa`	403
Deprecated declarations	66
Other linter messages and tactic suggestions	26
Exponentiation left unevaluated at a configured threshold	2
`Challenge.lean sorry`	1
Total	2,453

Most warnings concern proof-script cleanup or deprecated syntax. The placeholder warning is the separate reference stub described above; the audited solution proofs have no dependency on it.

At this point we would also like to express some concerns with the quality of the code in the Lean repository. There are 250,319 project declarations (including auto-generated auxiliary declarations), but only 100,168 (including auxiliary declarations) are used in the dependency tree of the main theorems. The difference is less drastic for source declarations (not including auxiliary declarations), but is still significant. We also remark that the OpenAI submission used unconventional naming for its declarations and files, such as `Farewell.dead` in TicketsChart.lean, `Cake` and `cookie` in Kitchen.lean, and `anxiety` in Parade.lean.

9.2 Comparator results

Comparator passed on 30 October 2026 for the submitted public theorem `YangMills.jointExistence`, and on 3 November for our independent combined theorem `YMReview.Acceptance.combined_acceptance`. The 3 November verification record preserves the checked target's identity, configuration, source-file hashes and logs. The corresponding Lean files in this release are identified by the review source manifest. The run passed statement and definition comparison, the permitted-axiom check, and the built-in Lean kernel. Four controls behaved as expected: a valid proof was accepted, while a changed statement, a forbidden axiom and a changed definition were rejected. [M2]

The combined run rebuilt its 40 required review modules in a new project and reused the previously built submission artifacts and dependency cache. The separate clean rebuild of the submission is documented above. All three runs used pinned, unmodified upstream Comparator and exporter revisions and Comparator's built-in Lean kernel. The project and exporter used Lean 4.32.0; the

Comparator kernel used Lean 4.34.0. On macOS, the runs used the upstream development launcher `fake-landrun.sh`, which runs the checks without Linux Landrun's process isolation. They therefore record the statement, definition, axiom and kernel checks, but do not test isolation of potentially hostile build code. An additional external kernel implementation was not used. Each verification record identifies the exact target, inputs and configuration that passed.

A further Comparator run on 5 November passed for the OPE statement-alignment theorem `YMReview.OPE.constructed`.

9.3 Additional review after the Lean kernel check

Lean kernel acceptance determines whether the proof follows from its formal definitions and permitted axioms. It does not settle whether those definitions express the intended natural language problem. For that reason, we did extensive (but not exhaustive) human review of the Lean definitions in the glossary below, checking them for alignment with the problem statement and with our expectations based on the discussion above.

We also compared the definitions with previous formalizations. In particular, we proved that the Schwinger functions of the centered stress trace in the selected phase satisfy the OS axioms and $E0'$ growth condition in the formulation of [DMY]. Section 10 describes the comparison of test spaces and axiom statements.

The scalar OS, lattice, field-content, translation Ward, leading trace and common-scale checks are assembled in `YMReview.Acceptance.combined_acceptance`, which compiled and passed a Comparator check. This theorem retains the stronger construction data supplied by `YangMills.constructed`, as well as the public existence conclusions. The retained data connect all the additional properties to the same construction. The OPE statement-alignment check in Section 11 is a separate endpoint, with its own Comparator run. [A7][A8][M2]

9.4 Identifying the constructed theory as Yang–Mills

The combined target retains the construction certificates `C.Makes` and `Spec.Final`, connecting the Wilson lattice fields to the continuum correlations, phase decomposition and reconstructed QFT described in Sections 4–6. These data fix the lattice fields' classical curvature-polynomial meaning and renormalization, identify continuum field words across both limits, and connect the selected phase's boundary distributions to reconstructed vacuum expectations. The comparison with the independent Wilson law is described in Section 11. [A2][A3][A7]

For each group and positive construction parameter, one Wilson family is fixed before the extraction order is chosen; one refinement, correlation table and final witness then satisfy all the combined checks. In particular, the Ward identity, scalar OS axioms and leading trace law concern the same stress tensor and selected phase. [A1][A4][A6] Together with the field-content and short-distance results of Section 7, these connections support identification with Yang–Mills theory. Universality, infrared uniqueness and full perturbative agreement remain open (Section 8).

Worked construction-chain audit. The commented `OctoberChain.lean` proves seven consequences of the retained construction data, without adding hypotheses or auditing the submitted proof's individual steps. Its boundary-uniqueness lemma concerns a fixed analytic continuation, not

uniqueness of the infrared limit. The compilation record and reproduction instructions document this supplementary Lean check, separate from the three Comparator runs. [A7][M2]

10 Independent verification of the OS axioms

The OS comparison uses the centered trace of the stress tensor Θ defined in equation (14), in the selected component ξ_*. The adapter between the two formalizations changes coordinates to match the [DMY] choice of time axis and proves the corresponding identities for kernels and measures. It preserves the field, the state, and the complex correlations at every order.

The domain of test functions for this check is `ZeroDiagonalSchwartz`, defined as

$$\mathscr{S}_0((\mathbb{R}^4)^n) = \{f \in \mathscr{S}((\mathbb{R}^4)^n) : \partial^\alpha f|_{\Delta_{ij}} = 0 \text{ for all } \alpha \text{ and } i \neq j\}, \tag{21}$$

where $\Delta_{ij} = \{\boldsymbol{x} : x_i = x_j\}$. These Schwartz tests vanish to every order on the coincidence diagonals, but need not vanish in a neighborhood of them or have compact support. They include the compactly supported smooth tests away from coincidences used in the lattice-to-continuum convergence statement. The Schwinger functional used in this independent verification is

$$S_n(f) = \int_{(\mathbb{R}^4)^n} K^{\xi_*}_{\Theta,n}(\boldsymbol{x}) f(\boldsymbol{x})\, d\boldsymbol{x}, \tag{22}$$

where $K^{\xi_*}_{\Theta,n}(\boldsymbol{x}) \coloneqq S^{\xi_*}_{\Theta,\dots,\Theta}(\boldsymbol{x})$.

The adapter proves that this integral is absolutely convergent and defines a continuous linear functional in the inherited Schwartz topology. It also proves the $E0'$ bound controlling growth with the number of insertions. [A1]

Several distinct distributional statements enter the construction. Lattice convergence first determines correlations away from coincidences. The independent OS comparison integrates the selected stress-trace kernels against joint Schwartz tests flat to all orders on every coincidence diagonal; these tests cannot detect contact terms supported on those diagonals. The submission's reconstruction separately supplies tempered Wightman distributions on unrestricted joint Schwartz tests. Euclidean contact extensions are additional constructions. The diagonal-flat OS comparison does not establish the uniqueness of these extensions or their satisfaction of OS positivity on unrestricted Euclidean test spaces.

10.1 Independence and scope of the check

The target definitions come from the formulation in the repo `xiyin137/OSreconstruction` [DMY], which is also in the Palomar registry. Its OS axioms and the corresponding Wightman axioms were formulated independently and before we learned of the OpenAI Yang–Mills construction. The adapter proves that this selected scalar hierarchy satisfies those target OS axioms and the associated growth condition, using the submission's analytic estimates and reconstruction theorems. This is a check against independently specified axioms; its analytic input comes from the submitted proof. The extracted target declarations were checked against the pinned upstream definitions, and the adapters compiled in the submission's Lean 4.32.0 environment. This comparison concerns the centered-trace hierarchy, rather than a separate translation of every tensor field into the upstream formalization.

11 Other independent checks and their implications

The independent lattice definitions come from `mrdouglasny/lgt`, at the pinned revision identified in reference [LGT]. The checked adapter reindexes sites, links, and plaquettes and proves equality of the normalized finite Wilson laws. In the conventions of that repository, $\beta_{\text{submission}} = m\,\beta_{\text{lgt}}$, where m is the matrix representation dimension. With that conversion, the action, partition function, probability measure, and integrals of integrable observables agree. The proof also matches the gauge actions and plaquette orientations. This comparison was done on Lean 4.32.0. [A3]

Our Ward-identity adapter derives equation (13) from the charge and differentiation laws in the OpenAI submission. It covers every local field label and every component, and proves existence of admissible support for the bump function χ, integrability, and independence of this support. This checks the formulation and translation normalization of the OpenAI submission's Ward identity. [A6]

One needs to ensure that different lattice cutoffs and box sizes use the same reference units and field normalizations:

1. The reference RG spacing is $a(0) = 1$ to fix a common unit of length.
2. For every cutoff N and volume index p, the RG trajectory reaches
$$\beta_{N,p}(0) = \beta_{\text{ref}} > 0.$$
In other words, the RG flow starting at smaller lattice spacings (with coupling $\beta_{N,p}(N)$) results in the same reference coupling at lattice spacing 1.
3. Kinetic normalization: the quadratic term in the expansion of the effective action is normalized to have identity matrix coefficients.
4. Insertion normalization: for a degree-d label v, its normalized coefficient at the fixed reference level $r(d)$ equals v, independently of cutoff and volume, whenever $N \geq r(d)$.

Our adapters extract these identities for the same family, establishing a common normalization convention across volumes. [A5]

A second check connects this calibration to a nonzero observable in the mixed theory. One can choose a fixed separated trace two-point test function f such that

$$\lim_{k\to\infty} \lim_{N\to\infty} |C_{N,p(q(k))}(\Theta, \Theta; f)|^{1/8} = |S_2^{\text{mix}}(\Theta, \Theta; f)|^{1/8} > 0. \tag{23}$$

Here the lattice insertion uses the renormalized lattice representative of Θ, which is centered by subtracting the selected component's expectation. Once chosen, the test and centering remain fixed at every cutoff and volume. [A5]

Our short-distance adapter expresses the OpenAI submission's trace asymptotic in equation (17), for approaches to the origin in all directions. It identifies the same centered trace used in the scalar OS check and the same stress tensor tested by the translation Ward identity. [A4]

We also check that the submitted OPE theorem implies the finite asymptotic expansion in Eq. (20), with explicit geometric conditions and bounds uniform over the phases. Its purpose is to check the meaning and scope of the formal statement. It does not compute the coefficients independently or compare their values with Yang–Mills perturbation theory. The analytic estimates are supplied by OpenAI's proof. [A8]

The review target `YMReview.OPE.CommonOPE` states two bounds directly, without invoking the source's `OperatorProductExpansion` predicate. For nearby fields of total degree w, it requires

$$|c_\alpha| \le r^{d_\alpha - w - \varepsilon}, \qquad |R_{D,s}| \le r^{D+1-w-\varepsilon}.$$

Here $R_{D,s}$ is the difference between the actual correlation and its finite OPE sum through degree D, evaluated in state s. The coefficients are independent of spectators, phases, and truncation orders. Our check still uses the OpenAI submission's definitions of fields, coefficients and correlations.

We perform additional checks on the consistency of the field decomposition. The two review modules compile without warnings, and all five audited declarations use only the permitted standard axioms. A separate Comparator run for `constructed` passed on 5 November. [A8]

12 Verification records and reproduction

The mathematical statements and their source definitions are collected in the Lean statement glossary. This section records which additional checks were proved, which versions were checked, and where the build and Comparator results can be inspected.

12.1 Versions and repository layout

The verification material is in `yang-mills-group/YM`. The October package is under `submissions/openai-20261028/`. Within that package, the submitted source is under `submission/`, the group's additional Lean code under `checks/`, the commented examples under `docs/examples/`, and the dated records under `evidence/`. Source and evidence links use this package layout; the assessment PDF is at the repository-level path `docs/assessment-1.pdf`.

The submitted files are preserved byte for byte. The provenance directory records their hashes, the imported review sources, and the upstream definition snapshots. The review source manifest identifies the included copies of the review files and their hashes, linking the recorded Comparator runs to the sources in this release.

12.2 Checks and their records

The identifiers M1–M2 are mechanical verification records, and A1–A8 are the group's additional checks. Their mathematical meaning is explained in Sections 9.4, 10 and 11. The table links each check to its specification, proof and verification record. Each A-entry also links a commented Lean example explaining its definitions, hypotheses and scope. Where a check uses a specific numerical coefficient or another result from the literature, the corresponding worked example gives the supporting references and explains the conventions used in the comparison. These references support particular checks; they are not a general bibliography of the construction. The examples distinguish informal literature comparisons from the statements verified in Lean.

Check	Specification, proof and worked example	Recorded verification
M1: Build and audits	The submitted project, its declaration inventory and the principal proof dependencies.	29 October clean build; axiom and declaration audit; kernel replay; warning summary. The 5 November packaged build reused the verified source build cache.
M2: Comparator	The public existence, combined acceptance and OPE theorems, each checked against its own frozen reference.	Passed: public theorem, 30 October; combined theorem, 3 November; OPE theorem, 5 November. See also the four control results and tool revisions.
A1: Scalar OS axioms	`CompleteOS.lean` and `ArityGrowth.lean` prove the selected trace's target OS axioms and $E0'$. *Worked example:* `OSAxioms-20261028.lean`.	Included in the combined theorem and the 5 November review audit. The upstream declarations and import provenance identify the independent definitions.
A2: Field content	`CompletenessAudit.lean`: field labels, bounded degree spaces, phase retention, density and faithfulness. This holds for the individual phases. *Worked example:* `OperatorContent-20261028.lean`.	Included in the combined theorem; the module and its printed axiom dependencies are covered by the 5 November review audit.
A3: Finite lattice law	`LGTWilsonBridge.lean` compares the actual finite Wilson measures. *Worked example:* `LatticeAndScale-20261028.lean`.	The interface provenance records the compared revision of `mrdouglasny/lgt` [LGT]. Included in the combined theorem and the 5 November review audit.
A4: Leading trace law	`AFSourceBridge.lean` proves the leading short-distance behavior for the same centered trace. *Worked example:* `LeadingTrace-20261028.lean`.	Frozen in the combined theorem's dependencies; compilation and axiom evidence are in the combined build record and the 5 November review audit.
A5: Common scale and probe	`CommonScaleAudit.lean` and `TwoPointPhysicalObservable.lean`. *Worked example:* `LatticeAndScale-20261028.lean`.	Included in the 5 November review audit.
A6: Translation Ward identity	`WardFormulationAudit.lean`: the fixed sign and coefficient, shell integrability, and window independence. *Worked example:* `TranslationWard-20261028.lean`.	Included in the combined theorem and the 5 November review audit.
A7: Combined acceptance	`AcceptanceSpec.lean` and `CombinedAcceptance.lean` bind the checks to one family and final witness, retaining `C.Makes` and `Spec.Final` and the connections explained in Section 9.4. *Worked example:* `OctoberChain.lean`.	2 November build and axiom audit; 3 November Comparator pass. The imported source versions are retained in the review manifest.
A8: OPE statement	`OPESpec.lean` and `OPECheck.lean`: Explicit-radius bounds uniform over phases derived from the submission's OPE and algebraic consistency checks. No perturbative coefficient comparison. *Worked example:* `CommonOPE-20261028.lean`.	Initial two-module audit; fresh review compilation; 5 November Comparator pass. This is a separate endpoint from the previously checked combined theorem.

The commented examples explain the existing checks without changing their targets. The initial compilation record covers `OctoberChain.lean` and `LeadingTrace-20261028.lean`; the additional

record covers the other five examples. All compiled without errors or warnings using only the permitted standard axioms. These ordinary Lean compilations are supplementary to the historical Comparator runs.

12.3 Reproducing the checks

The reproduction instructions explain how to reproduce the checks. From the release repository root, run

```
cd submissions/openai-20261028
python3 scripts/verify.py integrity
python3 scripts/verify.py all
```

The runner checks the imported files and OS-definition correspondence, builds the submission and review modules, audits axioms, builds the pinned tools, runs the controls, and checks all three Comparator endpoints. It writes a new dated record under `runs/`. The source and review use Lean `4.32.0` and Mathlib revision `81a5d257c8e410db227a6665ed08f64fea08e997`, and the pinned Comparator kernel uses Lean `4.34.0`. A fresh source rebuild requires a checkout without existing submission build artifacts. A run that reuses those artifacts is recorded as incremental.

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References

[JW] Arthur Jaffe and Edward Witten, *Quantum Yang–Mills Theory*, official Millennium Prize Problem statement. Also J. Carlson, A. Jaffe, and A. Wiles, eds., *The Millennium Prize Problems*, American Mathematical Society, Providence, RI; Clay Mathematics Institute, Cambridge, MA, 2006.

[OS] Konrad Osterwalder and Robert Schrader, *Axioms for Euclidean Green's Functions*, Communications in Mathematical Physics **31** (1973), 83–112; and *Axioms for Euclidean Green's Functions II*, **42** (1975), 281–305.

[DMY] Douglas–Mei–Yin OS axioms and reconstruction formalization, `xiyin137/OSreconstruction` and in the Palomar registry.

[LGT] *Lattice gauge theory formalization*, `mrdouglasny/lgt`, revision `b8793ccf6a51e00e9e2b1685ba191b8626e37137`.

A Lean statement glossary

The construction theorem `YangMills.constructed` supplies the data used to prove the public statement `YangMills.JointExistence`, displayed below. The 30 October Comparator run checked the proof of this public statement.

```
def JointExistence : Prop := ∀ g : GaugeGroup, g.IsSimple → ∃ C : WilsonFamily g,
  WilsonRegularity C ∧ ∀ p : ℕ → ℕ, StrictMono p → ∃ q : ℕ → ℕ, StrictMono q ∧
    ∃ T : Hierarchy C, T.Properties (p ∘ q)
```

`YangMills.constructed`; `YangMills.jointExistence`.

The purpose of this appendix is to explain each declaration that `JointExistence` depends on and how it appears in the construction. Each entry links to the submitted definitions or proofs. The ordering of entries displayed in this glossary is based on how they appear in the Lean code. The definitions of `GaugeGroup` and `IsSimple` can be found in `PublicTheory.lean`, but we do not provide extra descriptions of them here beyond the fact that `IsSimple` requires the Lie algebra to be noncommutative and have no proper nonzero ideals.

The mathematical notation below follows the main text. We write S^s when a statement applies to either the mixture ($s = \mathrm{mix}$) or a component ($s = \xi \in \mathcal{X}$). The selected Wightman theory has Hilbert space $\mathcal{H}_{\xi_*}$, domain $\mathcal{D}_{\xi_*}$, vacuum Ω_{ξ_*}, and fields $\Phi_v^{\xi_*}$, as in Section 6.1.

`YangMills.WilsonFamily`

Fix a closed matrix gauge group $G \subseteq U(m)$. The construction supplies these fields through `YangMills.wilsonFamily`.

`YangMills.WilsonFamily.period` Positive integers L_p giving the physical side lengths. The construction uses $L_p = 4(M + p + 1)$, where $M \in \mathbb{N}$ is fixed sufficiently large to satisfy some minimum-size and initial volume bounds.

`YM.RG.Supply.Program.book`; `YM.RG.Supply.Wait.engine`.

`YangMills.WilsonFamily.denominator` Positive integers D_N giving lattice spacing $a_N = D_N^{-1}$. The construction sets $D_N = \prod_{k<N} b_k$, where $b_N \geq 3$ is odd and $b_{N+1} \geq b_N$. It also ensures $\log b_N / \log D_N \to 0$. The lattice spacing ratios b_k are chosen in order to satisfy a rather complicated but explicit inequality in terms of constants appearing in the RG transformation.

`YangMills.WilsonFamily.periodStrict` Requires L_p to be strictly increasing. In the construction this is clear from the definition of L_p.

`YangMills.WilsonFamily.denominatorStrict` Requires D_N to be strictly increasing. In the construction this is clear from the definition of D_N.

`YM.Schedule`; `YM.Schedule.denominator`; `YM.Schedule.den_strict`; `YM.Schedule.Gentle`; `YM.RG.Calendar.Fits`.

`YangMills.WilsonFamily.beta` The coupling multiplying the Wilson energy in $e^{-\beta_{N,p}E(U)}$. For each cutoff N and volume p, the construction chooses the bare coupling $\beta_{N,p}$ so that, after N RG steps, the resulting coupling equals a fixed reference value $\bar{\beta}$.

`YangMills.wilsonLaw`; `YM.Family.beta`.

`YangMills.WilsonFamily.haar` A measure h on G. The construction uses the normalized Haar measure.

YangMills.WilsonFamily.probability Requires $h(G) = 1$ (satisfied by the normalized Haar measure).

YangMills.WilsonFamily.invariantHaar Requires h to be a Haar measure, in particular invariant under left translation (satisfied by the construction's Haar measure).

YM.Gauge.probHaar; YM.Gauge.probHaar_isProbability; YM.Gauge.probHaar_isHaar.

YangMills.WilsonFamily.Token An abstract common set of labels for lattice observables at all spacings and volumes. The construction uses pairs (d, c). Here, c is a coordinate vector in a chosen basis of V_d of a gauge-invariant polynomial in the curvature and its covariant derivatives at a point. The pair (d, c) records the degree bound and the polynomial before passing to the quotient F_d, defined below.

YM.Register.Token; YM.Labels.Frame.coords.

YangMills.WilsonFamily.insertion Assigns a complex value $I_{N,p}((d, c), x; U)$ to a token (d, c), a site $x \in \Lambda_{N,p} = (\mathbb{Z}/(L_p D_N)\mathbb{Z})^4$, and a configuration U ($U_{y,\mu} \in G$ for each positively oriented edge $(y, y + e_\mu)$, $\mu = 0, 1, 2, 3$).

For each degree bound d, the construction chooses a finite set of offsets $S_d \subset \mathbb{Z}^4$, shared by all labels in V_d and independent of N, p. It also chooses lattice polynomials $P_{d,1}, \dots, P_{d,k}$ representing the chosen basis $v_1, \dots, v_k$ of V_d. Each polynomial depends only on the matrix entries (and their complex conjugates) of the links whose endpoints lie in S_d.

The lattice polynomial $P_{d,j}$ is chosen to reproduce the continuum label v_j in the following sense. On link matrices obtained by parallel transport of polynomial test gauge potentials, its Taylor expansion in the lattice spacing agrees, through order d, with the corresponding scaled continuum polynomial v_j.

The construction also adjusts the token's coefficient vector c. For sufficiently fine lattice spacings, RG transport gives an invertible linear map from coefficients at lattice spacing a_N to coefficients at a fixed reference spacing $a_{r(d)}$, where $r(d) \le N$ and only depends on d. The construction applies its inverse to c, so that the transported coefficients equal c, and then applies coordinate-dependent rescaling factors. Write $\widetilde{c}_{N,p,j}$ for the resulting coefficients. The code defines this map and proves its invertibility, but does not provide explicit formulas for the inverse's matrix entries.

Writing $P_{d,j}(x; U)$ for the polynomial translated to x, the insertion is

$$I_{N,p}((d, c), x; U) = a_N^{-d} \sum_{j=1}^{k} \widetilde{c}_{N,p,j}\, P_{d,j}(x; U).$$

It therefore depends only on links with endpoints in $x + S_d$. For the token representing v, this is the insertion $I_{N,p}(v, x; U)$ in equation (5), where the spacing and normalization factors are included in the coefficients $c_j(N, p; v)$.

YM.Family.seen; YM.Family.O; YM.Source.SectionData; YM.Source.TransferData.cylinder.

YangMills.WilsonRegularity

This predicate imposes the following four conditions on a Wilson family. YangMills.regular proves them for the constructed family when G is simple.

YangMills.WilsonRegularity.running There is a positive slope b_* such that the normalized couplings converge to b_* uniformly over all volume indices:

$$\exists b_* > 0\ \forall \varepsilon > 0\ \exists N_0 \in \mathbb{N}\ \forall N \geq N_0\ \forall p \in \mathbb{N}, \qquad \left| \frac{\beta_{N,p}}{\log(a_N^{-1})} - b_* \right| < \varepsilon.$$

The construction uses $b_* = 2b_0 = 11C_{\mathfrak{g}}/(24\pi^2)$, where $C_{\mathfrak{g}}$ is the adjoint Casimir constant as above in the main text.

YM.Pert.Asymptotics.b0; YM.Gauge.casimir.

YangMills.WilsonRegularity.continuous For every N, p, v, x, the function $U \mapsto I_{N,p}(v, x; U)$ is continuous in the product topology on configurations. The construction proves this for its polynomial insertions.

YM.Family.continuous0.

YangMills.WilsonRegularity.invariant For every N, p, v, x, configuration U, and sitewise gauge transformation $t : \Lambda_{N,p} \to G$,

$$I_{N,p}(v, x; U^t) = I_{N,p}(v, x; U), \qquad U^t_{y,\mu} = t(y)\, U_{y,\mu}\, t(y + e_\mu)^{-1}.$$

The construction obtains this from the gauge-invariant local sections.

YM.Family.invariant.

YangMills.WilsonRegularity.localSupport Each token v has a finite set $S_v \subset \mathbb{Z}^4$, independent of N, p, x, such that the insertion depends only on links with both endpoints in the translated, periodically wrapped set $S = x + S_v$:

$$\forall U, V, \quad \Big[\forall y, \mu,\ y \in S\ \wedge\ y + e_\mu \in S\ \Longrightarrow\ U_{y,\mu} = V_{y,\mu}\Big]\ \Longrightarrow\ I_{N,p}(v, x; U) = I_{N,p}(v, x; V).$$

For a constructed token (d, c), S_v is the fixed neighborhood chosen for the polynomials $P_{d,j}$.

YM.Family.local0.

YangMills.Hierarchy

Further requirements for some of these fields are in YangMills.Hierarchy.Properties. The construction fills its fields through YangMills.hierarchy. Below, S^{mix} denotes the continuum Euclidean correlations obtained from the lattice limit, before selecting a phase.

YangMills.Hierarchy.H The complex Hilbert space $\mathcal{H}_{\xi_*}$ of the selected phase. The construction takes finite linear combinations of words of smeared fields, uses the selected phase's reconstructed Wightman functions to define their inner products, quotients by vectors with norm zero, and then takes the completion.

YM.Boundary.harvest; YM.Kernel.host.

YangMills.Hierarchy.labels The complex vector space $\mathcal{V}$ labeling the fields in the Wightman theory. The construction uses the following vector space. Given a token $t = (d, c)$, associate the function $R_t(n, b_1, \ldots, b_n; x_0, \ldots, x_n) = S^{\mathrm{mix}}(t, x_0; b_1, x_1; \ldots; b_n, x_n)$. Specifically, the input to R_t is $n \in \mathbb{N}$, n tokens $b_1, \ldots, b_n$, and $n+1$ distinct points $x_0, \ldots, x_n$. Then $\mathcal{V} = \mathrm{span}_{\mathbb{C}}\{R_{(d,c)} : d \in \mathbb{N},\ c \in V_d\}$.

YangMills.labelsOf; YM.Table.Union.

YangMills.Hierarchy.theory A bundled structure containing the field operators $\Phi_v^{\xi_*}$, their common domain $\mathcal{D}_{\xi_*}$ and vacuum Ω_{ξ_*}, adjoint operation on labels, translations, an energy–momentum measure, and Poincaré action. The construction uses the selected phase's reconstructed word representation: the empty word is the vacuum and each smeared field acts by prepending that field to a word, with the reconstructed symmetry actions and energy–momentum measure.

YangMills.view; YM.Boundary.Soil.starter.

YangMills.Hierarchy.Field A finite dimensional complex normed vector space F_d of fields for each degree bound $d \in \mathbb{N}$. The construction sets $F_d = V_d/N_d$, where V_d is the space of classical gauge-invariant polynomials of engineering degree at most d. Using a chosen basis of V_d, each polynomial is represented by a coefficient vector c, as used in the token (d, c). Here N_d consists of the token coefficient vectors c such that $R_{(d,c)}$ defined above is identically zero. We denote the equivalence class of c as $[c]$. The norm on F_d is the quotient norm inherited from the supremum norm on the coefficients of the basis of V_d, so $||c|| = \max_i |c_i|$.

YM.Table.null; YM.Table.Field.

YangMills.Hierarchy.toLabel A complex-linear map from F_d to the label space $\mathcal{V}$ defined above. The construction sends $[c]$ to $R_{(d,c)}$. This map is injective because N_d is precisely the kernel of the prequotient map $c \mapsto R_{(d,c)}$.

YM.Table.spectra; YM.Table.pin.

YangMills.Hierarchy.project Sends a Token to a Field. The construction uses just the quotient map by N_d: $(d, c) \mapsto (d, [c])$.

YM.Table.classTokens.

YangMills.Hierarchy.Character The space $\mathcal{X}$ indexing the phases. It is required to be a compact metric space, equipped with its Borel σ-algebra. Let $\mathcal{H}_{\mathrm{OS}}$ be the Hilbert space reconstructed from the mixed Euclidean correlations S^{mix}, with time semigroup $E(t) = e^{-tH}$. Let $\mathcal{H}_0 = \ker H$ be its zero-energy subspace, and let P_0 be the orthogonal projection onto $\mathcal{H}_0$. A *word* is a finite list of observable labels and their insertion points. For each word A with distinct points in a time slab $0 < t < s$, the construction supplies a bounded operator $B_{s,A}$: on vectors represented by positive-time words, it shifts the existing insertions forward by s and prepends the insertions in A. In the notation of Section 6.1, define

$$M_A = P_0 B_{s,A}\big|_{\mathcal{H}_0}, \qquad \mathcal{A} = C^*(M_A : s > 0,\ A \text{ contained in } 0 < t < s)\,,$$

where $M_A : \mathcal{H}_0 \to \mathcal{H}_0$, and $C^*(\cdots)$ denotes the norm-closed unital algebra generated by these operators and their adjoints. The construction proves that $\mathcal{A}$ is commutative and takes its Gelfand spectrum as the phase space $\mathcal{X}$, so $\mathcal{A} \cong C(\mathcal{X})$. Its points are the characters $\xi : \mathcal{A} \to \mathbb{C}$. Each character determines the phase's correlations by

$$S^{\xi}(A) = \xi(M_A),$$

after translating the entire word A into a positive-time slab. The result is independent of the translation and the slab width.

YM.OS.Semigroup.ground; YM.OS.Spread; YM.OS.Spread.algebra; YM.OS.Ground.at_any; YM.OS.Spread.Resolution.receives; YM.OS.Ground.Theta; YM.OS.Ground.resolution.

YangMills.Hierarchy.phaseMeasure A measure ν on $\mathcal{X}$. The construction uses the probability measure representing the vacuum state of the mixed theory (via the Gelfand representation of $\mathcal{A}$ and the Riesz-Markov-Kakutani representation theorem), satisfying

$$\int_{\mathcal{X}} \xi(a)\, d\nu(\xi) = \langle \Omega_0, a\Omega_0 \rangle \qquad (a \in \mathcal{A}),$$

where Ω_0 is the vacuum vector in the ground-state subspace $\mathcal{H}_0$ of the mixed theory.

YM.OS.Spread.Resolution.Out; YM.OS.Ground.resolution.

YangMills.Hierarchy.selected A distinguished phase $\xi_* \in \mathcal{X}$. The construction selects a phase with a one-dimensional ground-state subspace in which the shifted trace of the stress tensor defined below satisfies the two-point limit (also defined below). Then the quantum theory is reconstructed from that selected phase's Euclidean correlations. Write $\tau = \sum_{\nu=0}^{3} T_{\nu\nu}$ and $\tau_e = \tau - e\mathbf{1}$ where $e \in \mathbb{R}$ is arbitrary, and let $\mathcal{G}_\xi$ denote the ground-state space of phase ξ. The construction takes a sequence of distances $r_n > 0$, $r_n \to 0$ and shows that there are strictly increasing indices n_k such that the set

$$E = \Big\{ \xi \in \mathcal{X} : \dim \mathcal{G}_\xi = 1, \text{ and } \forall e \in \mathbb{R},\ \lim_{k\to\infty} r_{n_k}^8 S^\xi(\tau_e(0)\tau_e(r_{n_k} e_0)) = 0 \quad \Big\}$$

satisfies $\nu(E) = 1$.

Here $e_0 = (1, 0, 0, 0)$ is the Euclidean time unit vector. The same subsequence works for every $\xi \in E$ and every $e \in \mathbb{R}$. The construction chooses a $\xi_* \in E$.

YM.OS.Cluster.lifts; YM.Trace.submits; YM.Trace.Echo.retreats; YM.Trace.echo_tab.

YangMills.Hierarchy.correlation Euclidean n-point functions of fields (d_i, v_i) at positions $x_i \in \mathbb{R}^4$, with either no phase specified (`none`) or a character ξ (`some` ξ). These are written $S^{\mathrm{mix}}(\boldsymbol{v}; \boldsymbol{x})$ and $S^\xi(\boldsymbol{v}; \boldsymbol{x})$ in the main text above, respectively.

YM.OPE.spectator; YM.Table.useAt; YM.OS.Spread.Resolution.receives.

YangMills.Hierarchy.continuation A function that takes in n field labels, n complex times z_i, and n real spatial positions $\mathbf{x}_i \in \mathbb{R}^3$ and outputs a complex number, written as $F_n^{\xi_*}(\boldsymbol{v}; \boldsymbol{z}, \boldsymbol{x})$. The construction uses the analytic continuation of S^{ξ_*}, so that

$$S^{\xi_*}(\boldsymbol{v}; \boldsymbol{\tau}, \boldsymbol{x}) = F_n^{\xi_*}(\boldsymbol{v}; \mathrm{i}\boldsymbol{\tau}, \boldsymbol{x}), \qquad \tau_1 < \cdots < \tau_n.$$

YM.Boundary.Waves.Region; YangMills.hierarchy.

YangMills.Hierarchy.Channel An index set for the degree-d fields on the RHS of the operator product expansion (OPE). The construction uses the following set. Decompose F_d as $F_{d-1} \oplus E_d$. Then the index set is $\{0, \ldots, n_d - 1\}$ indexing a basis of E_d (the basis is chosen so each basis vector is invariant under conjugation), so $n_d = \dim E_d$. To match equation (20), combine a degree d and a channel $i \in \{0, \ldots, n_d - 1\}$ into one summation index $\alpha = (d, i)$, and set $d_\alpha = d$. The associated output field (defined below in `actor`) is $O_\alpha \in F_{d_\alpha}$.

YM.OPE.Orbits.Reader.cutTest; YM.OPE.Orbits.Reader.stratum; YM.OPE.Orbits.Reader.channels.

YangMills.Hierarchy.actor Assigns a field in F_d to each degree-d Channel. The construction uses the corresponding basis vector of E_d that is indexed by the Channel, so `actor`(d,i) gives the i-th basis element of $E_d \subseteq F_d$.

YM.OPE.Orbits.Reader.basis; YM.OPE.Orbits.Reader.actor.

YangMills.Hierarchy.coefficient For input fields $A_k = (d_k, v_k)$ at positions x_k and an expansion point o, `coefficient` gives the complex coefficient of each channel $\alpha = (d, i)$, written $c_\alpha(\boldsymbol{A}; \boldsymbol{x}, o)$, appearing on the RHS of the OPE. The construction uses finitely many correlations of the input fields with additional test insertions to form a field in F_d. It then averages over $O(4)$, symmetrizes under conjugation, removes lower-degree components, and takes the coordinates in the basis $O_{(d,i)}$. In equation (20), the inputs are A_i and rx_i, and $o = 0$, so the coefficients are abbreviated to $c_\alpha(r\boldsymbol{x})$.

YM.OPE.Prescription.ope; YM.OPE.shadow; YM.OPE.Orbits.averageV; YM.OPE.Orbits.checkReal; YM.OPE.Orbits.Reader.fresh; YM.OPE.Orbits.Reader.coef.

YangMills.Hierarchy.centeredTrace An element of F_4. The construction takes this element to be the trace of the Euclidean stress tensor and subtracts the one-point expectation in the selected phase:

$$\Theta = \sum_{\mu=0}^{3} T_{\mu\mu} - S^{\xi_*}\left(\sum_{\mu=0}^{3} T_{\mu\mu}\right)\mathbf{1},$$

where $\mathbf{1} \in F_4$ is the identity field. Thus $S^{\xi_*}(\Theta) = 0$ as in equation (14).

YM.TraceExtension.center; YM.TraceExtension.stressTrace; YM.TraceExtension.theta.

YangMills.Hierarchy.Properties

These `structure` fields are for the same hierarchy T as above and the selected volume indices $p(q(k))$ as in Theorem 5.1. We use S^{mix} and S^{ξ} for its correlations. The construction gives the proofs together in YangMills.constructed. The fields `reconstruction` and `nonfree` depend on structures that are further explained.

YangMills.Hierarchy.Properties.wilson Requires the full UV limit of the smeared Wilson correlations at every fixed volume, followed by the infinite-volume limit along $p(q(k))$:

$$\lim_{k\to\infty} \lim_{N\to\infty} C_{N,p(q(k))}(\boldsymbol{v}; f) = \int f(\boldsymbol{x})\, S^{\mathrm{mix}}(\overline{\boldsymbol{v}}; \boldsymbol{x})\, d\boldsymbol{x}.$$

Here v_i is a lattice token representing a classical label, and $\bar{v}_i = \mathsf{T}.\mathrm{project}(v_i)$. The main text suppresses the bar when passing from a label to its continuum field. The limit holds for every number of insertions, token list $\boldsymbol{v}$, and smooth test $f \in C_c^\infty(\{(x_1, \dots, x_n) \in (\mathbb{R}^4)^n : x_i \neq x_j \text{ for } i \neq j\})$. At fixed k, the support of f must lie strictly inside the finite-volume box (which for a given f holds for all sufficiently large k). In the notation of equation (6), Lean's `smear` is

$$C_{N,p}(\boldsymbol{v}; f) = a_N^{4n} \sum_{x_1,\dots,x_n \in \Lambda_{N,p}} f(a_N x_1, \dots, a_N x_n) \times \int \prod_{i=1}^{n} I_{N,p}(v_i, x_i; U)\, d\mu_{N,p}(U).$$

Here $\Lambda_{N,p} = (\mathbb{Z}/(L_p D_N)\mathbb{Z})^4$, $a_N = D_N^{-1}$, and $\mu_{N,p}$ is the normalized Wilson measure of equation (3). The sum runs over all ordered n-tuples of lattice sites. The construction obtains S^{mix} by these ordered limits and shows that if two tokens are equivalent under `project` then their limiting correlations are equal.

YangMills.WilsonFamily.smear; YangMills.WilsonFamily.finiteMoment; YangMills.WilsonFamily.place; YangMills.wilsonLaw; YangMills.WilsonFamily.Converges; YangMills.wilsonLimit.

YangMills.Hierarchy.Properties.ope Requires bounds on the OPE coefficients and the error after truncating at any degree D. For input fields $A_i \in F_{d_i}$ at positions x_i, set $w = \sum_i d_i$. Let B_ℓ denote the spectator fields at positions y_ℓ, and write $B = \prod_\ell B_\ell(y_\ell)$.

The expansion point $o \in \mathbb{R}^4$ is freely chosen. For fixed constants $M \geq 0$ and $\eta, c > 0$, the allowed configurations at scale $r > 0$ satisfy

$$\begin{aligned} \|x_i - o\| &\leq Mr \quad \text{for every } i, \\ \|x_i - x_j\| &\geq \eta r \quad \text{for } i \neq j, \\ \|y_\ell - o\| &\geq c \quad \text{for every } \ell, \\ \|y_\ell - y_m\| &\geq c \quad \text{for } \ell \neq m. \end{aligned}$$

Thus the input fields form a cluster near o, while the other fields remain separated from o and from one another. The theorem applies uniformly over all positions and expansion points satisfying these inequalities.

YangMills.Hierarchy.Within; YangMills.Hierarchy.Clear.

For each degree d and channel $i \in \mathsf{T}.\mathrm{Channel}(d)$, write $\alpha = (d, i)$ and $d_\alpha = d$. Then

$$\begin{aligned} O_\alpha &:= \mathsf{T}.\mathrm{actor}(d, i) \in F_d, \\ c_\alpha(\boldsymbol{A}; \boldsymbol{x}, o) &:= \mathsf{T}.\mathrm{coefficient}(\boldsymbol{A}, d, i, \boldsymbol{x}, o). \end{aligned}$$

Each OPE term replaces the input cluster by the field O_α inserted at o, multiplied by its complex coefficient $c_\alpha(\boldsymbol{A}; \boldsymbol{x}, o)$.

YangMills.Hierarchy.actor; YangMills.Hierarchy.coefficient.

For $s = \mathrm{mix}$ or $s = \xi \in \mathcal{X}$, define

$$R_{D,s} = S^s\left(\prod_i A_i(x_i)B\right) - \sum_{d_\alpha \leq D} c_\alpha(\boldsymbol{A}; \boldsymbol{x}, o)\, S^s(O_\alpha(o)B).$$

The bounds are

$$|c_\alpha(\boldsymbol{A}; \boldsymbol{x}, o)| \leq r^{d_\alpha - w - \varepsilon}, \qquad |R_{D,s}| \leq r^{D+1-w-\varepsilon}.$$

For every $\varepsilon > 0$, each bound holds for all sufficiently small r. The coefficient bound requires only the first two conditions above, and the remainder bound requires all four and holds uniformly over all states $s \in \{\mathrm{mix}\} \sqcup \mathcal{X}$. The coefficients are independent of the spectators, state, and truncation degree. Setting $o = 0$ and the positions of the input fields as rx_i gives the notation of equation (20).

YangMills.Hierarchy.OperatorProductExpansion; YM.OPE.Orbits.coefficient_separated; YM.OPE.Orbits.quantitative_separated; YangMills.publicOPE.

YangMills.Hierarchy.Properties.nonfree

Requires the following four conditions on the degree-four field $\Theta = \mathsf{T}.\mathrm{centeredTrace}$. The reconstructed vacuum is Ω_{ξ_*} and its smeared trace field is $\Phi_\Theta^{\xi_*}(f)$. In this section we abbreviate the correlation in equation (16) by

$$G_\Theta(r) = S^{\xi_*}(\Theta(-\tfrac{r}{2}e_0)\,\Theta(\tfrac{r}{2}e_0))$$

in the selected phase $\xi_* = \mathsf{T}.\mathrm{selected}$, where $e_0 = (1, 0, 0, 0)$ is the unit time direction. The construction proves these conditions together in YangMills.publicTrace.

YangMills.Hierarchy.TraceTest; YangMills.Hierarchy.traceTwo.

YangMills.Hierarchy.TraceTest.centered Requires the vacuum expectation to be zero for every complex Schwartz test function:

$$\langle \Omega_{\xi_*}, \Phi_\Theta^{\xi_*}(f)\Omega_{\xi_*}\rangle = 0 \qquad \forall f \in \mathscr{S}(\mathbb{R}^4;\mathbb{C}).$$

The construction uses the fact that by definition the Euclidean one-point function of Θ is zero. The construction then uses analytic continuation to get zero on the Minkowski side.

YM.TraceExtension.theta; YM.TraceExtension.vev_boundary; YM.TraceExtension.pinned_centered.

YangMills.Hierarchy.TraceTest.amplitudePositive Requires the short-distance coefficient to be strictly positive:

$$A_G := \frac{3\dim_{\mathbb{R}} \mathfrak{g}}{\pi^4} > 0.$$

The purpose is to make the proof of this constant being positive explicitly available to downstream proofs.

YM.Trace.Verse.award; YM.Trace.Over.force_b; YM.Trace.Verse.award_pos; YangMills.publicTrace.

YangMills.Hierarchy.TraceTest.shortDistance Requires the complex-valued correlation to satisfy

$$\lim_{r\downarrow 0} r^8 \big(\log(1/r)\big)^2 G_\Theta(r) = A_G.$$

Writing $c(r) = r^8 \log^2(1/r)$, the construction first shows the limit at approximated positions, using grid points on the lattice. The ultraviolet comparison, logarithmic normalization, and free two-point calculation give

$$c(r_n) G_\Theta(r_n \delta_n(s)) \longrightarrow A_G/s^8, \qquad \delta_n(s) \longrightarrow s.$$

The factors $\delta_n(s)$ arise from approximating the physical points separated by distance $r_n s$ by nearby lattice points (where s is a fixed number close to 1) which it then removes. The positive spectral representation makes G_Θ real-valued and nonincreasing. Choose $s_- < 1 < s_+$. Eventually,

$$G_\Theta(r_n\delta_n(s_+)) \le G_\Theta(r_n) \le G_\Theta(r_n\delta_n(s_-)),$$

and then multiply by $c(r_n)$, take $n \to \infty$, and let $s_\pm \to 1$.

YM.Trace.Verse.tagline; YM.Trace.Verse.monotone_axis; YM.Trace.Verse.border; YM.Trace.Verse.outside; YangMills.publicTrace.

YangMills.Hierarchy.TraceTest.excludes Requires G_Θ to disagree near zero with every finite function of the form

$$W(r) = \sum_{a=1}^{N} c_a \prod_{b=1}^{n_a} K_{ab}(r),$$

where $K_{ab} = K_{m_{ab},q_{ab},h_{ab}}$ is the kernel in equation (18); explicitly,

$$K_{ab}(r) = \int_{\mathbb{R}^3} \frac{E_{ab}(p)^{q_{ab}} h_{ab}(p)}{2E_{ab}(p)} e^{-rE_{ab}(p)}\, d^3p,$$
$$E_{ab}(p) = \sqrt{m_{ab}^2 + \|p\|^2}.$$

Here N, n_a are arbitrary nonnegative integers, $c_a \in \mathbb{C}$, $m_{ab} \ge 0$, and $q_{ab} \in \mathbb{N}_0$. Each $h_{ab} : \mathbb{R}^3 \to \mathbb{R}$ is continuous and homogeneous of some degree $j_{ab} \in \mathbb{N}_0$:

$$h_{ab}(\lambda p) = \lambda^{j_{ab}} h_{ab}(p) \qquad (p \in \mathbb{R}^3,\ \lambda > 0).$$

`YangMills.Hierarchy.Mode`; `YangMills.Hierarchy.Track`; `YangMills.Hierarchy.wickProfile`.

Precisely, the condition is

$$\forall W \text{ of this form}, \quad \forall \delta > 0, \quad \exists r \in (0, \delta) : \quad G_\Theta(r) \neq W(r).$$

The construction proves that the real part of each W admits an expansion around 0:

$$\operatorname{Re} W(r) = \sum_{\nu=1}^{M} a_\nu r^{\alpha_\nu} (\log r)^{k_\nu} + O(1), \qquad \alpha_\nu \in \mathbb{Z}, \quad k_\nu \in \mathbb{N}_0.$$

It then proves that any finite limit of

$$r^8 \big(\log(1/r)\big)^2 \operatorname{Re} W(r)$$

must be zero. From `YangMills.Hierarchy.TraceTest.shortDistance`, if it equaled G_Θ near zero this instead would give the strictly positive limit A_G, a contradiction.

`YM.TraceExtension.Wick.real_profile`; `YM.TraceExtension.Wick.AtOrder`; `YM.TraceExtension.Wick.no_powerlog`; `YM.TraceExtension.Wick.euclidean_excludes`.

`YangMills.Hierarchy.Properties.reconstruction`

Requires the following fifteen conditions on the hierarchy T. The selected phase has fields $\Phi_v^{\xi_*}(f)$, vacuum Ω_{ξ_*}, and analytic continuation of Schwinger functions $F_n^{\xi_*}$, and the measure on phases is denoted by ν. Labels $v \in \mathcal{V}$ are the images of quotient fields under `toLabel`. The construction proves these conditions together in `YangMills.publicReconstruction`.

`YangMills.Hierarchy.Reconstructed`.

`YangMills.Hierarchy.Reconstructed.axioms` Requires the hierarchy's QFT to satisfy the following Wightman axioms. Write $\mathcal{H}_{\xi_*}$ for its complex Hilbert space, $\mathcal{V}$ for its complex vector space of field labels, $\mathcal{D}_{\xi_*} \subseteq \mathcal{H}_{\xi_*}$ for the common field domain, and $\Omega_{\xi_*} \in \mathcal{D}_{\xi_*}$ for the vacuum. The smeared fields are complex-linear operators

$$\Phi_v^{\xi_*}(f) : \mathcal{D}_{\xi_*} \longrightarrow \mathcal{D}_{\xi_*}, \qquad v \in \mathcal{V}, \quad f \in \mathscr{S}(\mathbb{R}^4; \mathbb{C}),$$

depending complex-linearly on both v and f.

1. **W0: Translation spectrum and uniqueness of the vacuum.** This axiom requires the following three conditions.

   (a) Spacetime translations $a \in \mathbb{R}^4$ act through a strongly continuous unitary representation $U(a)$. Spectral theory provides a projection-valued energy–momentum measure E representing these translations, so that for every $\psi \in \mathcal{H}_{\xi_*}$, the finite positive measure

   $$\mu_\psi(B) = \langle \psi, E(B)\psi \rangle \quad \forall \text{ Borel } B \subseteq \mathbb{R}^4$$

   satisfies

   $$\langle \psi, U(a)\psi \rangle = \int_{\mathbb{R}^4} e^{\,ip\cdot a}\, d\mu_\psi(p), \qquad p \cdot a = \sum_{\nu=0}^{3} p_\nu a_\nu.$$

   (b) The energy–momentum measure is supported in the closed forward light cone:

   $$E(V_+^c) = 0, \qquad V_+ = \{p : p_{\text{time}} \geq \|p_{\text{space}}\|\}.$$

(c) Every vector invariant under all spacetime translations is a complex multiple of the vacuum:

$$[U(a)\psi = \psi \text{ for every } a \in \mathbb{R}^4] \quad \Longrightarrow \quad \psi = c\Omega_{\xi_*} \quad \text{for some } c \in \mathbb{C}.$$

2. **W1: Common domain, regularity, adjoints, and cyclicity.** The domain $\mathcal{D}_{\xi_*}$ is dense in $\mathcal{H}_{\xi_*}$, contains the normalized vacuum $\|\Omega_{\xi_*}\| = 1$, and is preserved by every smeared field. For each fixed label v and vector $\psi \in \mathcal{D}_{\xi_*}$, the map

$$f \longmapsto \Phi_v^{\xi_*}(f)\psi$$

is continuous from the Schwartz topology to the Hilbert norm topology. Write $v \mapsto v^*$ for the theory's conjugate-linear adjoint map on labels. Then

$$\langle \Phi_v^{\xi_*}(f)\psi, \chi\rangle = \langle \psi, \Phi_{v^*}^{\xi_*}(\overline{f})\chi\rangle \qquad (\psi, \chi \in \mathcal{D}_{\xi_*}).$$

Moreover, the common domain is exactly the span of the field words applied to the vacuum:

$$\mathcal{D}_{\xi_*} = \operatorname{span}_{\mathbb{C}} \left\{ \Phi_{v_1}^{\xi_*}(f_1)\cdots\Phi_{v_n}^{\xi_*}(f_n)\Omega_{\xi_*} : n \geq 0, \quad v_j \in \mathcal{V}, \quad f_j \in \mathscr{S}(\mathbb{R}^4;\mathbb{C}) \right\},$$

where the empty word is Ω_{ξ_*}.

3. **W2: Poincaré covariance.** The proper orthochronous Poincaré group acts on $\mathcal{H}_{\xi_*}$ through a strongly continuous unitary representation $U(g)$, whose restriction to translations is the representation in W0. It also acts on labels by complex-linear bijections $\rho(g)$, satisfying

$$\rho(1) = \mathrm{id}, \qquad \rho(gh) = \rho(g)\rho(h).$$

The state action preserves the common domain and fixes the vacuum:

$$U(g)\mathcal{D}_{\xi_*} = \mathcal{D}_{\xi_*}, \qquad U(g)\Omega_{\xi_*} = \Omega_{\xi_*}.$$

For $gx = \Lambda x + a$, where Λ is a Lorentz transformation, define

$$f_g(x) = f(\Lambda^{-1}(x - a)).$$

The fields transform according to

$$U(g)\Phi_v^{\xi_*}(f)\psi = \Phi_{\rho(g)v}^{\xi_*}(f_g)\, U(g)\psi \qquad (\psi \in \mathcal{D}_{\xi_*}).$$

4. **W3: Local commutativity.** If every point in the closed support of f is spacelike separated from every point in the closed support of h, then

$$\Phi_v^{\xi_*}(f)\Phi_w^{\xi_*}(h)\psi = \Phi_w^{\xi_*}(h)\Phi_v^{\xi_*}(f)\psi \qquad (v, w \in \mathcal{V},\ \psi \in \mathcal{D}_{\xi_*}).$$

Here spacelike separation means

$$(x_{\text{time}} - y_{\text{time}})^2 < \|x_{\text{space}} - y_{\text{space}}\|^2.$$

`YangMills.FieldSystem`; `YangMills.WightmanTheory.Axioms`.

The construction establishes these properties in the selected phase ξ_*. The Wightman boundary values of the analytically continued correlations define an inner product on finite words of smeared fields (after quotienting out by vectors with norm zero), and then taking the completion gives $\mathcal{H}_{\xi_*}$.

$\mathcal{D}_{\xi_*}$ is the subspace of finite linear combinations of images of words, and Ω_{ξ_*} is the empty word. Each smeared field can be seen as an operator $\Phi_v(f) : \mathcal{D}_{\xi_*} \to \mathcal{D}_{\xi_*}$ by prepending its label and test function, so

$$\Phi_v(f)\,[(v_1, f_1), \ldots, (v_n, f_n)] = [(v, f), (v_1, f_1), \ldots, (v_n, f_n)].$$

This proves the invariant domain and cyclicity conditions. Continuity of $f \longmapsto \Phi_v^{\xi_*}(f)\psi$ follows from continuity of the correlations with respect to the test function. The adjoint identity is done using the definition of the adjoint map on labels, reversing words, and conjugating test functions.

YM.Kernel.host; YM.Boundary.Soil.starter; YM.Boundary.Soil.sharp; YM.Boundary.Soil.usable; YM.Boundary.Soil.regional.

Poincaré transformations act by transforming every label and test function in a word. Invariance and continuity of the word inner products allow this action to extend to a strongly continuous unitary representation on $\mathcal{H}_{\xi_*}$, preserving $\mathcal{D}_{\xi_*}$, fixing the empty word, and satisfying field covariance.

YM.Boundary.Soil.mobilize; YM.Boundary.Soil.cultivates; YangMills.view.

The construction proved the existence of the energy-momentum measure and an isometric map from H_{ξ_*} into the selected phase's Osterwalder–Schrader Hilbert space. This isometric map sends the vacuum vector on the Wightman side to the OS vacuum vector and it intertwines translations. Next, nonnegative energy in the OS Hilbert space and Lorentz covariance are used to prove the forward light cone spectrum condition. The construction then proves that any translation-invariant vector on the Wightman side gets sent to a vector in the vacuum subspace on the OS side under the isometric map above. Injectivity of the isometric map means that this translation-invariant vector must also be in the vacuum subspace on the Wightman side in $\mathcal{H}_{\xi_*}$. These results supply the four public axioms through `YangMills.view_axioms`.

YM.Boundary.Outfield.daylight; YM.Boundary.harvest; YM.Boundary.Action.spectrum; YM.Boundary.Action.only.

`YangMills.Hierarchy.Reconstructed.faithful` Every nonzero label v has a test f with $\Phi_v^{\xi_*}(f) \neq 0$. In the construction, if all smeared Wightman correlations containing a label vanish, then all Euclidean correlations containing a representative of that label vanish, for arbitrary spectator insertions at pairwise distinct points. Euclidean correlations with this vanishing property correspond to exactly the labels that are quotiented out.

YangMills.FieldSystem.Faithful; YM.Work.C26.answer; YM.Boundary.Soil.sees.

`YangMills.Hierarchy.Reconstructed.injective` For each d, distinct fields in F_d have distinct images under `toLabel`. Again the quotient by the null set in the construction gives this.

YM.Table.faithfully; YM.Table.pin_inj.

`YangMills.Hierarchy.Reconstructed.acts` Some field in some F_d has a nonzero smeared operator. The construction applies `faithful` to the nonzero label of the identity field in F_0. The purpose of `acts` is to exclude models where `faithful` holds vacuously.

YangMills.publicReconstruction.

`YangMills.Hierarchy.Reconstructed.covers` Every label in $\mathcal{V}$ is T.toLabel(d, v) for some d and $v \in F_d$. In the construction, this is by the definition of $\mathcal{V}$.

YM.DiffPlan.covered.

YangMills.Hierarchy.Reconstructed.represents Every pair (d, v) with $v \in F_d$ is the image of a lattice token under `project`. This is clear in the construction since `project` is a quotient map.

YM.Table.representative; YangMills.publicReconstruction.

YangMills.Hierarchy.Reconstructed.normalized The phase measure has total mass one. The construction's representing measure has mass $\|\Omega_0\|^2 = 1$ because the unselected vacuum Ω_0 is normalized.

YM.OS.Ground.resolution.

YangMills.Hierarchy.Reconstructed.fullSupport Every nonempty open subset of $\mathcal{X}$ has positive ν-measure. The construction proves that no nonzero element of the commutative ground-state algebra annihilates Ω_0. This implies that ν assigns positive measure to every nonempty open subset of $\mathcal{X}$.

YM.OS.Ground.attains; YM.OS.Ground.supports.

YangMills.Hierarchy.Reconstructed.permutation Correlations (for both the mixed phase and individual phases) are invariant under simultaneously permuting fields and insertion points. The construction uses the fact that lattice correlations are integrals of products of complex-valued observables

$$\int \prod_{i=1}^{n} O_{v_i}(x_i; U)\, d\mu(U),$$

which is invariant under permuting v_i and x_i together. It then passes this symmetry to the continuum limit and then to each individual phase through the character construction.

YM.Table.class_perm; YangMills.publicReconstruction.

YangMills.Hierarchy.Reconstructed.smooth Correlations (for both the mixed phase and individual phases) are in $C^\infty(\{(x_1, \dots, x_n) \in (\mathbb{R}^4)^n : x_i \neq x_j \text{ for } i \neq j\}, \mathbb{C})$.

YM.Table.class_smooth; YangMills.publicReconstruction.

YangMills.Hierarchy.Reconstructed.characterContinuity For fixed fields at distinct points, correlations are continuous with respect to $\xi \in \mathcal{X}$. The construction writes each as $S^\xi(A) = \xi(M_A)$ for a fixed field word A, and character evaluation is continuous.

YM.OS.Spread.Resolution.Out; YangMills.publicReconstruction.

YangMills.Hierarchy.Reconstructed.mixture Averaging the phase correlations recovers the mixed correlations at distinct points:

$$\int_{\mathcal{X}} S^\xi(\boldsymbol{v}; \boldsymbol{x})\, d\nu(\xi) = S^{\mathrm{mix}}(\boldsymbol{v}; \boldsymbol{x}).$$

The construction applies the identity $\int_{\mathcal{X}} \xi(M_A)\, d\nu(\xi) = \langle \Omega_0, M_A \Omega_0 \rangle$ to M_A, where A is the word associated to $(\boldsymbol{v}; \boldsymbol{x})$ and $\langle \Omega_0, M_A \Omega_0 \rangle = \langle \Omega_0, P_0 B_{s,A} \Omega_0 \rangle = \langle \Omega_0, B_{s,A} \Omega_0 \rangle = S^{\mathrm{mix}}(A)$.

YM.OS.Spread.Resolution.barycenter; YangMills.publicReconstruction.

YangMills.Hierarchy.Reconstructed.analytic For each fixed list of fields and spatial positions, the continuation $F_n^{\xi *}(\boldsymbol{v}; \boldsymbol{z}, \boldsymbol{x})$ is holomorphic in $\boldsymbol{z}$ on $\operatorname{Im} z_1 < \cdots < \operatorname{Im} z_n$.

YM.Work.C03.defined_exists; YM.Work.C03.defined_region.

YangMills.Hierarchy.Reconstructed.root At purely imaginary times, the analytic continuation equals the selected-phase Euclidean correlation: for $\tau_1 < \cdots < \tau_n$,

$$F_n^{\xi_*}(\boldsymbol{v}; \mathrm{i}\boldsymbol{\tau}, \boldsymbol{x}) = S_n^{\xi_*}(\boldsymbol{v}; (\tau_i, \mathbf{x}_i)_{i=1}^n).$$

YM.Work.CO2.answer; YangMills.publicReconstruction.

YangMills.Hierarchy.Reconstructed.boundary Approaching real times from any fixed strictly ordered imaginary direction recovers the smeared Wightman vacuum expectation. For $h_1 < \cdots < h_n$, Schwartz tests f_i, and quotient fields $v_i \in F_{d_i}$, write $\widehat{v}_i = \mathsf{T}.\mathrm{toLabel}(d_i, v_i)$ for their images in $\mathcal{V}$. Then

$$\begin{aligned}\lim_{\varepsilon \downarrow 0} \int F_n^{\xi_*}(\boldsymbol{v}; \boldsymbol{t} + \mathrm{i}\varepsilon \boldsymbol{h}, \boldsymbol{x}) \prod_{i=1}^n f_i(t_i, \mathbf{x}_i)\, d^{4n}(\boldsymbol{t}, \boldsymbol{x}) \\ = \langle \Omega_{\xi_*}, \Phi_{\widehat{v}_1}^{\xi_*}(f_1) \cdots \Phi_{\widehat{v}_n}^{\xi_*}(f_n) \Omega_{\xi_*} \rangle \\ = W_n^{\xi_*}(\widehat{v}_1, f_1; \ldots; \widehat{v}_n, f_n).\end{aligned}$$

YM.Boundary.Waves.Opens; YM.Boundary.Soil.relishes; YangMills.publicMoment; YangMills.publicReconstruction.