#### PRIMA: What, Why, How, and So What?

#### Zaikun Zhang

#### The Hong Kong Polytechnic University

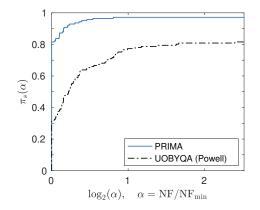
ICIAM 2023, Tokyo, Japan

Derivative-Free Optimization (DFO): Optimization without using derivatives

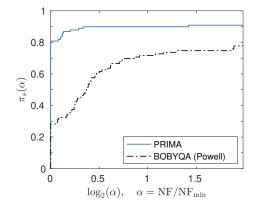
Topics of this talk:

- A new DFO solver named PRIMA
- 2 The mathematics behind it

Let us start with some tests of PRIMA based on the CUTEst problems.

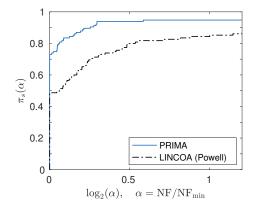


# PRIMA v.s. UOBYQA (unconstrained problems, at most 100 variables)



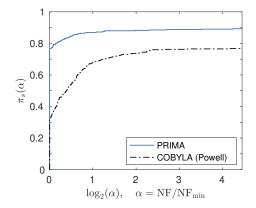
#### PRIMA v.s. BOBYQA

(bound-constrained problems, at most 200 variables)



#### PRIMA v.s. LINCOA

(linearly constrained problems, at most 200 variables, 20,000 constraints)



#### PRIMA v.s. COBYLA

(nonlinearly constrained problems, at most 100 variables, 10,000 constraints)

Topics of this talk:

A new DFO solver named PRIMA

PRIMA is not a new solver but a re-implementation of Powell's solvers.

2 The mathematics behind it

There is no mathematics in this talk.

# PRIMA: Reference Implementation for Powell's Methods with Modernization and Amelioration

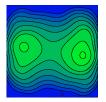
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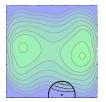
#### Dedicated to the late Professor M. J. D. Powell FRS (1936–2015)

Funding: Hong Kong RGC grants 253012/17P, 153054/20P, and 153066/21P.



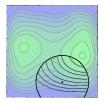
 $x_{k+1} \approx x_k + \operatorname*{arg\,min}_{\|d\| \le \Delta_k} M_k(x_k + d)$ 

- $M_k$  is the trust-region model (surrogate)
  - $M_k(x) \approx f(x)$  around  $x_k$
  - $M_k$  interpolates f on a set  $\mathcal{X}_k$  consisting of previous iterates
- $||d|| \leq \Delta_k$  is the trust-region constraint
  - If "things work well", increase  $\Delta_k$
  - Otherwise, decrease  $\Delta_k$



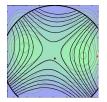
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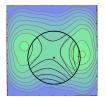


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#### Maintenance of the interpolation set

The interpolation set  $\mathcal{X}_k$  must be updated with care.

- $\mathcal{X}_k$  must reuse previous iterates as much as possible.
- The geometry of  $\mathcal{X}_k$  must ensure the well-conditioning of the problem

$$M_k(x) = f(x), \quad x \in \mathcal{X}_k.$$

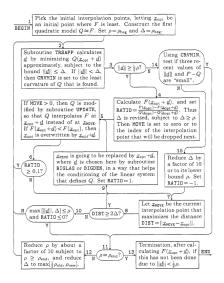
- Normally,  $\mathcal{X}_{k+1} = (\mathcal{X}_k \cup \{x_{k+1}\}) \setminus \{a \text{ "bad" point}\}.$
- Take geometry-improving steps if the geometry of  $\mathcal{X}_k$  deteriorates.

## Powell's trust-region DFO algorithms and software

- COBYLA: solving general nonlinearly constrained problems using linear models; code released in 1992; paper published in 1994
- UOBYQA: solving unconstrained problems using quadratic models; code released in 2000; paper published in 2002
- NEWUOA: solving unconstrained problems using quadratic models; code released in 2004; paper published in 2006
- BOBYQA: solving bound-constrained problems using quadratic models; code released and paper written in 2009
- LINCOA: solving linearly constrained problems using quadratic models; code released in 2013 but no paper written

#### What do these algorithms look like?

The NEWUOA software 259



NEWUOA

The development of NEWUOA has taken nearly three years. The work was very frustrating ...

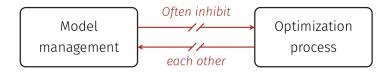
— M. J. D. Powell

The NEWUOA software for unconstrained optimization without derivatives, 2006

N.B.

- NEWUOA was Powell's third trust-region DFO solver, COBYLA and UOBYQA being the first two.
- Mathematically speaking, NEWUOA and UOBYQA are essentially the same except for the ways they construct the model.
- Given the experience with UOBYQA (and COBYLA), Powell still spent three frustrating years on the development of NEWUOA.

## The central difficulty



#### Powell's implementation

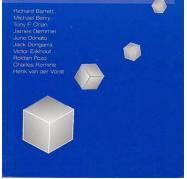
- Powell implemented these five methods into publicly available solvers.
- The solvers are widely used by scientists and engineers.
- They are often used as benchmarks when designing new algorithms.
- However, the implementation was in Fortran 77, with plenty of GOTOs: in total, 7939 lines of code with 249 GOTOs!

A modernized implementation is greatly needed.

## Why should ${\bf I}$ work on a modernized implementation

- Professor Powell, April 2015: "It would be a relief to me if you would kindly continue to look after my optimisation software (NEWUOA, BOBYQA and LINCOA). Also I would like you to add COBYLA and TOLMIN if you do not have them already."
- Stefan Wild, ICCOPT 2016, Tokyo: People do not want interfaces. They want implementations that they can understand and play with.
- Jeff Larson, ISMP 2018, Bordeaux: Numerical linear algebra people have standard implementations for standard algorithms, e.g., LAPACK, whereas we all work on our own implementation of interpolation, model improvement, ...

Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods



## Isn't it a perfect project for an engineer or a student?

- Given that Powell spent three frustrating years on the development of his own algorithm NEWUOA despite his abundant experience, where could I find this genius engineer who can learn all the five algorithms from scratch and implement them in a reasonable amount of time?
- Assume that I am lucky to find the abovementioned genius engineer and he/she happens to be my Ph.D. student. How should I persuade him/her to be fully devoted to a project for three years (as I did) without producing a single publication? Am I even allowed to do so?



libprima.net

PRIMA is an acronym for

"Reference Implementation for Powell's Methods with Modernization and Amelioration",

"P" for Powell.

## An overview of PRIMA

- The solvers are implemented in a structured and modularized way so that they are understandable, maintainable, extendable, fault-tolerant, and future-proof.
- The code has no GOTO and uses matrix-vector procedures instead of loops whenever possible.
- The implementation is mathematically equivalent to Powell's except for the bug fixes and improvements we introduce intentionally.
- The implementation of PRIMA in modern Fortran (F2008 or above) has been finished.
- Versions in MATLAB, Python, Julia, R, ... will be implemented using the modern Fortran as a reference.
- A MATLAB interface is provided to use the modern Fortran version.
- The inclusion of PRIMA into SciPy is under discussion, and the major SciPy maintainers are positive about it.

#### Why do I start with modern Fortran?



Fortran? Are you a caveman?

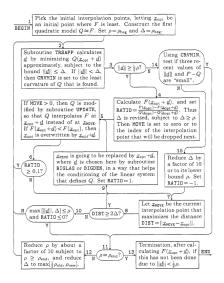
- The syntax and style of modern Fortran are very similar to MATLAB.
- I start with modern Fortran, so that I can systematically verify the bit-to-bit faithfulness of PRIMA, as the original code is Fortran.
- With other languages, the verification is hard, if not impossible.

Ultimate goal of PRIMA:

Make Powell's methods available to everyone in her/his favorite languages.

## Powell's description of NEWUOA (recapped)

The NEWUOA software 25



NEWUOA

## The original implementation of NEWUOA: a snippet

117	
118	100 KNEW=0
119	CALL TRSAPP (N,NPT,XOPT,XPT,GQ,HQ,PQ,DELTA,D,W,W(NP),
120	1 W(NP+N),W(NP+2*N),CRVMIN)
121	DSQ=ZER0
122	DO 110 I=1,N
123	110 DSQ=DSQ+D(I)**2
124	DNORM=DMIN1 (DELTA, DSQRT(DSQ))
125	IF (DNORM .LT. HALF*RHO) THEN
126	KNEW=-1
127	DELTA=TENTH*DELTA
128	RATIO=-1.0D0
129	<pre>IF (DELTA .LE. 1.5D0*RHO) DELTA=RHO</pre>
130	IF (NF .LE. NFSAV+2) GOTO 460
131	TEMP=0.125D0*CRVMIN*RHO*RHO
132	<pre>IF (TEMP .LE. DMAX1(DIFFA,DIFFB,DIFFC)) GOTO 460</pre>
133	GOTO 490
134	END IF
135	
136	120 IF (DSQ .LE. 1.0D-3*XOPTSQ) THEN
137	TEMPQ=0.25D0*X0PTSQ
138	DO 140 K=1,NPT
139	SUM=ZERO
140	DO 130 I=1,N
141	130 SUM=SUM+XPT(K,I)*XOPT(I)
142	TEMP=PQ(K)*SUM
143	SUM=SUM-HALF*X0PTSQ
144	W(NPT+K)=SUM
145	DO 140 I=1,N
146	GQ(I)=GQ(I)+TEMP*XPT(K,I)
147	<pre>XPT(K,I)=XPT(K,I)=HALF*XOPT(I)</pre>
148	VLAG(I)=BMAT(K,I)
149	W(I)=SUM*XPT(K,I)+TEMPQ*XOPT(I)
150	IP=NPT+I
151	DO 140 J=1,I
152	<pre>140 BMAT(IP,J)=BMAT(IP,J)+VLAG(I)*W(J)+W(I)*VLAG(J)</pre>
153	

# Faithful pseudocode of NEWUOA in PRIMA

Pick  $\mathcal{X} \subset \mathbb{R}^n$  and  $\rho > 0$ . Let M interpolate f on  $\mathcal{X}$ .  $x_o := \arg \min_{x \in \mathcal{X}} f(x)$ .  $\Delta := \rho$ . 1: while not converged **do** 

- 2: Calculate a trust-region trial point  $x_{tr} \approx \arg \min\{M(x) : ||x x_o|| \le \Delta\}$
- 3: if  $M(x_o) M(x_{tr})$  is too small or  $||x_{tr} x_o||$  is too short then
- 4: Reduce  $\Delta$  subject to  $\Delta \ge \rho$
- 5: else

8:

- 6: Evaluate the reduction ratio and update  $\Delta$  accordingly subject to  $\Delta \geq \rho$
- 7: **if** it is proper to replace a point  $x_{drop} \in \mathcal{X}$  with  $x_{tr}$  **then** 
  - Set  $\mathcal{X} = ig(\mathcal{X} \cup \{x_{\mathsf{tr}}\}ig) \setminus \{x_{\mathsf{drop}}\}$ , and then update M and  $x_{\mathsf{o}}$
- 9: end if
- 10: end if
- 11:  $improve_geo := x_{tr}$  is bad & the geometry of  $\mathcal{X}$  is inadequate
- 12: reduce\_rho :=  $x_{tr}$  is bad & the geometry of  $\mathcal{X}$  is adequate &  $\Delta$  is small
- 13: if improve\_geo then
- 14: Decide a point  $x_{drop} \in \mathcal{X}$  to drop and a geometry-improving point  $x_{geo}$
- 15: Set  $\mathcal{X} = (\mathcal{X} \setminus \{x_{drop}\}) \cup \{x_{geo}\}$ , and then update M and  $x_o$
- 16: end if
- 17: **if** reduce\_rho **then** reduce  $\rho$  and reduce  $\Delta$  subject to  $\Delta \geq \rho$
- 18: end while

**N.B.**: The updates keep M interpolating f on  $\mathcal{X}$  and  $x_{o} = \arg \min_{x \in \mathcal{X}} f(x)$ .

## PRIMA NEWUOA: trust-region phase (In. 2–10)

```
161
162 do tr = 1, maxtr
        call trsapp(delta, gopt, hq, pq, trtol, xpt, crvmin, d)
        dnorm = min(delta, norm(d))
        shortd = (dnorm < HALF * rho)
166
        qred = -quadinc(d, xpt, gopt, pq, hq)
168
        if (shortd .or. .not. ared > 0) then
            delta = TENTH * delta
170
            if (delta <= gamma3 * rho) then</pre>
                delta = rho ! Set DELTA to RHO when it is close to or below.
           end if
        else
            x = xbase + (xpt(:, kopt) + d)
           call evaluate(calfun, x, f)
            nf = nf + 1
178
            dnorm rec = [dnorm rec(2:size(dnorm rec)), dnorm]
179
            moderr = f - fval(kopt) + gred
180
            moderr_rec = [moderr_rec(2:size(moderr_rec)), moderr]
            ratio = redrat(fval(kopt) - f, gred, eta1)
            delta = trrad(delta, dnorm, eta1, eta2, gamma1, gamma2, ratio)
184
            if (delta <= gamma3 * rho) then
                delta = rho ! Set DELTA to RHO when it is close to or below.
186
            end if
188
            ximproved = (f < fval(kopt))
189
            knew_tr = setdrop_tr(idz, kopt, ximproved, bmat, d, delta, rho, xpt, zmat)
190
            if (knew_tr > 0) then
                xdrop = xpt(:, knew_tr)
                xosav = xpt(:, kopt)
                call updateh(knew_tr, kopt, d, xpt, idz, bmat, zmat)
194
                call updatexf(knew_tr, ximproved, f, xosav + d, kopt, fval, xpt)
                call updateq(idz, knew_tr, ximproved, bmat, d, moderr, xdrop, xosav, xpt, zmat, gopt, hq, pq)
196
                call tryqalt(idz, bmat, fval - fval(kopt), ratio, xpt(:, kopt), xpt, zmat, itest, gopt, hq, pq)
            end if
198
        end if ! End of IF (SHORTD .OR. .NOT. ORED > 0). The normal trust-region calculation ends here.
199
```

## PRIMA NEWUOA: improve\_geo, reduce\_rho (In. 11, 12)

```
199
200
        accurate mod = all(abs(moderr rec) <= 0.125 \times \text{crymin} \times \text{rho} \times 2).and, all(dnorm rec <= rho)
201
        distsg = sum((xpt - spread(xpt(:, kopt), dim=2, ncopies=npt))**2, dim=1)
202
        close_itpset = all(distsg <= 4.0 * delta**2)</pre>
        adequate geo = (shortd .and, accurate mod) .or, close itpset
204
        small_trrad = (max(delta, dnorm) <= rho)</pre>
205
        bad_trstep = (shortd .or. (.not. gred > 0) .or. ratio <= eta1 .or. knew_tr == 0)</pre>
206
207
        improve_geo = bad_trstep .and. .not. adequate_geo
        bad_trstep = (shortd .or. (.not. gred > 0) .or. ratio <= 0 .or. knew_tr == 0)</pre>
208
        reduce_rho = bad_trstep .and. adequate_geo .and. small_trrad
209
```

## PRIMA NEWUOA: post-processing phase (In. 13–17)

```
209
210
        if (improve_geo) then
211
            knew_geo = int(maxloc(distsq, dim=1), kind(knew_geo))
212
            delbar = max(min(TENTH * sqrt(maxval(distsq)), HALF * delta), rho)
213
            d = geostep(idz, knew geo, kopt, bmat, delbar, xpt, zmat)
214
            x = xbase + (xpt(:, kopt) + d)
215
           call evaluate(calfun, x, f)
216
            nf = nf + 1
217
218
           dnorm = \min(\text{delbar}, \text{norm}(d))
219
            dnorm_rec = [dnorm_rec(2:size(dnorm_rec)), dnorm]
220
            moderr = f - fval(kopt) - quadinc(d, xpt, gopt, pq, hq)
221
            moderr_rec = [moderr_rec(2:size(moderr_rec)), moderr]
222
223
            ximproved = (f < fval(kopt))
224
            xdrop = xpt(:, knew_geo)
225
            xosav = xpt(:, kopt)
226
            call updateh(knew_geo, kopt, d, xpt, idz, bmat, zmat)
227
            call updatexf(knew_geo, ximproved, f, xosav + d, kopt, fval, xpt)
228
            call updateg(idz, knew geo, ximproved, bmat, d, moderr, xdrop, xosay, xpt, zmat, gopt, hg, pg)
229
        end if ! End of IF (IMPROVE GEO). The procedure of improving geometry ends.
230
231
        if (reduce rho) then
232
            if (rho <= rhoend) then
233
                info = SMALL_TR_RADIUS
234
                exit
            end if
235
236
            delta = HALF * rho
237
           rho = redrho(rho, rhoend)
238
            delta = max(delta, rho)
239
           dnorm rec = REALMAX
240
            moderr_rec = REALMAX
241
        end if ! End of IF (REDUCE RHO). The procedure of reducing RHO ends.
242
243
        if (sum(xpt(:, kopt)**2) >= 1.0E2 * delta**2) then ! 1.0E2 works better than 1.0E3 on 20230227.
244
            call shiftbase(kopt, xbase, xpt, zmat, bmat, pq, hq, idz)
245
        end if
246 end do ! End of DO TR = 1. MAXTR. The iterative procedure ends.
247
```

## Issues in the Fortran 77 implementation: an example

72 C
73 C If KNEW is zero initially, then pick the index of the interpolation
74 C point to be deleted, by maximizing the absolute value of the
75 C denominator of the updating formula times a weighting factor.
76 C
77 C
78 IF (KNEW .EQ. 0) THEN
79 DENMAX=ZERO
80 DO 100 K=1,NPT
81 HDIAG=ZERO
82 D0 80 J=1,NPTM
83 TEMP=ONE
84 IF (J.LT. IDZ) TEMP=-ONE
85 80 HDIAG=HDIAG+TEMP*ZMAT(K,J)**2
86 DENABS=DABS(BETA*HDIAG+VLAG(K)**2)
87 DISTSQ=ZERO
88 DO 90 J=1,N
89 90 DISTSQ=DISTSQ+(XPT(K,J)-XPT(KOPT,J))**2
90 TEMP=DENABS*DISTSQ*DISTSQ
91 IF (TEMP.GT. DENMAX) THEN
92 DENMAX=TEMP
93 KNEW=K
94 END IF
95 100 CONTINUE
96 END IF
97 C
98 C Apply the rotations that put zeros in the KNEW-th row of ZMAT.
99 C
101 IF (NPTM .GE. 2) THEN
102 DO 120 J=2,NPTM
103 IF (J.EQ. IDZ) THEN
104 JL=IDZ 105 ELSE IF (ZMAT(KNEW.J) .NE. ZERO) THEN
105 ELSE IF (ZMAT(KNEW, J) .NE. ZERO) THEN

The above code may crash, as KNEW may be used uninitialized.

## Issues in the Fortran 77 implementation

- The Fortran 77 solvers may crash with memory violations (segfaults). Reason: Some indices are only initialized under conditions that can never be met because of NaN resulted from floating point exceptions.
- The Fortran 77 solvers may get stuck in infinite loops. Reason: Some loops are only terminated under conditions that can never be met because of NaN resulted from floating point exceptions.

#### **N.B.**:

- The problems are due to floating point exceptions in the Fortran 77 code rather than flaws in the algorithms.
- The problems affect all implementations or wrappers of these solvers based on the Fortran 77 code, including SciPy (COBYLA), NLopt, ...

## How to ensure PRIMA does not have similar issues?

#### Strategy 1. Programming by contract



- The preconditions and postconditions are checked only in the debug mode. In the code that users receive, they are disabled by default.
- In the debug mode, if some subroutine receives strange inputs or produces strange outputs, the program will raise an error so that the developer (i.e., Zaikun Zhang) can check the issue and fix it.

## How to ensure PRIMA does not have similar issues?

#### Strategy 2. TOUGH (Tolerance Of Untamed and Genuine Hazards) tests

```
651 function f = noisy(f, x, noise_level)
652 if nargin < 3
       noise_level = 2e-1;
653
654 end
655 r = cos(1.0D6 * sin(1.0D6 * (abs(f) + 1.0D0) * cos(1.0D6 * sum(abs(x)))));
656 f = f*(1+noise\_level*r);
657 if (r > 0.9)
658
       error('Function evaluation fails!');
659 elseif (r > 0.75)
660 f = inf:
661 elseif (r > 0.5)
      f = NaN:
662
663 elseif (r < - 0.999)
664 f = -1e30;
665 end
666 return
667
```

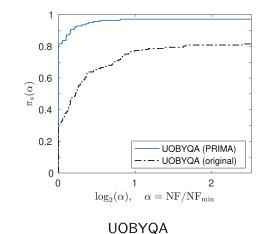
- In TOUGH tests, objective functions are corrupted as above with NaN, Inf, evaluated failures, ..., and then fed to the solvers.
- PRIMA works properly even if the objective functions are corrupted in this severe way. (What about your solvers?)

## How to ensure PRIMA does not have similar issues?

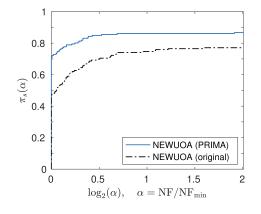
#### Strategy 3. Automated and randomized tests using GitHub Actions

- Every day, extensive TOUGH tests and other tests are conducted automatically on randomized variants of CUTEst problems.
- The longer time passes, the more reliable PRIMA is, automatically.
- As of June 2023, > 42,000 workflows have been successfully run.
- Each workflow consists of  $\sim$  5 (sometimes more than 150) randomized tests, each test taking from tens of minutes to several hours.
- In other words, PRIMA has been verified by more than 200,000 hours (or more than 20 years) of randomized tests.

#### Code must be battle-tested before it becomes software.

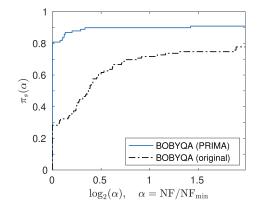


(unconstrained problems, at most 100 variables)



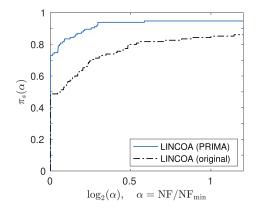
#### NEWUOA

(unconstrained problems, at most 200 variables)



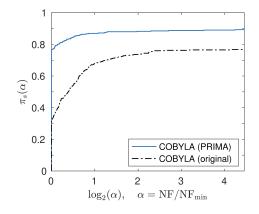
#### BOBYQA

(bound-constrained problems, at most 200 variables)



#### LINCOA

(linearly constrained problems, at most 200 variables, 20,000 constraints)



#### COBYLA

(nonlinearly constrained problems, at most 100 variables, 10,000 constraints)

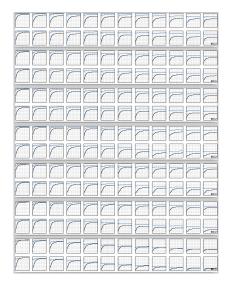
#### How to ensure the improvements are not by luck?

#### Take COBYLA as an example.

The PRIMA implementation of COBYLA is tested on 359 nonlinearly constrained CUTEst problems with at most 100 variables and 10,000 constraints. Seven tests are made.

- A plain test
- A test that permutes the variables randomly
- A test that perturbs the starting point randomly
- A test based on single-precision objective & constraint values
- A test using only 5 significant digits of the objective & constraints
- A test contaminating the objective & constraints by deterministic noise
- A test contaminating the objective & constraints by random noise

#### How to ensure the improvements are not by luck?



168 performance profiles of the new and old implementations of COBYLA

# A "fun" fact ...

- Working on PRIMA, I have spotted a dozen of bugs in reputable Fortran compilers and two bugs in MATLAB.
- Each of them represents days of bitter debugging.
- From an unusual angle, they reflect how intensive the coding is.
- The bitterness behind this fun fact is exactly why I work on PRIMA:
  - I hope all the frustrations that I have experienced will not happen to any user of Powell's methods anymore.
  - I hope I am the last one in the world to decode a maze of 244 GOTOs in 7939 lines of Fortran 77 code I did this for three years and I do not want anyone else to do it again.

## But it is not quite rewarding in terms of career and life ...

- You may write 3 good papers in 1 year, but not 1 good package in 3 years, especially if you start with a nontrivial Fortran 77 codebase.
- Internet: "Writing software is a low-status academic activity."
- Internet: "A general problem is that ... professors are usually rewarded for publications, not their software."
- Comments on my grant proposal: The PI's expertise seems in software development, but he may not be a good mathematician.
- As a "not-so-good mathematician", I much prefer spending my time on proofs, which are a lot easier and much more enjoyable for me.
- Sometimes we do things that are not enjoyable but have to be done.
- Who translated Euclid's Elements to modern languages?
- You probably do not know (and do not care). But we cannot live in a world without such a translation.

## Concluding remarks

- Implementation of model-based DFO solvers is intrinsically hard
- PRIMA provides the reference implementation of Powell's DFO solvers
- The modern Fortran version and a MATLAB interface is finished
- PRIMA will also be implemented in MATLAB, Python, Julia, R, C++
- PRIMA fixes issues in the original Fortran 77 code
- PRIMA is tested extensively to ensure its correctness & robustness
- PRIMA outperforms the original implementation of the solvers



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Thank you!